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ARTICLE

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## Air-coupled PMUTs array with residual stresses: experimental tests in the linear and non-linear dynamic regime

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### ABSTRACT

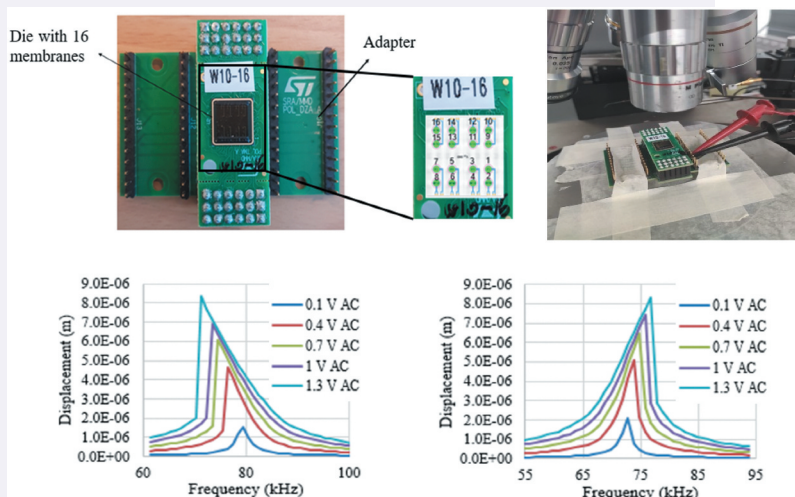
The mechanical characterization of a  $4 \times 4$  air-coupled array of Piezoelectric Micromachined Ultrasonic Transducers (PMUTs) is presented. The experimental campaign consists of three set of experimental tests, namely: topography measurements, small signal dynamic measurements, and vibrometry in the non-linear dynamic regime. The behavior of three different kinds of PMUT are reported. They differ according to the thermo-electrical treatment that has been applied to the piezoelectric material. The presence of the fabrication induced residual stresses is investigated and the treatment effect is evaluated in terms of the initial deflected configuration. The results reported in this paper represent an experimental mechanical investigation useful for the design of PMUT structures with advanced functionalities in the linear and non-linear regime.

### ARTICLE HISTORY

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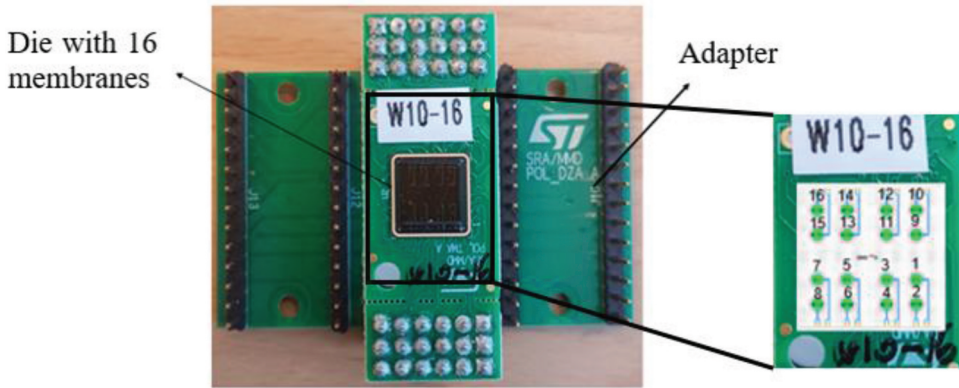
### KEYWORDS

acoustic-structure interaction; array; experimental characterization; non-linear dynamics; Piezoelectric Micromachined Ultrasonic Transducers (PMUTs); ultrasound



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**Figure 1.** PMUTs die on PCB with the custom-made adapter for characterization (left). Die layout detail with membranes numbering (right).

## 1. Introduction

Piezoelectric Micromachined Ultrasonic Transducers (PMUTs) are layered diaphragms with a piezoelectric active layer to emit and receive ultrasonic pressure waves [1,2]. They are used for many purposes like in-air range-finding [3], finger-printing recognition [4], and sonography [5,6] in which the in-water propagation is involved.

This work deals with a  $4 \times 4$  array of circular transducers (see a detail in Figure 1) for in-air applications with linearized resonance frequency of the order of kHz.

The piezoelectric thin film, constituted of Lead Zirconate Titanate (PZT), is deposited by a sol-gel technique [7,8].

The performing natural PMUTs' eigen-frequency is about 88 kHz, it decreases up to 75 kHz upon applying the thermal-electrical poling treatment to the PZT layer, as it is presented in Section 4.

In literature, several examples of air-coupled transducers are present, characterized by linear resonance frequency above 200 kHz [9]. Considering circular plates, the resonance frequency decreases as the radius increases. The difference in the performing frequency introduces the role of non-linearities [10], activated by the very high diameter/thickness aspect ratio, related to the measured transducers.

The primary aim of this paper is to present the experimental mechanical characterization of different kinds of PMUTs, in the static regime and in the linear and non-linear dynamic regime. To this scope, the experimental activity has been conducted on three types of samples, which differ according to the thermo-electrical treatment that has been applied to the piezoelectric material. The devices have been measured as pristine, which have not undergone any treatment, activated and poled, which have been treated at temperatures of 20°C and 150°C, respectively, under the application of a static voltage of 20 V. The applied DC voltage has been kept for 24 hours in the case of the activation treatment while for 90 minutes in the case of the poling one. The experimental tests have been carried out in order to evaluate the impact of the treatment on the overall performance of the device.

The characterization campaign conducted in the non-linear regime shows the presence of non-linear softening and hardening phenomena in the frequency response functions (FRFs)

curves of the system. Such a behavior is strictly related to the influence of the fabrication induced residual stresses, applied DC voltage bias, reference equilibrium configuration and transversal oscillation amplitude on the stiffness of the PMUT [10].

Considering the activated membranes, a transition between these two types of non-linear behavior is shown.

The main novelties of the paper are the completeness of the experimental campaign carried out in static and dynamic conditions and in the linear and non-linear regimes and the presentation and discussion of the mechanical behavior of PMUTs in the linear and non-linear regimes in relation to the described electro-thermal treatments.

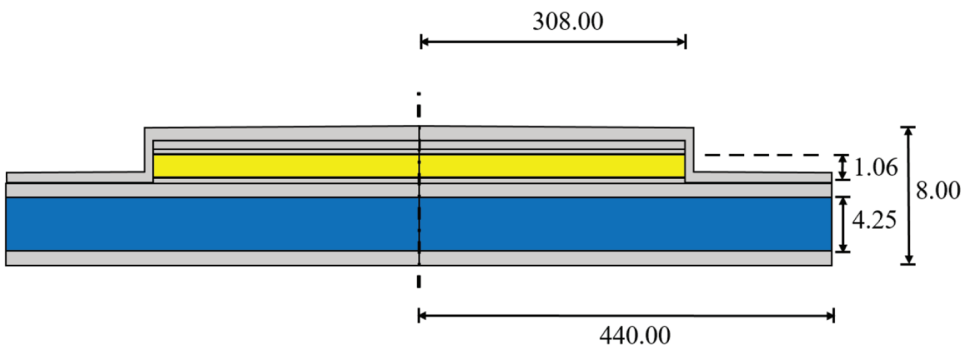
The paper is organized as follows. In the second section, the sample description and the mechanical characterization procedures are presented together with the Polytec MSA-500, interferometer, and Laser Doppler Vibrometer (LDV), involved in the static and dynamic measurements. In the third section, the experimental tests, performed in the static regime, are reported in terms of PMUT center transversal displacement at different applied DC voltage bias. In Section four, the experimental tests, carried out in the linear dynamic regime, are shown in terms of linear frequency response functions due to the small signal analysis, considering the three kinds of transducers. Section five, is devoted to the experimental tests, performed in the non-linear dynamic regime. Pristine, activated and poled transducers behavior are presented and discussed. In the end, Section five collects some closing remarks.

## 2. Mechanical characterization

In this section, the tested PMUTs array description is provided together with the description of the characterization procedures and setup. The mechanical characterization campaign included measurements in the static and dynamic linear and non-linear regimes.

### 2.1. Description of the sample

The measured transducers array is characterized by 16 PMUTs arranged in a  $4 \times 4$  configuration belonging to a silicon (Si) die, with in plane dimensions  $7.2 \times 7.2 \text{ mm}^2$  and a thickness of 0.4 mm (refer to Figure 1).



**Figure 2.** PMUT layered configuration with PZT in yellow, Si in blue and other layers in gray. The reported dimensions are in  $\mu\text{m}$ .

The single diaphragm has a radius equal to  $440\ \mu\text{m}$ , an overall thickness of  $8\ \mu\text{m}$ , which gives an aspect ratio diameter/thickness of 110. The structural thickest layer consists of silicon with a thickness of  $4.25\ \mu\text{m}$ . The circular active PZT thin film is deposited in a hat configuration on the Si layer, coaxial with the structural plate.

The piezoelectric layer has a thickness equal to  $1.06\ \mu\text{m}$  and a radius of  $308\ \mu\text{m}$ . Additionally, each PMUT is placed on top of a closed cylindrical air cavity, with height of  $400\ \mu\text{m}$  and the same radius of the upper transducer.

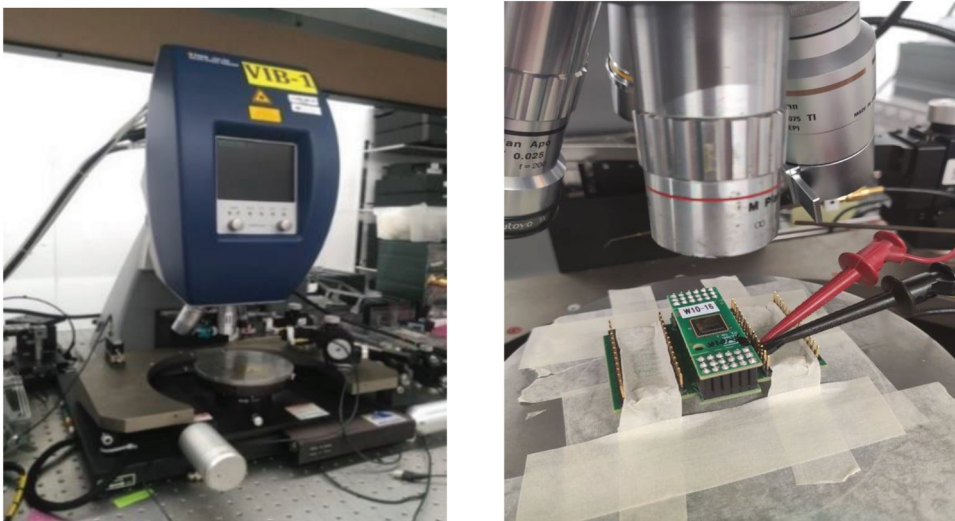
Each layer of the PMUT is characterized by a certain amount of fabrication induced residual stresses. Such initial stresses are highly heterogeneous, in the different layers, in terms of magnitudes and signs. Based on the employed micromachining process, only the silicon layer is characterized by a negligible initial stress. The described PMUT stratification is schematically reported in [Figure 2](#).

## 2.2. Characterization procedures

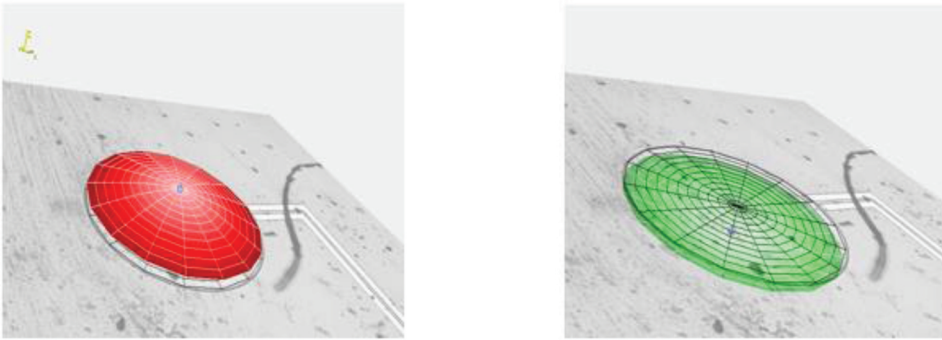
Initially, the surface topography of the pristine, activated and poled PMUT, has been measured by means of the Polytec MSA-500, adopting the configuration of white light interferometer, as reported in [Figure 3](#). At this stage, the effect of the application of the residual stresses and the DC voltage bias on the static equilibrium configuration of the system has been investigated. In particular, the DC voltage bias has been varied in the range of 0–20 V.

The dynamic linear small signal characterization has been performed in order to evaluate the resonance frequency of the membranes at various static bias voltages. Hence, a periodic chirp from 20 kHz to 200 kHz has been considered as the excitation signal, with an AC voltage input amplitude of 0.1 V and a DC voltage offset varying from 0 to 20 V.

The instrument acquires the transversal displacement of each scanning point, for all the frequencies of the selected bandwidth, in order to generate the frequency spectrum



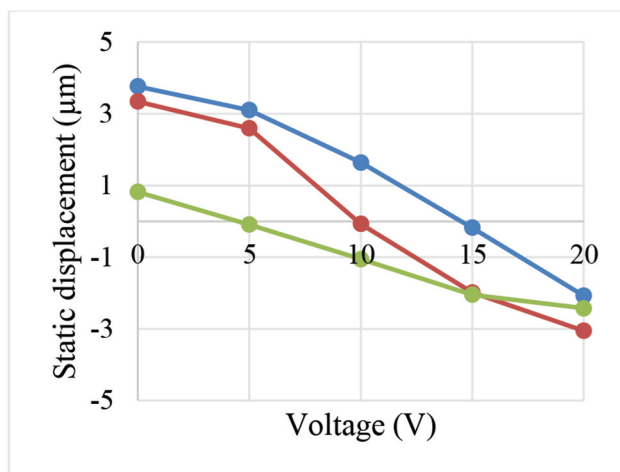
**Figure 3.** Polytec MSA-500 scanning head (left). Sample under the objective lens (right).



**Figure 4.** Out of plane movement time lapses of an actuated membrane, corresponding to the fundamental resonance eigen-mode, obtained by means of the Polytec MSA-500.

and the plate surface configuration. As a matter of fact, the fundamental vibrational eigen-mode that corresponds is presented in [Figure 4](#) for a representative case. The out of plane movement time lapses are obtained by means of the post-processing Polytec MSA-500 software. They correspond to the virtual reconstructions of the transversal oscillation based on the acquired amplitudes at the grid nodes. The two pictures show the shape of the fundamental mode.

The second part of the experimental campaign consisted of vibrometric measurements in the time domain with an aim to investigate the nonlinear behavior of the PMUTs. The samples have been excited by an all-positive sinusoidal voltage excitation with varying offset, amplitude, and frequency. A DC voltage offset in the range of 2–10 V has been applied, while the input frequency has been swept with a step of 1 kHz in a 40 kHz band centered at the linearized resonance frequency of each diaphragm, previously obtained by the small signal measurements. To optimize the acquisition, a subroutine has been used in which the parameters of the sinusoidal signal, such as the frequency and its



**Figure 5.** Membrane nr. 11. Central static displacement vs Voltage, with pristine (blue line), activated (red line) and poled material (green line).



amplitude have been chosen. On the other hand, the bias voltage has been supplied with an external generator. The amplitude of the actuation signal has been selected in the range of 0.1–1.3 V. At the end of the acquisition the post-processed displacement amplitude FRFs are obtained.

Since the transversal deflection of the diaphragm is the highest at its center, in the following sections regarding the experimental results, the central point of the PMUT has been considered.

### 3. Experimental results in the static regime

This section is devoted to the static measurements made to characterize the transducers topography. The static transversal PMUT displacement is presented as a function of the applied DC voltage bias in [Figure 5](#), considering the membrane nr.11 as a representative PMUT. The measurement has been performed through the Polytec MSA-500 by means of the white light interferometry.

The diaphragm shows an upward initial deflected dome-shaped configuration, with the maximum value at the transducer center, due to the fabrication process residual stresses, as it is reported in [Figure 5](#) in correspondence of 0 V. Such a pre-deflected PMUT configuration is already reported in the paper [1] by the authors, together with its numerical comparison, obtained through a suitable finite element model of the transducer. After the activation and poling treatment, this initial displacement decreases. Furthermore, the static voltage load induces a deformation recovery [11]. As a matter of fact, the system tends to the flat undeformed configuration. This happens at 15 V for the pristine PMUT, at 10 V for the activated piezoelectric material and at 5 V for the poled one. Going beyond this threshold, the diaphragm shows the reverse deformation and the dome-shaped configuration changes concavity. Hence, the application of the DC voltage bias produces an internal stress distribution. This is characterized by a stress resultant, associated with an internal bending moment that has the opposite sign of the initial one, related to the residual stress distribution. Therefore, the reduction of the deflection and then the reversed configuration occur, due to the DC voltage increments. This behavior affects the stiffness of the system and subsequently the linear resonance frequency as it is reported in [Figure 7](#).

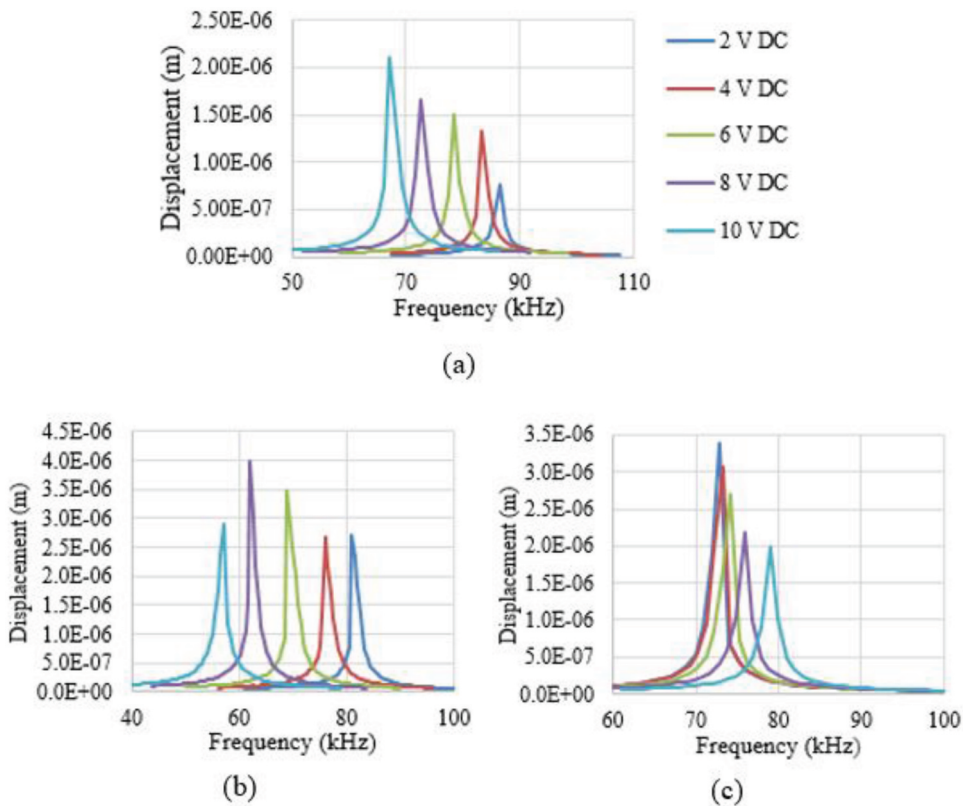
### 4. Experimental results in the linear dynamic regime

In this section the experimental results obtained from measurements carried out in the linear dynamic regime, are presented.

In [Figure 6](#), the obtained linear FRFs, with pristine, activated and poled piezoelectric material, are shown for the membrane nr. 11, around its fundamental resonance frequency, considering the applied different bias voltages.

It is worth noting that, after the poling treatment the resonance frequencies dependency on the DC voltage bias decreases, as it reported in [Figure 6](#) where they are closer to each other.

The numerical modeling of the presented phenomenon is reported in Authors' work [11], where the PMUTs array eigen-modes and frequencies are obtained by means of a proper finite element modeling applied to the device. The value of the applied voltage bias influences the stiffness of the device, through affecting the internal stress state in the



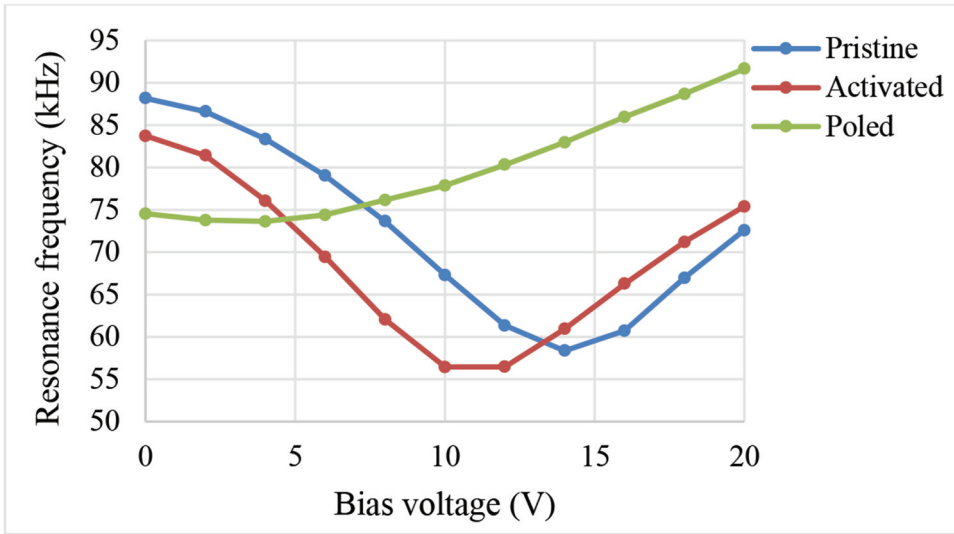
**Figure 6.** Membrane nr. 11. Displacement vs Frequency measured in the time domain, at fixed  $V_{AC} = 0.1$  V and  $V_{DC} = 2\text{--}10$  V, with pristine (a), activated (b) and poled material (c).

diaphragm. When no bias is applied, the initial stress state at the PMUIT center has a compressive stress resultant. This becomes larger as the applied bias voltage increases and brings the system to the flat un-deformed configuration. During this process, the stiffness and therefore the resonance frequency decreases. Further increase of the bias voltage leads to an inversion of the membrane concavity, which results in reversal of the internal deformation state with respect to the initial one. Therefore, increasing the DC voltage beyond the flat threshold, the stress resultant magnitude at the center of the PMUT decreases and subsequently a higher resonance frequency is detected [12,13].

The threshold for reversal of the deformation state and the obtainment of the un-deformed configuration corresponds to a minimum of the Resonance frequency vs Bias voltage curve. Such a value has been experimentally assessed. Hence, for pristine membranes, it is 15 V, while after activation and poling it decreases to 10 V and 5 V, respectively. The resonance frequencies are extracted from Tables 1–3, considering the pristine, activated, and poled membrane 11. The trend as a function of the applied voltage bias is reported in Figure 7.

The experimental tests have been performed on eight diaphragms and the results show the described trend. Additionally, average values and standard deviations, corresponding to the resonance frequency at different applied static voltage bias, are reported in Tables 1–3. The results are organized in the following way: three different tables refer to





**Figure 7.** Membrane nr. 11 resonance frequency vs Bias voltage, with pristine (blue line), activated (red line) and poled material (green line).

**Table 1.** Measured resonance frequency [kHz] vs Bias voltage [V] for pristine membranes. Average values and standard deviations.

	1	2	3	4	5	7	9	11	Average	St. dev.
0 V	87.01	90.53	88.11	94.85	89.36	87.29	87.81	88.18	89.14	2.40
2 V	85.85	88.55	87.23	92.95	87.72	85.80	86.10	86.61	87.60	2.21
4 V	83.62	85.88	84.72	90.16	85.23	83.57	83.52	83.336	85.00	2.13
6 V	80.39	82.69	80.93	86.68	81.79	80.53	80.20	79.02	81.53	2.20
8 V	76.59	78.00	76.43	81.75	77.07	76.50	76.03	73.65	77.00	2.14
10 V	72.41	72.33	70.75	75.30	71.62	72.14	70.96	67.30	71.60	2.09

**Table 2.** Measured resonance frequency [kHz] vs Bias voltage [V] for activated membranes. Average values and standard deviations.

	1	2	3	4	5	7	9	11	Average	St. dev.
0 V	83.31	84.67	85.07	89.63	84.75	83.48	83.04	83.71	84.71	1.99
2 V	79.97	80.99	81.06	85.44	81.16	80.09	79.37	81.40	81.19	1.74
4 V	75.63	76.06	76.03	79.81	76.19	75.67	74.67	76.05	75.26	1.41
6 V	70.61	69.97	69.91	72.59	70.22	70.88	69.17	69.44	70.32	0.94
8 V	66.51	63.64	63.64	63.66	63.56	63.82	63.01	62.04	64.11	1.31
10 V	64.54	59.02	59.36	54.15	58.75	62.60	60.96	56.43	59.49	3.10

**Table 3.** Measured resonance frequency [kHz] vs Bias voltage [V] for poled membranes. Average values and standard deviations.

	1	2	3	4	5	7	9	11	Average	St. dev.
0 V	82.53	76.28	83.85	70.09	Failed	74.48	80.90	74.53	77.52	4.65
2 V	82.05	75.96	81.27	67.58		74.64	81.20	73.76	76.63	4.87
4 V	81.77	76.24	79.00	65.97		75.53	82.10	73.63	76.32	5.14
6 V	81.97	77.26	76.99	65.76		77.07	83.62	74.37	76.72	5.36
8 V	82.64	78.83	75.73	67.13		79.32	85.39	76.15	77.88	5.41
10 V	83.90	80.91	75.28	69.68		81.80	87.46	77.86	79.56	5.44

the three different analyzed conditions, namely, pristine, activated and poled piezoelectric material; each row contains the values of the resonance frequency in kHz at each considered static bias voltage  $V$  and the corresponding average values and standard deviations (St. Dev. in Tables 1–3) in kHz. In order to identify the tested PMUTs on the die, the adopted numbering is reported (see the numbering detail in Figure 1). It is worth noting that only PMUT 5 failed after poling.

## 5. Experimental results in the non-linear dynamic regime

The aim of this section is to present the obtained results from the experimental characterization performed in the time domain, in order to thoroughly investigate the non-linear dynamic behavior of the measured samples, preliminarily reported in the authors work [10].

In this part of the measurement campaign, the non-linearity limit has been determined. Exceeding this threshold, resonance frequency jump/drop phenomena appear and non-linear softening/hardening behavior are reported in the FRFs.

The non-linear phenomena are described and correctly simulated in Authors' paper [10], subsequently, a proper finite element large displacement numerical modeling of the system has shown the perfect match between the experimental and numerical results. Hereinafter, the mechanical interpretation is exhaustively reported in order to explain the physical phenomenon fully investigated in [10].

These non-linear phenomena are related to the diaphragm vertical oscillation of on their stiffness and consequently on the resonance frequency.

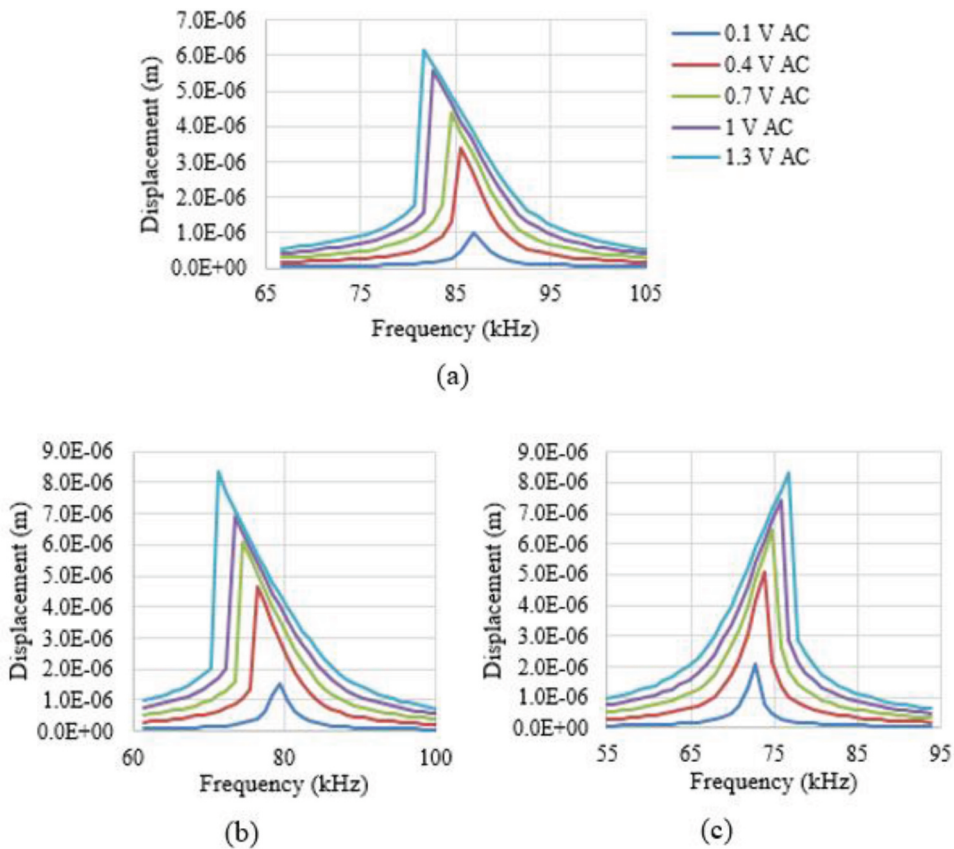
Furthermore, the synergy of residual stress and static bias voltage induces a membrane force, that affects the linearized resonance frequency and influences the initial deformed reference equilibrium configuration, around which the vibrations occur. Their impact on the stiffness of the system determines the governing non-linear phenomena.

In Figures 8 and 9 are shown the FRFs curves corresponding to a pristine, activated, and poled PMUT, for different bias voltages in the range of 2–6 V. Moreover, the reported FRFs are obtained enforcing five different AC input amplitudes of the excitation signal in the range of 0.1–1.3 V.

Upon application of 2 V, pristine and activated transducers show a softening behavior, while considering the poled one a non-linear hardening trend, for every value of the AC voltage is reported. This mechanical interpretation is related to the reference equilibrium configuration effect on the stiffness with respect to the vertical oscillation one. As a matter of fact, the highest initial deflection is associated with the pristine piezoelectric material. Consequently, without any treatment its effect on the stiffness is the dominant one and is related to the softening trend.

On the other hand, after the poling treatment the starting configuration is characterized by the smallest initial deflection. Hence, its stiffness contribution is the lowest. In this case, the governing non-linear term is related to the displacement oscillation amplitude one, which leads to the hardening trend.

Increasing the value of the applied bias voltage to 6 V, the same considerations as for 2 V tests regarding pristine and poled transducers can be adopted. On the other hand, after activation, concerning the case of the DC voltage of 2 V, the measured non-linearity corresponds to softening. Then, as the amplitude of the excitation signal increases, the



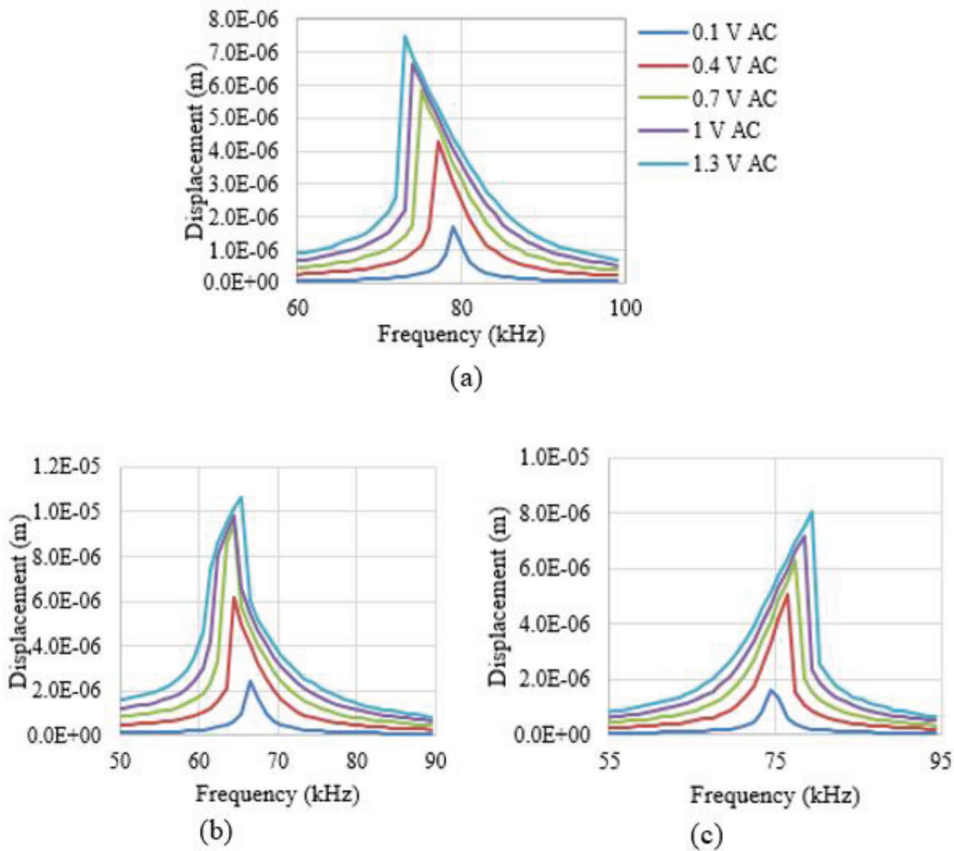
**Figure 8.** Membrane nr. 11 Displacement vs Frequency measured in time domain, at fixed VDC = 2 V and VAC = 0.1–1.3 V, with the pristine (a), activated (b) and poled material (c).

non-linear hardening becomes the governing phenomenon, as reported in [Figure 9](#) where the results of the application a DC bias of 6 V are shown. This means that for small AC voltages, the influence of the stiffness contribution related to the equilibrium reference deflection is the highest and it determines the observed non-linear softening. As the AC voltage amplitude signal increases, consequently the effect of the vibration amplitude on the stiffness becomes the dominant one; therefore, a transition behavior from non-linear softening to hardening is reported [14–16].

## 5. Conclusions

In this paper, the results of a complete experimental characterization campaign performed on PMUTs have been presented and discussed.

The linear behavior is investigated through a set of small signal measurements. The static voltage bias effect on the internal stress state of the PMUTs and consequently on the stiffness and linearized resonance frequency is reported.



**Figure 9.** Membrane nr. 11 Displacement vs Frequency measured in time domain, at fixed  $V_{DC} = 6$  V and  $V_{AC} = 0.1$ – $1.3$  V with the pristine (a), activated (b) and poled material (c).

The static voltage bias threshold, for reaching a flat configuration that corresponds to the minimum resonance frequency, becomes lower after the application of the thermo-electrical treatment to the piezoelectric material.

The carried out FRFs show the linear behavior threshold of the diaphragm. Overcoming it, non-linear softening and hardening phenomena are observed, depending on the simultaneous effect of the static equilibrium configuration and the oscillation amplitude on the stiffness.

As far as poled membranes are concerned, the only measured non-linearity corresponds to hardening. On the other hand, considering the activated and pristine PMUTs, the non-linear trends are, respectively, characterized by a softening-hardening transition and a pure softening behavior.

The results collected and discussed in this work represent an experimental investigation useful for the design and development of new PMUT structures with advanced functionalities in the linear and non-linear regime.

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