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Native Signal Self-Mix Interferometer has less than 1-nm Noise-Equivalent-Displacement

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Self-Mixing Interferometer (SMI), also variously called Optical Feedback Interferometry, Induced Modulation Interferometry, etc., is a measurement configuration which takes advantage of interaction of the laser cavity field with the small fraction of power travelled to the remote target and returned back into the cavity [1]. The interaction process induces modulations both in amplitude and frequency (AM and FM) of the cavity field, with driving terms cos 2ks for AM and sin 2ks for FM, where 2ks is the phase shift of optical pathlength s to the target and back, and $k=2\pi/\lambda$ is the wavevector. As the process is coherent, the SMI can work well even with very minute returns (e.g. down to 10^{-8} of emitted power) and this feature, coupled to the simplicity of the setup (no external optical parts required in principle) has led to the development of a number of applications of SMI, in the fields of mechanical metrology, biomedical signal sensing, physical quantity measurements and consumer products, see e.g. Ref. [2,3] for reviews.

About detection and processing of the modulated signal, usually the AM component is preferred because readily available on the laser beam power, and conveniently detected by the monitor photodiode normally mounted by the manufacturer on the rear mirror of the laser in the same package. Using the AM modulation, we can make digital or analogue processing of the SMI signal, respectively to count fringes of half-wavelength for displacement measurement and or to sense vibrations with an output analogue replica of the signal s(t) waveform, down to fraction of the wavelength and even much less with appropriate circuits.

A summary of typical performance obtained with SMI in vibration and displacement measurements by analogue and digital processing is provided by Fig.1, [2], where we can see that the range of covered amplitudes reach 6...8 decades in a displacement measurement and 5...6 in vibration measurements, with a minimum detectable amplitude (or NED, noise equivalent displacement) reaching down to about 0.1-nm, not far away from the quantum limit of detected signal $NED_q=(2k\ SNR)^{-1}$, where $SNR=[I_{ph}/2eB]^{1/2}$ is the signal-to-noise ratio of the (photodiode) detected current I_{ph} and B is the measurement bandwidth [4].

Of course, to obtain such high-performance results, we need appropriate circuits to process the plain signal supplied by the SMI. For example, to ensure linearity and wide dynamic range operation of the vibrometer, a solution is to set up a feedback loop locking the quiescent point of operation at half fringe, as described in Ref. [5].

On the other hand, many other applications and especially the consumer ones, require the simplest possible setup, with the minimum possible electronic processing circuits as well as the minimum optical components external to the laser package.

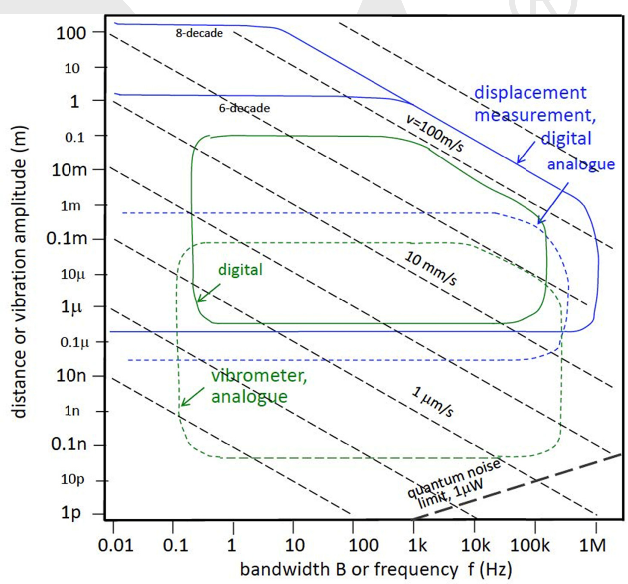

Fig. 1. State-of-the-art performance of displacement-measuring SMI instruments (blue lines) and of vibrometers (green lines) obtained with digital (full lines) and analogue (dotted lines) processing.

In this context, Choi et al. have presented in a recent paper [6] a minimum part-count vibrometer based on measuring the 'native SMI signal' without the need of any processing.

Measurement was on either the optical frequency of the laser or the voltage across the laser diode terminals.

Observation in the *optical domain* of the FM self-mixing signal was performed in Ref. [6] by an Optical Spectrum Analyzer, and a few additional optical components were also required, that is: an optical isolator, a beam splitter and a pair of collimating lenses. The reported NED, or minimum detectable displacement, achieved with the 1550-nm laser was $\lambda/130$ or 12 nm [6].

In this Letter we report a NED of 0.72 nm, or λ/1870 at the laser wavelength of 1310 nm, obtained by the bare laser diode package (that includes a monitor photodiode) and two biasing resistances plus a single collimating lens, observed in the *electrical domain* without any electronic processing circuit, just by measuring the AM signal with a plain oscilloscope, as shown in Fig. 2. Surprisingly, this result has never been reported before, to the best of our knowledge.

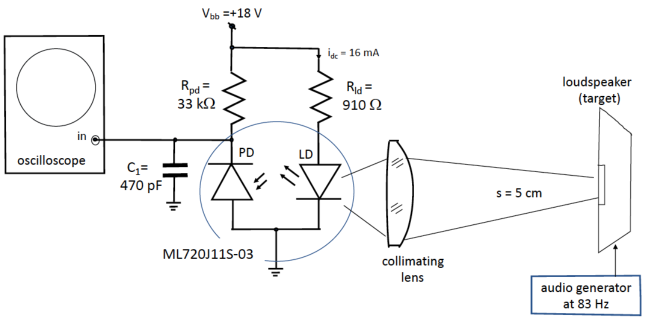

Fig. 2. Minimum part-count SMI for observing the AM signal in the time domain with an oscilloscope.

The diode laser we used is the ML720J11S-03 from Mitsubishi, and we biased it at 18 mA, not much in excess of the 12-mA threshold, so as to obtain a large SMI signal, about 90-mV across the load resistance of 33 kΩ, at an I_{pd} = 480 µA quiescent current detected by the photodiode. Emitted power at 1310-nm was around 5 mW. Capacitance C_1 was inserted to limit noise bandwidth at 15 kHz.
As the target we employed a loudspeaker, placed at 5-cm distance, with the central part covered by plain white paper. The laser spot, ≈0.5-mm diameter, was projected on the target by means of a 4.5-mm focal length, 0.55-NA collimation lens.

Note that the scheme of Fig. 2 is the *minimum part-count* configuration of the SMI interferometer, using just two resistances for biasing, the device and a single collimating lens as optical part. So, it is even simpler than the scheme considered by Choi et al. [6] for the analysis of the FM native SMI signal.

In our case, we consider the AM native signal and analyze its performance in the electrical signal time domain, by means of an inexpensive oscilloscope (Rhode& Schwarz RTDB 2004, 10-bit, 50-Mz, and noise 34 µV when filtered at 15 kHz). The AM SMI signal picked at the photodiode cathode (Fig.2) is shown in Fig. 3. Here, the drive applied to the loudspeaker is a sine wave of 83-Hz frequency and with ≈3.5-µm peak-to-peak amplitude. The SMI signal cos 2kΔs had a peak-to-peak amplitude of $2S_0$=90 mV and shows little waveform distortion, and we can estimate for it a feedback factor C≈0.25 [7].

Now, to determine the minimum detectable signal or NED of our minimum part-count scheme, a simple way is to gradually decrease the drive signal to the loudspeaker until the SMI signal is hidden in noise. After a calibration of the loudspeaker transfer characteristics, made at large Δs counting the λ/2-periods so as to find the nm/V ratio, we can trace the measurement down to nm amplitudes.

Indeed, doing so, a NED around 1-nm is found, but not very precisely because the disappearance of signal is not sharp.

For a better measurement, let us we write the expression of the SMI signal amplitude S as:

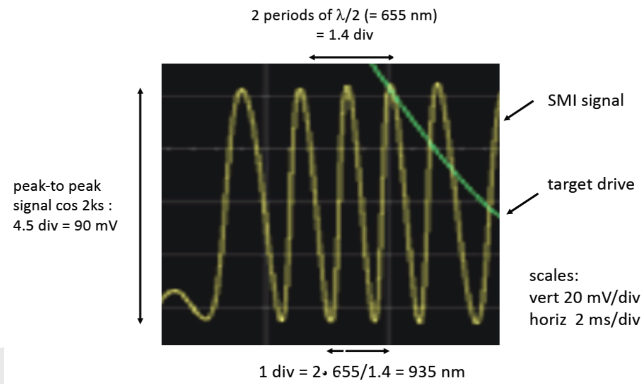

Fig. 3. Waveform of the AM SMI signal supplied by the circuit of Fig. 2, as seen at the oscilloscope. A period of the SMI signal corresponds to a displacement Δs of half wavelength or 655 nm.

$$S = S_0 \cos [2k(s_0+NED)] \qquad (1)$$

where S_0 is the peak amplitude, s_0 is the average distance to the target, and NED is the noise term reported to an equivalent displacement quantity. The linear response of the cos function is obtained at the bias $2ks_0$ =-π/2, and substituting this value in Eq.1 we obtain:

$$S = S_0 \cos (-\pi/2+2kNED) = S_0 \sin (2kNED)$$

and for small 2kNED<<1

$$S = S_0\, 2kNED \qquad (2)$$

Now, the amplitude corresponding to NED is just the noise N we measure on the waveform (Fig.4), i.e. N= S_0 2k NED, and then we can solve for NED as

$$NED = N/ (S_0\, 2k) = (\lambda/4\pi)\, N/ S_0 \qquad (3)$$

By removing the excitation to the target, so that only noise is left, and expanding the scale of the oscilloscope screen as shown in Fig.4, we get experimentally:

$$NED = (104.3 \text{ nm})\ 0.31 \text{ mV}/45 \text{ mV} = 0.72 \text{ nm} \qquad (4)$$

In Eq.4, we have used λ /4π = 104.3 nm, S_0 =½ 90 mV=45 mV from Fig.3 and read from Fig.4 N=1.0 $mV_{(p-p)}$ or 1.0/3.2 = 0.31 mV for the rms amplitude, 3.2 being the ratio of full-width at 90% enclosed area to rms value for a Gaussian noise. The standard deviation directly measured by the digital oscilloscope was about 0.3 mV, confirming the hypothesis of Gaussian noise.

In addition, on another check we looked at the noise when the beam is stopped by an absorber and found that it was the same as in Fig. 4: thus, the noise limiting NED comes from the shot noise of the detected (mean) current I_0 (about 450 μA) and from the thermal noise of the photodiode load resistance.

Another feature worth be discussed is the different slope of the leading and trailing edges of the SMI signal (Fig.3). Working at moderate C (typ. 0.25) like in this measurement, the slopes are nearly the same, but going to a large C (e.g., C=0.5-0.8) the slopes become in the ratio 3:1 or even larger. As the amplitude noise is the same for both, and a small phase signal is converted in an amplitude by the SMI process with a proportionality to slope, the NED depends on whether the interferometer quiescent point is placed, on the middle of either the leading or trailing edge. By computing rms NED as rms amplitude noise divided by slope [4], we obtain the NED values of 0.41 nm (leading edge) and 1.23 nm (trailing edge). So, the NED=0.72 nm calculated above is actually the value for the low C of the weak interaction regime, in which the SMI waveform is sinusoidal. For C not too small and less than one we have a ratio (1+C)/(1-C) of slopes and of NEDs, centered around the C<<1 value of NED = 0.72-nm.

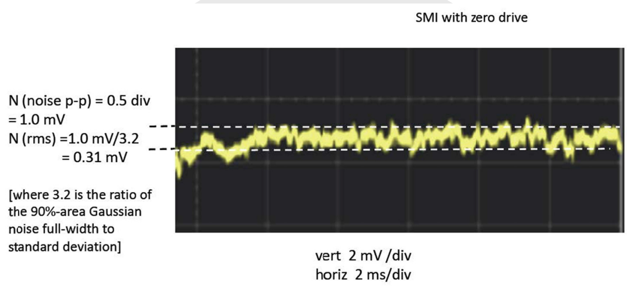

Fig. 4. Noise of the AM SMI waveform after a μ (x10 vertical) to evaluate amplitude noise.

However, attempts to improve the NED using the steeper response of leading edge is not advisable, because a reasonably constant C factor is difficult to achieve in practice. For the operation to be independent from C, we shall either stay at C<<1 (for short distance) where the signal is a sine wave, or at C>1 and C<4.7 (for large distance), where the trailing edge fills the full period of the SMI signal, and in both cases work with the SMI signal locked at half-fringe, as described in Ref. 8. As the NED depends on signal amplitude, it will normally vary with target distance, worsening quadratically at higher distances as reported in Ref. [9].

About quantities useful to compare noise performance, it is customary to normalize the NED respect to the bandwidth, introducing the *noise equivalent spectral density* NED_B= NED/√B.

Indeed, both in the quantum- and thermal-noise regimes of detection [10], contributions to NED are white noises. The NED increases as √B, and therefore it is not completely meaningful to speak of a pure NED, nor of a fractional-λ minimum detectable signal, without specifying bandwidth.

For example, the noise in of the SMI scheme of Fig.2 is given by the quadratic sum of shot noise of the detected mean current I_0 and of the Johnson noise associated to load resistance R, multiplied by an excess noise factor F:

$$i_n = F\,[2e\,I_0\,B + 4kTB/R]^{1/2} \qquad (5)$$

Inserting in Eq.5 the values for Fig.2, I_0 = 480 μA, R=33 kΩ and B=15-kHz for the circuit (Fig.2) bandwidth, we obtain v_n=Ri_n = 0.16 F mV (rms) and letting F=2 for the excess noise factor we get exactly the experimental data of Fig.4.

Then, for our native signal SMI, we have observed a noise equivalent spectral density

$$NED = 0.72\text{nm}/\sqrt{15\text{ kHz}} = 5.9\text{ pm}/\sqrt{\text{Hz}}. \qquad (6)$$

a value to be compared with the 10...100 pm/√Hz range of values of spectral NED obtained by state-of-the-art vibrometers [8] based on analogue processing of the SMI signal and half-fringe locked loop, a technique that ensures a wide dynamic range of operation, on amplitudes up to a fraction of millimeter with good linearity, as required for mechanical transfer function applications.

Usually the minimum detectable signal or NED of an SMI instrument is not the sole parameter of performance, because dynamic range, and also linearity and bandwidth of operation are quite important. Yet, pushing the limit of detectability down to picometer and below is sometimes (even though rarely) required in applications.

From this point of view, the frequency (or FM) channel of the SMI is quite interesting because it can provide a large gain in signal amplitude as well as a better SNR, as demonstrated in Refs.[11-13]. SNR and NED improvement of the converted FM signal have been analyzed in Ref. [13], while in Refs. [12-14] we have shown experimentally that the FM-converted SMI can attain 1.3 pm/√Hz respect to the 220 pm/√Hz of the plain AM-channel of the particular diode laser source employed in the SMI.

For the bandwidth B=1-kHz of an audio signal, these figures translate into a NED of 6-nm for the AM and 40-pm [13] for the converted FM, both values measured experimentally.

Still another possibility for making a minimum part-count SMI measurement is to use the voltage V_{LD} across the laser diode terminals as the output signal, as considered in [6]. However, the laser-diode impedance is low (a few Ω's respect to the typical hundreds kΩ of the photodiode load) and thus the noise performance is not that good, typically it is two or three decades worse [15] than obtained by the PD output of Fig.2. Yet, for an SMI detector operating at THz frequency where detectors haven't the good performance as in the visible, the V_{LD} output is a practicable choice [16].

In conclusion, we have demonstrated for the first time at the best of our knowledge that the native SMI electrical signal, picked across the photodetector available in the laser diode package, achieves a minimum detectable amplitude of vibration or of displacement of 0.72 nm (rms at SNR =1) at B=15-kHz without the need of any electronic processing nor additional expensive optical instrumentation.

Disclosure. Authors declare no conflict of interest.

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