A Switching Nonlinear MPC Approach for Ecodriving

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***Abstract*— In recent years many works focusing on improved vehicle fuel efficiency through advanced control have been carried out, reflecting the high interest in ecodriving of vehicles. Although many studies have shown the potential that optimal control based ecodriving can offer, these solution are often difficult to be translated into online control strategies, one of the reasons being the complexity of the optimal control problem and therefore the computational burden.**

To cope with this a novel online approach, based on switching Nonlinear Model Predictive Control (NMPC), is proposed. The NMPC strategy is developed for the case of conventional vehicles, where gear shifting and longitudinal dynamics are controlled. It is shown that our proposal can operate in real time, while recovering most of the performance achievable by an offline optimal solution.

The development of the method is described in detail and its performance is analyzed. The results show that the proposed NMPC can successfully solve the ecodriving task and seems a good compromise between computational burden and performance suitable for field implementation.

I. Introduction

As the number of vehicles on the road continue to rise, so does the energy consumption and pollution caused by them. The rise of electrified vehicles is a clear sign for the interest in cleaner transportation, however many transportation task are currently and will still be carried out by conventionally powered vehicles. Therefore, the efficient operation, often referred to as ecodriving, for conventional vehicles is an important topic, as also seen by the numerous publications, e.g., [1], [2] and [3] to name just a few of them.

One of the earliest works on fuel optimal vehicle control can be found in [4], while fuel optimal control of a rocket dates back almost one hundred years [5].

Application of optimal control for ecodriving can be found, e.g., in [2], where a vehicle control problem, called Ecodriving Optimal Control Problem (OCP), is formulated and its solution is used for advising or replacing the driver. An optimal control ecodriving approach for an electric vehicle is presented in [6], where a Dynamic Programming (DP) based solution is combined with a tracking Model Predictive Control (MPC) to realize an online ecodriving system. Since fuel efficiency is especially important for heavy duty vehicles,in [7] the authors study fuel optimal control using road grade look ahead for that type. Therein special attention is paid to the gear shift modeling and the differences between energy based optimal control formulation and classic dynamical one are investigated.

Among the strategies proposed in the literature for energy efficient operation of vehicles, MPC is an effective approach thanks to its capability to minimize some cost function subject to state and input constraints for a certain prediction horizon and deal with economic objectives (see e.g., [8], [9]). In the literature, for instance in [10], MPC has been applied to the energy efficient operation of trains, where the optimization problem consists in controlling the train velocity, while fulfilling constraints on maximum velocity and total journey time. More recently, a switched Nonlinear Model Predictive Control (NMPC) has been proposed in [11], where an optimization among a set of operating modes is performed, subject to predefined driving sequences in order to enforce ecodriving in a collaborative fashion.

In this paper, inspired by [11], we recast the conventional vehicle ecodriving control problem as a nonlinear switching OCP. The latter relies on a switching system, which typically consists of a family of subsystem dynamics specified at each sampling time by a switching signal. Significant results for stability and stabilization of this type of systems have been presented in [12]–[14]. In the case of vehicle dynamics, under suitable modeling assumptions, it is possible to prove that the solution of the ecodriving OCP consists only of four possible modes of operation, i.e., acceleration, cruising, coasting and braking (see [2] and the references therein). The restriction to these modes make the vehicle model switching and therefore motivates the use of a switching NMPC strategy [15]–[19].

The contribution of the paper is a novel online capable control, based on switching NMPC, which solves an extended ecodriving control task considering road properties such as curves, grade and velocity limits. The problem is defined as a multiobjective optimal control problem, where the minimization of the fuel consumption, number of gear shifts and travel time is sought. Note that the problem at hand is an extension of standard ecodriving due to the consideration of lateral acceleration limits. Since many features such as curves, road grade and velocity limits are space-dependent, a space domain representation is adopted and minimum time based precomputation of velocity limits is performed to improve the feasibility of the proposed NMPC.

The rest of the paper is organized as follows: in Section II some preliminaries on NMPC and DP are recalled; in Section III the vehicle model is described and the control problem is formulated; the proposed NMPC algorithm is discussed in Section IV, while simulation carried out on a realistic case study are reported in Section V; some conclusions are gathered in Section VI.

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II. Preliminaries

In this section, some preliminary elements will be introduced: the main notation used in the paper is reported, and basics of switching predictive control and dynamic programming are recalled.

A. Notation

The notation adopted is standard. Let $\mathbb{N}$ denote the set of natural numbers and $\mathbb{R}$ denote the set of real numbers. Let x be a vector then x_i is its entry. Given a signal w, then $w_{p|w}$ denotes its prediction trajectory with initial condition w, so that at current sampling time instant k, $w_{k|w} = w$. Moreover, let $\mathbf{w}_{[k,k+p]}$ be the signal w defined from k to $k+p$.

B. Switching Finite-Horizon Optimal Control Problem

Consider the following discrete time switched nonlinear system

$$x_{k+1} = f_{\sigma_k}(k, x_k), \quad \forall k \in \mathbb{N} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $\sigma_k \in \mathbb{N}$ is the switching rule and x_0 is the initial condition. The active model at the time instant k is determined by the integer $\sigma_k \in \mathcal{S}$ with $\mathcal{S} = \{1, \cdots, M\}$. Letting $\mathcal{W}$ be the set of all possible sequences, consider now the following Finite-Horizon Open-Loop Optimal Control Problem (FHOCP) which consists in minimizing at each sampling instant k with respect to the sequence $\mathbf{w}_{[k,k+N_\mathrm{p}-1|k]}$ a predefined prediction cost, i.e.,

$$\min_{\mathbf{w}\in\mathcal{W}} J_{\mathbf{w}}(x) = \sum_{p=k}^{k+N_\mathrm{p}-1} l_{\sigma_k}(x_{p|x}) + V_\mathrm{f}(x_{p+N_\mathrm{p}|x})$$

subject to

$$\begin{aligned}
\mathbf{w}_{[k,k+N_\mathrm{p}-1|k]} &= [\sigma_{k|k}, \ldots, \sigma_{k+N_\mathrm{p}-1|k}] \\
x_{p+1|x} &= f_{\sigma_p}(p, x_{p|x}) \\
x_{k|x} &= x \\
x_{p|x} &\in \mathcal{X}_p, \quad \forall p \in [k, k+N_\mathrm{p}] \\
x_{k+N_\mathrm{p}|x} &\in \mathcal{X}_\mathrm{f}\,,
\end{aligned} \tag{FHOCP}$$

where $x_{p|x}$ in turn depends on the predicted switching strategy $\mathbf{w}$ over the prediction horizon N_p, V_f is the terminal cost, $\mathcal{X}_p \subset \mathbb{R}^n$ is the state constraint set and the terminal constraint set is $\mathcal{X}_\mathrm{f} \subset \mathbb{R}^n$. At each sampling instant k, the optimal switching policy, denoted by $\mathbf{w}^\mathrm{o} = [\sigma^\mathrm{o}_k, \ldots, \sigma^\mathrm{o}_{k+N_\mathrm{p}-1}]$ will be inside the set of feasible sequences $\mathcal{F} \subseteq \mathcal{W}$. Finally, only the first element of the resulting optimal control switching strategy is used at each step, while the remaining entries are discarded. In the following sections it will be illustrated how to make the formulation of the vehicle dynamics under consideration fit the structure (1), and be therefore eligible to be solved via a NMPC. The solution of the problem (FHOCP) can be computed following Algorithm 1.

Algorithm 1 Switching NMPC - Pseudo Algorithm

Require: k, x_k
1: **for each** $\mathbf{w}_i \in \mathcal{W}$ with index i **do**
2: compute $J_{\mathbf{w}_i}$ through simulation of the system using the sequence $\mathbf{w}_i$, time k and initial state x_k
3: **if** $\mathbf{w}$ is feasible **then**
4: add i to the set of feasible indexes $\mathcal{I}_\mathcal{F}$
5: **end if**
6: **end for**
7: compute the index i^o optimal sequence

$$i^\mathrm{o} = \arg\min_{i\in\mathcal{I}_\mathcal{F}} (J_{\mathbf{w}_i})$$

8: **return** the first element of the optimal sequence $\mathbf{w}^\mathrm{o} = \mathbf{w}_{i^\mathrm{o}}$

C. Dynamic Programming

Following the theory within [20], DP can be used to solve problems of the form

$$\begin{aligned}
\min_{\pi=\{u_1,\ldots,u_{N-1}\}} \; & \sum_{k=1}^{N-1} g_k(x_k, u_k) \\
\text{s.t.} \quad & x_{k+1} = f_k(x_k, u_k) \\
& u_k \in \mathcal{U}_k(x_k) \\
& x_k \in \mathcal{X}_k.
\end{aligned} \tag{DPOCP}$$

The optimal policy $\pi^\mathrm{o} = \left\{u^\mathrm{o}_0(\cdot), \ldots, u^\mathrm{o}_{N-1}(\cdot)\right\}$ is computed through solving (2)

$$J^\mathrm{o}_k(x_k) = \min_{u_k\in\mathcal{U}_k(x_k)} \left\{J^\mathrm{o}_{k+1}(x_{k+1}) + g_k(x_k, u_k)\right\} \tag{2}$$

for each $x_k \in \mathcal{X}_k$, starting from $k = N-1$ with a given $J^\mathrm{o}_N(\cdot)$ running backward until $k = 1$.

In this work DP will be used to generate an optimal solution against which the prosed algorithm will be benchmarked. For all the computations we use a constant state grid and linear interpolation of the cost-to-go.

III. Problem Setting

In this work we study the ecodriving problem of a conventional vehicle, where the aim is to minimize the use of fuel, gear shifting and time to perform a prescribed trip. We start with stating the model used to describe the dynamical behavior of the vehicle and use this model to define what we consider to be optimal vehicle operation. Although we mainly concentrate on the longitudinal dynamics within this work, we will also consider the influence of curves through limits to the lateral acceleration.

A. Vehicle modeling

The model is divided into the parts: driveline, resistance, engine, route and gear shifting.

1) Driveline model: The effective propulsion force F_p caused by the engine torque τ_e and braking force F_b is given by

$$F_\mathrm{p} = \frac{\gamma^{(j)}\eta^{\mathrm{sign}(\tau_\mathrm{e})}}{r}\tau_\mathrm{e} - F_\mathrm{b}, \tag{3}$$

where η is the drivenline efficiency, r the wheel radius, $j \in \left\{1, \ldots, N_\mathrm{gear}\right\}$ the selected gear and $\gamma^{(j)}$ is the gear dependent transmission ratio.

Since no wheel slip is assumed, the vehicle speed v and the rotational speed of the engine are related through

$$\omega_\mathrm{e} = \frac{\gamma^{(j)}}{r} v. \tag{4}$$

2) Resistance forces: The resistance force F_r of a vehicle is modeled as follows

$$F_\mathrm{r}(s,v) = mg\left(c_\mathrm{r} + \sin(\alpha(s))\right) + \frac{c_\mathrm{d}\rho_\mathrm{air}A}{2}v^2 \tag{5}$$

Here m is the vehicle mass, g the gravitational constant, $\sin(\alpha)$ is the road grade, ρ_air the density of air, A the front area of the vehicle, c_r the rolling resistance and c_d the drag coefficient.

By equating the propulsion force (3) with the resistance force (5) we get the differential equation for the longitudinal velocity v

$$\tfrac{\mathrm{d}}{\mathrm{d}t} v = a_\mathrm{x} = \frac{1}{\lambda^{(j)}m} \cdot \left(\frac{\gamma^{(j)}\eta^{\mathrm{sign}(\tau_\mathrm{e})}}{r}\tau_\mathrm{e} - F_\mathrm{b} - F_\mathrm{r} \right) \tag{6}$$

with λ being the factor accounting for the rotational inertia and a_x being the longitudinal acceleration.

3) Engine: The engine will be modeled through a static fuel flow function q_fuel, which is assumed to only depend on the rotational speed ω_e and torque τ_e.

4) Route: The signals defining the model of a route with length s_f, are characterized by three functions:

- road grade $\alpha(s) : [0, s_\mathrm{f}] \mapsto \mathbb{R}$,
- curvature $c(s) : [0, s_\mathrm{f}] \mapsto \mathbb{R}$,
- velocity limit $v_\mathrm{max}(s) : [0, s_\mathrm{f}] \mapsto \mathbb{R}$.

It is assumed that the vehicle perfectly follows a given route, under this assumption that lateral acceleration is defined through the curvature of the route and the velocity. The lateral accelerations a_y is given by

$$a_\mathrm{y}(s,v) = v^2 \cdot c(s). \tag{7}$$

5) Gear Shifting: The gear j is modeled as a state variable which can be influenced by the gear shift command u_j.

$$\tfrac{\mathrm{d}}{\mathrm{d}t} j = u_j\,. \tag{8}$$

Since the gear j is a value discrete variable, the continuous input is assumed to be a sum of dirac impulses. As the control problems will be solved in discrete domain, this assumption will not cause issues. More specifically in this work we assume that the only three gear shifting actions are possible: upshift, down shift and stay.

B. The Ecodriving Control Problem

We define the following ecodriving problem

$$\begin{aligned}
&\min_{\tau_\mathrm{e},u_j,F_\mathrm{b}} \int_0^{t_\mathrm{f}} \left(q_\mathrm{fuel} + \mu_j\left|u_j\right|\right)\cdot \mathrm{d}t \\
\text{s.t.}\quad & \tfrac{\mathrm{d}}{\mathrm{d}t}s = v \qquad\qquad \tfrac{\mathrm{d}}{\mathrm{d}t} j = u_j \\
& \tfrac{\mathrm{d}}{\mathrm{d}t}v = a_\mathrm{x}(\tau_\mathrm{e}, F_\mathrm{b}, j, v) \\
& v(t) \le v_\mathrm{max}(s(t)) \qquad s(t_\mathrm{f}) = s_\mathrm{f} \\
& (\omega,\tau) \in \mathcal{P} \qquad\qquad (a_\mathrm{x}, a_\mathrm{y}) \in \mathcal{A}.
\end{aligned} \tag{EcoCPt}$$

More specifically, the control aim is to minimize a multi-objective cost function consisting of fuel consumption $m_\mathrm{fuel} = \int_0^{t_\mathrm{f}} q_\mathrm{fuel}\mathrm{d}t$ and the weighted number of gear shifts $\#_\mathrm{GS} = \int_0^{t_\mathrm{f}} |u_j|\mathrm{d}t$. Furthermore the minimization problem is subject to previously described dynamics of the vehicle, space dependent velocity limits, a constraint enforcing the vehicle to reach the destination s_f on time and other two constraints which will be explained more detailed:

- limits to the engine operation, given by $(\omega_\mathrm{e}, \tau_\mathrm{e}) \in \mathcal{P}$,
- limits to accelerations, given by $\left(a_\mathrm{x}, a_\mathrm{y}\right) \in \mathcal{A}$.

The first constraint is due to physical limitations within the engine and the second one mimics the typical behavior of drivers, which only accept certain accelerations due to comfort.

Since many of the features of the road are expressed as function of space we transform (EcoCPt) into space domain, i.e.,

$$\begin{aligned}
&\min_{\tau_\mathrm{e},u_j,F_\mathrm{b}} \int_0^{s_\mathrm{f}} \tfrac{1}{v}\left(q_\mathrm{fuel} + \mu_j\left|u_j\right| + \mu_{t_\mathrm{f}}\right)\cdot \mathrm{d}s \\
\text{s.t.}\quad & \tfrac{\mathrm{d}}{\mathrm{d}s} j = u_j \qquad\qquad \tfrac{\mathrm{d}}{\mathrm{d}s}v = \tfrac{1}{v}a_\mathrm{x}(\tau_\mathrm{e}, F_\mathrm{b}, j, v) \\
& (\omega_\mathrm{e},\tau_\mathrm{e}) \in \mathcal{P} \qquad \left(a_\mathrm{x}, a_\mathrm{y}\right) \in \mathcal{A} \\
& v(s) \le v_\mathrm{max}(s).
\end{aligned} \tag{EcoCPs}$$

Note that the cost function now contains an additional term $\frac{1}{v}\mu_{t_\mathrm{f}}$, which is the weighted time to reach the final position s_f. This term appears due to the fact that within distance domain the requirement on final time $t(s_\mathrm{f}) = t_\mathrm{f}$ is in fact an isoperimetric constraint, which can be lifted to the cost function by introducing the constant value μ_{t_f}.

IV. Recasting the Ecodriving Problem into the NMPC Framework

In this section previous ecodriving control problem (EcoCPs) is recast according to the NMPC formulation introduced in Section II-B. Therefore, the continous dynamics are discretized, thus leading to a discrete model in the form

$$x_{k+1} = \bar{f}_k(x_k, u_k), \tag{9}$$

where k is the space discretization index, $x = [v, j]^\top$ and $u = [\tau_\mathrm{e}, F_\mathrm{b}, u_j]^\top$, with $u_j \in \{+1, 0, -1\}$. Through discretizing also cost function and constraints one obtains a form of the ecodriving problem perfectly fitting (DPOCP). Thus an optimal solution can be computed through applying the DP algorithm, as in [1]. From here on the solutions obtained using this approach will be refereed to as (EcoDPc).

A. The switching vehicle model

Motivated by the fact that under suitable assumptions, the optimal operation of a conventional vehicle can be described by a combination of $m = 4$ operating modes (acceleration, cruising, coasting, braking) [2], it seems reasonable to model the vehicle dynamics as a switched system, switching among these four operating modes. Additionally gear shifting can be viewed as operating among three modes, therefore in total we have $M=12$ possible modes of operation.

Given $\sigma_k \in \{1,\ldots,12\}$, the mode of operation is realized based on the following control laws for the input variables

$$u_j(k,x_k,\sigma_k) = \begin{cases} +1, & \sigma_k \in \{1,4,7,10\} \\ 0, & \sigma_k \in \{2,5,8,11\} \\ -1, & \sigma_k \in \{3,6,9,12\} \end{cases}$$

$$\tau_{\mathrm{e}}(k,x_k,\sigma_k) = \begin{cases} \tau_{\mathrm{e,max}}(k,x_k), & \sigma_k \in \{1,2,3\} \\ \tau_{\mathrm{e,cruise}}(k,x_k), & \sigma_k \in \{4,5,6\} \\ \tau_{\mathrm{e,min}}(k,x_k), & \sigma_k \in \{7,8,9\} \\ \tau_{\mathrm{e,min}}(k,x_k), & \sigma_k \in \{10,11,12\} \end{cases}$$

$$F_{\mathrm{b}}(k,x_k,\sigma_k) = \begin{cases} 0, & \sigma_k \in \{1,2,3\} \\ F_{\mathrm{b,cruise}}(k,x_k), & \sigma_k \in \{4,5,6\} \\ 0, & \sigma_k \in \{7,8,9\} \\ F_{\mathrm{b,max}}(k,x_k), & \sigma_k \in \{10,11,12\} \end{cases},$$

where $\tau_{\mathrm{e,max}}(k,x_k)$ is the maximum engine torque, $\tau_{\mathrm{e,cruise}}(k,x_k)$ as well as $F_{\mathrm{b,cruise}}(k,x_k)$ are the engine torque and braking force leading to constant speed, $\tau_{\mathrm{e,min}}(k,x_k)$ is the minimum engine torque and $F_{\mathrm{b,max}}(k,x_k)$ is the maximum braking force. All of these values are computed such that they comply with the constraint sets $\mathcal{P}$ and $\mathcal{A}$, and such that the v may not surpass $v_{\max}$ in the next step.

Letting $u_{\sigma_k}(k,x_k) = [\tau_{\mathrm{e}}(k,x_k,\sigma_k), F_{\mathrm{b}}(k,x_k,\sigma_k), u_j(k,x_k,\sigma_k)]^\top$ be the input of the discrete vehicle model (9), it becomes evident that the controlled dynamics are now equivalent to the following ones

$$f_{\sigma_k}(k,x_k) = \bar{f}(k,x_k,u_{\sigma_k}(k,x_k)). \tag{10}$$

Now we are in a position to formulate the (FHOCP) problem exactly as stated in Section II-B. Regarding V_{f} it would be best to use the optimal cost to go as terminal cost, but since it is not available $V_{\mathrm{f}}(\cdot)=0$ is used. Furthermore $l_{\sigma_k}(\cdot)$ is found as the discretized version of the cost and $\mathcal{X}_p$ as the discretized version the constraints in (EcoCPs). Finally $\mathcal{X}_{\mathrm{f}}$ is not used.

Note that, using the modes based model (10) within the DP algorithm a solution refereed to as (EcoDPm) can be found in a similar fashion to (EcoDPc) defined above.

B. Simplifications and improvements to the NMPC

The NMPC as presented above could be particularly difficult to be applied in field as the computation time can become prohibitively high. For this reason we propose improvements of the NMPC through three modifications. We introduce blocking, a simplified gear shifting strategy to reduce the computational complexity and improve constraint compliance by doing a velocity limit precomputation.

1) Blocking mechanism: The cardinality of $\mathcal{W}$ is strictly related to the computational complexity of solving the NMPC problem, since it is the number of possible sequences, that need to be evaluated during the optimization. In our case it is given by $\mathrm{card}(\mathcal{W}) = M^{N_{\mathrm{p}}}$. Due to the exponential growth, this number can become prohibitively large for long prediction horizon, therefore the common blocking strategy [21] is hereafter adopted. Let N_{b} be the number of blocks used over the horizon, so that the new cardinality becomes $\mathrm{card}(\mathcal{W}) = M^{N_{\mathrm{b}}}$.

2) Gear shifting strategy: Since complexity reduction achieved by blocking may be not sufficient, the following simplified gear shift logic is adopted. Specifically, we only allow a controlled gear shift at the first prediction step, while an auxiliary gear shift control law $\bar{u}_j$ is used afterwards:

$$u_j(p,x_p,\sigma_p) = \begin{cases} u_j(p,x_p,\sigma_p), & p=k \\ \bar{u}_j(p,x_p), & p\neq k \end{cases}. \tag{11}$$

The auxiliary law is defined as

$$\bar{u}_j(p,x_p) = \begin{cases} +1, & \omega_{e,p+1} > \omega_{\mathrm{e,max}} \\ -1, & \omega_{e,p+1} < \omega_{\mathrm{e,min}} \\ 0, & \text{otherwise} \end{cases}. \tag{12}$$

This strategy further reduces the cardinality of $\mathcal{W}$ to $\mathrm{card}(\mathcal{W}) = M\cdot m^{(N_{\mathrm{b}}-1)} = 12\cdot 4^{(N_{\mathrm{b}}-1)}$.

3) Velocity limit precomputation: It is possible to precompute an improved velocity limit $\bar{v}_{\max}$ relying on a minimum time solution to a simplified problem. This solution is guaranteed to be faster than any solution obtainable by solving (EcoCPs) and therefore can be used as a less conservative limit especially helpful for short prediction horizon. In fact in the latter case, the presence of curves ahead, which can only be driven with a certain maximum speed, will be taken into account when using $\bar{v}_{\max}$ even if not immediately appearing in the prediction horizon.

The considered reduced system dynamics are

$$\tfrac{\mathrm{d}}{\mathrm{d}s}v = \tfrac{1}{v}a_{\mathrm{x}} \quad . \tag{13}$$

On a route with curvature $c(s)$ the lateral acceleration is given through

$$a_{\mathrm{y}} = v^2\cdot c(s). \tag{14}$$

Using the relations $\mathrm{d}t = \frac{1}{v}\mathrm{d}s, u=a_{\mathrm{x}}, x=v^2$, the minimum time solution which fulfills the acceleration constraint can be found by solving the following convex problem

$$\begin{aligned} \min_{u(s),x_0} & \int_0^{s_{\mathrm{f}}} \frac{1}{\sqrt{x(s)}}\mathrm{d}s \\ \text{s.t.} \quad & \tfrac{\mathrm{d}}{\mathrm{d}s}x = 2u \qquad x(0) = x_0 \\ & x(s) \leq v_{\max}(s)^2 \quad A\cdot\begin{bmatrix} u \\ c\cdot x\end{bmatrix} \leq \mathbb{1} \end{aligned}, \tag{15}$$

where A represents the constraint set $\mathcal{A}$ and $\mathbb{1}$ is a vector with all ones. The problem (15) can be solved efficiently through numerical optimization and is not bound to be performed online.

V. Assessment of the Ecodriving NMPC

In this section the ecodriving NMPC developed throughout this work is assessed by means of a simulation case study.

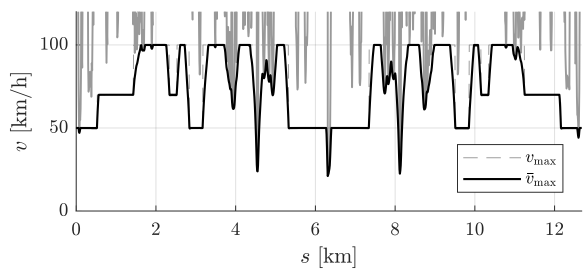

Fig. 1. Solution of problem (15), where the black line is the optimal velocity profile, the solid gray line is the maximum speed due to the limit on lateral acceleration, while the dashed gray line is the limit given by law. Using a discretisation of 1 m, the computation takes 96 s.

A. Description of the case study

All the results presented hereafter are generated based on the same settings in terms of route, discretisation size and vehicle parameters, which are described in the following.

The route model used is based on a trip from Johannes Kepler University (JKU) Linz to Altenberg, a near by village, and back. The parameters and a validation of the considered road and vehicle model can be found in [22].

For discretisation an equally spaced grid with sampling distance $\Delta s = 5\,\mathrm{m}$ is used. The result of the velocity limit precomputation for this setting is presented in Fig. 1.

B. Impact of the restriction to longitudinal modes

In this section we discuss the effect of longitudinal modes onto the optimization potential and indicate that it is not very restrictive. To do so we compute a Pareto front from the multi objective control problem using (EcoDPc) as discussed in Section IV-A. We compare the resulting Pareto front with the one obtained when using (EcoDPm), which corresponds to the optimal solution when only using discrete modes. We focus on the time t_f versus fuel consumption m_fuel tradeoff and therefore do not show the number of gear shifts $\#_\mathrm{GS}$. To make a fair comparison the number of gearshifts is kept close to 60, which is a typical number for the considered trip.

Fig. 2 shows that the performance achieved through (EcoDPm) is only slightly worse than that of (EcoDPc). This indicates that, even though the assumptions [2] necessary for the theoretical justification of modes are not given, the modes based solution (EcoDPm) can perform similarly, while being a strong simplification of (EcoDPc).

C. Performance of the proposed NMPC

Now we discuss the performance of NMPC depending on the prediction horizon. Therefore multiple Pareto fronts for different settings of N_p with $N_\mathrm{b} = 4$ are computed. Fig. 2 shows that the performance obtained with the proposed NMPC can be made very close to optimal one, achieved via (EcoDPm) and (EcoDPc). Specifically the higher prediction horizon the better the performance. The numerical results for the performances and cost function weights used are reported in Table I.

TABLE I

CONTROL PERFORMANCES FOR THE CASE STUDY, USING THE PROPOSED NMPC WITH $N_\mathrm{b} = 4$

N_p	#	m_fuel[kg]	t_f [s]	$\#_\mathrm{GS}$[1]	μ_{t_f} [-]	μ_j [-]
10	1	0.914	641.0	51	1.0e-01	2.5e-04
	2	0.764	662.1	56	3.0e-02	1.4e-04
	3	0.692	701.2	59	1.5e-02	9.0e-05
	4	0.667	745.2	64	1.0e-02	8.0e-05
20	1	0.849	643.0	59	3.0e-02	3.4e-04
	2	0.729	667.2	62	1.3e-02	2.4e-04
	3	0.670	709.3	63	6.0e-03	1.1e-04
	4	0.652	742.7	58	4.0e-03	9.5e-05
30	1	0.851	643.2	61	3.0e-02	3.4e-04
	2	0.734	661.7	62	1.3e-02	2.4e-04
	3	0.673	693.6	60	6.0e-03	1.1e-04
	4	0.643	749.4	61	4.0e-03	9.5e-05

Finally Fig. 3 illustrates the time evolution of the velocity, gear, fuel and travel time when using the NMPC with $N_\mathrm{p} = 30$, $N_\mathrm{b} = 4$ and different cost function weights. It can be observed that rising the weight on time μ_{t_f} leads solutions with higher velocities, as expected, and higher fuel consumption. The figure also shows that the NMPC fulfills all the constraints such as velocity limits, curving speed and gear shifting.

D. Computational Complexity

Finally the computational complexity of the NMPC, which is important in face of field implementation, is evaluated. Since the computation time t_comp is not constant over time, we compute the average and standard deviation of t_comp occurring during the case study simulations for different settings of N_p and N_b. The resulting values and settings of N_p and N_b are reported in Table II.

One can notice that, as expected the computation time rises with both higher N_p and N_b. Even in the most demanding setting the average computation time for the NMPC is below 0.2 s, which makes the strategy eligible to be implemented even in practice.

VI. CONCLUSIONS

In this paper a switching NMPC has been proposed to deal with ecodriving of conventional vehicles. The merit of

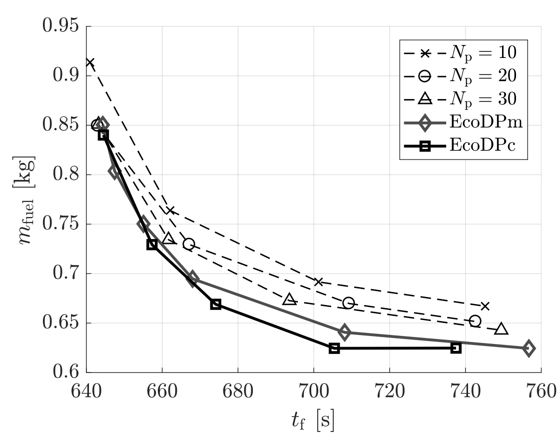

Fig. 2. Pareto front when using (EcoDPc) solution with fine discretisation of the longitudinal input compared to the Pareto front when using (EcoDPm) with longitudinal modes and NMPC with $N_\mathrm{b} = 4$ and $N_\mathrm{p} \in \{10, 20, 30\}$

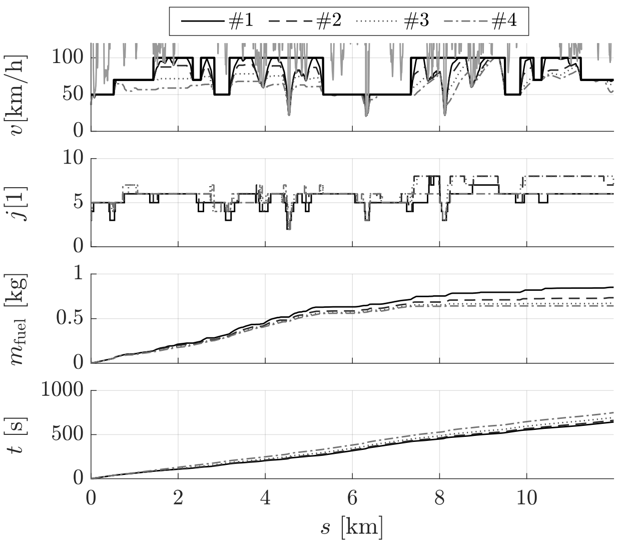

Fig. 3. Time series of the simulation results when using the NMPC with $N_\text{p} = 30$ and settings corresponding to the results shown in the Table I

the proposal is to provide an online solution for the complex ecodriving task, which consists of controlling the vehicle speed and gear according to route based constraints onto the vehicle behavior.

The realistic case study, carried out to validate our proposal, shows that the restriction to a mode based vehicle operation does not significantly influence the achievable performance. Indeed it simplifies the control task such that a computational demanding switching NMPC strategy can be adopted. Further do the results show that the proposed method is comparable to an offline optimal solution in terms of achievable control performance, while also being efficient in terms of computational complexity, thus being in principle eligible to field implementation. In particular, this is evident since even in the case of a prediction horizon $N_\text{p} = 40$ steps combined with input blocks $N_\text{b} = 6$ a computation time less than $0.2\,\text{s}$ can be achieved.

In future works the focus could be on reducing the frequency of switching among modes, improving the simplified gear shifting strategy within the prediction model, studying the computational complexity in a formal way and on an extension to more realistic driving scenarios with time driven disturbances like traffic.

TABLE II

DEPENDENCY OF COMPUTATION TIME ON THE PARAMETERS N_b AND N_p WITH WEIGHTS $\mu_{t_\text{F}} = 0.005$ AND $\mu_j = 0.00022$

	t_comp [ms]			
	N_p			
N_b	10	20	30	40
3	6.3 ± 1.79	12.7 ± 3.70	16.7 ± 2.10	22.1 ± 1.56
4	8.6 ± 0.35	17.1 ± 2.48	23.1 ± 2.26	29.9 ± 3.80
5	24.1 ± 2.59	41.6 ± 10.57	49.7 ± 9.81	60.1 ± 11.53
6	81.3 ± 13.45	130.9 ± 29.52	167.4 ± 37.59	191.7 ± 43.32

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