

Selective maintenance planning of a steel truss bridge based on the Markovian approach

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ABSTRACT: Structures of strategic importance, such as bridges, require careful planning in terms of reliability, durability and safety, qualities which must be guaranteed throughout the entire life cycle of the structure. However, due to the ageing of materials and to aggressive environmental actions which cause deterioration, the response of these structures, just like others, changes over time, resulting in a loss of performance. Yet it is important to maintain a satisfactory level of performance in a bridge throughout its service. To ensure such a performance it is important to apply properly planned maintenance strategies. Appropriate maintenance strategies require knowledge of the process of deterioration and the consequent damages to be expected in order to schedule proper maintenance procedures. It would be fundamental to define a selective maintenance plan which may involve only some parts of the structure, thus allowing bridge viability even during the maintenance activity.

This paper proposes the study of strategies of selective maintenance for a steel bridge immersed in an aggressive environment, starting from the simulation of each individual member. Simulation of deterioration is obtained through the application of an appropriate damage law implemented with a Monte Carlo methodology, while the time prediction of occurrence of the deterioration is obtained through the application of a Markovian probabilistic approach. The results of the Markovian approach were the starting point for choosing strategies of selective maintenance, as the Markov process allowed the identification, in probabilistic terms, of the structure members with the highest risk of collapse and the timing for achieving levels of damage related to the possible collapse of compromised members. This timing was used to identify possible intervals of maintenance. Proposed scenarios are compared with each other both in terms of associated risk, and in terms of life-cycle cost effectiveness.

1. INTRODUCTION

In the last few decades scientific research has pointed out how it is important, when designing a structural system, to guarantee compliance of the performance requirements during the entire life cycle of the structure. This change of thought has led to the consideration, during planning, of factors of uncertainty such as the increase in demand for performance, the presence of aggressive environmental factors, the inevitable human errors, etc, which, over time, may compromise the safety of the entire structural system (Bontempi and Giuliani 2010, Sebastiani et al. 2015). Recent studies have addressed the issue of uncertainty in structural planning, paying adequate attention to problems of reliability (Zhang et al. 2010, Zhu and Frangopol 2013, Barone and Frangopol 2014), robustness (Ghosn et al. 2010, Olmati et al. 2013), safety (Malerba et al. 2012, Arangio and Bontempi 2014) and developing study approaches based on statistical and probabilistic methods that consider the random aspects present in the potential causes of risk for the structure (Barbato et al. 2013, Beer et al. 2013, Barbato et al. 2014, Olmati et al. 2014).

Among the major causes of risk for structures such as bridges, designed to be immersed in the environment while showing their entire structural system, is certainly damage due to aggressive environmental factors (Strauss et al. 2008, Strauss et al. 2009, Malerba 2014). In fact, it is well known that during its life cycle, the characteristics of a structure are modified due to aging; such changes are often accelerated by aggressive environmental actions which, causing a rapid deterioration of structural elements, lead to the inevitable and progressive loss of performance capabilities of the entire system (Ceravolo et al. 2009, Cavaco et al. 2013, Malerba and Sgambi 2014, Sgambi 2014). Such actions occur with a force and in a time period which cannot be predicted, therefore they become difficult to model. On this topic, the scientific community is facing the development of not only probabilistic-type, but also semi-probabilistic, combinatorial or fuzzy-type intervention methods, making it possible to take due account of the uncertainties which afflict the occurrence of aggressive environmental phenomena and the resulting uncertainties in structural response (Ma et al. 2013, Garavaglia and Sgambi 2014, Li et al. 2014).

In strategic structures such as bridges, it is clear how important it is to maintain a satisfactory level of performance throughout their service life. To ensure this performance, it is essential to apply properly planned maintenance strategies.

Appropriate maintenance strategies require knowledge of the process of deterioration over time and the consequent damages to be expected in order to schedule proper maintenance procedures. In literature, different authors have tackled the problem by developing probabilistic-type methods (Bocchini and Frangopol, 2011; Dong and Frangopol, 2015; Timashev and Bushinskaya, 2015; Dao and Zuo, in press) also associated with appropriate cost-benefit analyses (Kong and Frangopol 2003, Dao et al. 2014).

Clearly, major maintenance interventions on bridges involve considerable discomforts and disruptions to large areas of the territory. Certainly selective maintenance interventions, which may involve only some parts of the structure, would be more adequate, thus allowing bridge viability even during maintenance. In literature, the most common approach to the issue of selective maintenance is the use of Monte Carlo simulations which imply a remarkable computational expenditure (Liu and Frangopol, 2005; Frangopol et al., 2009; Marseguerra, 2013).

The scenario just described is one of the new frontiers in scientific research of interest for civil engineering and it is within this context that this work fits. Indeed, this work studies strategies of selective maintenance for an existing steel bridge immersed in an aggressive industrial environment. The starting point is simulation of the damage which each member of the bridge may suffer, over its useful life, due to aggressive environmental attacks and changes in performance demand.

The choice to proceed with the adoption of a rather flexible damage law, and the simulation of damage to structural members over time is necessary. Structures are rarely monitored frequently, even though monitoring would be capable of providing sufficient experimental data to ensure a reliable statistical interpretation of the temporal evolution of deterioration.

Deterioration is simulated through the application of a time-dependent damage law whose parameters, assumed as random variables and associated with proper functions of probability, can grasp all uncertainties inherent to phenomena of environmental aggressiveness and the possible variations in performance demand required by the structural system. The damage law is implemented with a methodology of Monte Carlo simulation, which generates different and possible laws of damage to be applied to the structure. Once certain levels of damage are established and which are significant for the structure under consideration, the structural analysis, performed for each damage law generated, gives back, at any given time, the level of damage reached by each member and the time to reach and overcome same damage. In this way the process of deteri-

oration is interpreted as a process of transition through different performance stages, characterised by different significant levels of damage.

The time prediction of transition from a state of damage and the next one is obtained through the application of a Markovian-type probabilistic approach (Cox, 1962). Indeed, starting from the current performance condition, the Markovian approach is able to assess the possible evolution of the damage to the structural system, and evaluate the transition probability for the system throughout different performance states.

The results of the Markovian approach, applied to the large sample size of transition times obtained from the Monte Carlo simulation, are the starting point for choosing strategies of selective maintenance.

As already mentioned, a thorough maintenance plan can definitely guarantee protection of the features of structure reliability. However, every maintenance intervention involves a cost in terms of intervention on the structure, the use of handling means and discomfort for the partial or total unavailability of the structure during the maintenance operations. The situation becomes particularly delicate for bridges, since their total or partial closure would result in remarkable inconveniences for the traffic flow the entire area. For all these reasons, it would be important to define a selective maintenance plan which may involve only some parts of the structure, thus reducing the inconveniences and guaranteeing, at the same time, the appropriate reactivation of the reliability features of the bridge itself.

Whenever selective maintenance is carried out, each member involved in the maintenance renews its own performance capabilities, which leads to an increase in its service-life and in the service-life of the whole structural system.

In this work, appropriate scenarios of selective maintenance have been studied which involve only members with a high probability of failure. Each maintenance operation is intended as an action which renews the performance of the concerned member. It is an action that can be repeated at more or less regular intervals, which depend on how structural member deterioration progresses after every maintenance operation. The process as intended is a renewal process which can adequately be modelled as a Markovian Renewal Process (MRP).

The processes of Markov renewals are suitable for modelling repetitive phenomena which renew their features after every event. For example, MRP processes are used when modelling the waiting times of strong earthquakes, in fact, this process is considered a renewal process: during a violent earthquake there is a total release of energy but such energy is recharged again during the entire period of suspension, renewing itself, to

be then released in the next event (Shimazaki and Nakata, 1980 Alvarez, 2005, Garavaglia and Pavani, 2011, Masala, 2011, Votsi et al., 2012). Recently MRP processes have been applied in predicting the return periods of hurricanes and tornadoes, obtaining good results (Masala, 2012).

The MRP is able to predict, with different levels of probability, the transition from the current state of the system, i.e. at the time t_0 , in a previous or next state. One special form of such processes, called semi-Markovian processes, predicts the next transition taking into account the time already spent by the system in its current state, in other words: the time interval between the previous transition and the current instant t_0 (Garavaglia and Pavani, 2011, Masala, 2011, Votsi et al., 2012). The application of this type of process allows the probabilistic prediction of the future state, and its likely time of achievement, starting from the actual instant of investigation. The Markov approach proposed in this paper refers to a renewal semi-Markov process.

The transition times modelled in the MRP are those obtained from the application of Monte Carlo simulation. Members involved in any action for selective maintenance are suggested by the structural analysis performed for every damage law generated during the Monte Carlo simulation.

The approach proposed in this paper combines two methodologies that are well known in literature but hardly used contemporaneously in the solution of a problem: the Monte Carlo simulation and the Markov approach at the time when the occurrence of the damage is predicted. In this case, the authors demonstrate how the synergistic use of the two approaches could lead to the definition of satisfactory maintenance strategies, reducing the computation time. In fact, in the proposed methodology, the Monte Carlo simulation is applied only once, unlike other approaches where a new simulation is required after each maintenance action (Biondini et al., 2008).

To show the possible application proposed, different maintenance scenarios have been studied:

- three scenarios of non-selective maintenance which involve the whole structure (scenarios 1, 2, 3) with different maintenance times;
- two scenarios of selective maintenance which keep the structure:
 - a) at the initial undamaged state;
 - b) in a state of slight damage.

Finally, the scenarios of maintenance are compared with each other in a first assessment of costs-advantages which evaluates the sustainability of each scenario, taking into account the costs and the risks associated with each of the possible interventions assumed.

2. THE STEEL BRIDGE IN NORTHERN ITALY

The bridge is located in the municipality of Bellinzago Lombardo, on the western side of Milan, and it connects a commercial centre to an important extra urban road. The bridge consists of two structures: the former, of 6.4 meters, is used to cross a bike path while the latter, of 28.8 meters, oversteps the artificial channel Naviglio Martesana. The bridge deck is composed of IPE600 steel beams and a slab of reinforced concrete that is 20 cm high. Figure 1 shows the longitudinal profile of the bridge.

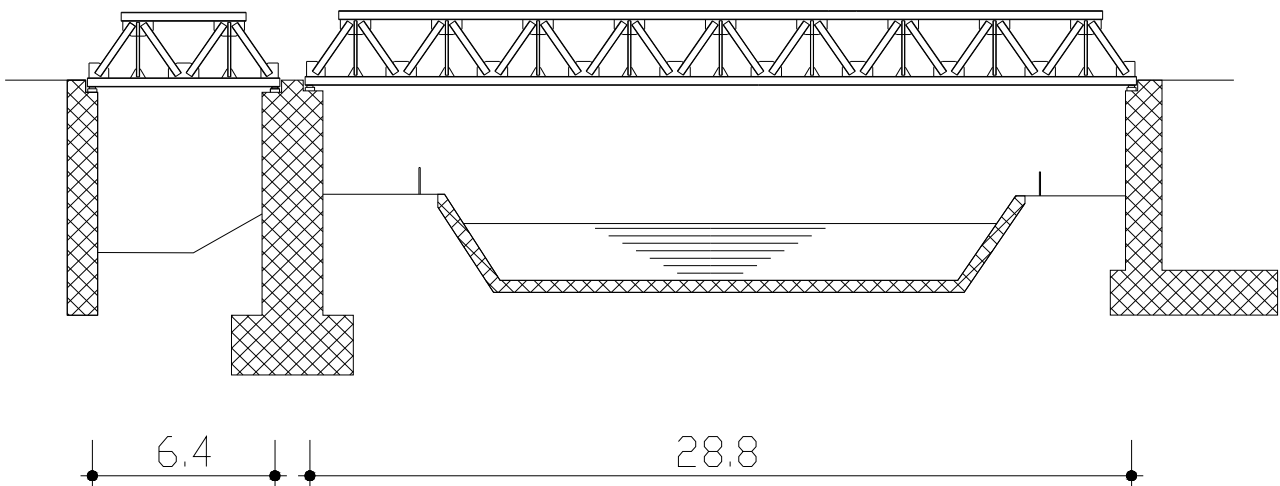


Figure 1. Longitudinal profile of the bridge.

From a structural point of view, the bridge consists of two main beams composed of a Warren truss structure supported by reinforced concrete piles. All members of the structure are connected with welded joints. Figure 2 shows an overview of the bridge structure during construction works



Figure 2. Overview of the bridge structure during the construction works.

To withstand the torsional action transmitted from the deck to the beams, each Warren truss beam is composed of a three-dimensional structure. In the design of the bridge, all internal actions were taken into account by using a three-dimensional numerical model. For simplicity, and without loss of generality, the authors used the equivalent two-dimensional finite element model showed in Figure 3 to perform the deterioration simulations and maintenance described in this paper.

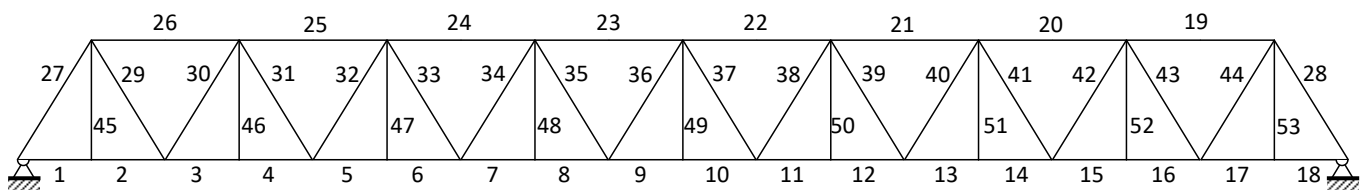


Figure 3. Statically indeterminate truss bridge schema.

The bridge structure is a statically indeterminate truss. Each component consists of members with an I or H profile whose geometric characteristics are reported in Table 1.

Table 1. Truss dimensions

Member	height	area	inertia	volume
	cm			
1-18	35	630	150062	100800
19-26	35	630	150062	201600
27-28	35	630	150062	192333
29-40	30	540	94500	164857
41-53	30	450	78750	117000

The allowable material stress is assumed to be $\bar{\sigma} = 380$ MPa. The load history is characterised by the dead load and by the live loads of the structure summarised in forces Q_1 , Q_2 and Q_3 that are applied on the bottom of the truss as shown in Figure 4.

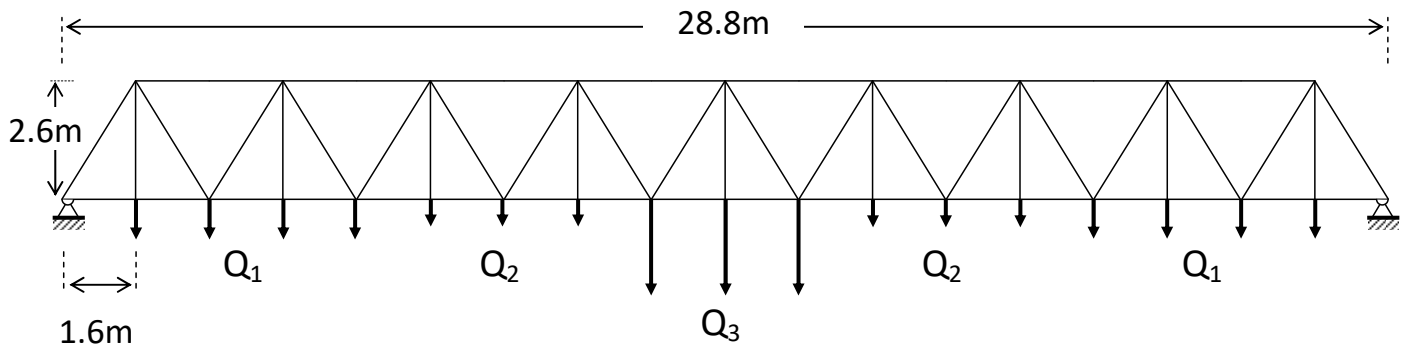


Figure 4. Bridge loads history. The bridge is subject to its own dead load and to live load schematised by the concentrated forces

$$Q_1=290\text{kN}, Q_2=200.7\text{kN} \text{ and } Q_3=700\text{kN}.$$

3. DETERIORATION PROCESS

The deterioration process in structures, which occurs due to environmental aggressiveness, leads to a progressive loss of their own load bearing capacities (Malerba, 2014; Basso et al., 2014). An example is the progressive reduction of section resistance in members affected by deterioration and the consequent increase of stress in the material.

Whenever the member reaches certain levels of deterioration, system performance reduces, so that the system passes from the current state to a subsequent state characterised by lower performance capabilities.

In this way the deterioration process (failure process) may be defined as a *transition process* (Garavaglia et al., 2004).

Every transition depends on:

- the magnitude of the attack (stress cycle);
- the ability of the system to withstand this attack.

Both these parameters depend on a large number of time dependent and random variables (r.v.); therefore it is right to consider the transition process as a stochastic process. The r.v. *service lifetime* τ_i is suitable for describing this process. It is defined as: “the waiting time spent by the material in the performance state i before

a transition” (Garavaglia et al., 2004).

In a process of transition, transitions can occur both from a state characterised by high quality performance to a state characterised by lower performance, and, vice-versa, from a state characterised by low quality performance to a state characterised by a higher quality performance (Fig. 5). In this case we say that the system renews its own performance capabilities. In case of a deteriorated system, the renewal occurs whenever a maintenance action is carried out; maintenance in fact, improves the qualities of the system by bringing it back to a better state of health, sometimes even similar to the initial one, thus extending its service life.

If the process of deterioration is assumed to be a process of down-up transition, same process can be modelled in a satisfying way by the Markovian Renewal Processes (Howard, 1971; Limnios and Oprisan, 2001; Biondini and Garavaglia, 2005; Garavaglia et al., 2012). In fact, the MRP seem to be suitable for describing the development of the system life among different service states with different waiting times; it also takes into account the age t_0 of the system, i.e. the time already spent by the system in the current service state before the prediction is made. This aspect is very important in reliability analyses when maintenance must be planned.

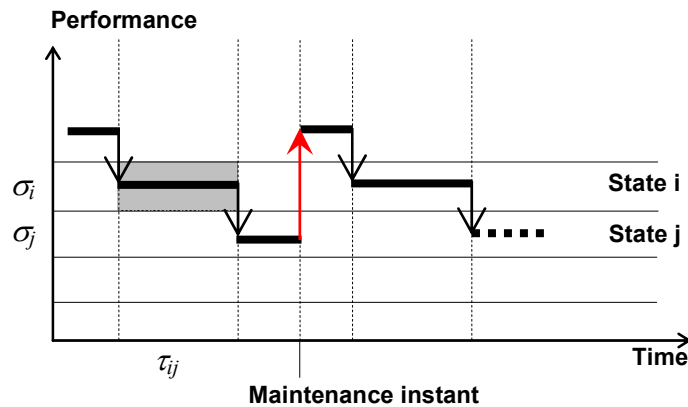


Figure 5. Schematic transitions between different states of performance.

3.1 Markov Renewal Processes

Let us recall the definition and some properties of a Markov renewal process with finite state space.

In what follows $E = \{1, \dots, N\}$ with $N \in \mathcal{N}$; $\mathbf{F} = (F_{ij}(\cdot))_{ij \in E}$ denotes a matrix of distribution functions on \mathfrak{R}_+ , $\mathbf{P} = (p_{ij})_{ij \in E}$ denotes a transition matrix on E and $\mathbf{a} = (a_1, \dots, a_N)$ a probability distribution on E (i.e. $a_i \geq 0$ and $\sum_{i=1}^N a_i = 1$).

Let us consider a two-dimensional stochastic process $(J_n, \tau_n)_{n \geq 0}$ defined on a complete probability space satisfying (Limnios & Oprisan, 2001):

1. Initial conditions: initial state J_0 , i.e. the state occupied by the system at the starting time of the prediction. Defining the time t_0 already spent by the system in the initial state J_0 before the starting time of the prediction is also needed (Howard, 1971).
2. $\Pr(J_0 = i) = a_i$ for every $i \in E$, where $E = \{1, \dots, N\}$ is the set of all possible states, this probability distribution represents the probability that the initial state J_0 will be i .
3. $\Pr(J_j, \tau_{ij} \leq t \mid J_i) = p_{ij} F_{ij}(t)$ for every $t \in (0, +\infty)$ and $i, j \in E$. (1)

Assuming J_i as present state, point (3) (Eq. 1) describes the probability that transition into the next state J_j , occurs by time t ; within it p_{ij} are the transition probabilities of the Markov chain $(J_n)_{n \geq 0}$ and $F_{ij}(t)$ are the distribution functions associated with waiting times in state J_i before moving to state J_j .

Under these three assumptions, the process $(J_n, \tau_n)_{n \geq 0}$ is called the Markov renewal process determined by the space E , $\mathbf{F} = (F_{ij}(\cdot))_{ij \in E}$ the matrix of distribution functions on \mathfrak{R}_+ , $\mathbf{P} = (p_{ij})_{ij \in E}$ the transition matrix on E and \mathbf{a} the initial probability distribution.

Let us recall some consequences of the definition:

- i. $(J_n)_{n \geq 0}$ is an E -valued Markov chain with transition matrix \mathbf{P} and initial distribution \mathbf{a} ;
- ii. for every $n > 1$, τ_1, \dots, τ_n are conditional independent, given $(J_n)_{n \geq 0} \quad \forall n \geq 0$ and

$$\Pr(\tau_1 \leq t_1, \dots, \tau_n \leq t_n \mid J_n, n \geq 0) = \prod_{i=1}^n F_{(i-1),i}(t_i) \quad \text{with } t_i \geq 0 \quad (2)$$

Eq. (2) is the marginal distribution function of the waiting times τ_i describing the probability of transition into each state i by time t_i .

The Markov chain $(J_n)_{n \geq 0}$ represents the states successively visited by the system, while the process $(\tau_n)_{n \geq 0}$ represents the successive waiting times in each visited state. In our application, the states are the different levels of performance which structural systems subject to deterioration may present during their useful life. Such levels are measured in terms of structural response to induced stresses. The τ_n 's are the times of permanence of the structural system during the performance state n before being subject to damage and before passing to the following state, which will be characterised by lower performance.

If the MRP is defined as shown in points 1-3, assuming J_i as the present state and t_0 as the time already passed by the system in the i -state, the probability of moving to state j , J_j , can be defined as follows:

$$\Pr(J_j, \tau_{ij} \leq t_0 + \Delta t \mid J_i, \tau_i > t_0), \quad (3)$$

where i is the state of the present performance; j is the state of the next lower performance; τ_{ij} is the waiting time spent by the system in the state J_i before moving to J_j , under the limiting condition that no transition has already happened (defined through the condition: state i , $\tau_{ij} > t_0$); Δt is the discrete time in which the prediction should be obtained.

If the distribution functions $F_{ij}(t)$ are defined and $\tau_{ij} = t_0$, the probability (3) can assume the following form (Garavaglia & Pavani 2011):

$$P_{\Delta t | t_0}^{(ij)} = \frac{[F_{ij}(t_0 + \Delta t) - F_{ij}(t_0)] p_{ij}}{\sum_{k=1}^S [1 - F_{ik}(t_0)] p_{ik}}. \quad (4)$$

The probability p_{ij} can be obtained by experimental observations through the ratio:

$$p_{ij} = \frac{\text{observed transitions from } i \text{ to } j}{\text{observed transitions from } i}. \quad (5)$$

Equation (5) describes the probability p_{ij} as the ratio between the transitions from state i to those of state j (e.g.: transition *State 0-State 1*), as observed in the experimental sample, and all the transitions from state i to any other considered state k , with $k = 1, \dots, N$ (ex.: transitions *State 0-State 1*; *State 0-State 2*, ..., *State 0-State k*, ..., *State 0-State N*).

Of course, the probability (4) can be evaluated both for the system and for each member which composes the

system; in the following dissertation the member probability will be indicated with: $P_{\Delta|t_0}^{(m,ij)}$, where m is the number identifying the structural member analysed.

3.2 The deterioration process as a Markovian Process

Different deterioration processes connected with aggressive environmental attacks can affect every single member of a structure. The failure of one or more members involves the loss of performance of the whole system.

When modelling a phenomenon, it is essential to pinpoint an appropriate variable which may properly describe the evolution of the process. In the present case a correct risk index can be the evolution of the value σ (of the matter stress) which, as it is well known, depends also on the resistant area of the damaged member.

Therefore, assuming the stress value σ , the ratio between the internal forces and the deteriorated area of the member cross sections as a random variable in the deterioration process, and using an appropriate computer code for structural analysis, the performance decrease in load-bearing capacity can be evaluated.

In order to model the time evolution of the variable within a Markov renewal model, the following assumptions are introduced (Biondini and Garavaglia; 2005):

- the structure, as well as each component member, are undamaged at the initial time $t_0 = 0$;
- at each instant t the members are considered to occupy a given state i , with $i > 0$, when $\sigma_i \leq \sigma \leq \sigma_{(i+1)}$, where σ_i and $\sigma_{(i+1)}$ are the lower and upper thresholds respectively, which characterise the state i ;
- the member performance moves from a state i , $i > 0$, to another state j , with $j > i$, characterised by a lower level of performance, σ_j , when $\sigma_i < \sigma_j$, during a time interval τ_{ij} . Of course, condition $j < i$, with $\sigma_i > \sigma_j$, is also possible if some maintenance is carried out.

In MRP, each waiting time τ_{ij} must be modelled by choosing an appropriate probability density function (PDF). This is not a simple choice and it can be “not unique”. It should be made on the basis of physical knowledge of the phenomenon and knowledge of the theoretical behaviour of the distributions in their tails, well described by the hazard rate function $\lambda(t)$.

Physical knowledge, based on experimental evidence, suggests that the deterioration of building materials increases over time, therefore the hazard rate must also increase over time. The distributions obeying this law are, for example, the Weibull distributions where, if $t \rightarrow \infty$, the hazard rate $\lambda(t)$ increases and tends to an infi-

nite value too, and Gamma distributions where, if $t \rightarrow \infty$, the hazard rate $\lambda(t)$ increases and tends to an asymptotic value. In this paper, starting from the previous considerations, Weibull distributions were chosen in order to model structural member deterioration. The parameters involved in the Weibull distributions are estimated by the maximum likelihood criterion using a computer code where the routine ROSE of IMSL Fortran Library, which is part of Rosenbrock's optimisation method, was implemented.

4. DAMAGE LAW

In this paper the Markov renewal approach is applied to evaluate the transition process of each structural member of the steel bridge shown in Figure 2. Our aim is to be able to program maintenance interventions on one or multiple members based on the most likely instants of transition between states evaluated using MRP.

As mentioned above, the deterioration of building materials due to the action of aggressive environment agents is a random process which depends on a certain number of variables. The uncertainty, in modelling these phenomena, derives from the fact that we do not always have sufficient data to accurately simulate the variations of mechanical behaviour in structures that have deteriorated over time. In fact, this would be possible only if there were a monitoring action for a sufficiently long period of time so as to guarantee a rather wide collection of data (Garavaglia e Sgambi, 2014). However, even if there were the possibility of carrying out a data-campaign, there would be a complete picture of the behaviour over time only after several years of structure service. For strategic structures, such as bridges, it is important to immediately plan proper maintenance, therefore it is necessary to schedule a simulation methodology which, starting from already-known and readily available data, can reliably evaluate how the structural behaviour develops in the presence of deterioration.

The randomness of the deteriorating actions and the uncertainties in loads or in constraints must be modelled through proper probabilistic distributions. Some of these distributions are already present in literature, others, such as the environmental ones, are not easy to establish and their wrong definition might compromise the whole model. The authors are well aware of this limitation; as a consequence, the choices shown here are certainly not complete and still require a further in-depth analysis.

To simulate the structural behaviour, taking into account some uncertainties and stochastic cases present in the actions of deterioration, the Monte Carlo methodology is used. The results obtained from such methodol-

ogy are the basis for the application of the Markovian probabilistic model in order to foresee the length of the system lifecycle in a given state.

The Monte Carlo simulation requires an appropriate deterioration model. Biondini et al. (2008) proposed a suitable damage model which seems to be effective in order to simulate the deterioration process investigated. This model has been implemented into our computer code in order to perform Monte Carlo simulations and structural analyses (MCS).

By denoting Θ as a generic material property, the material deterioration over time t can be evaluated as follows:

$$\Theta(t) = \Theta_0[1 - \delta(t)] \quad (6)$$

where “0” denotes the initial undamaged state, and the deterioration over time t is measured by a time-variant damage index $\delta = \delta(t) \in [0;1]$. The following damage model is assumed (Biondini et al., 2008):

$$\delta(\tau) = \begin{cases} \omega^{1-\rho} \tau^\rho & , \tau \leq \omega \\ 1 - (1-\omega)^{1-\rho} (1-\tau)^\rho & , \omega < \tau < 1 \\ 1 & , \tau \geq 1 \end{cases} \quad (7)$$

where $\tau = t/T_C$, T_C is the normalised time instant when the failure threshold $\delta=1$ is reached, and ω and ρ are shape parameters of the damage curve.

The damage parameters ρ and ω must be chosen according to the actual damage process development. Damage rates may be associated with environmental aggressiveness, as well as with the level of acting stress.

The following linear relationship is assumed (Biondini et al., 2008):

$$\rho = \rho_a + (\rho_b - \rho_a) \xi \quad (8)$$

$$\omega = \omega_a + (\omega_b - \omega_a) \xi \quad (9)$$

where the subscript “a” refers to the damage associated with environmental aggression, the subscript “b” refers to the damage associated with loading effects, and ξ refers to the ratio between the level of acting stress and the limit state value $\bar{\sigma}$, $\xi = \sigma / \bar{\sigma}$.

Thereby, the proposed damage law is able to represent damage mechanisms induced by environmental deterioration, such as steel corrosion or material fatigue. These mechanisms are usually present and interact with

each other. Therefore, based on experimental observations and/or laboratory accelerated test data, a proper calibration of the damage parameters is required.

5. APPLICATION OF MRP TO AN EXISTING STEEL BRIDGE

The bridge in Figure 2 is subject to deterioration involving a reduction of both the cross-sectional area A and the material stress σ of each structural member. Without any loss of generality, in this study it is assumed that such properties suffer from the same damage process:

$$A(t) = A_0[1 - \delta(t)] \quad (10)$$

where “0” denotes the initial undamaged state. The damage model assumed is the model in Eq. (7).

The approach is applied to the statically indeterminate truss bridge in Figure 1. Buckling failures are assumed to be avoided. The initial value of the cross-sectional area A of the member and of material stress $\bar{\sigma}$, as well as the forces Q_i , are assumed as deterministic.

Each probabilistic approach requires several samples on which to base the prediction. In this study, the samples are obtained using the Monte Carlo simulation, which is applied to the formulation of time-dependent damage law, then implemented in the structural analysis of the system. In this simulation process deterioration laws are simulated, member by member, and the damage parameters ρ_a , ρ_b , ω_a , ω_b , and T_C of every law are modelled, member by member, as random variables with prescribed probability distribution. In the proposed procedure, the damage laws and the random variables of each member can be assumed to be independent from one another, as well as correlated with each other with a different degree of correlation, as explained in 5.1. Each distribution is chosen on the basis of physical knowledge of phenomena investigated, and phenomenon modelling starts by using mean and standard deviation (Ciampoli, 1999; Gupta and Lawsirirat, 2005), (Tab. 2). In the case study presented, damage laws are assumed to be independent, member by member, while random parameters of every damage law, generated by each member of the structure, are correlated (see 5.1). The mean values and standard deviation reported in Table 2 are assumed to be equal for each member of the structural system.

Table 2. Distribution used in the Monte Carlo procedure.

Variable	Units	Distribution	mean value	standard deviation
ρ_a	dimensionless	Normal	1.20	0.2
ρ_b	dimensionless	Normal	1.20	0.2
ω_a	dimensionless	Normal	0.35	0.2
ω_b	dimensionless	Normal	0.35	0.2
T_C	Years	Weibull	60	10

5.1 The procedure of the Monte Carlo simulation for the MRP

This paragraph tries to give a better explanation of the simulation procedure used in this work (Figure 6 and 7).

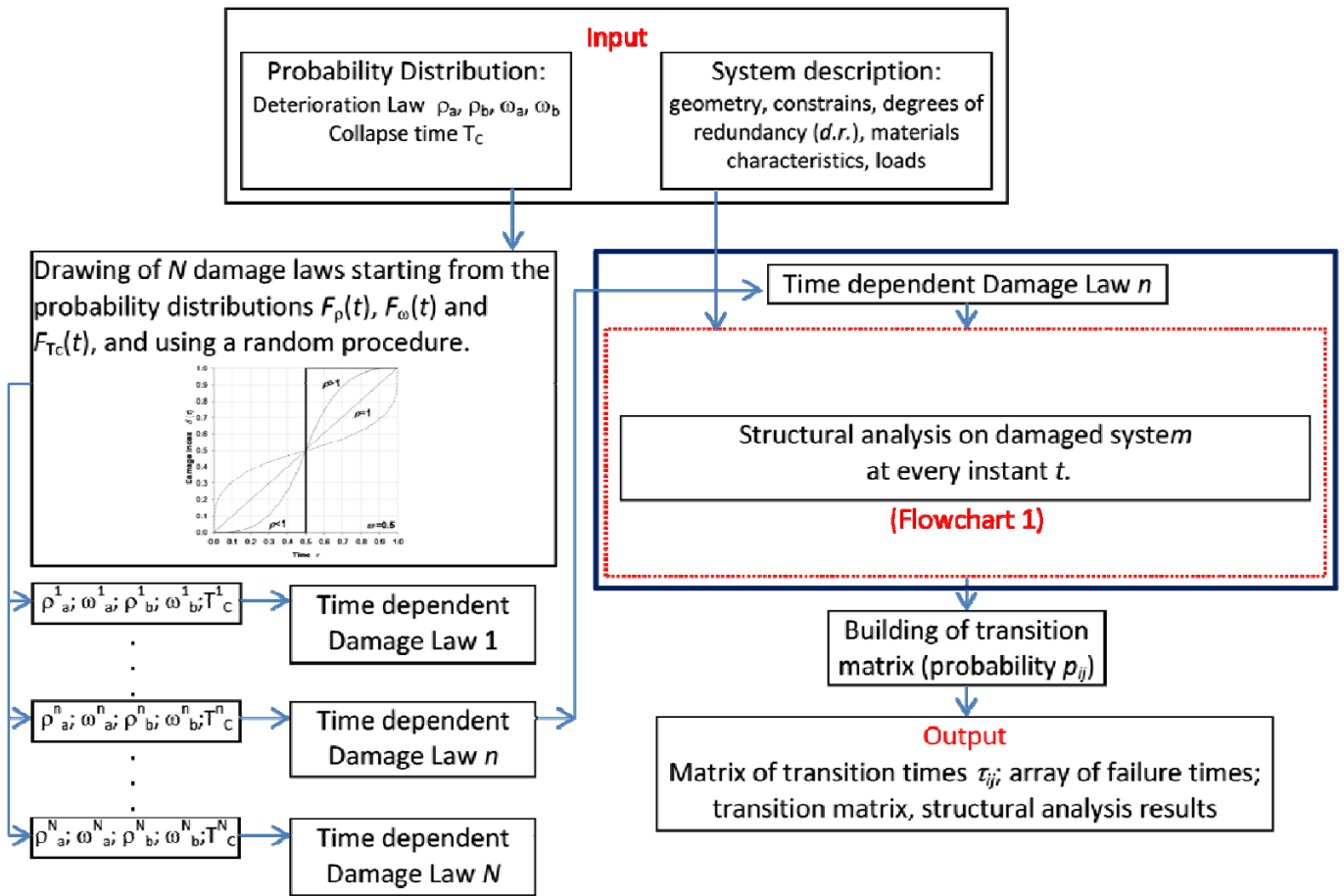


Figure 6. Procedure proposed: simulation process.

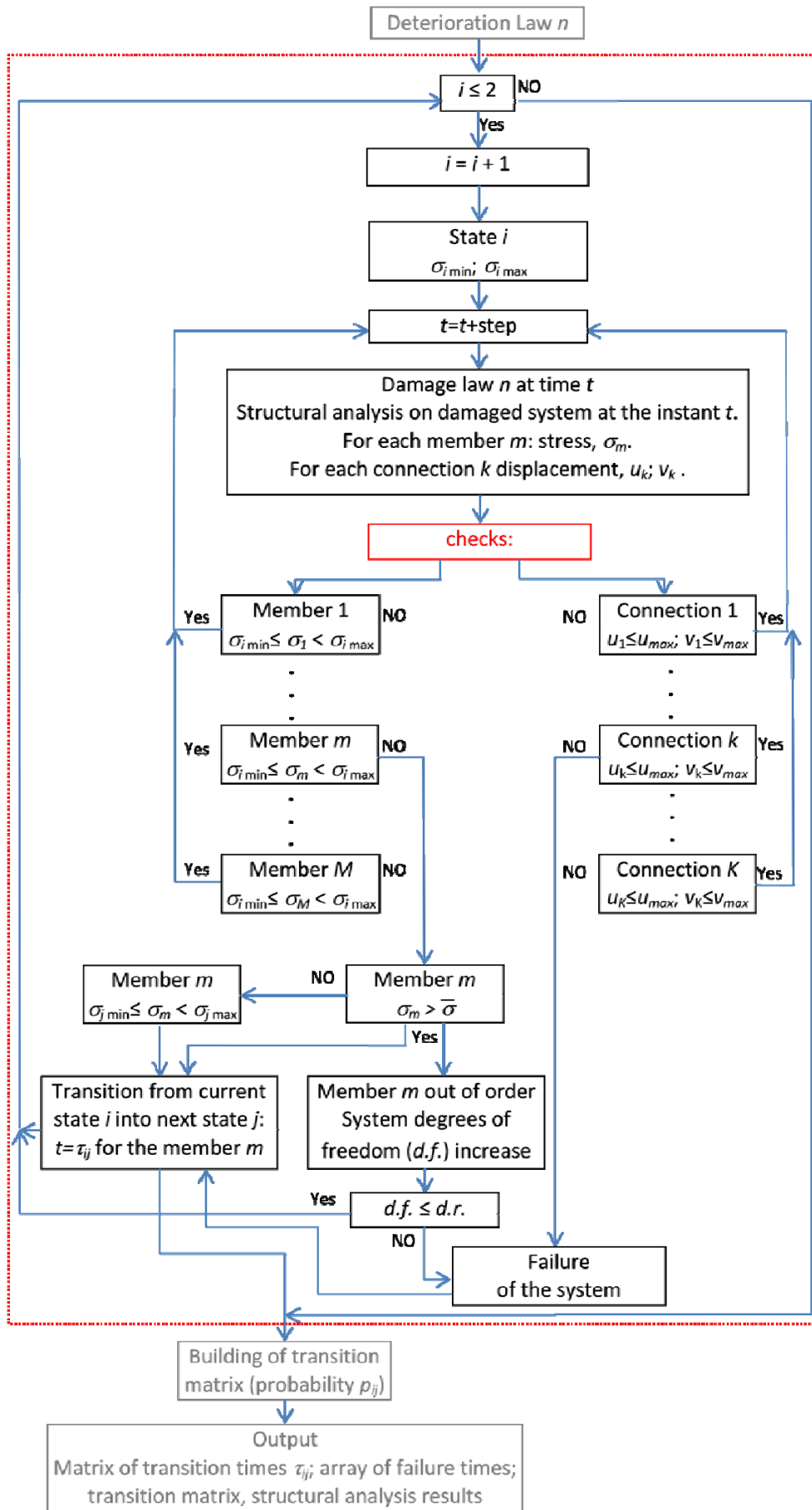


Figure 7. Development of the red-box in Flowchart 1.

Once distributions appropriate for describing the random variables in Table 2 are chosen, and after having set their average values and standard deviations, and provided that these choices are supported by experimental data regarding deterioration of the structure in question or of similar structures reported in literature, for each simulation cycle, starting from the average values and standard deviation, the developed procedure generates the probability distribution to be attributed to every random variable. Subsequently, through a procedure of random selection (crude technique), a value taken from the assigned probability function is then attributed to variables, and is then inserted in the law (7) to define its shape. The selection process provided for in the calculation code allows the introduction of different levels of correlation among random variables at stake. Three types of correlation are provided:

- Grade 0 represents a zero correlation, i.e. each random variable is independent from all others, both in terms of member and of structure. This degree of correlation is obtained by performing, for each random variable, a random selection in the Monte Carlo simulation process.
- Level 1 identifies a correlation among random variables of each member and is realised through a single selection of the random number in the Monte Carlo process; this number is then used for all random variable generation operations relative to the member concerned.
- Level 2 reproduces a complete correlation with the whole structure, i.e. all the random variables are generated with the same probability of occurrence.

In the present case, the correlation adopted for parameters of damage laws generated member by member is Level 1.

Each damage law so constructed is applied to time-dependent structural analysis (Flowcharts 1 and 2). At each instant of time t , and for each member of the system, the damage law provides a certain value $\delta(t)$ which, applied to the initial section area, A_0 , of each member, quantifies its deterioration as area reduction (Eq. 10). Therefore, at each instant t , each member of the deteriorated system, subjected to stresses Q_i invariant over time, always provides a different structural response, quantified in terms of effort σ and of nodal displacements. The increase of the effort recorded in each member, step by step, describes its performance and places it, at that precise instant, in a determinate service state.

We can therefore say that during its life the structure, as well as its single members, move into different service states, therefore a transition can involve two or more states for each step. To simulate the behaviour for

the bridge studied, three different states are assumed: *State 0* relates to low damage, *State 1* relates to moderate damage and *State 2* relates to heavy damage. The state boundaries are chosen on the basis of expert judgment and are reported in Table 3. They reflect about 51% of the allowable value for $\bar{\sigma}$, previously given to be 380 MPa and 65% of the same value. The choice of these limits is sensitive and certainly not unique. It is an interesting field of research for evaluating the impact that different thresholds may have on the choice of maintenance strategies to be undertaken. The topic, however, goes beyond the context of this paper, therefore the authors have chosen to entrust them to expert judgment.

Table 3. State boundaries.

State	σ_{\min} (MPa)	σ_{\max} (MPa)	Damage level
<i>State 0</i>	>0.00	≤ 195	Low (undamaged)
<i>State 1</i>	>195	≤ 250	Moderate
<i>State 2</i>		>250	Heavy

The time-variant structural analysis of each simulation cycle is carried out by updating, step-by-step, the stiffness matrix of the deteriorated structural members and the matrix of the nodal displacements. Referring to the limit state i $|\sigma| \leq \sigma_{\max}^i$ member transition occurs when the member reaches the upper boundary σ_{\max}^i of the state i . During every simulation, the transition time for each member is recorded and is defined as the waiting time τ_{ij} spent by the member in state i before moving to a successive state j .

Considering the limit state $(|\sigma| - \bar{\sigma}) = 0$, whenever a member of the system reaches this state, it is considered out of order. In the following structural analysis the degrees of system freedom will increase and distribute the stress on the members that are still active. System failure is reached when the number of failed members activates a certain mechanism. The time during which the failure occurs is called *failure time*. Whenever a member comes to a given damage threshold, the time taken to reach this damage state is called *crossing time*.

The proposed approach is applied to investigating, from a probabilistic point of view, the next transition of each member of the system in Figures 3 and 4, from the current state i , since t_0 with $t_0 > 0$, to the state j in the next interval Δt . Using the Monte Carlo simulation, a group of 5000 samples of the same structural system have been built which differ from each other for the deterioration suffered due to the application of generated damage laws (7) and to which their members are submitted. These laws are constructed by generating random parameters as shown in Flowchart 1 and subsequently introduced into the structural time-dependent analysis

(Flowchart 2). From the structural analysis, step by step, of each of the 5000 samples, the simulation returns the state of effort, every instant, of each system member, the nodal displacements and the members' transition times connected with each damage threshold and the failure time of the whole system. The transition times between the different damage thresholds and the time of failure are useful outputs for setting the Markovian approach to maintenance. They will be modelled with Weibull distributions.

Considering the structure always at the initial time $t_0=0$ and using Eq. (4), the probabilities $P_{\Delta t|t_0}^{(ij)}$ and $P_{\Delta t|t_0}^{(m,ij)}$ are evaluated for the whole system and for each m member of the system ($m=53$). When the probability $P_{\Delta t|t_0}^{(ij)}$ (or $P_{\Delta t|t_0}^{(m,ij)}$), with $i \neq j$, reaches a value greater than or equal to $r \cdot 10^{-4}$ ($r =$ positive number) the risk of transition from i to j is assumed as being upcoming; therefore maintenance could be planned in instants close to which the probability is recorded.

If the structure has already spent a certain period of time in a certain state, t_0 is to be considered different from zero and the transition probability to the following stage changes. Figure 8 shows how transition probabilities change when t_0 varies.

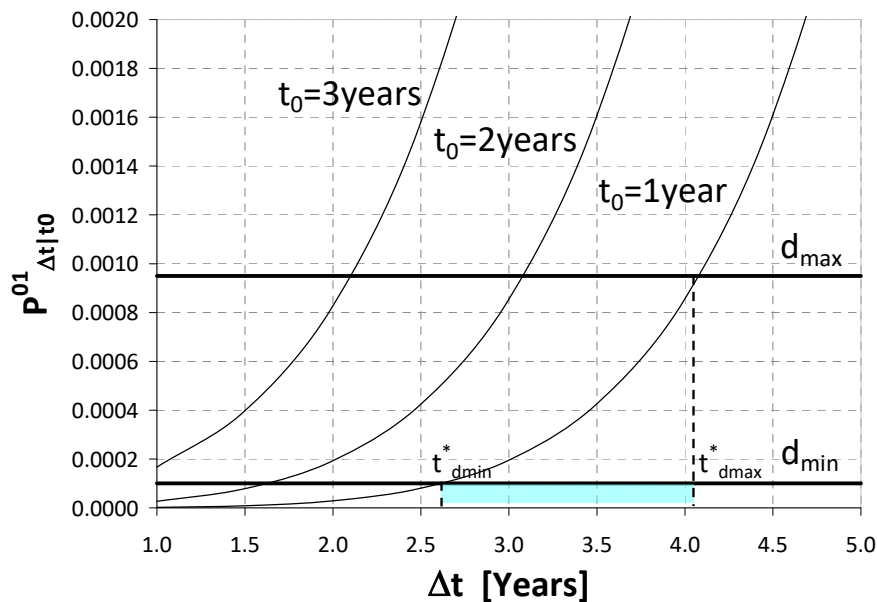


Figure 8. System's probability of transition $P_{\Delta t|t_0}^{(ij)}$ vs Δt ($i=0$, vs. $j=1$) evaluated at different t_0 . Damage $d=n \cdot 10^{-4}$ are the damage thresholds; instants t^* are the instants when the member exceeds d_{min} and d_{max} for different time t_0 spent in state $i=0$. The shadowed area is the period during which the risk increases.

6. MAINTENANCE PLANNING BASED ON MRP EVIDENCE

The application of the MRP to the whole system can lead to identifying *when* to carry out maintenance, but not *where* to do it.

Figure 9 shows the period during which it is likely that the structural will fail in the next Δt , if the system has already passed t_0 of its service life. If the risk threshold assumed is $r \cdot 10^{-4}$ the probable dangerous interval for each t_0 is the shadowed interval $t_{\min}^*; t_{\max}^*$.

Figure 10, instead, shows the period during which the system is likely to cross the first threshold in the next Δt , passing from *state 0* to *state 1*, if the system has been in service at *state 0* for one year (P_{01}) and the period during which it is likely that the system crosses the second threshold, during the next Δt , passing from *state 1* to *state 2*, if it has been in *state 1* for one year (P_{12}).

In both cases, a decision could be made to take a reasonable risk and carry out maintenance on the structure at a given instant of the shadowed period (*when*), but without knowing exactly *where*.

In fact, using this approach, maintenance can be scheduled following three different scenarios:

scenario 1) $t_0=1$ year: renew/replace the *whole* system during the interval [$t_{\min}^*=21.0$ years; $t_{\max}^*=25$ years], before the system fails. This heavy maintenance should be cyclically repeated every 21.00-25 years.

scenario 2) $t_0=1$ year: repair the *whole* system during the interval [$t_{\min}^*=3.0$ years; $t_{\max}^*=4.0$ years], before transition of the system into *state 1*. This light maintenance should be repeated cyclically every 3-4 years to keep the system in *state 0*.

scenario 3) $t_0>3$ year (after the first transition): repair/replace the *whole* system during the interval [$t_{\min}^*=6.0$ years; $t_{\max}^*=7.0$ years], before transition of the system from *state 1* into *state 2*. This maintenance will be repeated cyclically every 9-10 years to prevent transition of the system into the *state 2*.

Scenarios 1-3 involve the *whole* system in the maintenance plan, without identifying what system members are the most damaged. As this could be quite expensive, it becomes interesting to survey if the MRP approach can also suggest possible selective maintenance scenarios.

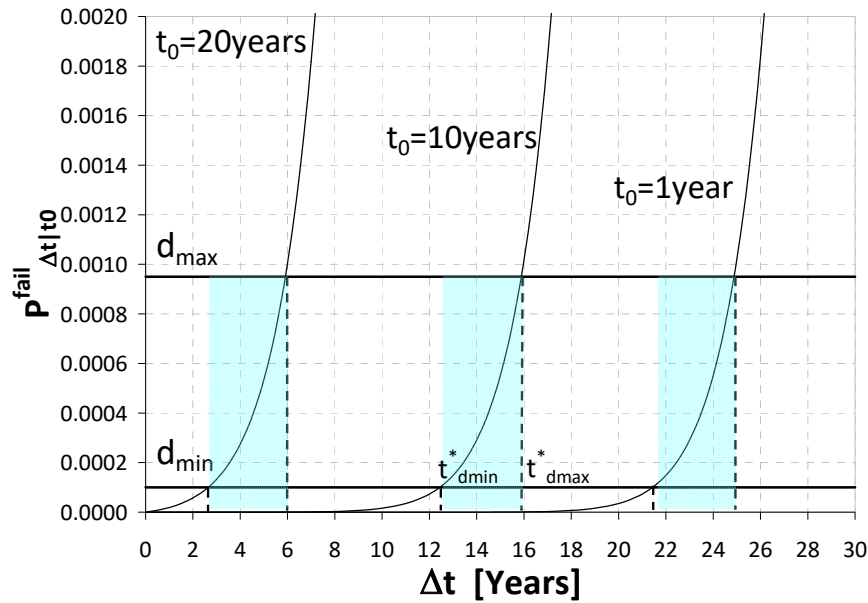


Figure 9. Probability of system transition $P_{\Delta t|t_0}^{(fail)}$ vs Δt evaluated at different t_0 . The shadowed area is the period during which the risk increases.

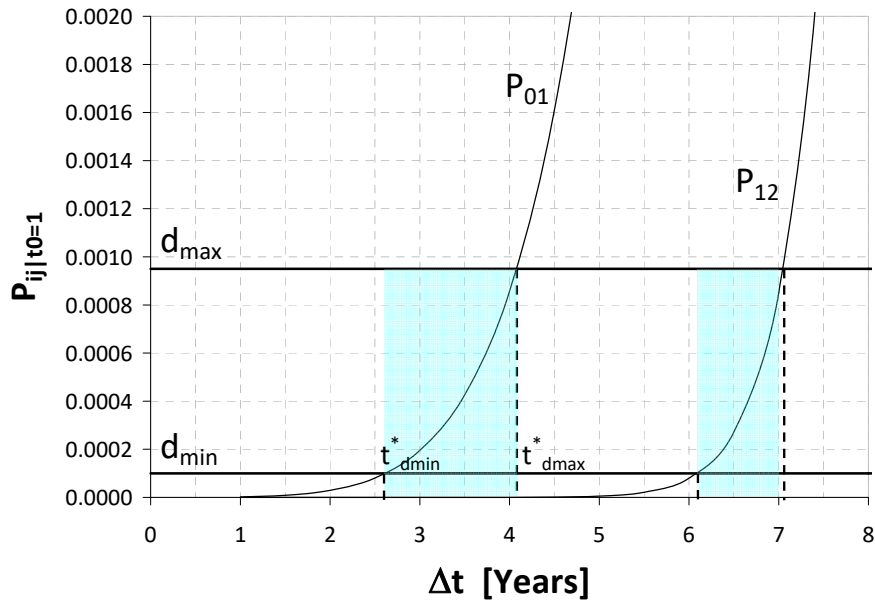


Figure 10. Probability of system transition $P_{\Delta t|t_0}^{(ij)}$ vs Δt (i vs. j) evaluated at $t_0=1$ year and for $i=0, j=1$ and $i=1$ and $j=2$.

Following this aim, MRP was applied to evaluate the probability of transition to a different service state, both member by member and the whole system.

Figure 11 shows the period during which it is likely that the 53 members of the system will pass from *state 0* to *state 1*, during the next Δt , if each member has been in *state 0* for one year. In Figure 11 and in the following figures, the probabilities related to members 26-53 are identified as “*other members*”.

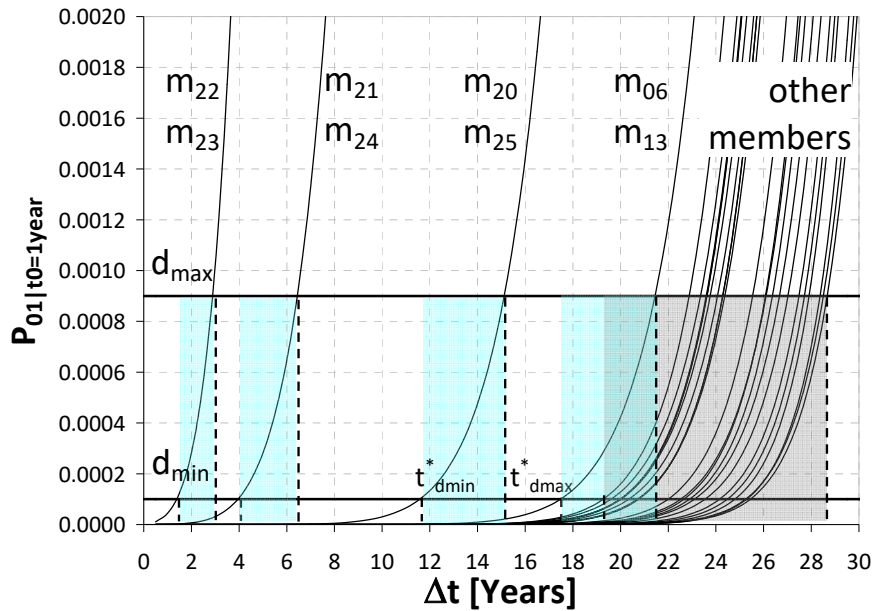


Figure 11. Probability of member transition $P_{\Delta t | t_0}^{(m,ij)}$ vs Δt (i vs j) evaluated at $t_0=1$ year, $i=0, j=1$.

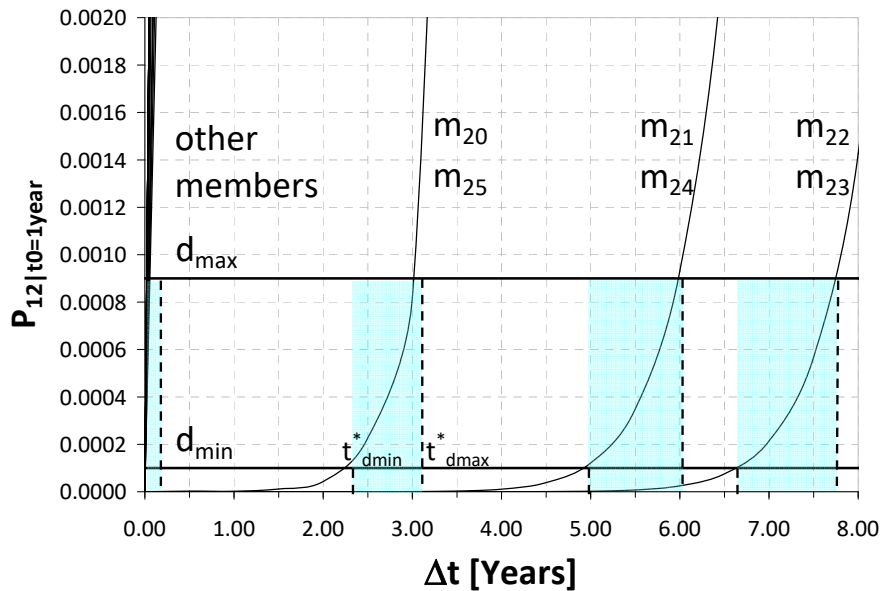


Figure 12. Probability of member transition $P_{\Delta t | t_0}^{(m,ij)}$ vs Δt (i vs j) evaluated at $t_0=1$ year, $i=1, j=2$.

Figure 11 shows the period during which it is likely that the 53 members of the system will pass from *state 1* to *state 2* during the next Δt , if each member has been in *state 1* for one year. In the case studied, for each member and for the whole system, the transition into *state 2* occurs always m years after the transition into *state 1* has happened; the transition 0-2 has never occurred. Figure 13 and Table 4 sum up the transitions from *state 0* to *state 1* of each of the 53 members.

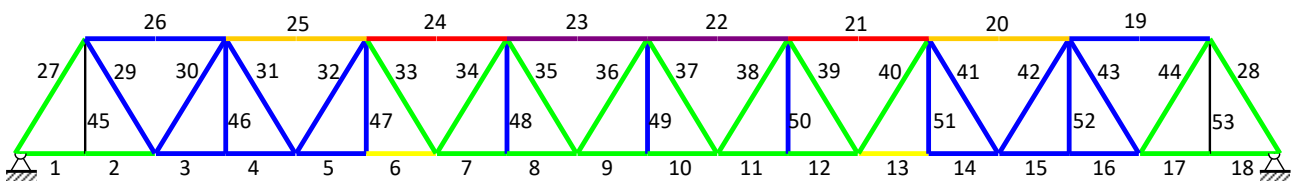





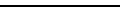


Figure 13. Scheme of the progressive passage of the members from state 0 to state 1 (Colours refer to the range of crossing times listed in table 4).

Table 4. Probable passage times from state 0 to state 1

	crossing time τ_{01} (range in years)
	1.35 – 2.95
	4.00 – 6.40
	11.60 – 15.10
	17.50 – 21.50
	19.30 – 24.75
	22.00 – 28.65

According to these last figures, maintenance could be scheduled following these other three scenarios:

scenario a) $t_0=1$ year: repair of the most deteriorated members every 3 years. Precisely, every 3 years an intervention on members 22 and 23 should be carried out, every 6 years the intervention involves also members 21 and 24, and every 12 years the intervention involves also members 20 and 25. These interventions of light maintenance, cyclically repeated, can keep the structure in *state 0*, which is not damaged. Every 21 years the repair intervention should concern the entire bridge (Fig. 14).

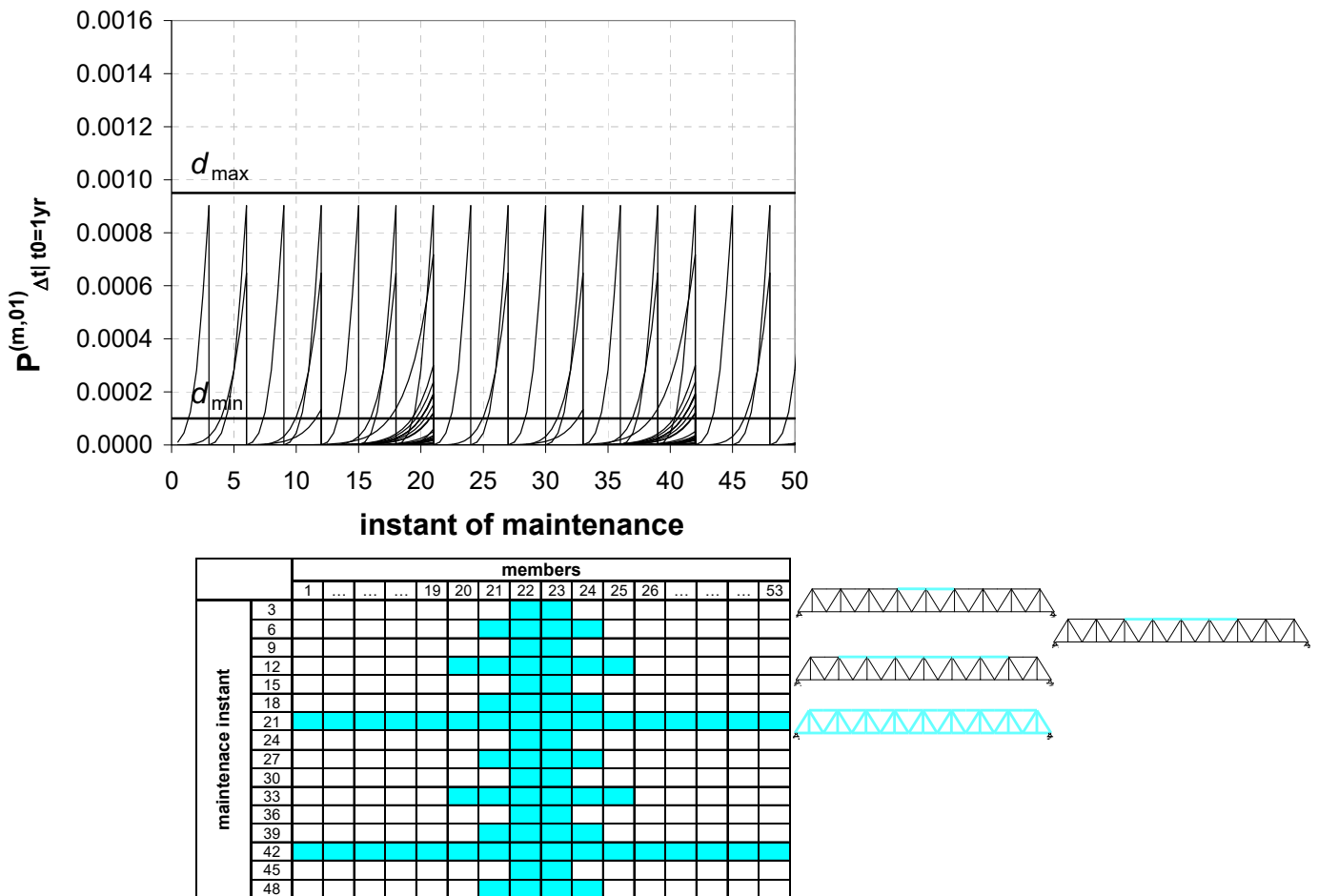


Figure 14. Selective maintenance scenarios: a). Considering maintenance instants, the figure shows how the system leans towards the initial level of performance when damaged members are repaired; the table shows which members are repaired and the instant when maintenance occurs (the instant is shown in the table in the first column on the left, it is expressed in years and is measured from the year of construction). The selection of the time of maintenance and of the members to be submitted to maintenance is suggested by the transition probability threshold 1, evaluated for each system member with the MRP application.

If the choice involves the assumption of a certain degree of risk (i.e. accepting that the transition probability for some members is higher than the value $d_{\max} = 9 \times 10^{-6}$) the scenario could be:

scenario b) $t_0=1$ year: the repair of members 20, 21, 22, 23, 24, 25 every 11 years, then the repair/replacement of *whole* members every 22 years. Also this maintenance, cyclically repeated, would keep the system in *state 1* for a long time (Fig. 15).

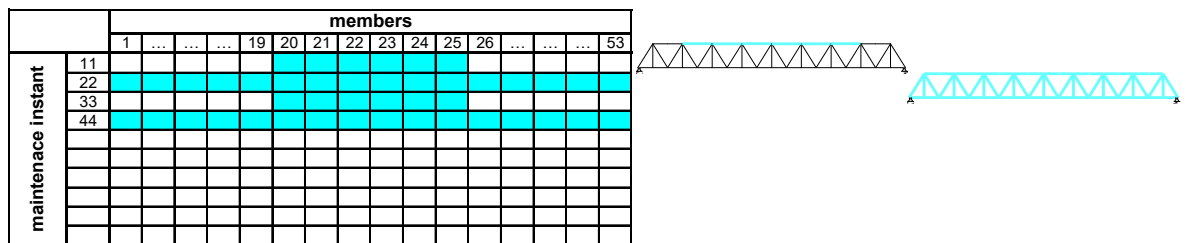
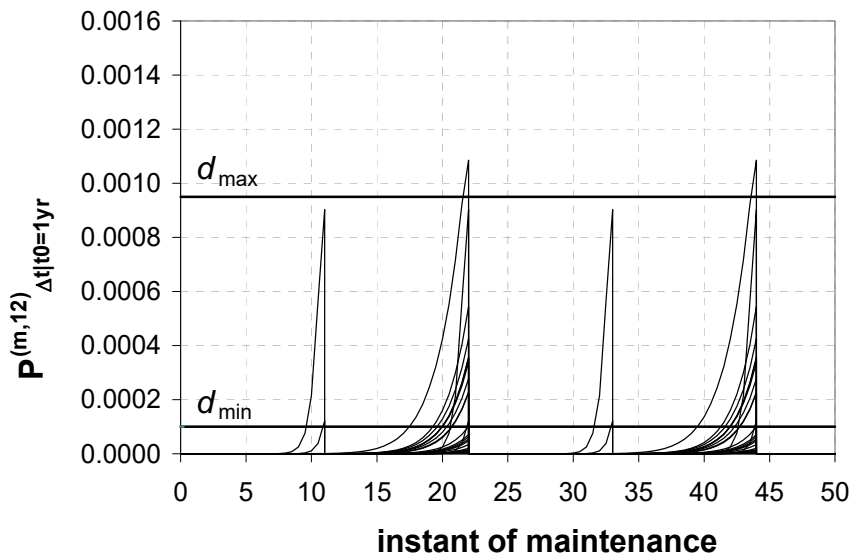


Figure 15. Selective maintenance scenarios: b). Considering maintenance instants, the figure shows how the system leans towards the initial level of performance when damaged members are repaired; the table shows which members are repaired and the instant when maintenance occurs (the instant is shown in the table in the first column on the left, it is expressed in years and is measured from the year of construction). The selection of the time of maintenance and of the members to be submitted to maintenance is suggested by the transition probability threshold 2, evaluated for each system member with the MRP application.

In conclusion, the MRP methodology can lead us to choose the best maintenance strategy to keep the system healthy and in service for a long time, but it can also affect the initial planning choices of the structural system in order to achieve longer maintenance intervals. Redesigning members 20-25 could maybe benefit the whole system in terms of probability of damage.

Of course all scenarios proposed must be evaluated from an economical point of view. This important aspect is introduced in the next section.

7. LIFE-CYCLE COST ANALYSIS

The maintenance of a system involves costs which do not include only the actual cost of repair, but also the construction site costs and those directly related to the duration of the maintenance and to inconveniences caused by a possible temporary unavailability of construction.

If the construction is a bridge, the temporary unavailability will cause inconveniences to the traffic in the entire surrounding area. Thus, when calculating the costs of every maintenance choice, even these aspects must be taken into account by giving them the right importance.

The life-cycle cost C_T over the expected lifetime T can be computed as the sum of the initial cost C_0 and the maintenance cost C_m :

$$C_T = C_0 + C_m \quad (11)$$

The initial cost can sometimes be associated to the cost of the material volume:

$$C_0 = \sum_k c_{0k} V_k \quad (12)$$

where c_{0k} and V_k are the volume unit cost and the material volume of member k , respectively.

For a prescribed maintenance scenario, the total cost of maintenance C_m can be evaluated by summing the costs C_q of the individual interventions (Frangopol et al., 1997; Kong and Frangopol, 2003):

$$C_m = \sum_q \frac{C_q}{(1+\nu)^{t_q}} \quad (13)$$

where the cost C_q of the q^{th} rehabilitation was referred to the initial construction time by taking a proper discount rate of money ν into account. The cost C_q of the individual intervention is assumed as follows (Biondini et al., 2008):

$$C_q = C_{\alpha q} + \sum_k \delta_{kq} (\chi c_{kq}) V_k \quad (14)$$

where $C_{\alpha q} = \alpha_q C_0$ is a fixed cost computed as a percentage α_q of the initial cost C_0 , δ_{kq} is the damage index of member k , c_{kq} is the volume unit cost for restoring the member k . Finally, with regard to the expected structural lifetime T , the annual cost C can be computed as follows (Flanagan et al., 1989):

$$C = C_T \frac{\nu(1+\nu)^T}{(1+\nu)^T - 1} \quad (15)$$

Based on this cost model, alternative maintenance scenarios can be compared, and the optimal maintenance strategy associated to the minimum life-cycle cost can be selected (Biondini et al., 2006a; 2006b).

The weight χ is introduced here (Eq. 14) as a multiplier of the unit volume cost c_{kq} able to consider some matters usually involved in the maintenance process and which, inevitably, affect the maintenance costs. For example if frequent but light maintenance involves little site work and partial unavailability of the construction, the weight χ can be assumed to be 1. When the maintenance involves the whole construction and total unavailability for a long time, the incidence of maintenance costs is significant and the weight χ can reach a higher value.

7.1 Application to the case examined

As said in the previous sections, the MRP applied to the entire system can foresee when a transition between states occurs but not what members may be involved in the damage in a more serious way, therefore maintenance scenarios 1), 2) and 3) can arise. On the contrary, applying MRP to all the structure members can lead to selective maintenance which, time by time, involves only the most deteriorated members. In this case scenarios a) and b) are possible.

For each of the scenarios proposed in section 4, the yearly cost C was analysed (Eq. 15) and a comparison between them was made. The comparison was carried out in terms of normalised cost $c_k=C_k/C_i$ where C_i indicates the cost of the scenario taken as reference and C_k the cost of the scenario it is to be compared with.

In Figure 16a the comparison is made between scenarios (2) and (a) aimed at maintaining the structure always in *state 0 (low damage)* but with a different maintenance approach: in scenario (2) it is done on the entire system with frequency, scenario (a) is selective and foresees interventions only on the most damaged members. Costs are normalised at the cost of scenario (1) which imposes maintenance of the whole system only when the structure has already reached *state 2 (heavy damage)*, hence: $c=c_2=C_2/C_1$ e $c=c_a=C_a/C_1$.

In Figure 16b the comparison is made between scenarios (3) and (b) aimed at maintaining the structure always in *state 1 (moderate damage)*. Also in this case the scenarios face the problem in a different way: non selective scenario (3), selective scenarios (b). The costs are normalised at the cost of scenario (1) which imposes maintenance on the entire system only when the structure has already reached *state 2 (heavy damage)*, hence: $c=c_3=C_3/C_1$ e $c=c_b=C_b/C_1$.

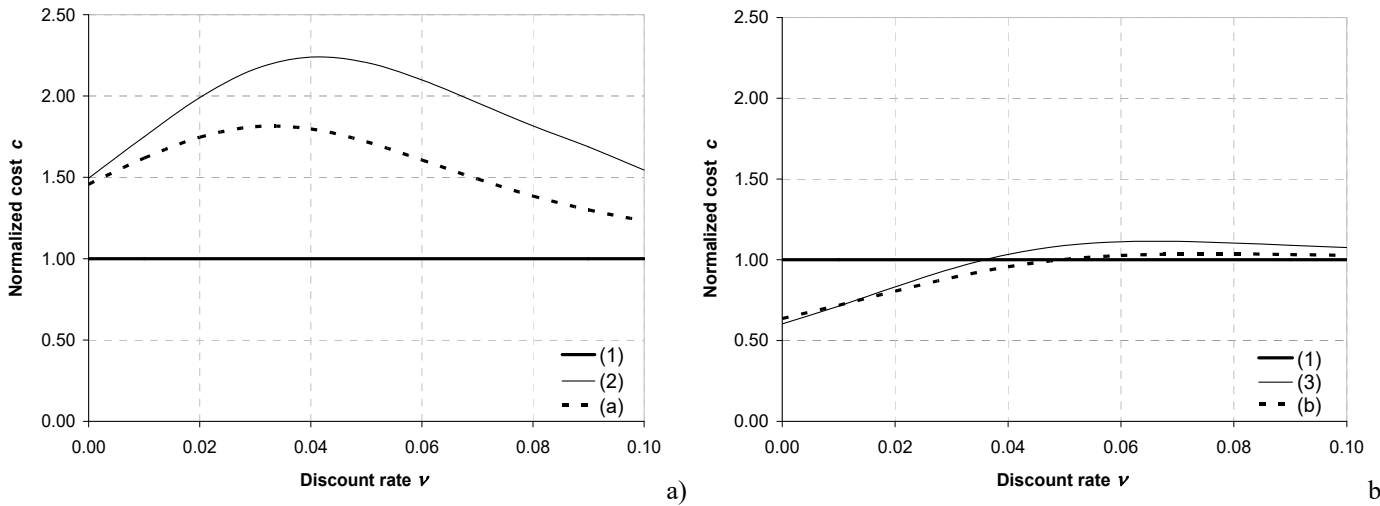


Figure 16. Total costs of maintenance: comparison between selective scenario (a) and non-selective scenario (2), and between selective scenario (b) and non-selective scenario (3). All the costs are normalised to non-selective scenario (1). In Fig. 16a) the cost associated with the selective scenario (a) and the non-selective scenario (2) are compared. In Fig. 16b) the cost associated with the selective scenario (b) and the non-selective scenario (3) are compared.

For the scenarios which foresee a thorough intervention on the entire system with subsequent higher site work and inconvenience for its users, the cost of maintenance c_{kq} is multiplied by $\chi=3$. For maintenance interventions which involve a limited number of members, c_{kq} is multiplied by $\chi=1$.

From Figure 16 it is clear how the scenarios (a) and (b), which foresee selective maintenance, are to be preferred to the scenarios which involve maintenance of all the members. However, Figure 16a shows how maintenance that is too frequent and prudential, even if it involves a few members, is always more expensive than maintenance carried out on the entire system when it is already in *state 2*, even if it is associated with a higher risk of failure. Higher costs are foreseen for $\nu = 0.03-0.04$. Figure 16b shows, instead, that for values of ν between 0.00 and 0.05 a selective intervention carried out over longer times, and which is able to keep all members of the system in *state 1*, with a low probability of immediate failure is preferable. For higher values of ν scenario (b) results as being slightly more expensive than scenario (1), however it has a lower risk of possible failure.

The total yearly cost was evaluated taking into account the different incidence of the fixed cost $C_{aq} = \alpha_q C_0$ on the maintenance cost C_q as the value of α_q changes (Eq. 14).

Figure 17 shows the incidence of the value of α_q on a total cost C for both scenarios; the reference cost for normalisation is the cost with $\alpha_q = \alpha = 0.00$.

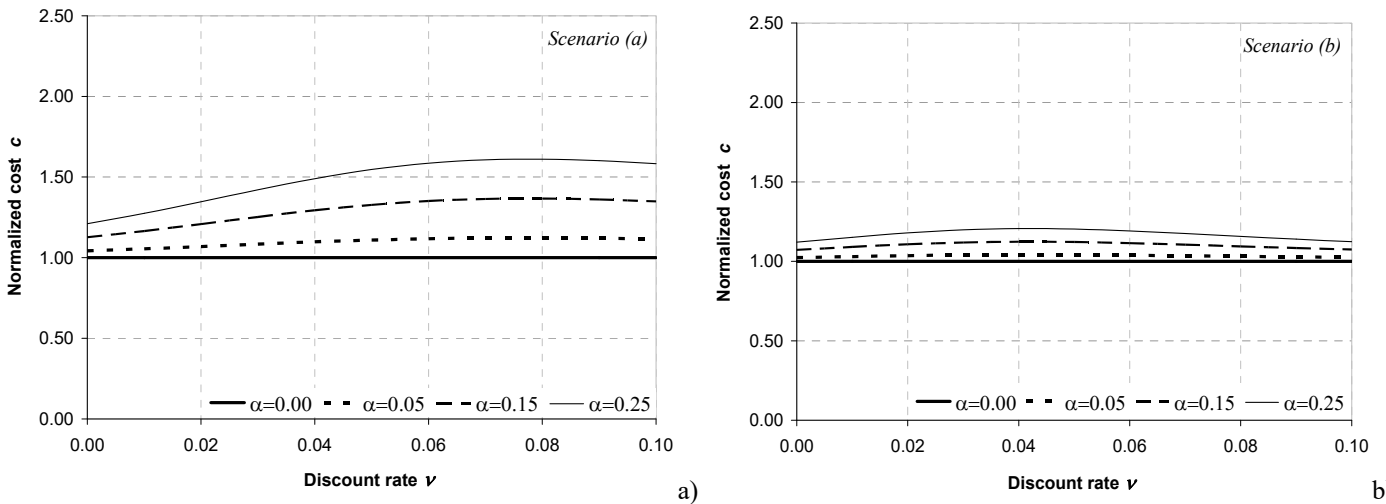


Figure 17. Influence of the parameter α on the trend of the total yearly costs: a) Scenario (a); b) Scenario (b). The cost with $\alpha=0.00$ was considered as a reference for the normalisation of both scenarios.

Figure 17 shows how parameter α affects the total yearly cost for scenario (b) much less if compared to (a). This is reasonable, since the reduced number of maintenance interventions involve less computation of fixed costs added to the costs of maintenance.

Figure 18 compares scenarios (a) and (b) with the normalisation scenario (1) for values $\alpha=0.00$ and $\alpha=0.20$.

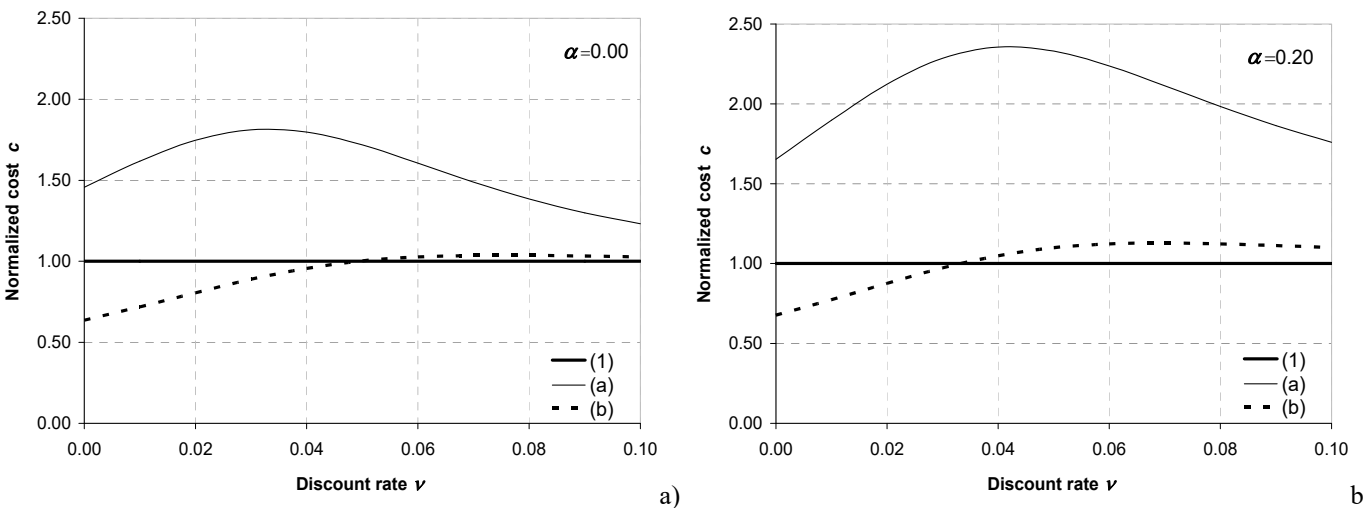


Figure 18. Comparison between scenarios (a) and (b), and the normalisation scenario (1) for two different values of α : (a) $\alpha=0.00$; (b) $\alpha=0.20$.

Figure 18 shows the great influence which fixed costs can have on the total yearly cost, above all in scenarios where frequent maintenance is expected. Therefore, the influence of the fixed cost as a percentage of the initial cost of construction must be carefully evaluated because it could allow for less frequent maintenance scenarios even if at a higher risk of failure.

CONCLUSIONS

In this work the authors propose the application of the Markovian Renewal Processes (MRP) as a predictive model for planning selective maintenance.

The choice is due to the interpretation given to the deterioration process caused by environmental aggressiveness.

As deterioration is seen as a dynamic process which leads the system to change its performance whenever a significant threshold is reached, then the process can be described as a process of transition and an appropriate method of modelling for this process is represented by the use of MRPs. In fact, MRPs are able to predict, in probabilistic terms, the instant of system transition from its current state of service to another state of service characterised by different levels of performance.

In this work it has been shown that if MRP is applied to the entire system, it is able to predict the instant in which the system probably leaves the current state and passes to the following one. If, instead, it is applied to all members of the system it is able to predict, for each member, the probable instant of transition. This highlights:

- 1) the weakest members of the system, therefore, during the design phase it could eventually be used to change the mechanical characteristics of such members and guarantee a longer useful life.
- 2) the members which are the first to require possible maintenance.

So that such model can be reliably applied to evaluation of the useful life of a structure, it requires an appropriate quantity of experimental data which may describe the performance evolution of the system over time. For this purpose, the authors suggest the use of the Monte Carlo method, which simulates structure behaviour depending on certain parameters, which simulate the changes in environment aggressiveness, and on a law which simulates the subsequent structural deterioration.

From application of the Monte Carlo and the MRP approach, a methodology was developed which led to the planning of some scenarios of selective maintenance whose effectiveness has been assessed also in terms of total annual costs of maintenance.

The selective maintenance problem has been dealt in the literature using the Monte Carlo methodology associated to the development of different indexes of maintenance able to identify the damage or the decrease in the reliability of the system components. Yet these methods, which are equally reliable, require several appli-

cations of the Monte Carlo methodology with a remarkable use of computational time. The methodology proposed, instead, requires only an MC application, which may allow the user to obtain, for each member, 5000 transition times for each of the thresholds established. This set of times, characterised by a high sample size, represent the base for the development of an MRP and for the calculation of the conditioned probabilities necessary for planning possible maintenance scenarios without further iterations of simulations.

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