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A novel incentive-based demand response model for Cournot competition in electricity markets

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Abstract This paper presents an analysis of competition between generators when incentive-based demand response (DR) is employed in an electricity market. Thermal and hydropower generation are considered in the model. A smooth inverse demand function is designed using a sigmoid and two linear functions for modeling the consumer preferences under demand response rebate program. Generators compete to sell energy bilaterally to consumers and system operator provides transmission and arbitrage services. The profit of each agent is posed as an optimization problem, then the competition result is found by solving simultaneously Karush–Kuhn–Tucker conditions for all generators. A Nash-Cournot equilibrium is found when the system operates normally and in peak times when DR is required. It is shown that DR diminishes the energy consumption and improves the net consumer surplus due to incentives received for participating in DR program. However, the generators decrease their profit due to the reduction of energy traded and retail price.

Keywords Demand response · game theory · Cournot equilibrium · baseline

1 Introduction

Demand Response (DR) is a program to motivate changes in electricity usage by customers in response to changes in the price signal. DR is a mechanism implemented by system operator (SO) to balance the load with power generation in a smart grid. Advanced metering is required to implement a DR

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program (Aketi and Sen, 2014). The main applications are to decrease the load at peak times in order to guarantee the reliability, and the security of the power assets (Zhu et al., 2013; Bloustein, 2005; Su and Kirschen, 2009). The aim is to control noncritical loads at residential, commercial and industrial levels for matching supply and demand.

There are some DR programs implemented as part of strategies to reduce peak power. In (Vardakas et al., 2015; Deng et al., 2015; Siano, 2014; Albadi and El-Saadany, 2008; Madaeni and Sioshansi, 2013) are shown some complete summaries regarding mathematical models, pricing methods, optimization formulation and future extensions. An interesting mechanism to induce DR is via incentive payment using a technique called Peak Time Rebate (PTR) (Vuelvas and Ruiz, 2017, 2015; Mohajeryami et al., 2016a; Severin Borenstein, 2014), where customers receive electricity bill rebates by not consuming (relative to a previously established, household-specific baseline) during peak periods. In PTR, the baseline is a vital concept since the payment depends on the calculation of estimated consumption, namely, a counterfactual model must be developed. In (Mohajeryami et al., 2016a) some methods are explained to estimate the customer baseline. In (Mohajeryami et al., 2016b) is developed a randomized controlled trial method in order to establish customer baseline load, applied to aggregated forms of the consumption load. (po Chao, 2011) shows critical facts on the selection of customer baseline, they design a suitable baseline focusing on administrative and contractual approaches in order to get an efficient DR. Furthermore, in (Faria et al., 2013; Wijaya et al., 2014; Antunes et al., 2013) the performance of DR baseline estimation models is studied and new methods are regarded as establishing the reasonable compensation for the consumer.

In addition, (Genc and Thille, 2008) shows the competition between hydro and thermal electricity generators under uncertainty over demand and water flows. The authors in (Garcia et al., 2005) analyze the price-formation in an oligopoly model where hydroelectric generators are involved in dynamic Bertrand competition. Furthermore, in (Villar and Rudnick, 2003) is built a model to understand a hydrothermal electric power market based on simple bids to the SO. On the other hand, (Zhu et al., 2013) illustrates by means of Stackelberg game what is the value that DR management can bring to generation companies and consumers in the smart grid. In (Su and Kirschen, 2009) is devised a method for quantifying the effect of the demand response for the market as a whole. The co-existence of a variety of generation technologies is an interesting problem from gaming point of view and even more with the integration of DR into the electricity market.

The agents involved in an electricity market in competition with DR are shown in Fig. 1. In this paper, SO is responsible for arbitrage services in order to establish a proper environment for competition and gaming. The generators have different technologies, costs, revenues, and each firm seeks to maximize its profit (the difference between producers' revenue and costs). Furthermore, the aggregators carry out the request to users of reducing energy consumption, namely, DR process. The main goal is to estimate the equilibrium price under gaming environment. This competition is less than perfect, some firms are able to influence the market price through their actions. Such interacting optimization problems form what is called in game theory a non-cooperative game (Vega Redondo, 2003; Gabriel et al., 2013; Tirole, 1988; Osborne, 1995; Varian, 1992). The solution of such a game is called a Nash equilibrium and represents a market equilibrium under imperfect competition.

In this paper, a game among generators with different technologies in an electricity market is analyzed whether DR is required when the demand side exceed an aggregated baseline or a threshold estimated a priori by SO in order to guarantee the objectives of a smart grid. This threshold is calculated from all customer baseline load. Then a novel demand curve is proposed in order to understand the effect of electricity market behavior when an incentive-based DR program, like PTR, is held to diminish the energy consumption at the peak periods. The customers receive an incentive payment for reducing load requirement during peak demand.

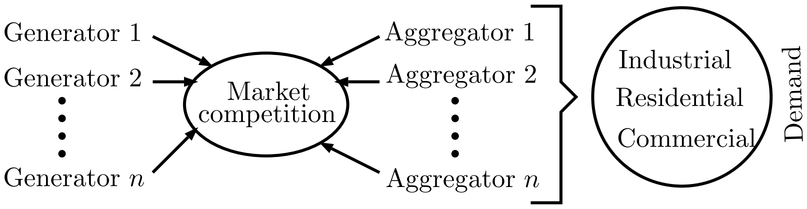


Fig. 1 Gaming in the electricity market

The contributions of this paper are summarized as follows:

- A novel incentive-based DR model is proposed. Demand function is formulated by using a sigmoid function between two linear polynomials to depict the energy threshold or aggregated baseline when DR is required. This formulation is a continuous function with finite marginal value in the demand curve. In particular, it is an alternative modeling of DR to (Su and Kirschen, 2009), where, the demand curve has two parts: perfect inelastic behavior and price responsive consumers. The inconvenience of (Su and Kirschen, 2009) is that demand does no have perfect inelastic role since the consumers have a limited willingness to pay.
- A Nash-Cournot equilibrium is formulated as a complementary problem in the presence of DR (Gabriel et al., 2013). The generators compete without a centralized mechanism. Cournot gaming is compared when an electricity market operates normally and in peak demand when the DR is active.

The rest of this paper is organized as follows. Section II describes the agent models in an electricity market. Section III presents the problem formulation as Cournot Competition in the presence of DR. The discussion and numerical results are presented in Section IV. Finally, conclusions are drawn in Section V.

2 Agent models

In this section, a novel demand model is proposed when an incentive-based DR is required in an electricity market. This formulation illustrates the wholesale market behavior when it is applied a PTR mechanism to demand side during the day of DR event. In addition, generator models are posed under Cournot Competition.

2.1 Demand response model

The most important decision unit of microeconomic theory is the demand (Varian, 1992; Mas-Colell et al., 1995). In this section, a new approach for modeling the demand is posed when consumers participate in a incentive-based DR program. Let the set $T = \{1, 2, ..., n_t\}$, where n_t is the amount of periods to take into account in the horizon time. An aggregated demand is considered for this DR rebate model. The decision maker's preferences are specified by giving smooth utility function $G(q_t)$, where q_t is the energy consumption at time t. $G(q_t)$ depicts the level of satisfaction obtained by the demand as a function of the total power consumption.The utility function satisfies the following properties:

Property 1: Demand has a risk-averse behavior. Then, $G(q_t)$ is concave (Vega Redondo, 2003; Osborne, 1995). This implies that the marginal benefit of users is a nonincreasing function. That is,

$$\frac{d^2G(q_t)}{dq_t^2} \leq 0 \, \forall t \in T$$

Property 2: Utility function is nondecreasing. Therefore, the marginal benefit is nonnegative

$$\frac{dG\left(q_t\right)}{dq_t} \geq 0 \, \forall t \in T$$

Property 3: $G(q_t)$ is zero when the consumption level is zero,

$$G\left(0\right) = 0 \, \forall t \in T$$

The energy market price is p_t^* at the time t. The superscript star indicates the equilibrium price. The cost function is assumed increasing with respect to the total offered energy capacity. In addition, the cost function is strictly convex. Then, other definitions are considered as follows.

Definition 1 The demand energy total cost is $\pi(q_t) = p_t^* q_t$.

Definition 2 $G(q_t)$ can be approximated by a second order polynomial around $\overline{q}_t \, \forall t \in T$. In general, a quadratic function is considered. That is,

$$G\left(q_t\right) = -\frac{\gamma_t}{2}\left(q_t - \overline{q}_t\right)^2 + p_t^*\left(q_t - \overline{q}_t\right) + k \, \forall t \in T$$

being $k = \overline{q}_t\left(\frac{\gamma_t}{2} + p_t^*\right)$ a constant value, obtained by the Property 3.

Definition 3 The payoff function is defined as $U_t\,(q_t) = G(q_t) - \pi(q_t)$, which indicates the user benefit of consuming q_t energy during the interval t.

Basically, incentive-based DR programs request customers to curtail demand in response to a price signal or economic incentive. Typically the invitation to reduce demand is made for a specific time period or peak event. There are three main concepts in order to define DR rebate program:

Definition 4 Baseline: the amount of energy the user would have consumed in the absence of a request to reduce (counterfactual model) (Deng et al., 2015). This quantity can not be measured, then it is estimated from the previous consumption of the agent, i.e., the baseline takes into account $q_{t-1}, ..., q_{t-n}$. Where n defines the historical consumer behavior, i.e., n corresponds to the periods taken into account within the baseline function. In this work, the aggregated baseline corresponds to the sum of all customer baseline loads $j \in J$ in order to determine the DR threshold required in the electricity market. Being $J = \{1, 2, ..., n_j\}$, where n_j is the total number of users participating in DR.

$$b = \sum_{j \in J} baseline(q_{t-1}, ..., q_{t-n}) \tag{1}$$

Definition 5 Actual Use (q_t): The amount of energy the aggregated demand actually consumes during the event period.

Definition 6 Load Reduction ($\triangle_t\,(b(\cdot\,), q_t)$): The difference between the baseline and the actual use.

$$b - q_t = \triangle_t\,(b(\cdot\,), q_t)$$

In incentive-based DR programs, the rebate is only received if there is an energy reduction. Otherwise, the user does not get any incentive or penalty. Mathematically,

Definition 7 Let p_2 the rebate price received by the demand due to energy reduction in peak periods. The DR incentive π_2 is

$$\pi_2\,(q_t; p_{2t}, b(\cdot\,)) = \begin{cases} p_{2t}\,(\triangle_t\,(b(\cdot\,), q_t)) = p_{2t}(b(\cdot\,) - q_t) & q_t < b \\ 0 & q_t \geq b \end{cases} \;\forall t \in T$$

The demand payoff function with DR rebate program is

$$\hat{U_t}\,(q_t; p_{2t}, b(\cdot\,)) = G(q_t) - \pi(q_t) + \alpha\pi_2(q_t; p_{2t}, b(\cdot\,))\;\forall t \in T \tag{2}$$

where α is a binary variable representing whether DR is required.

In this paper, the inverse demand function is needed to formulate the Cournot's model of oligopoly. The inverse demand function is given by $p_t\,(q_t) = \frac{dU_t(q_t)}{dq_t}$. Where U_t is the demand payoff function, q_t is the amount of energy and $p_t\,(q_t)$ is the price function at the time t.

Accordingly, the inverse demand function without DR is obtained from definition 3. The linear inverse demand function is:

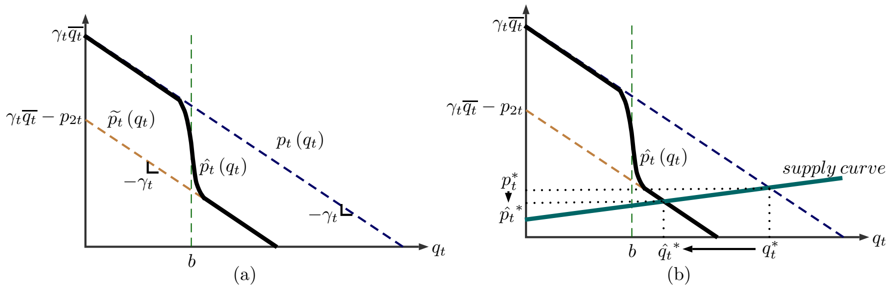
$$p_t\left(q_t\right) = -\gamma_t q_t + \gamma_t \overline{q_t}\, \forall t \in T \tag{3}$$

Whether the demand payoff function with DR is considered when $q_t < b$. The inverse demand function is:

$$\widetilde{p_t}\left(q_t\right) = -\gamma_t q_t + \left(\gamma_t \overline{q_t} - p_{2t}\right)\ \forall t \in T \tag{4}$$

In order to model the electricity market with DR in peak times, a sigmoid function between both inverse demand function 3 and 4 is formulated which it is illustrated in fig. 2 (a). The new inverse demand function is:

$$\hat{p_t}\left(q_t\right) = -\gamma_t q_t + \left(\gamma_t \overline{q_t} - \frac{p_{2t}}{1+e^{-\alpha\left(q_t+b\right)}}\right)\ \forall t \in T \tag{5}$$



(a) (b)

Fig. 2 Inverse demand function

where α is a constant value which represents the smoothness of the sigmoid function that joins the two straight lines.

Notice that this demand model depicts a preference alteration of consumers. Fig 2 (b) shows the case when the supply curve is intersected by demand curve for $q_t > b$. The equilibrium price $\hat{p_t}^*$ is less than the energy price given by the inverse demand function $p_t\left(q_t\right)$. In addition, the energy consumption decreases to $\hat{q_t}^*$ owing to the incentive price π_2, which it is the DR request made it from SO by setting the baseline b. The incentive is paid by the SO when the DR program is required. Besides, SO determines the threshold b according to the available energy and historic consumption. Then aggregators request to the customers the energy reduction.

In (Su and Kirschen, 2009), the demand model has two parts, consumers that have perfect inelastic behavior which they are represented by an infinite marginal value and users that participate in a DR program can place price responsive bids with a finite marginal value in the demand curve. However, the drawback of the model (Su and Kirschen, 2009) is that demand does no have perfect inelastic behavior because the consumers have a limit willingness for energy payment. In this formulation, demand always has a finite marginal value, hence, this model is a valuable alternative to represent the DR behavior in the market.

2.2 Supply model

The relationship between total energy from all generators and price lead to make the supply curve. In this work, producers try to anticipate the results of their actions on the market price, then the market experiences imperfect competition. SO has arbitrage services and manages the transmission assets as its functions into the electricity market. Therefore, each generator seeks independently to maximize its own economic benefits. The profit is given by its revenue from sold energy minus the cost of generating it. Two kinds of power suppliers are considered: thermal generators are represented with quadratic costs and hydropower are formulated with fixed costs (Genc and Thille, 2008).

2.2.1 Thermal generation modeling

The thermal cost is given by an increasing quadratic function. Let r_{ta} be the power generated by producer $a \in A$ at the period t, where A is the set of thermal generators. Thus, the costs have the following form: $c1_a r_{ta} + \frac{c2_a}{2} r_{ta}^2 + c3_a$, being $c1_a$, $c2_a$ and $c3_a$ constant values that represent private information. These quadratic costs are stated because thermal power has a expensive economic behavior (Genc and Thille, 2008). In this sense, each generator uses its knowledge of the inverse demand function ($p_t\left(q_t\right)$ or $\hat{p_t}\left(q_t\right)$) to anticipate its own effect on the market price in order to maximize its profit. Then, the optimization problem for thermal generator is posed as follows.

$$\begin{array}{rl} \max & \sum_{t \in T} \left[p_t\left(q_t\right) r_{ta} - \left(c1_a r_{ta} + \frac{c2_a}{2} r_{ta}^2 + c3_a \right) \right] \\ \text{s.t.} & r_{ta} \leq r_a^+ \; : \; \mu_{ta}^T \; \forall t \in T \\ & r_{ta} \geq 0 \; \forall t \in T \end{array} \tag{6}$$

where r_a^+ is the maximum value of the energy that each thermal power can generate in each period. μ_{ta}^T is dual variable for the first constraint. This model does not consider ramp constraint, minimum uptime and downtime, among other constraints.

2.2.2 Hydropower modeling

Hydropower is considered in gaming into the electricity market. The hydro has a production function $H_{tb}(w_{tb})$ that converting water release to energy and w_{tb} is the water release of hydro reservoir for each generator $b \in B$. For this kind of producer, a fixed cost $c4_b$ is formulated. Hence, the optimization problem is to maximize the profits by each hydropower:

$$\begin{array}{rl} \max & \sum_{t\in T}\left[p_t\left(q_t\right)H_{tb}(w_{tb})-c4_b\right] \\ \text{s.t.} & w_{tb}\le w_{tb}^{+}\ :\ \mu_{tb}^{H}\ \forall t\in T \\ & w_{tb}\ge 0\ \forall t\in T \end{array} \tag{7}$$

where w_{tb}^{+} is the maximum value of the water release at the time t for the generator b. μ_{tb}^{H} is dual variable for the first constraint.

3 Icentive-based demand response in Cournot competition

A Cournot competition is considered for studying the DR model that was described in section 2. This model assumes that generators cannot collude or form a cartel, and they seek to maximize their own profit based on demand model. This section describes the game between market participants in order to settle the energy price by solving simultaneously the optimization problems (6) and (7), which, it is explained in (Gabriel et al., 2013). Let the definition of Nash equilibrium is stated as follows.

Definition 8 Considering the game $G=\left\langle I,\{S_i\}_{i=1,2,..n_i.},\{\xi_i\}_{i=1,2,...,n_i.}\right\rangle$, where I is the players set, S_i is the strategies set of each player and ξ_i : $\prod_{i\in I}S_i$ is the utility function of each generator. $(s_1^*,...s_i^*)$ is a Nash equilibrium whether $\forall i$ player it is true that: $\xi_i\left(s_i^*,s_{-i}^*\right)\ge\xi_i\left(s_i,s_{-i}^*\right)$, $\forall s_i\in S_i$, being s_{-i} all strategies except the player i (Vega Redondo, 2003).

Remark 1 Nash equilibrium has two interpretations: s_i^* is the best response to s_{-i}^* or it does not exist unilateral incentives to deviate from Nash equilibrium. Furthermore, an equilibrium problem can be solved using Karush-Kuhn-Tucker (KKT) conditions of several interrelated optimization problem (Gabriel et al., 2013).

The aim is to solve the equations (6) and (7) in the cases when no demand response is requiered, i.e, using the demand model $p_t\left(q_t\right)$ given by (3) and the situation when DR is requested, then the model used is (5) given by the baseline set by SO. In order to find the solution, the KKT conditions of each agent are solved simultaneously.

In this paper, a duopoly is assumed for understanding the effect of the proposed DR model. In particular, two generators are employed to find the Nash-Cournot equilibrium: one thermal energy producer and one hydropower according to the suggested supply curve from Section 3. For simplicity, the subscript a and b from equations (6) and (7) are eliminated because there is one generator per techology. Therefore, the balance constraint is $q_t=r_t+H_t(w_t)$. The KKT conditions with and without DR are presented as follows.

3.0.3 Electricity market without demand response

First, considering the case when DR is not requiered in the market. Then, the KKT conditions rewritten as complementary model by using equations (3) are shown below:

$$\begin{array}{c} 0 \leq r_t\left(2\gamma_t + c_2\right) + \left(\gamma_t H_t\left(w_t\right) + c_1\right) - \gamma_t \overline{q_t} + \mu_t^T \perp r_t \geq 0 \,\forall t \in T \\ 0 \leq \mu_t^T \perp r^+ - r_t \geq 0 \,\forall t \in T \end{array} \tag{8}$$

$$\begin{array}{c} 0 \leq \frac{dH_t(w_t)}{dw_t}\left[\gamma_t r_t + 2\gamma_t H_t\left(w_t\right) - \gamma_t \overline{q_t}\right] + \mu_t^H \perp w_t \geq 0 \,\forall t \in T \\ 0 \leq \mu_t^H \perp w_t^+ - w_t \geq 0 \,\forall t \in T \end{array} \tag{9}$$

where (8) and (9) are the KKT conditions rewritten as complementary model for thermal generation and hydropower, respectively.

3.0.4 Electricity market with demand response

On the other hand, the KKT conditions when the market has an incentive by reducing energy given by the demand model (5).

$$\begin{array}{c} 0 \leq p_{2t}\frac{e^{\alpha(r_t+H_t(w_t))}\left[e^{\alpha(r_t+H_t(w_t))}+e^{\alpha b}(\alpha r_t+1)\right]}{\left[e^{\alpha(r_t+H_t(w_t))}+e^{\alpha b}\right]^2} + r_t\left(2\gamma_t + c_2\right) \\ + \left(\gamma_t H_t\left(w_t\right) + c_1\right) - \gamma_t \overline{q_t} + \mu_t^T \perp r_t \geq 0 \,\forall t \in T \\ 0 \leq \mu_t^T \perp r^+ - r_t \geq 0 \,\forall t \in T \end{array} \tag{10}$$

$$\begin{array}{c} 0 \leq \frac{dH_t(w_t)}{dw_t}\left[p_{2t}\frac{e^{\alpha(r_t+H_t(w_t))}\left[e^{\alpha(r_t+H_t(w_t))}+e^{\alpha b}(\alpha H_t(w_t)+1)\right]}{\left[e^{\alpha(r_t+H_t(w_t))}+e^{\alpha b}\right]^2}\right] \\ + \frac{dH_t(w_t)}{dw_t}\left[\gamma_t r_t + 2\gamma_t H_t\left(w_t\right) - \gamma_t \overline{q_t}\right] + \mu_t^H \perp w_t \geq 0 \,\forall t \in T \\ 0 \leq \mu_t^H \perp w_t^+ - w_t \geq 0 \,\forall t \in T \end{array} \tag{11}$$

where (10) and (11) are the KKT conditions for thermal generation and hydropower when there is DR, respectively.

4 Numerical results

The analysis involves three aspects: the effect of demand response, the study of consumers and generators surplus and the incentive effect. The simulation is developed in GAMS 24.7.4 using PATH as the solver. In table 1. are shown the parameters of the simulations in order to show the new approach of demand model with DR given by equation (5). The data used in the simulation are based on (Forouzandehmehr et al., 2014; Genc and Thille, 2008; Cunningham et al., 2002).

First, the Cournot competition between generators is shown under this approach. Fig. 3 depicts the results of gaming between thermoelectric and hydroelectric when the inverse demand function is (3). The energy required by demand versus hours in a day are depicted in Fig. 3 according to the generator technology. All simulations are made in a 24-hour horizon. Hydropower has the main participation in the market due to it does not have the variable cost, therefore, it is cheaper than the thermal generation. The peak time is between 19 to 21 hours.

On the other hand, in Fig. 4 is presented the competition case when there is an incentive if the energy consumption is greater than the threshold or the

$T=\{1,2,3,...,24\}\ h$	$b=1000\ MWh,\ H_t(w_t)=w_t,\ \alpha=0.1$
$c1=10\ \$$	$\gamma_t=\{0.065,0.067,0.063,0.063,0.06,0.065,0.062,$ $0.068,0.065,0.067,0.063,0.067,0.068,0.069,0.062,0.061,$ $0.067,0.067,0.055,0.054,0.055,0.065,0.063,0.061\}\ \frac{\$}{MWh^2}$
$c2=0.025\ \$$	$\gamma_t\overline{q_t}=\{108.4,103.82,105.67,109.2,105.32,104.56,110.56,$ $111.14,110.19,112.23,111.45,115.7,113.45,103.13,102.54,$ $108.87,109.95,115.23,120.19,120.35,120.23,108.4,105.67,105.67\}\ \frac{\$}{MWh}$
$c3=0\ \$$	$p_{2t}=\{0,0,0,0,0,0,0,0,$ $0,0,0,0,0,0,0,0,$ $0,0,20,20,20,0,0,0\}\ \frac{\$}{MWh}$
$c4=0\ \$$	$r^+=500\ MWh,\ w_t^+=1000\ \frac{acre-ft}{h}\ \forall t\in T$

Table 1 Simulation parameters

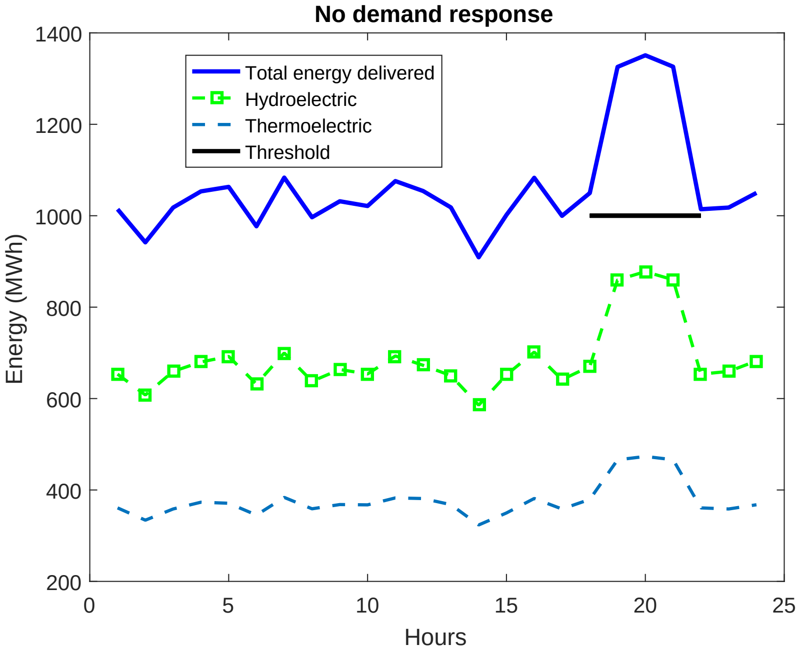


Fig. 3 Cournot competition without demand response

baseline $b=1000\ MWh$. Below this threshold, the DR does not apply. For instance, notice that the total energy delivered at the hour 20 is about 1351 MWh, i.e., above the baseline. As long as, in Fig. 3, the energy value at the same time is around 1119 MWh, therefore, the energy reduction is about 232 MWh since the demand behavior is altered by the incentive payment given by the definition 7. In addition, the reduction proportion is the same for each technology.

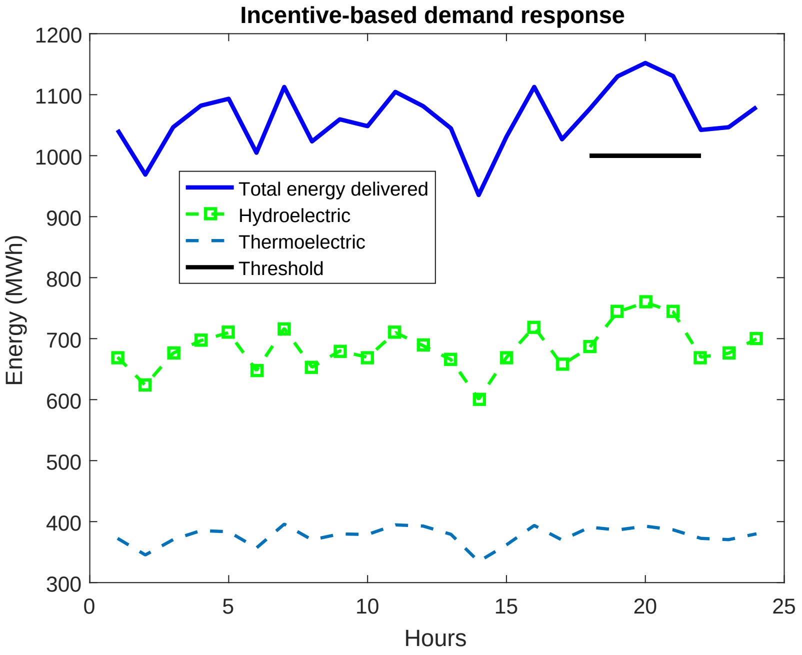


Fig. 4 Cournot competition with demand response

4.1 The effect of demand response

In Fig. 5. is shown the effect of DR in term of energy. Whether the energy is higher than the threshold value, that is, $b = 1000\, MWh$ then the DR model stimulates the consumer to reduce the energy consumption pattern. This behavior is found because is introduced the incentive p_{2t} in the inverse demand function. Thus, this incentive is understood as an alteration of consumer preferences made by SO, achieving reliability and alleviation in contingency situation where a reduction is required in the grid operation. In this case, the energy reduction for all periods is about 2.71%. This percentage changes according to the baseline selected by SO.

Even though, in Fig. 6 is depicted the results of the DR in term of prices. The fashion in which a consumer reduces his energy is through from economic stimulus or incentives. Under this mechanism, consumers are rewarded by a reduction of energy in peak hours or peak period times. Fig. 6 shows that DR reduces the retail energy prices since obeying the law of supply and demand. However, for obtaining the energy reduction, SO must pay an economic incentive in order to motivate the load curtailment by consumers. Therefore, in certain events, the incentive-based DR is a reasonable alternative to overcome contingency scenarios in the electric power system. At these times, it is more cost-effective to diminish demand than increase supply to maintain power balance.

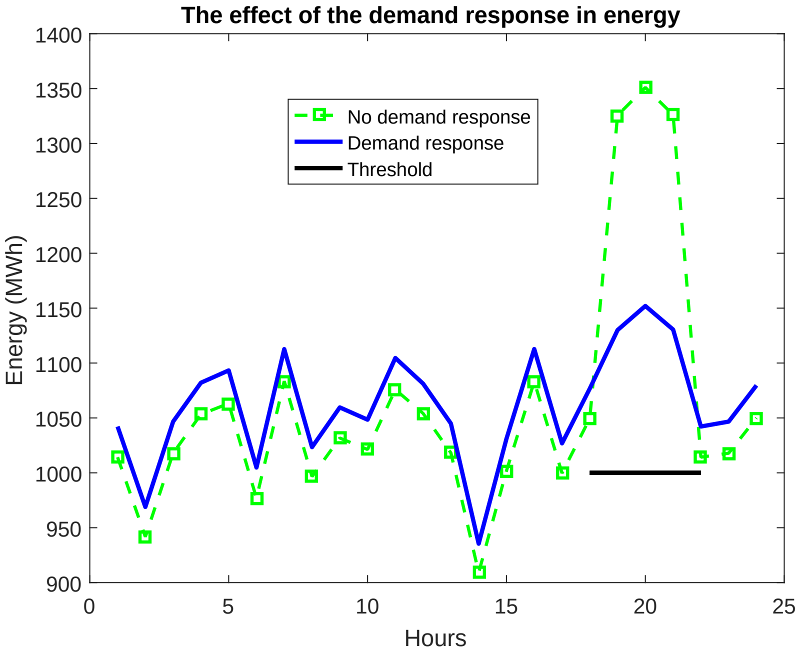


Fig. 5 The effect of the demand response in energy

4.2 Consumer and producer surplus

In Fig. 7. is shown the surplus of the consumers and generators. The producer surplus is calculated from the sum of the objective functions of optimization problems (6) and (9). Whereas that the consumer surplus is obtained by replacing directly the inverse demand function (5) in $\int_0^{q_t} p_t\left(E'\right) dE' - p_t^* q_t$. An important feature of the incentive-based DR mechanism is that the generators decrease their profit or surplus when DR is required. This effect is due to the reduction performed by users, in which, the prices are affected by the inverse demand curve stated when the energy exceeds the baseline. On the other hand, the consumers are rewarded by a reduction of the energy bill whether they reduce their consumption. Therefore, users have a greater economic surplus with DR program than by not participating in the program, taking into account the previous definitions for this model.

In Fig. 8. is illustrated the generators surplus by technology. Hydropower has the main participation in the electricity market, therefore, it suffers the greatest reduction in its benefits. Whereas that thermoelectric slightly reduced their profits. In general, the energy reduction is proportional and depends on the participation of each generator in the energy market.

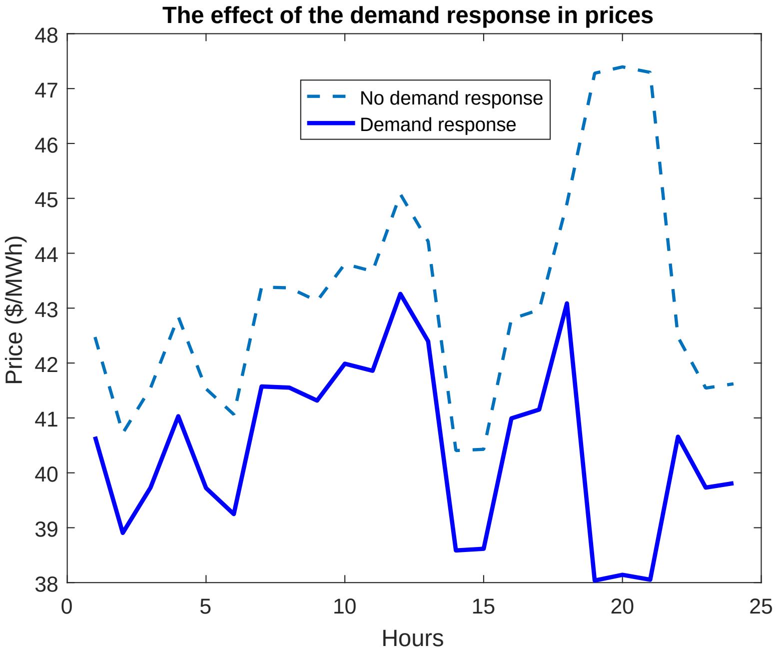


Fig. 6 The effect of demand response in prices

4.3 Incentive effect in demand response

For analyzing the incentive effect in this kind of DR program, the simulation parameters are set in $\gamma_t = 0.054 \frac{\$}{MWh^2}$ and $\gamma_t \overline{q_t} = 120.35 \frac{\$}{MWh}$ for all periods. The previous values guarantee that DR would be required at all times given the baseline $b = 1000\, MWh$. Hence, the aim is to change the incentive price in order to understand what happens to the energy reduction, retail price, and market participant surplus. In Fig. 9 is shown the energy reduction in percentages and the retail price according to the incentive. The immediate effect of DR is the decrease of electric power requirement and, by the law of supply and demand, the retail price declines as increases the incentive signal. For instance, whether the incentive price p_{2t} is equal to $10 \frac{\$}{MWh}$, then the market price is $43.67 \frac{\$}{MWh}$ and the energy reduction is 8.6%.

In Fig. 10 is illustrated the generation and demand surplus as increases the incentive price. The amount of energy to be dispatch is less when DR is required. Then, the generation profit diminishes also caused by decreasing retail prices. Therefore, the main objectives of this incentive-based DR program are to guarantee the reliability of power assets in peak events or a solution in a contingency situation, e.g., low water levels in reservoirs of hydroelectric power. Furthermore, the consumers perceive more economic benefits when they are participating in the DR program since the net price is cheaper whether they reduce their consumption. For example, if the incentive price is $10 \frac{\$}{MWh}$ then

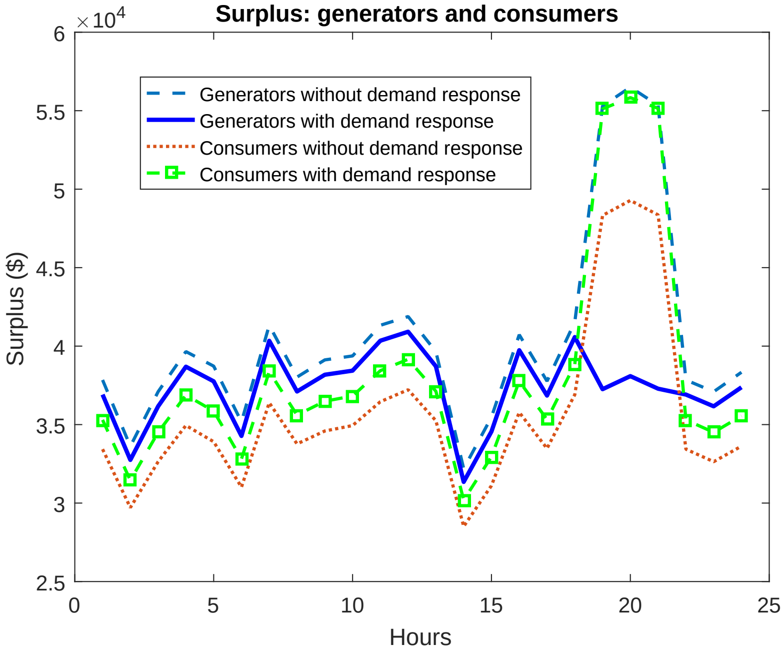


Fig. 7 Consumer and producer surplus

the users notice an increase about 23.9% of their surplus, while, the generation have a decrease around 24.51% of the profit.

5 Conclusions

In this work was developed an analysis of Cournot competition in an incentive-based DR program. A new demand curve was proposed for modeling consumer preferences in order to include DR in the electricity market. Incentives for consumers was considered as the DR mechanism. The demand model was devised as a composition of two linear functions and a sigmoid, which, it represents an energy threshold for requesting the load reduction. It was found that the incentive-based DR is a cost-effective solution to reduce energy consumption. However, this mechanism affects negatively the generator surplus and SO.

Incentive-based DR is an appropriate program for the reliability grid system. Notwithstanding, the generators and SO are affected from the economic point of view. A key question: how far is suitable reducing consumption seeking the social welfare under this kind of mechanism?

For future works, transmission, and intertemporal constraints can be incorporated into the model in order to take into account all characteristic of dispatch problem. Additionally, this work can be reformulated as a centralized problem, i.e., without competition. Finally, a significant improvement would

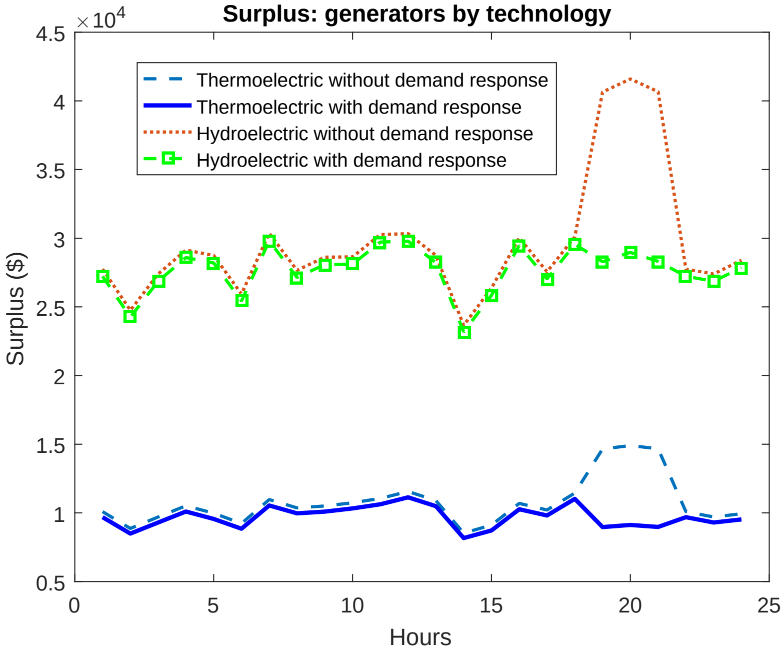


Fig. 8 Generators surplus by technology

be to model the demand curve as a random process to integrate renewable energies in the distributed system of the electrical grid.

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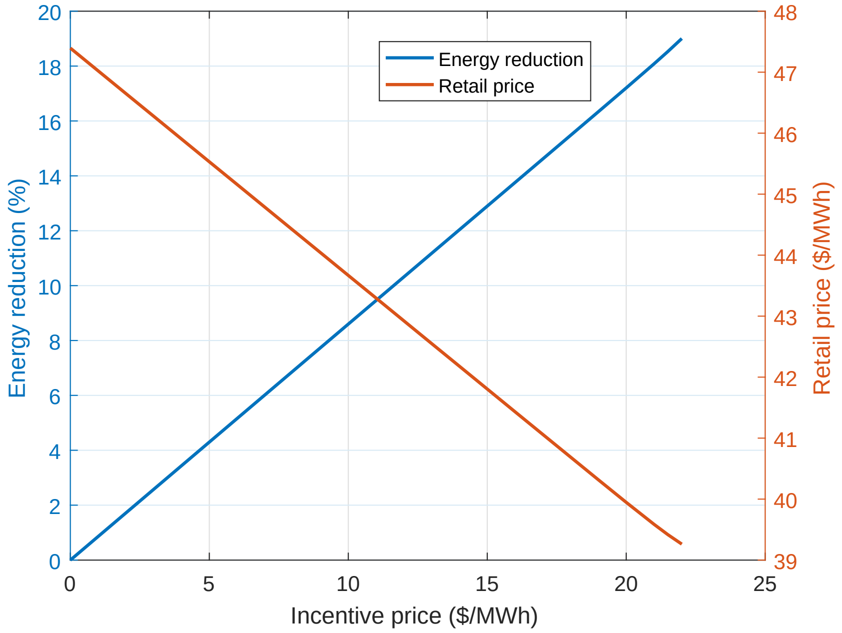

Fig. 9 Energy reduction and retail price affected by the DR incentive

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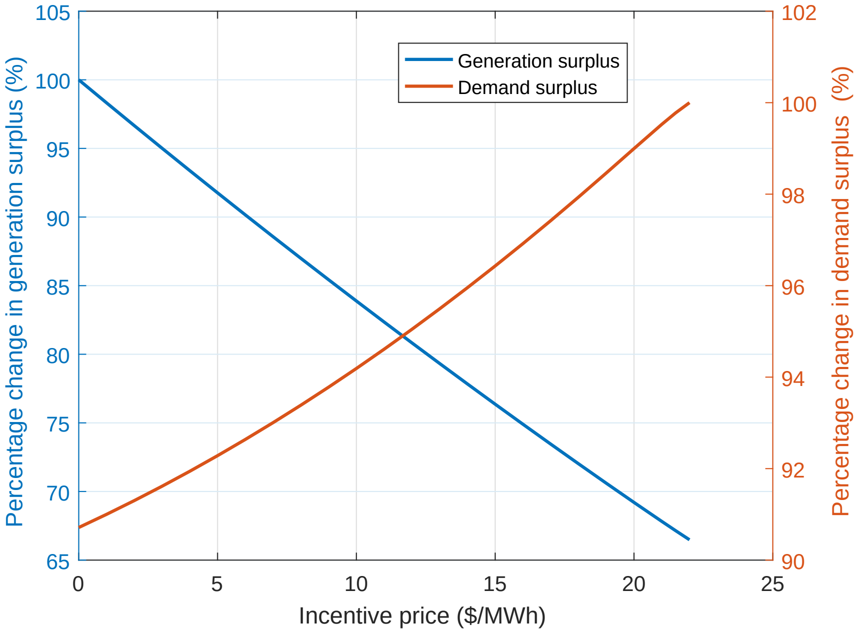

Fig. 10 Generation and demand surplus behavior according to the incentive price

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