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Abstract: The new Database of Italy's Seismogenic Sources (DISS Working Group, 2007) identifies areas with a degree of homogeneity in earthquake generation mechanism judged sufficiently high. Nevertheless, their seismic sequences show rather long and regular interoccurrence times mixed with irregularly distributed short interoccurrence times. Accordingly, the following question could naturally arise: do sequences consist of nearly periodic events perturbed by a kind of noise; are they Poissonian; or short interoccurrence times predominate like in a cluster model? The relative reliability of these hypotheses is at present a matter of discussion (Faenza et al. 2003, Corral 2005 and 2006, ...).
In our regions a statistical validation is not feasible because of the paucity of data. Moreover the classical tests do not clearly suggest which one among different proposed models must be favoured.
In this paper we adopt a model of interoccurrence times able to interpret the three different hypotheses, ranging from exponential to Weibull distributions, in a scenario of increasing degree of predictability.

In order to judge which one of these hypotheses is favourite, we adopt, instead of the classical tests, a more selective indicator measuring the error in respect to the chosen panorama of possible truths.

The earthquake prediction is here simply defined and calculated through the conditional probability of occurrence depending on the elapsed time t0 since the last earthquake. Short and medium term predictions are performed for all the Italian seismic zones on the basis of datasets built in the context of the National Projects INGV-DPC 2004-2006, in the frame of which this research was developed.

The mathematical model of interoccurrence times (mixture of exponential and Weibull distributions) is justified in its analytical structure. A dimensionless procedure is used in order to reduce the number of parameters and to make comparisons easier. Three different procedures are taken into consideration for the estimation of the parameter values: in most of the cases, they give comparable results. The degree of credibility of the proposed methods is evaluated. Their robustness as well as their sensitivity are discussed.

The comparison of the probability of occurrence of a Maw>5.3 event in the next 5 and 30 years from January 1st, 2003, conditional to the time elapsed since the last event, shows that the relative ranking of impending rupture in five years is roughly maintained in a 30-years perspective with higher probabilities and large fluctuations between sources belonging to the same macro region.

Response to Reviewers: The response to reviewer is attached to the manuscript

Renewal Models for Earthquake Predictability

E. Garavaglia[1], *E. Guagenti*[1], *R. Pavani*[2] *& L. Petrini*[1]

Abstract

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In order to judge which one of these hypotheses is favourite, we adopt, instead of the classical tests, a more selective indicator measuring the error in respect to the chosen panorama of possible *truths*.

The earthquake prediction is here simply defined and calculated through the conditional probability of occurrence depending on the elapsed time t_0 since the last earthquake. Short and medium term predictions are performed for all the Italian seismic zones on the basis of datasets built in the context of the National Projects INGV-DPC 2004-2006, in the frame of which this research was developed.

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The comparison of the probability of occurrence of a Maw>5.3 event in the next 5 and 30 years from January 1st, 2003, conditional to the time elapsed since the last event, shows that the relative ranking of impending rupture in five years is roughly maintained in a 30-years perspective with higher probabilities and large fluctuations between sources belonging to the same macro region.

Key words: Renewal Process, Hazard rate, Earthquake Predictability, Credibility and Robustness of the Model, Italy

Introduction

One of the main issues in seismology and seismic engineering is reliable earthquake prediction. However, since earthquake occurrence depends on many random variables whose characteristics are often not well defined, its prediction is still a difficult matter. Several approaches have been presented, but it seems that none is clearly superior.

The degree of uncertainty, epistemic and statistic, contained in the models leads to statements of this kind. "Although seismology has intensively analyzed the characteristics of single earthquakes with great success, a clear description of the properties of seismicity …is still lacking" (A. Corral 2006); "…test based on the likelihood ratios should be approached with caution…" and "physical significance (of cyclic effects and clustering) therefore remains questionable." (D.Vere-Jones et al. 1982); "…these results… could be very relevant in the debate of seismic gap models versus clustering occurrence" (A. Corral 2005).

One of the major challenges is to interpret two apparently opposite characteristics in recurrence time models: temporal clustering and nearly periodic events. In A. Corral (2006) the Author quotes "…results turn out to be in contradiction with each other, from the paradigm of regular cycles and characteristic earthquakes to the view of totally random occurrence" and proposes a unifying description of seismicity.

We do not have the pretension to enter in the debate, but we propose an elementary model that could be a first step to incorporate quasi-periodic events as well as poissonian or clustered events. Moreover we adopt an indicator, called credibility, to compare the competing models.

The mathematical tool of the model consists in a renewal process, already proposed by many authors in order to overcome the Poisson models.. The renewal process is not free from criticism: 1) all past history, not only the last event, influences the earthquake generation; 2) the complexity of Italian context makes debatable a complete energy release after each earthquake followed by a new cycle; 3) the recurrence times are independent identically distributed (i.i.d.); 4) the sum of many causes of seismicity can suggest the Poisson process (this criticism being feeble because the renewal process includes the Poisson process as particular case).

Nevertheless, in the frame of the renewal processes, the role of the specific survivor function (complementary distribution function) with the implied hazard rate is crucial. The mixture exponential-Weibull distribution function we propose has the capacity to incorporate in a probabilistic way three different behaviours (with different kind of energy release): to collapse into a Poisson process, to model characteristic or clustering events.

The existence of characteristic or clustering events needs sound physical basis to be asserted; here the terms are used, for shortness, to indicate respectively the presence of rather regular temporal sequence (even if mixed with irregular short interoccurrence times) or the dominance of short interoccurrence times.

The main responsible to make the mixture suitable in the three mentioned cases is the shape parameter α of Weibull component. In general its behaviour is as follows.

If α is greater than the unity the hazard rate comes out to be increasing until a peak value: it models a kind of seismic gap. If α is lesser than the unity the hazard rate comes out to be decreasing like in clustering events. If α is equal to the unity the proposed renewal process weakly differ from Poisson process. Moreover the renewal process can collapse precisely in a Poisson process, as shown in the next sections.

In *SIS Intermediate Conference 2007 Risk and Prediction,* held in Venice in June 2007, such a model was already presented (Garavaglia et al. 2007) in a preliminary way; here it is studied and applied to the Italian seismic zones.

During the last twelve years and at the present Italian researchers are improving the earthquake catalogues and proposing different seismogenetic models of the seismic source. Seismic zoning, as well as the seismic catalogues, are again under revision.

In the context of a research plan proposed by Italian Department of Civil Protection (DPC) and National Institute of Geophysics and Vulcanology (INGV) (National Projects INGV-DPC 2004-2006) in the frame of which this work has been developed, new databases were released with the aim of calculating the probability of occurrence of strong events in the next 30 years. Earthquake sub-catalogues were obtained by the association of the events of the Italian historic catalogue CPTI04 (Catalogo Parametrico dei Terremoti Italiani–Versione 2004 - CPTI Working Group, 2004 INGV, Bologna) with seismogenetic source areas (SA) (DISS03, INGV - DISS Working Group, 2006) that are sources having homogeneous behaviour. For a lot of SAs the number of events in the sub-catalogues is inadequate for a significant statistical analysis; therefore the predictability could not be applied to the whole Italian context. In order to overcome this problem, the SAs have been grouped in macro regions (MR) (Basili, 2007) with the following characteristics: similar style of faulting and amount of deformation, This last association permitted to have adequate interoccurrence times datasets. The approach herein proposed was applied to these catalogues, as described in the following.

1. The interoccurrences time model

This paper is aimed at providing a model for earthquakes prediction, which takes into account interoccurrence time both for nearly periodic, characteristic earthquakes, and for clusters and other randomly occurring earthquakes having a magnitude greater than a selected threshold.

The procedure is purely probabilistic, but at least three physical hypotheses are included:

1) a double behaviour in earthquake generation, as already said;
2) a dependence on the past history, expressed with the use of a renewal process;
3) a stationary asymptotic behaviour of the hazard: a very long seismic silence in a zone could be reasonably interpreted as an energy release having occurred somewhere else, rather than a continuous enormous accumulation of energy in progress in the zone; so even in the case of increasing hazard rate its limiting value is a finite quantity.

More precisely, property 2) means that the probability of an earthquake occurrence depends on the elapsed time t_0 from the last event. Property 3) means that, in case of $\alpha>1$ the hazard increases with t_0 but, after a peak value, if the earthquake doesn't occur, it decreases and reaches a stable value, describing for every time increment a constant probability of an immediate occurrence.

In our regions an exponential-Weibull (ex-w) mixture, with the Weibull's shape parameter $\alpha>1$ appears to be suitable: it generally shows, after a first phase of weakly decreasing hazard rate, a central phase of increasing hazard followed by a stationary one. The exponential distribution mainly models the short interoccurrence times, related to clustering or random events, while the Weibull distribution mainly models the interoccurrence times related to the characteristic earthquake. Nevertheless, both distributions are defined in $[0, \infty)$ without a threshold of separation between the two families of events. The weaker the characteristic earthquake is expressed, the more the tails of the two distributions overlap.

In any case the mixture distribution overcomes the drawback of a single distribution that in general is not able to fit the entire class of interoccurrence times. In fact what in general happens is that a distribution that fit well the short interoccurrence times does not fit the long interoccurrence time and viceversa.

Moreover note that the entire set of interoccurrence times has often coefficient of variation (COV) near to 1. So this property does not indicate a propensity towards characteristic earthquake nor towards clustering. The preliminary check of the global COV simply would indicate that the Poisson hypothesis is not rejectable. A better fitting is obtained through a mixture. The coefficient of aperiodicity (Matthews et al. 2002) of the resulting process can adequately fit sequences with different degrees of regularity.

The choice of the two component distributions here proposed for the mixture is the result of investigations on different mix of distributions and on correspondence between the theoretical characteristic of the mixture and the physical aspect of phenomenon modelled (Guagenti Grandori et al. 1990).

The ex-w survivor function, $\Im_\tau(t_0) = 1 - F_\tau(t_0)$, and the probability density function, $f_\tau(t_0)$, are respectively (Cox, 1962):

$$1 - F_\tau(t_0) = (1-p)\exp\left[-bt_0\right] + p\,\exp\left[-(\rho\, t_0)^\alpha\right] \tag{1}$$

$$f_\tau(t_0) = (1-p)\left\{b\exp\left[-bt_0\right]\right\} + p\left\{\alpha\rho(\rho\, t_0)^{\alpha-1}\exp\left[-(\rho\, t_0)^\alpha\right]\right\}. \tag{1'}$$

Let us call τ the interoccurrence time random variable, t_0 being its generic value, i.e. the time since the last event. The parameters involved in Eqs. (1) are: p, which can be considered the weight of the characteristic earthquake portion of the model; α, whose value governs the Weibull coefficient of variation $\mathrm{COV_W}$:

$$\mathrm{COV_W} = \left\{\frac{\Gamma\left(1+\frac{2}{\alpha}\right)}{\left[\Gamma\left(1+\frac{1}{\alpha}\right)\right]^2} - 1\right\}^{\frac{1}{2}}; \tag{2}$$

b and ρ, that can be defined as functions of μ_1 and μ_2, which are the two return period components of the short interoccurrence times and of the characteristic earthquake interoccurrence times, respectively:

$$\mu_1 = \frac{1}{b} \qquad\qquad \mu_2 = \frac{\Gamma\left(1+\frac{1}{\alpha}\right)}{\rho}. \tag{3}$$

Moreover the COV of the mixture in the resulting process, called coefficient of aperiodicity, comes out to be

$$\mathrm{COV}_{\mathrm{ex-w}} = \left\{(1-p)\cdot 2k_1^{\,2} + pk_2^{\,2}\frac{\Gamma\left(1+\frac{2}{\alpha}\right)}{\Gamma^2\left(1+\frac{1}{\alpha}\right)} - 1\right\}^{\frac{1}{2}}. \tag{2'}$$

The information about aperiodicity contained in (2') differs from the information contained in the sample COV.

The mixture (1) models a wide class of earthquake interoccurrence times; its hazard rate (HR), $\lambda(t_0)$, is defined as:

$$\lambda(t_0) = \frac{f_\tau(t_0)}{1-F_\tau(t_0)} \tag{4}$$

and it is shaped as required for property 3). Because parameter p can assume different values, the distribution (1) can explore different degrees of characteristic earthquake evidence: increasing values of p means larger and larger evidence of characteristic earthquake; p=1 means lack of the irregular short interoccurrence times. In the two extreme cases p=0 and p=1 the mixture distribution (1) becomes, respectively, exponential and Weibull and property 3) fails: when p=0 the HR is constant in time and the predictability is missing; when p=1 the HR is continuously increasing until the certainty of an immediate occurring earthquake.

Eq. (4) furnishes the prediction in a very short (infinitesimal) next time interval *dt:*

$$\lambda(t_0) = \lim_{\Delta t_0 \to 0+} \frac{\Pr\left\{ t_0 < \tau \le t_0 + \Delta t_0 \middle| \tau > t_0 \right\}}{\Delta t_0} = \frac{f_\tau(t_0)}{1-F_\tau(t_0)} \tag{5}$$

while the medium term Δt prediction is the probability, at each t_0 of an earthquake in the next Δt:

$$\mathrm{P}_{\Delta t|\,t_0} = \frac{F_\tau(t_0+\Delta t) - F_\tau(t_0)}{1-F_\tau(t_0)}. \tag{6}$$

In this context, prediction means, at each instant, the conditional probability of an earthquake with magnitude greater than a specific value in the next time interval (*dt* or Δ*t*), given the length of the preceding seismic gap.

The presence of four parameters is the main drawback of Eq. (1) in estimation procedure. To avoid this shortcoming, we propose the following time scale transformation:

$$h = \frac{t_0}{\mu} \tag{7}$$

where

$$\mu = (1-p)\,\mu_1 + p\,\mu_2 \tag{8}$$

is the global return period of the earthquake process. Eq. (7) transforms Eq. (1) into dimensionless terms as follows:

$$1 - F(h) = (1-p)\exp\left[-\frac{h}{k_1}\right] + p\exp\left\{-\left[\Gamma\left(1+\frac{1}{\alpha}\right)\frac{h}{k_2}\right]^{\alpha}\right\} \tag{9}$$

where $k_1 = \mu_1/\mu$, $k_2 = \mu_2/\mu$ are the two return period components, μ_1 and μ_2, expressed in dimensionless terms.

Eq. (9) offers the advantage of presenting the range of a source's possible earthquake behaviour independently of the numerical value of the respective return periods. The Eq. (8) becomes:

$$1 = (1-p)\,k_1 + p\,k_2\,. \tag{8'}$$

The parameter *p* now can be estimated through Eq. (8') as follows:

$$p = \frac{1-k_1}{k_2 - k_1}\,. \tag{10}$$

Three parameters remain to be estimated in Eq. (9). In order to make the estimate procedure easier and more stable, we proceed assuming $\alpha =$ *constant* with different trial values; then we choose the best one in respect to likelihood function. In this way only k_1 and k_2 have to be estimated: they offer the advantage of being directly suggested by the sample, at least if the ratio $r = k_2/k_1$ is large enough to ensure weak overlap of the two component densities. In this case the mean values of the interoccurrence times < 1 and the interoccurrence times > 1 give empirical estimates of $k_1$and k_2, respectively.

Varying k_2 and k_1 we obtain different families of earthquake processes, representing conjectural "truths". In Figures 1-5 they are shown in a compact form through the dimensionless HR $\widetilde{\lambda}$.

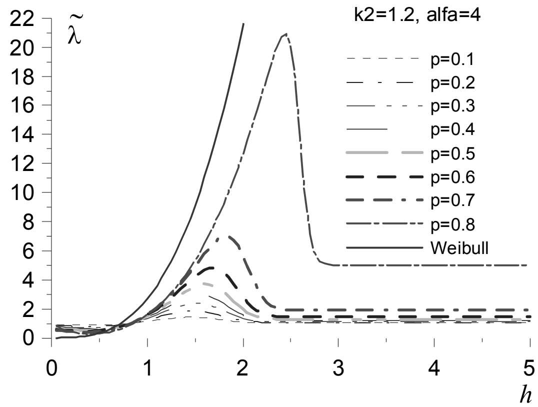

Figure 1. Dimensionless HR for conjectural "truths" with k_2=1.2 and α=4.

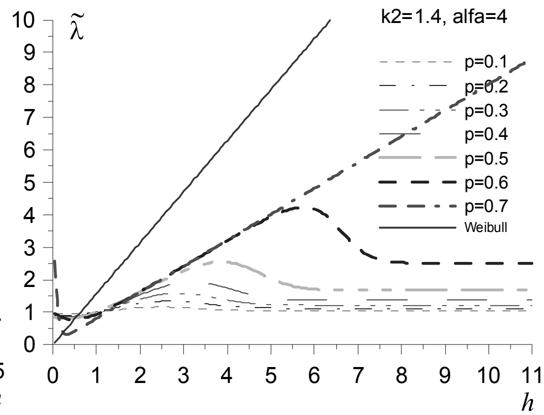

Figure 2. Dimensionless HR for conjectural "truths" with k_2=1.4 and α=4

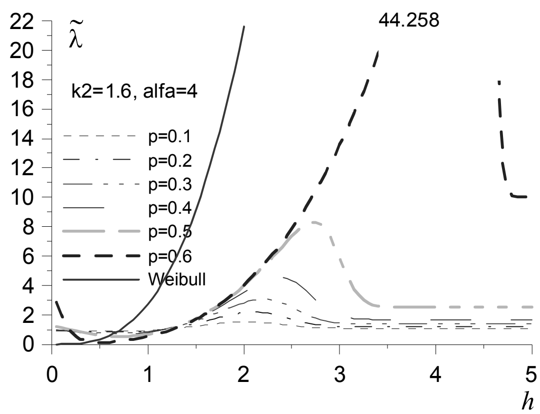

Figure 3. Dimensionless HR for conjectural "truths" with k_2=1.6 and α=4.

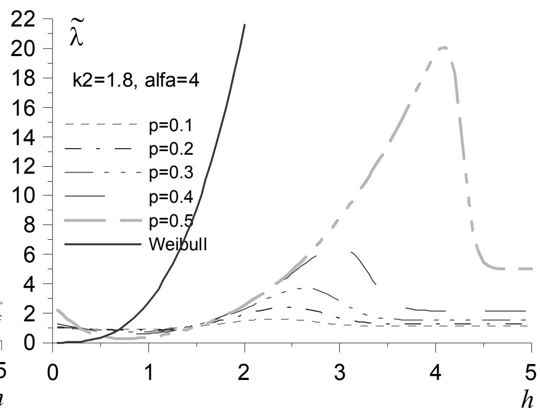

Figure 4. Dimensionless HR for conjectural "truths" with k_2=1.8 and α=4

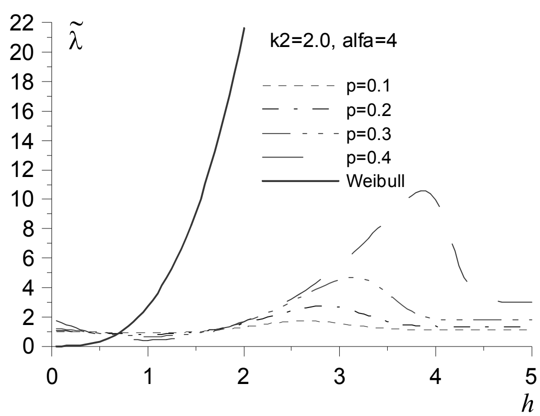

Figure 5. Dimensionless HR for conjectural "truths" with k_2=2.0 and α=4.

The assumed range [1.2-2] for k_2 can be considered exhaustive for a wide class of Italian seismic zones. The values of k_1 are chosen between 0 and 1 to give, from Eq. (10), equally spaced p values ($0.1 \leq p < 1/k_2$ with increment 0.1).

In the above figures the value of α is 4. Our large set of numerical simulations show that larger or smaller values of α lead to similar behaviours with more or less pronounced peak relative to characteristic earthquake.

Let us remember that all distributions (Eq. 9) have mean value =1. If we want the prediction in time scale we will use the relationships

$$\tilde{\lambda}(h) = \mu\, \lambda(t_0)\,; \qquad \mathrm{P}_{\Delta t|\, t_0} = \mathrm{P}_{\Delta h|\, h} \qquad \text{with} \qquad \Delta t = \mu\, \Delta h \tag{11}$$

$$\tilde{F}(h) = F_\tau(t_0) \qquad \text{and} \qquad \tilde{f}(h) = \mu\, f_\tau(t_0)\,. \tag{12}$$

Moreover, the asymptotic value of hazard rate is given by: $\lambda_\infty = 1/\mu_1$ and its dimensionless form is given by $\tilde{\lambda}_\infty = 1/k_1$.

2. Comparison between Estimation Procedures

We consider four different estimation procedures for conditional earthquake occurrence probability.

The first one assumes the mathematical model (Eq. 9) and it uses the classical method of maximum likelihood (ML) for the estimation of the parameters k_1 and k_2 (Maybeck, 1979,...).

A second method, proposed by us and called threshold method (TR), assumes again the mathematical model (Eq. 9) with the means of interoccurrence times less than and greater than 1 taken as estimators of k_1 and k_2, respectively. As we will see in the following, if r assumes values around 4 or more, the TR method is better than the ML method: indeed it is able to catch the contribution of the few interoccurrence times in the tail better than the ML, which is sensitive to the total contribution of the interoccurrence times. On the other hand, the TR fails for low values of r, i.e. if the tails of the two components of the mixture overlap.

A third method (ME) is based on the maximum entropy principle that implies the use of a generalized exponential distribution (Jaynes, 1957, Akaike, 1977, Tagliani, 1989 and 1990). Indeed, this is the distribution function that incorporates only the information inherent in dataset (so maximizing the entropy of the distribution). This method does not agree with our choice of limited λ_∞, but it is useful in a qualitative sense, because it is very sensitive to the dataset and is free from the characteristic earthquake or other interpreting hypothesis (Vere-Jones et al. 1982).

Finally, an empirical method (EMP) consists in reading directly on the cumulative frequency polygon F^* the values of λ^* and $\mathrm{P}_{\Delta h|h}$. Specifically the empirical value λ^* is:

$$\lambda^*(h) = \frac{(tg\theta)_h}{1 - F^*(h)} \tag{13}$$

being $(tg\theta)_h$ the tangent of that polygon F^* side which h belongs to (Fig. 6). Eq. (13) becomes meaningless at the end of the polygon, but, before the end, it offers an empirical rough prediction directly coming from the dataset, free from mathematical model (apart the piecewise linearity of the polygon). Moreover the dataset can suggest possible variability of the hazard that the structure of the chosen mathematical model cannot incorporate; typically, a possible multimodality of the hazard will be suggested from the empirical method. Obviously, this rough empirical method EMP cannot be considered as a reliable non-parametric procedure, but it may furnish complementary qualitative information.

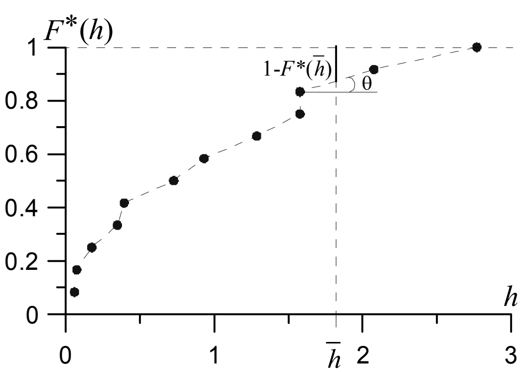

Figure 6 Building of empirical value λ^* as exposed in (13)

3. Procedure Credibility

In previous papers (Grandori et al. 1998, 2003, 2006; Guagenti et al. 2003, 2004) the credibility of a procedure has been introduced: a measure of the error, with respect to a conjectural truth $F°$, when a target quantity A is estimated with that procedure. Let $A°$ be the true value of A (in the conjectural truth) and $\hat{A}_s$ its estimator with the procedure s. The credibility Δ_s° of the procedure s relative to the truth $F°$

(that here is simply a given distribution) is defined as follows:

$$\Delta_s^\circ = \Pr\left\{ A^\circ - kA^\circ \le \hat{A}_s \le A^\circ + kA^\circ \right\}. \tag{14}$$

In other words, Δ_s° is the probability to have (as absolute value) a fractional (or relative) error not larger than a given value k in the estimate of the quantity A. In this research the target quantity A is the prediction (λ or $\mathrm{P}_{\Delta t | t_0}$).

The merit of credibility Δ_s° is to focus attention on the target quantity A of interest, instead of on the data fitting. This makes the credibility specifically useful to the aim of the research and more selective than classic tests. For example, exponential, lognormal, gamma and Weibull distributions, largely used for interoccurrence time model, can lead, with the classic tests, to similar levels of significance; on the other hand they lead to predictions drastically different.

It is remarkable that the credibility Δ_s° is not based on the single sample given by the catalogue but on "all samples" (i.e. 1000 samples) that can be drawn from $F°$.

Because of definition (14) of credibility, competing models can be compared in the frame of the chosen conjectural truths. The values of Δ are obtained with Monte Carlo simulation.

In Table 1. and 2. the Δ values are shown, relative to the estimated quantity $\hat{\lambda}$ or $\hat{P}$ in respect to some conjectural $F°$ identified through their single parameter k_2°, being the comparison done with the same values of p and α. The values p=0.5 and α=4 are assumed because they are frequently observed in the studied zones; the parameter k_1° is evaluated through Eq. (8').

k_2° \ h	methods	0.5	1	1.5	2	2.5	h_{max}
1.2	ML	0.99	0.89	0.94	0.49	0.87	0.890
	TR	1.00	0.02	0.22	~0.0	~0.0	~0.0
1.4	ML	0.96	0.93	0.81	0.74	0.45	0.50
	TR	1.00	0.76	0.67	0.93	~0.0	0.09
1.6	ML	0.92	0.93	0.76	0.73	0.74	0.82
	TR	1.00	0.94	0.84	0.83	0.88	0.87
1.8	ML	0.61	0.80	0.65	0.64	0.64	0.92
	TR	0.57	0.83	0.71	0.71	0.71	0.80

Table 1. Values of $\Delta = \Pr\left\{ \left| \frac{\hat{\lambda}(h) - \lambda°(h)}{\lambda°(h)} \right| < 0.3 \right\}$ with samples size ν=100

h / k_2	methods	0.5	1	1.5	2	2.5	h_{max}
1.2	ML	0.99	0.91	0.96	0.59	0.89	0.90
	TR	1.00	0.03	0.51	~0.0	~0.0	0.01
1.4	ML	0.97	0.93	0.86	0.80	0.53	0.530
	TR	1.00	0.75	0.75	0.98	~0.0	0.11
1.6	ML	0.92	0.91	0.79	0.84	0.85	0.88
	TR	1.00	0.93	0.87	0.90	0.95	0.97
1.8	ML	0.61	0.77	0.68	0.71	0.75	0.98
	TR	0.54	0.82	0.73	0.75	0.80	0.94

Table 2. Values of $\Delta = \Pr\left\{\left|\frac{\hat{\mathrm{P}}(h)-\mathrm{P}°(h)}{\mathrm{P}°(h)}\right| < 0.3\right\}$; Δh=0.1 and with samples size ν=100

Table 1. gives the credibilities of the estimate of $\tilde{\lambda}$ with the procedures ML and TR. Table 2 gives the same credibilities for the estimate of $\mathrm{P}_{\Delta h|h}$. The sample size is rather numerous in this first analysis: it is ν = 100. The threshold k is taken equal to 0.3; a series of preliminary tests in various hypothetical conditions confirmed a good sensitivity of the method of comparison based on k=0.3.

The credibilities are time dependent, hence their values are calculated for different time instant h with steps of 0.5, i.e. 0.5 times the return period μ. These step values allow determining in which periods the prediction is more or less reliable. The last column gives the mean value of the credibilities if the prediction is made when the elapsed time h is equal to the maximum observed interoccurrence times.

In some cases the tables show large differences. Which one must be judged more credible? On the basis of many numerical simulations, we can say that the TR method is reliable when the ratio $r = k_2/k_1$ is large ($r > 4$), as in the case of third or fourth row. This behaviour can be explained thanks to the meaning of the ratio r: its large values (third and fourth case) mean a weak overlap of the tails of the two component densities (ex and W) and the information coming from the larger IT is really interpreted as Weibull contribution, while small values of r mean that the role of the two components is not clearly separate in the interpretation of short and long interoccurrence times.

The seismic zones in this study, have much fewer data than in this first simulation. So, the next Table 1'. and Table 2'. give the credibilities when the sample has the more realistic size of 20 elements (ν=20). The credibility values are obviously rather small, but they confirm the preceding trend.

h / k_2°	methods	0.5	1	1.5	2	2.5	h_{max}
1.2	ML	0.71	0.57	0.57	0.19	0.50	0.49
	TR	0.91	0.21	0.37	~0.0	~0.0	~0.0
1.4	ML	0.67	0.61	0.46	0.38	0.25	0.36
	TR	0.91	0.65	0.54	0.68	0.14	0.70
1.6	ML	0.58	0.59	0.41	0.39	0.34	0.69
	TR	0.87	0.70	0.54	0.54	0.58	0.69
1.8	ML	0.33	0.41	0.36	0.34	0.34	0.43
	TR	0.38	0.49	0.38	0.38	0.40	0.57

Table 1'. Values of $\Delta = \Pr\left\{ \left| \frac{\hat{\lambda}(h) - \lambda^\circ(h)}{\lambda^\circ(h)} \right| < 0.3 \right\}$ with samples size ν=20

h / k_2	methods	0.5	1	1.5	2	2.5	h_{max}
1.2	ML	0.78	0.58	0.59	0.24	0.51	0.49
	TR	0.90	0.20	0.37	~0.0	~0.0	~0.0
1.4	ML	0.70	0.58	0.53	0.45	0.28	0.28
	TR	0.92	0.63	0.55	0.68	0.14	0.70
1.6	ML	0.58	0.58	0.44	0.48	0.48	0.77
	TR	0.88	0.69	0.50	0.53	0.57	0.63
1.8	ML	0.32	0.41	0.34	0.34	0.38	0.79
	TR	0.41	0.50	0.36	0.35	0.36	0.59

Table 2'. Values of $\Delta = \Pr\left\{ \left| \frac{\hat{\mathrm{P}}(h) - \mathrm{P}^\circ(h)}{\mathrm{P}^\circ(h)} \right| < 0.3 \right\}$; Δh=0.1 and with samples size ν=20

With regard to the ME method, its credibility is almost always around 0.25: a reliable prediction cannot be based on it. Nevertheless, as already noticed, from a qualitative point of view, it offers useful information directly coming from data, free of modelling error; therefore it must be taken into consideration, especially when it appears largely different from the assumed model.

All the methods are significantly reliable in deciding whether the Poissonian model could be rejected. Indeed, in case of Poisson model is HR=1. If the credibility is aimed at testing the hypothesis HR=*constant* against any other time varying HR, it becomes:

$$\Delta_\delta = \Pr\left\{\hat{\lambda}(h) - 1 < k_\delta\right\}. \tag{15}$$

Eq. (15) is the probability of an error not larger than a given value k_δ accepting the hypothesis

HR=*constant.* Table 3 gives its complementary values Δ'_{δ} with k_{δ}=2. Therefore the table shows the credibility of an HR more than three times the poissonian constant HR. All the three methods, even if the probability can be different, agree in judging the difference from the Poisson hypothesis, when it exists.

k_2 \ h	methods	0.5	1	1.5	2	2.5	h_{max}
1.2	ML	0.00	0.00	0.42	0.46	0.25	0.47
	TR	0.00	0.00	~0.0	0.79	0.68	0.71
	ME	0.01	0.08	0.28	0.21	0.24	0.15
1.4	ML	0.00	0.00	0.29	0.67	0.54	0.77
	TR	0.00	0.00	~0.0	0.74	0.84	0.88
	ME	0.02	0.04	~0.0	0.50	0.64	0.99
1.6	ML	0.00	0.00	0.12	0.54	0.74	0.87
	TR	0.00	0.00	~0.0	0.45	0.84	0.88
	ME	0.02	0.01	0.09	0.35	0.81	0.99
1.8	ML	0.00	0.00	~0.0	0.32	0.64	0.86
	TR	0.00	0.00	~0.0	0.21	0.58	0.76
	ME	0.01	0.01	0.04	0.18	0.50	1.00
0(exp)	ML	0.00	0.00	0.00	0.00	0.00	0.00
	TR	0.00	0.00	0.00	0.00	0.00	0.00
	ME	0.05	0.04	0.04	0.05	0.04	0.00

Table 3. Values of $\Delta'_{\delta} = \Pr\left\{ \hat{\lambda}(h) - 1 > 2 \right\}$ with samples size ν=20

4. Robustness

The credibility procedure is now applied to investigate the robustness of the mixture procedure. In particular, assumed as true distribution of the interoccurrence times the exponential process, we drew from it several random samples using Monte Carlo method. Considered as competing models the ex-w mixture distribution and the exponential distribution, from each sample we derived, with the ML method, the models parameters and we evaluated the hazard rate curves $\widetilde{\lambda}$. Finally, we calculated the credibilities (in $\widetilde{\lambda}$ estimate) of the mixture procedure, $\Delta_{\text{ex-w}}^{\text{exp}}$, i.e. when the mixture is a wrong model being the truth is exponential. From the obtained results it is evident that the ex-w mixture model has the advantage of being robust, at least in respect to the Poissonian hypothesis. Indeed, the mixture model gives a reliable prediction even if the true distribution is exponential, as shown in Table 4, because of its adaptability. Note that the credibility values are rather high, even if a realistically small size of the samples (ν=20) was chosen.

Moreover, it was verified that the above credibility $\Delta_{\text{ex-w}}^{\text{exp}}$ is about of the same order of the credibility $\Delta_{\text{exp}}^{\text{exp}}$: it means that the degree of uncertainty of the wrong (but robust) mixture procedure is similar to the degree of uncertainty due to statistical variability of the samples in the true exponential model.

Repeating the same procedure assuming the exp-w mixture process as true distribution, we verified the lack of robustness of the exponential model. In fact its credibility, when the truth is a mixture, is near to zero.

h	0.5	1	1.5	2	2.5	h_{max}
$\tilde{\lambda}$	0.92	0.95	0.70	0.57	0.48	0.42
$P_{0.1\|h}$	0.94	0.95	0.71	0.59	0.50	0.44

Table 4. Values of $\Delta_{\text{ex-w}}^{\text{exp}}$ for estimate with ML of $\tilde{\lambda}$ and $P_{0.1|h}$; with samples size ν=20

5. Application to Italian seismic sources

The catalogues taken into consideration for the application of the proposed procedures are obtained by the association of the events of the Italian historic catalogue CPTI04 (CPTI Working Group, 2004) with the seismogenetic source areas (SA) defined in DISS3.0.2 (Database of Individual Seismogenetic Sources, DISS, DISS Working Group, 2006). In DISS, SAs are identified on the basis of geological and geophysical characteristics (Basili, 2007).

These new sub-catalogues have been made available by the members of Task 1.1 of the INGV-DPC Project S2 (Slejko & Valensise, 2007). Each analysed catalogue involves events with magnitude M defined in CTPI04 as Maw > 5.3 and having occurred from 1600 to 2002. To better understand the results presented below, we have provided some supplementary electronic material (the label “SEM”, included in table and figure references, will be used to identify further supplementary electronic material associated with this paper). In Figure 01SEM a plot of the the SA’s labelled with the identification number used in DISS3.0.2 is given.

As explained in the previous sections, the sample size ν plays an important role on the reliability of a probabilistic analysis. Initial inspection of the catalogues of the 74 SA Sources reveals immediately that the methods of this paper cannot be applied successfully: many of the SA sources have too small a sample size ν (0-5 events and 0-4 interoccurrence times). We obtained a set of sources having a sufficiently large set of interoccurrence times, using the new association of source areas and related seismicity, grouping the SA's into eight Macro Regions (MR), on the basis of geophysical aspects (Basili, 2007). These are shown in Figure 02SEM; the MR coordinates are listed in Table 01SEM.

The combining of SAs in MR can lead to lose the proper identity of each SA. To maintain unaltered the characteristics of the interoccurrence of each SA, the interoccurrence times of each MR are built using the interoccurrence times obtained for each SA; as an example, Table 5 shows the data set of MR6 built using the data set of SA24, SA34, SA38 and SA63. The implicit assumption is that the constituent sources in a region behave similarly in their seismic cycles. The choices made during this phase of the project were discussed and decided with other components of the INGV-DPC Project S2 with the aim that the results obtained by the tasks involved into the project could be comparable each other.

Interoccurrence times for each seismogenetic area of macro region MR6 (in years)						
SA24						
13.76984	30.71288	72.65607	100.3346	56.73634		
SA34						
27.18319	4.688642	122.936				
SA38						
122.937	5.881749	161.8029				
SA63						
215.7441						
Interoccurrence times of macro region MR6 (in years)						
MR6						
13.76984	30.71288	72.65607	100.3346	56.73634	27.18319	4.688642
122.936	122.937	5.881749	161.8029	215.7441		

Table 5. Building of a MR interoccurrence time dataset using the interoccurrence time datasets of the constituent SA's.

The three methods, ML, TR and ME, proposed in the previous sections were applied to the eight Italian MR. For each MR, the data were organized as shown in Table 6 and Figure 7 for MR6. In Table 6 are collected: the MR6 catalogue in dimensional and dimensionless forms, the MR6 return period μ, the parameter *p,* representative of the evidence of characteristic earthquake, the asymptotic hazard rate value λ_∞ and its dimensionless form $\tilde{\lambda}_\infty$. Figures 7*a* and 7*b* show, respectively, the MR6 $\hat{F}(h)$ and $\hat{\lambda}(h)$

estimations with the three methods in dimensionless form. In the figures the experimental HR λ* (EMP) and the exponential HR (EXP) are also presented. The behaviour of EMP is useful for investigating anomalies in the data sets not captured by ML, TR and ME.

MR6_ *interoccurrence times in years*							
4.688642	5.881749	13.76984	27.18319	30.71288	56.73634	72.65607	100.3346
122.9360	122.937	161.8029	215.7441				
μ = 77.94861 $\quad$ p = 0.4167 $\quad$ λ_∞ = 0.033077 $\quad$ $\widetilde{\lambda}_\infty$ = 2.5783							
MR6_ *interoccurrence times in dimensionless form*							
0.060150	0.075457	0.176653	0.348732	0.394014	0.727869	0.932102	1.287189
1.577142	1.577154	2.075764	2.767774				

Table 6. MR6: Seismic catalogue in dimensional and dimensionless form

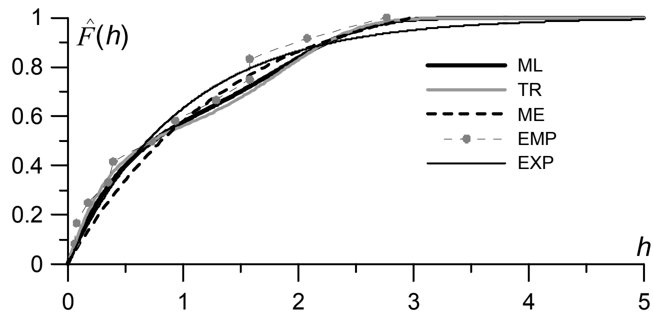

Figure 7a MR6: Estimation of $\hat{F}(h)$ in dimensionless form

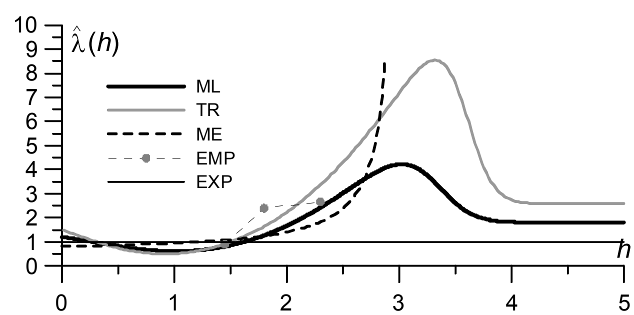

Figure 7b MR6: Estimation of $\hat{\lambda}(h)$ in dimensionless form

All the results produced are collected in the SEM file of this paper (Figures 03SEM-10SEM; Tables 02SEM-09SAM).

In all the implementations, the exponential distribution seems inadequate to model the seismic renewal process, while the mixture ex-w, supported by the proposed credibility analysis, seems more adequate.

The next step was the evaluation of the probability $\hat{P}_{\Delta t|t_0}$: it is the probability of occurrence of an event of magnitude Maw>5.3 (according with the events considered in the project) in the next interval of time Δt if a time t_0 is passed from the last event occurring in each SA and the last year reported in CPTI04 (2002) assumed as date of reference. Different interval of time Δt have been considered: 5, 10, 20, 30 50 and 100 years. In particular, the probability obtained for Δt equal 5 and 30 years are collected, both in tables and in maps, in the SEM of this paper (Figures 11SEM-16SEM; Tables 01_PSEM–08P_SEM). As example, Table 7 shows the probability of occurrence connected with the SA classified in MR6.

MR6		Δt=***5years***	Δt=***10years***	Δt=***20years***	Δt=***30years***	Δt=***50years***	Δt=***100years***
SA24	t_0=40						
ML		0.039451	0.096306	0.175295	0.237261	0.351327	0.627267
TR		0.04145	0.088838	0.147305	0.195926	0.281847	0.580433
ME		0.042291	0.106876	0.204251	0.286431	0.44201	0.721479
SA34	t_0=22						
ML		0.056713	0.107703	0.207961	0.277258	0.392574	0.612508
TR		0.058281	0.108403	0.200431	0.259055	0.350627	0.559674
ME		0.049064	0.096063	0.198182	0.277916	0.428859	0.700014
SA38	t_0=4						
ML		0.06687	0.136225	0.243931	0.323475	0.445231	0.636549
TR		0.080004	0.148542	0.272828	0.349374	0.456243	0.607427
ME		0.047855	0.101151	0.193301	0.271068	0.413136	0.682776
SA63	t_0=92						
ML		0.038474	0.083398	0.16832	0.249702	0.435952	0.845912
TR		0.032162	0.078694	0.150611	0.231663	0.429369	0.867464
ME		0.057749	0.122077	0.233283	0.327149	0.504588	0.823986

Table 7. MR6: Probability of occurrence $\hat{P}_{\Delta t|t_0}$ for different Δt and different approach of each SA

Since the year 2002 (the end of our catalogue) until today, 5 years are already passed. The occurrence probabilities shown in Table 7 are low and the earthquake did not occur. Is this a confirmation of the model goodness? Not at all. Only it can be taken as a kind of consolation. The degree of goodness of the model is measured by the above shown credibility that had explored the “complete” (1000 samples) sequence of possible events.

Figure 8 and 9 collect and compare the probability obtained in each SA with the 3 different methods proposed for Δt=5 years and Δt=30years (in the SEM file Figure 8 and 9 are reported in larger format and labelled as Figure 17SEM and Figure 18SEM respectively). We may observe that in most of the cases the three methods give comparable results. There are few cases in which the ML and ME methods produce lower probabilities than the TR method. It could be due to the frame of TR method more able to take into account the tail of the distribution when the ratio r is large, as previously discussed: accordingly, high value of TR probability may indicate peaks in the hazard curve not caught by ML and ME methods. In any case, these situations should be investigated. However, when the gap t_0 is large (i.e. SA40 and SA11) the TR prediction can not be done because the asymptotic behaviour of hazard rate has been reached. ME results mainly confirm the results of ML. On the other hand, being ME method strictly related to datasets, in few cases it gives differences or even lack of prediction. Considering ML results, for Δt=5 years , the highest hazard of occurrence are find in SA 23 (MR1 - Western Alps), in SA 80 (MR7 – Calabrian Arc) and in the SA's of the Central Northern Apennines (MR4). Increasing the interval of time up to 30 years, the hazard increases maintaining mainly the same ratio between the different areas. It indicates that not significant changes should be expected in the hazard of the country increasing the expected time from 5 to 30 years, according with ML and ME methods. On the contrary, TR method underlines some differences.

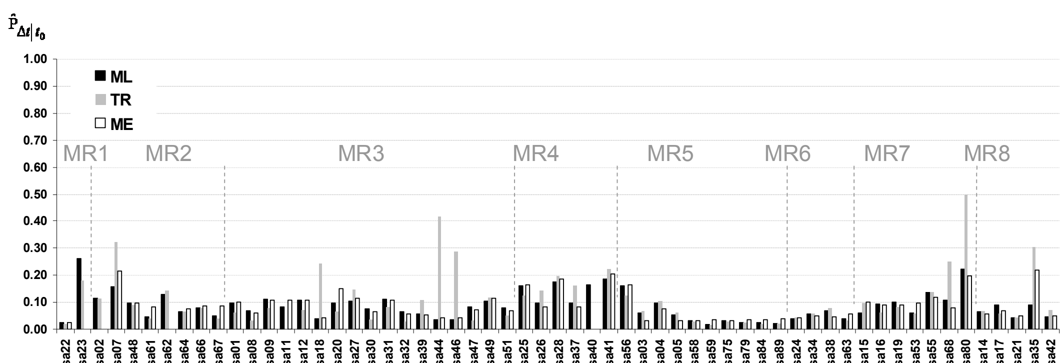

Figure 8. Italian SA Sources: conditional occurrence probability for Δt=5 years, given elapsed time t_0, different for each SA. Comparison between the ML, TR and ME methods. The vertical dotted lines define the eight MR.

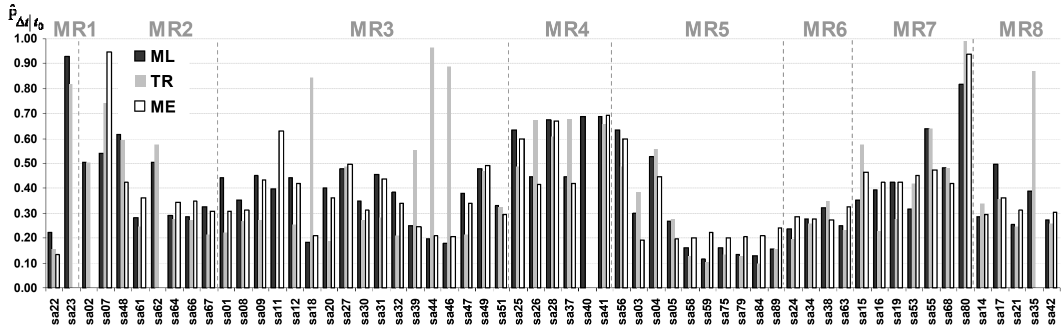

Figure 9. Italian SA Sources: conditional occurrence probability for Δt=30 years, given elapsed time t_0, different for each SA. Comparison between the ML, TR and ME methods. The vertical dotted lines define the eight MR.

6 Discussion and conclusions

Within the context of earthquake prediction, if it is admissible to assume that the generation of strong events depends primarily on the time elapsed since the last event, at least in some seismic areas, and if, in this reduced context, it is reasonable to accept an asymptotic stationary prediction, an exponential–Weibull mixture renewal model, associated with an empirical criterion, leads to conditional probabilities of occurrence having a rather good degree of credibility, at least for the Italian sources considered.

The proposed model appears flexible enough to interpret different behaviours of earthquake occurrence, whose identification is up to day questionable. It is a robust model, in particular it can give a statistic symptom of the dominant behaviour in the analyzed regions: clustering, or seismic gap, or poissonian one.

The shape parameter α of Weibull component is responsible of increasing or decreasing hazard rate, hazard rate being nearly constant when α is near to 1.

It is remarkable that the proposed mixture spontaneously fits the alternative behaviours with a good credibility. As an example let us consider samples drawn from two known distributions with increasing or decreasing hazard rate respectively; precisely two Weibull distributions respectively with $\alpha>1$ and $\alpha<1$ are assumed as conjectural truths. Figures 10 and 11 show the estimated hazard rate with the ex-w mixture model. It is evident that, even if the mixture is a wrong model in this case, it is able to catch the true increasing or decreasing behaviour of the hazard rate in the two assumed conjectural truths.

In the regions analyzed in this paper the model leads to estimate values of $\alpha>1$.

On the other hand we have to stress that the robustness of the mixture model refers only to the explored distributions F°. The panorama of conjectural truths F° and the target quantities A should be properly enriched; other models can enter in competition. Nevertheless all the numerical experiments carried out up to now showed a remarkable robustness of the model, i.e. its adoption reduces the epistemic uncertainties.

The credibility Δ_S° can be a good tool to judge a model in its structure in a panorama of conjectural truths, rather than to check a hypothesis upon a single catalogue.

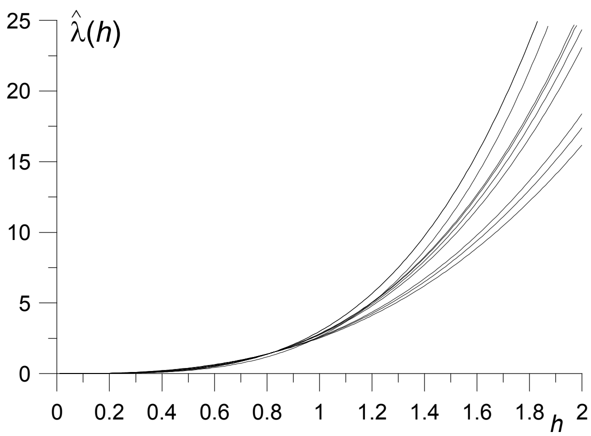

Figure 10. Mixture-estimated $\hat{\lambda}(h)$ (in dimensionless form) for samples drawn from known distribution with increasing hazard rate (Weibull with α>1).

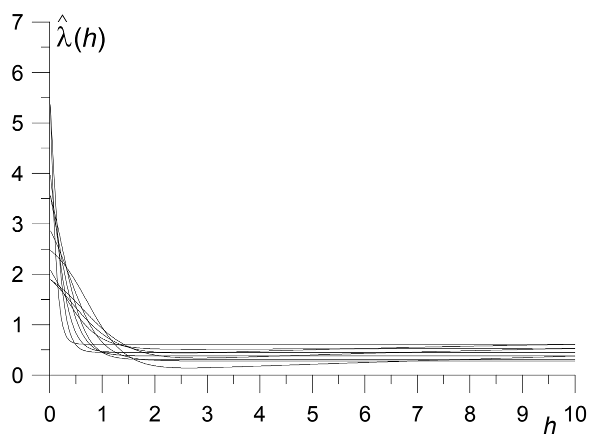

Figure 11. Mixture-estimated $\hat{\lambda}(h)$ (in dimensionless form) for samples drawn from known distribution with decreasing hazard rate (Weibull with α<1).

Acknowledgements

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Protezione Civile (DPC). Scientific paper funded by DPC do not represent its official opinion and policies. The authors would also like to deeply thank both reviewers for their valuable comments and suggestions in relation to the shortcomings of the initially submitted work.

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Response to Reviewer #1

Firstly, we would like to deeply thank reviewer #1 for his valuable comments and suggestions in relation to the shortcomings of the initially submitted work, thanks to which it was certainly possible to now increase considerably the quality of the paper.

In what follows, a point-by-point response to the specific comments made by this reviewer is given.

Questions 1 and 4

In the case that the paper aims to be more than a theoretical exercise, some reason has to be given for the exclusion of the more plausible recurrence model for seismicity in Italy. It is well known that three alternative views exist: "more than poissonian" time clustering, Poissonian time clustering, "less than poissononian" clustering (Characteristic earthquake). So far, the first model only has found some empirical support (Faenza et al., 2003). Despite of this, Authors ignore this model and concentrate their analyses on the remaining ones. In their answer, Authors state that their methodology is in principle able to manage the "clustering" model but, anyway, no attempt is made in this direction and no argument is given in support of this choice. This is more than an "incompleteness". Since the "credibility" analysis aims at a relative evaluation of the competing models, the lack of one of these makes the results useless: what does it mean that CE model works better than the poissonian one if both are wrong?

In the new version of the paper the three alternative views have been discussed and an attempt has been made to show how our methodology is in principle able to manage all the three alternatives. Moreover, in the text the following sentence has been added: “On the other hand we have to stress that the robustness of the mixture model refers only to the explored distributions *F*°. The panorama of conjectural truths *F*° and the target quantities *A* should be properly enriched; other models can enter in competition. Nevertheless all the numerical experiments carried out up to now showed a remarkable robustness of the model, i.e. its adoption reduces the epistemic uncertainties.” This means that the necessity of enriching the panorama of conjectural truths *F*° and target quantities *A* is recognized by the authors, but in any case the performed credibility analysis highlights the robustness of the considered model and the great advantages of reducing the epistemic uncertainties with its employment. This means that the author recognize the necessity of enriching the panorama of conjectural truths *F*° and target quantities *A,* but at the same time they consider useful the performed credibility analysis because it allows to highlight the robustness of the considered mixture model and hence the great advantages of reducing the epistemic uncertainties with its employment. For example, the added figure 10 and 11 show that, “even if the mixture is a wrong model in this case, it is able to catch the true increasing or decreasing behaviour of the hazard rate in the two assumed conjectural truths.”

Question 2

Authors state that "credibility" is a better measure of model performances than an "economy" principle. In this position, no importance is given to the number of parameters to be estimated for each model. This can be considered a possible choice, but it is not in line with the current practice.

From the point of view of the authors the interesting characteristic of the credibility (which could suggest preferring this measure than an “economy” principle) is that “it allows to judge a model in its structure in a panorama of conjectural truths, rather than to check a hypothesis upon a single catalogue.”

Question 3

To the third question, (i.e. to the possibility to apply the CE model to relatively small earthquakes) no answer has been given but a purely formal one (it depended on the position of the research group) that does not rely on the physical problem I have posed. At least an attempt to support with scientific arguments this position should be considered as mandatory.

The following sentence has been added in the paper: “The existence of characteristic or clustering events needs sound physical basis to be asserted; here the terms are used, for shortness, to indicate respectively the presence of rather regular temporal sequence (even if mixed with irregular short interoccurrence times) or the dominance of short interoccurrence times.” This sentence wants to clarify the meaning of the term “characteristic” for the authors, which is not strictly connected with the magnitude of the earthquake but rather to the time frequency.

Final comment

In summary, I have not found technical errors in the paper, but it lacks of an open discussion about its limitations. In my opinion the paper can be considered for publication if at least some discussion is included in the text (e.g., in the form of a Discussion section) about the questions stated above to make readers aware of possible critical aspects of the proposed procedure.

A paragraph devoted to the discussion about the limitation of the proposed methodology has been added.

Supplementary electronic material (SEM) of the paper:

Renewal Models for Earthquake Predictability

E. Garavaglia[1], E. Guagenti[1], R. Pavani[2] & L. Petrini[1]
Politecnico di Milano

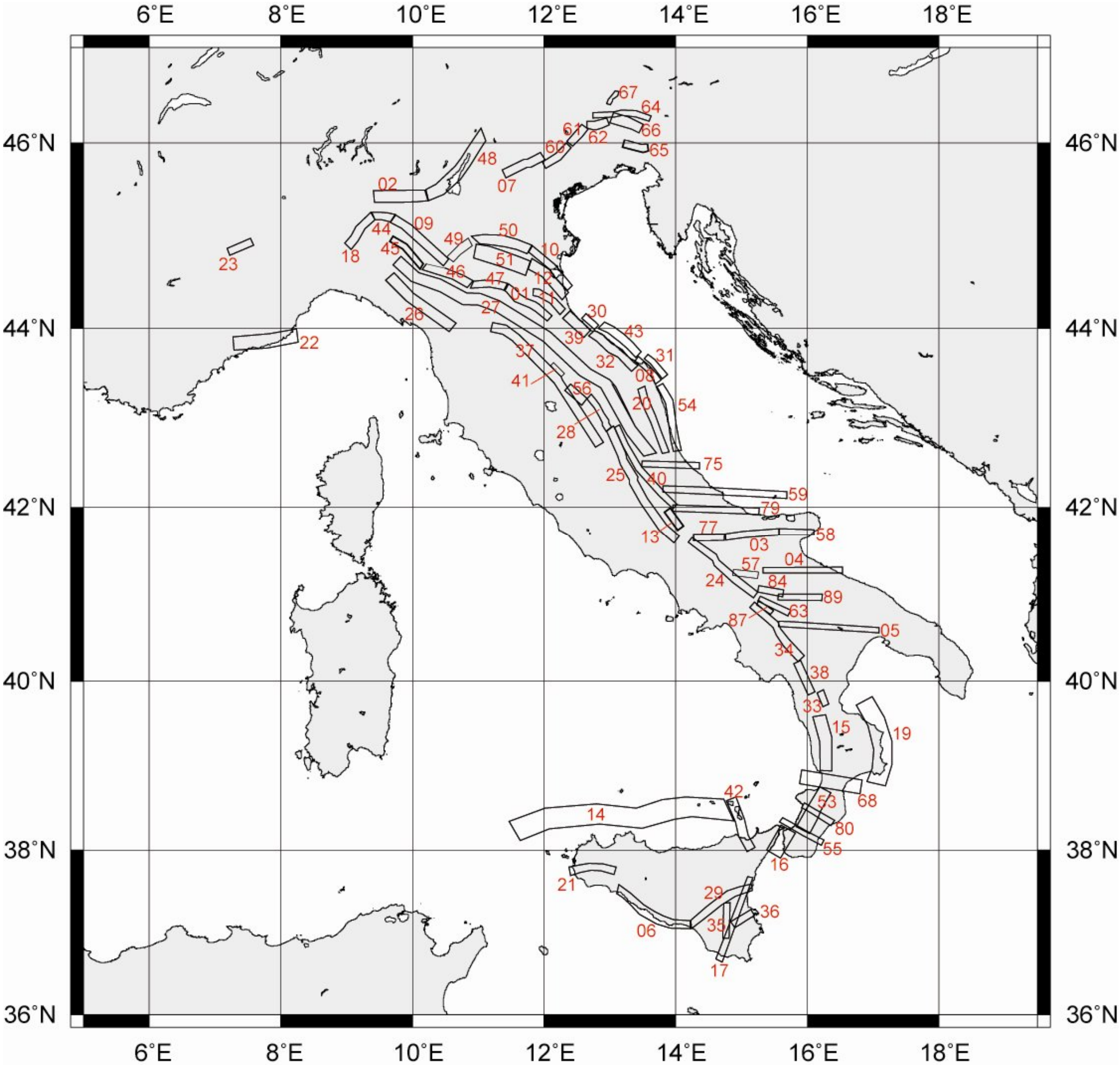

Figure 01SEM Seismogenetic Source Areas (SA) labelled with the identification number used in DISS3.0.2.

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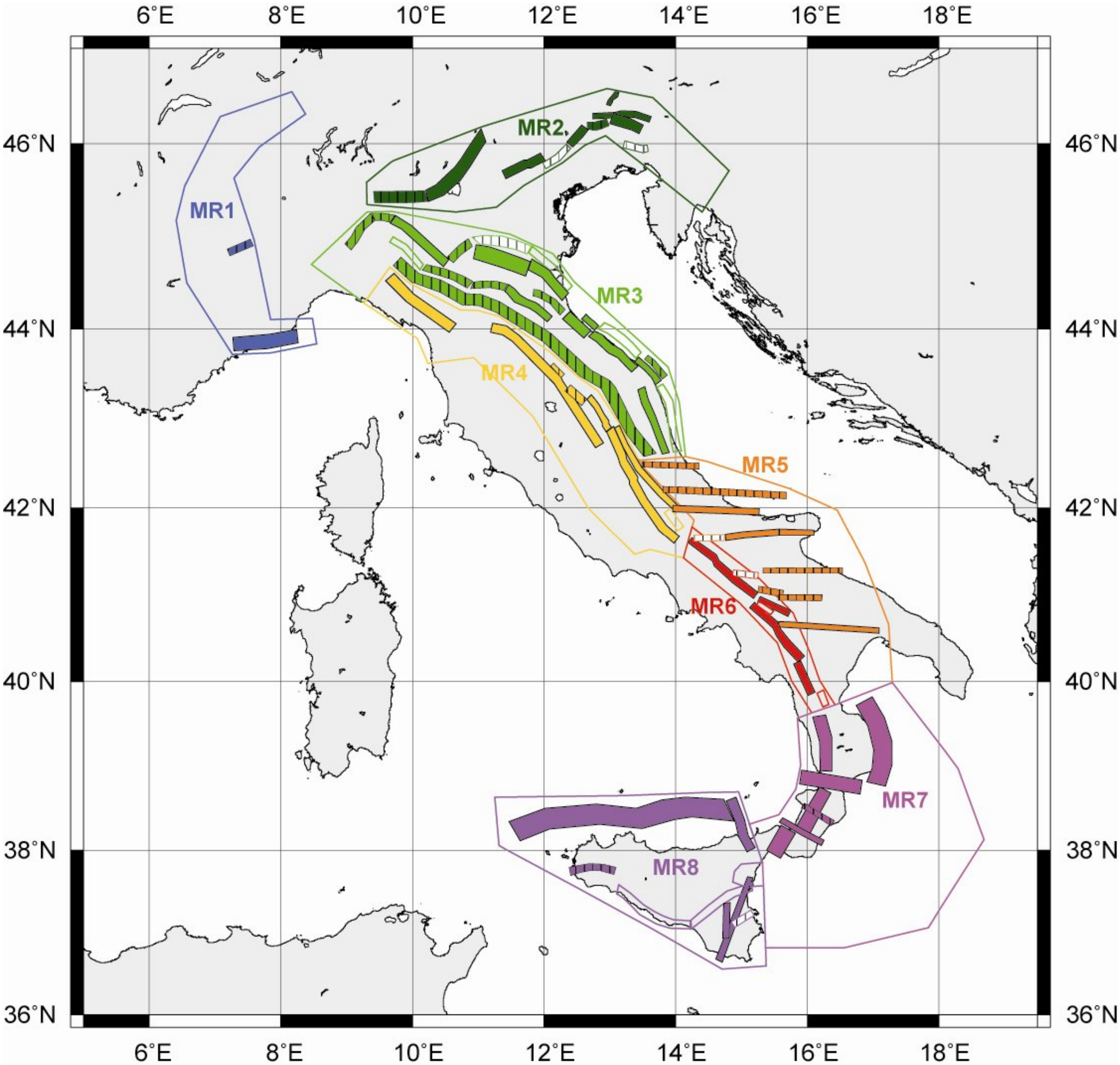


Figure 02SEM The eight Italian Macro Regions (MR) including all the 74 SA

MR1		MR2		MR3		MR4		MR5		MR6		MR7		MR8	
Lon.	Lat.	Lon.	Lat.	Lon.	Lat.	Lon.	Lat.	Lon.	Lat.	Lon.	Lat.	Lon.	Lat.	Lon.	Lat.
8.16	46.55	9.31	45.35	13.45	42.54	9.26	44.29	17.29	39.98	14.11	41.43	15.36	36.82	11.25	38.63
7.08	46.29	10.67	45.27	13.44	42.55	9.66	44.67	17.28	39.99	14.12	41.43	15.33	37.58	13.21	38.65
6.55	45.55	11.27	45.32	13.15	42.94	9.96	44.50	16.42	39.73	14.25	41.78	15.32	37.85	14.95	38.70
6.41	45.16	11.69	45.56	12.82	43.31	10.73	44.21	16.19	40.01	15.24	41.21	15.12	38.31	15.32	37.85
6.57	44.50	12.28	45.80	12.52	43.47	10.99	44.20	16.03	40.34	15.79	40.79	15.57	38.43	14.99	37.83
7.26	43.72	12.62	45.98	12.09	43.75	11.31	44.14	15.79	40.80	16.03	40.35	15.83	38.73	14.89	37.75
7.85	43.74	12.93	46.08	11.31	44.14	12.09	43.75	15.24	41.21	16.19	40.01	15.90	39.02	14.87	37.62
8.54	43.84	13.47	45.80	10.99	44.21	12.52	43.47	14.26	41.78	16.42	39.73	15.85	39.57	15.06	37.56
8.48	44.12	14.40	45.27	10.73	44.21	12.82	43.31	14.25	41.78	16.07	39.63	16.07	39.63	15.33	37.58
7.84	44.11	14.82	45.72	9.96	44.50	13.15	42.94	14.28	41.86	15.76	40.01	16.07	39.63	15.37	36.60
7.59	44.97	13.65	46.49	9.66	44.67	13.44	42.55	13.53	42.44	15.54	40.45	16.42	39.73	14.71	36.56
7.29	45.63	12.96	46.58	9.29	44.31	13.53	42.44	13.44	42.55	14.97	40.91	17.29	39.99	11.31	38.06
7.68	45.97	11.06	46.18	9.26	44.29	14.28	41.86	13.45	42.54	14.11	41.43	18.29	38.96	11.25	38.64
8.38	46.32	9.67	45.81	8.47	44.70	14.12	41.43	14.15	42.58			18.69	38.13		
8.16	46.55	9.30	45.59	9.31	45.27	13.61	41.52	14.47	42.53			17.84	37.07		
		9.31	45.35	9.69	45.28	13.38	41.47	15.72	42.22			16.55	36.82		
				11.48	45.03	12.68	42.00	16.45	41.98			15.36	36.82		
				12.14	44.82	11.83	43.03	16.87	41.38						
				12.45	44.52	10.93	43.69	17.23	40.67						
				13.89	43.58	10.23	43.62	17.29	39.98						
				14.05	43.19	10.07	43.90								
				14.15	42.58	9.26	44.29								
				13.45	42.54										

Table 01SEM Macro Regions MR coordinates

Macro Region 1 (MR1)

MR1_ interoccurrence times in years							
0.874467	12.37671	23.59352	32.15384	174.0233			
$\mu = 48.60436$ $p = 0.2155$ $\lambda_\infty = 0.057246$ $\widetilde{\lambda}_\infty = 2.7824$							
MR1_ dimensionless interoccurrence times							
0.017992	0.254642	0.48542	0.661542	3.580404			

Table 02SEM. MR1: interoccurrence time dataset using the interoccurrence time datasets of the SA composing MR1

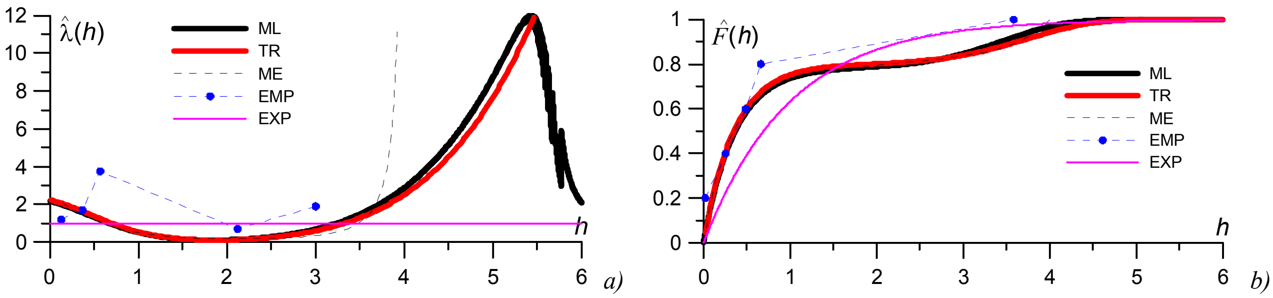


Figure 03SEM MR1: a) estimation of $\hat{\lambda}(h)$ in dimensionless form; b) estimation of $\hat{F}(h)$ in dimensionless form. ML= method of maximum likelihood; TR = threshold method; ME = maximum entropy method; EMP = empirical method; EXP = exponential model

		Δt					
MR1		**5 years**	**10 years**	**20 years**	**30 years**	**50 years**	**100 years**
SA22	t0=115						
ML		2.42E-02	5.55E-02	0.123149	0.2249110	0.4674350	0.96663
TR		1.65E-02	3.99E-02	9.00E-02	0.1560858	0.3411289	0.8898991
ME		2.35E-02	3.96E-02	8.33E-02	0.1347154	0.3073436	out range
SA23	t0=194						
ML		0.262917	0.4778	0.770954	0.927526	0.996741	1.00
TR		0.179589	0.343982	0.6157831	0.8178208	0.9758246	1.00
ME		out range	out range	out range	out range	out range	out range

Table 01P_SEM MR1: Probability of occurrence $\hat{P}_{\Delta t|t_0}$ for different Δt

The method ME, based on the maximum entropy principle, suffers for the truncation of the modelling on the last value present in the catalogue; therefore, the method is not able to give a prediction for values of Δt out of the range of estimation of the method.

Macro Region 2 (MR2)

MR2_ interoccurrence times in years							
0.359444	3.290192	16.42364	44.36438	57.36636	60.67586	63.30458	88.23068
119.7216	125.9131	141.2944					
μ = 65.540373	p =0.3636	λ_∞ = 0.028466	$\tilde{\lambda}_\infty$ = 1.8657				
MR2_ dimensionless interoccurrence times							
0.005484	0.050201	0.250588	0.676902	0.875283	0.925778	0.965887	1.346204
1.826684	1.921153	2.155836					

Table 03SEM. MR2: interoccurrence time dataset using the interoccurrence time datasets of the SA composing MR2

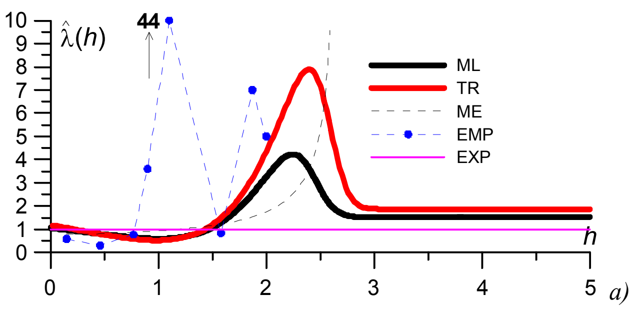


Figure 04SEM MR2: a) estimation of $\hat{\lambda}(h)$ in dimensionless form; b) estimation of $\hat{F}(h)$ in dimensionless form. ML= method of maximum likelihood; TR = threshold method; ME = maximum entropy method; EMP = empirical method; EXP = exponential model.

		Δt					
MR2		**5 years**	**10 years**	**20 years**	**30 years**	**50 years**	**100 years**
SA02	t0=200						
ML		0.114447	0.215815	0.375595	0.502973	0.689655	0.9022
TR		0.114461	0.215896	0.3851	0.502927	0.78673	0.903689
ME		out range	out range	out range	out range	out range	out range
SA07	t0=66						
ML		0.158078	0.269989	0.43255	0.541493	0.70936	0.909814
TR		0.321277	0.495745	0.659574	0.739362	0.845745	0.96383
ME		0.21459	0.350154	0.723573	0.947063	out range	out range
SA48	t0=101						
ML		0.097604	0.189295	0.412469	0.614987	0.881123	0.97791
TR		0.088018	0.185602	0.402292	0.595472	0.894155	0.993447
ME		0.095892	0.156473	0.323342	0.423213	0.63837	out range
SA61	t0=66						
ML		0.046522	0.086649	0.188084	0.283234	0.53926	0.966194
TR		0.03828	0.076316	0.158063	0.243755	0.494248	0.983021
ME		0.082384	0.13443	0.277791	0.363595	0.54844	0.878493
SA62	t0=226						
ML		0.128002	0.203876	0.385098	0.503002	0.685044	0.902293
TR		0.144231	0.259615	0.451923	0.576923	0.759615	0.942308
ME		out range	out range	out range	out range	out range	out range
SA64	t0=25						
ML		0.064062	0.11414	0.210998	0.28877	0.398698	0.776872
TR		0.064237	0.10656	0.209174	0.277682	0.365149	0.742312
ME		0.073819	0.120454	0.228635	0.344005	0.506115	0.787165

SA66	t0=26						
ML		0.077873	0.112654	0.208371	0.285504	0.395937	0.785124
TR		0.077312	0.104475	0.195526	0.272838	0.367076	0.751409
ME		0.085219	0.123454	0.231635	0.348005	0.509115	0.790165
SA67	t0=74						
ML		0.049144	0.086881	0.20383	0.326123	0.618982	0.970412
TR		0.040839	0.084217	0.183693	0.212682	0.590012	0.98808
ME		0.084818	0.138399	0.285992	0.308792	0.56463	0.904426

Table 02P_SEM MR2: Probability of occurrence $\hat{P}_{\Delta t|\, t_0}$ for different Δt.

The method ME, based on the maximum entropy principle, suffers for the truncation of the modelling on the last value present in the catalogue; therefore, the method is not able to give a prediction for values of Δt out of the range of estimation of the method.

Macro Region 3 (MR3)

MR3_ interoccurrence times in years							
0.247982	0.286035	0.501886	0.515518	3.714619	6.824474	12.61874	13.72396
18.150685	19.08252	21.23771	24.29092	25.32855	26.28320	27.48978	28.17743
29.07295	32.18060	34.87820	40.30768	45.14968	51.01707	57.10596	61.22063
69.90959	88.22727	92.85134	112.2265	114.6950	139.3373	226.8711	343.7824
μ=55.22835 p = 0.3125 λ_∞ = 0.047715 $\tilde{\lambda}_\infty$ = 2.6352							
MR3_ dimensionless interoccurrence times							
0.004490	0.005179	0.009087	0.009334	0.067259	0.123568	0.228483	0.248495
0.328648	0.345520	0.384544	0.439827	0.458615	0.475900	0.497748	0.510199
0.526414	0.582683	0.631527	0.729837	0.817509	0.923748	1.033997	1.108500
1.265828	1.597500	1.681226	2.032045	2.076741	2.522931	4.107873	6.224745

Table 04SEM. MR3: interoccurrence time dataset using the interoccurrence time datasets of the SA composing MR3

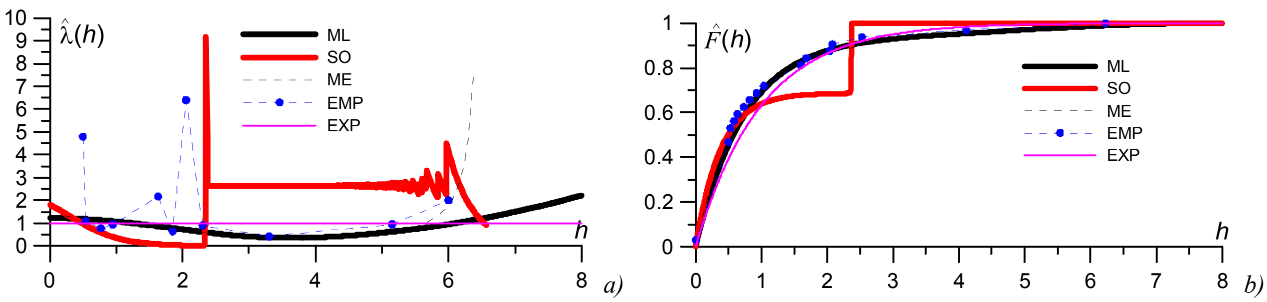


Figure 05SEM MR3: a) estimation of $\hat{\lambda}(h)$ in dimensionless form; b) estimation of $\hat{F}(h)$ in dimensionless form. ML= method of maximum likelihood; TR = threshold method; ME = maximum entropy method; EMP = empirical method; EXP = exponential model.

		Δt					
MR3		**5 years**	**10 years**	**20 years**	**30 years**	**50 years**	**100 years**
SA01	t0=39						
ML		0.097136	0.155752	0.334919	0.441766	0.606052	0.790995
TR		0.060704	0.09108	0.170151	0.224741	0.325826	0.690268
ME		0.101924	0.101924	0.164919	0.307479	0.412297	0.564775
SA08	t0=85						
ML		0.068794	0.145291	0.260374	0.351829	0.489361	0.657236
TR		0.033601	0.068969	0.166828	0.267182	0.492241	0.925766
ME		0.059416	0.12626	0.228966	0.313581	0.446366	0.640212
SA09	t0=31						
ML		0.10992	0.18795	0.336037	0.452739	0.617338	0.809933
TR		0.078931	0.129461	0.214149	0.273656	0.369558	0.663746
ME		0.108195	0.18366	0.324212	0.432985	0.585567	0.776203
SA11	t0=314						
ML		0.0807	0.143167	0.269105	0.397791	0.608181	0.901755
TR		Indeterminate	Indeterminate	Indeterminate	Indeterminate	Indeterminate	Indeterminate
ME		0.108338	0.197557	0.394052	0.629315	out range	out range
SA12	t0=35						
ML		0.108889	0.186193	0.332904	0.442808	0.61168	0.803073
TR		0.071571	0.117655	0.196069	0.253169	0.352314	0.671131
ME		0.105843	0.179818	0.317926	0.419943	0.576562	0.767946

SA18	t0=174						
ML		0.038135	0.066846	0.123666	0.182167	0.2903	0.531044
TR		0.244295	0.444243	0.679979	0.84388	0.9764	1
ME		0.043343	0.076289	0.141507	0.207804	0.325277	0.558601
SA20	t0=59						
ML		0.096164	0.169474	0.309698	0.403487	0.561684	0.740425
TR		0.065783	0.030664	0.125568	0.187595	0.345312	0.792662
ME		0.151949	0.151949	0.277931	0.363193	0.512205	0.704592
SA27	t0=4						
ML		0.105332	0.198818	0.363088	0.478811	0.656339	0.852444
TR		0.14839	0.241956	0.392762	0.487295	0.591478	0.723327
ME		0.115908	0.215478	0.382951	0.49534	0.659692	0.837468
SA30	t0=86						
ML		0.076278	0.144136	0.258339	0.349129	0.482961	0.654875
TR		0.034266	0.079779	0.170561	0.272975	0.51385	0.934332
ME		0.065894	0.125216	0.227317	0.311422	0.44375	0.639332
SA31	t0=30						
ML		0.110247	0.188512	0.329682	0.454086	0.626229	0.811018
TR		0.081529	0.1336	0.220552	0.281018	0.376094	0.654307
ME		0.108956	0.184954	0.31937	0.435564	0.595158	0.777673
SA32	t0=72						
ML		0.066097	0.158425	0.289704	0.38262	0.526976	0.701678
TR		0.028749	0.064834	0.133591	0.211139	0.408489	0.864575
ME		0.05778	0.13883	0.255654	0.340407	0.478975	0.672883
SA39	t0=127						
ML		0.058057	0.099848	0.185191	0.248238	0.35562	0.540899
TR		0.107143	0.193066	0.381088	0.55247	0.820965	0.995826
ME		0.054645	0.095011	0.180328	0.246254	0.364446	0.568923
SA44	t0=216						
ML		0.037089	0.066417	0.128039	0.196347	0.332166	0.635779
TR		0.417526	0.628866	0.860825	0.963918	1	1
ME		0.041511	0.073485	0.139304	0.209611	0.34368	0.663052
SA46	t0=184						
ML		0.036477	0.060891	0.120398	0.179568	0.292278	0.551265
TR		0.285114	0.465236	0.742417	0.888941	1	1
ME		0.042245	0.070604	0.138722	0.204941	0.324222	0.573454
SA47	t0=73						
ML		0.083336	0.15739	0.287834	0.380181	0.526787	0.696951
TR		0.032685	0.065433	0.135485	0.214743	0.415692	0.877162
ME		0.072746	0.137734	0.25379	0.338106	0.479253	0.670184
SA49	t0=6						
ML		0.10497	0.198134	0.354213	0.477168	0.657778	0.850691
TR		0.117069	0.232688	0.377818	0.46904	0.731742	0.708144
ME		0.114446	0.212894	0.371241	0.490386	0.657513	0.832779
SA51	t0=93						
ML		0.080047	0.137052	0.251217	0.328324	0.461556	0.633179
TR		0.049414	0.102624	0.19574	0.324121	0.966505	0.951065
ME		0.069352	0.119479	0.222445	0.294863	0.428614	0.625052

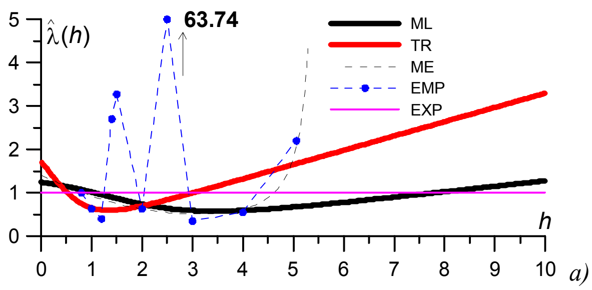
Table 03P_SEM MR3: Probability of occurrence $\hat{P}_{\Delta t|t_0}$ for different Δt

In the TR method “Indetermimate” means that probability of occurrence reach an indeterminate value [(1.-1.)/0.]

Macro Region 4 (MR4)

MR4_ interoccurrence times in years							
0.051265	1.403851	1.841407	2.175973	3.153834	4.277055	4.631491	5.863488
6.118125	6.824168	10.87455	10.88431	12.26855	13.88837	15.13162	16.39599
22.08228	22.26318	22.32155	24.58904	24.67123	29.21877	31.78788	40.20788
41.59272	42.28641	43.96712	46.34520	60.64055	77.54435	122.9986	159.6148
$\mu = 28.99736$	$p = 0.3438$	$\lambda_\infty = 0.090629$	$\tilde{\lambda}_\infty = 2.6280$				
MR4_ dimensionless interoccurrence times							
0.001768	0.048413	0.063503	0.075040	0.108763	0.147498	0.159721	0.202208
0.210989	0.235338	0.375017	0.375355	0.423092	0.478953	0.521827	0.565431
0.761527	0.767766	0.769779	0.847975	0.850810	1.007636	1.096234	1.386605
1.434362	1.458285	1.516246	1.598256	2.091244	2.674186	4.241717	5.504459

Table 05SEM. MR4: interoccurrence time dataset using the interoccurrence time datasets of the SA composing MR4



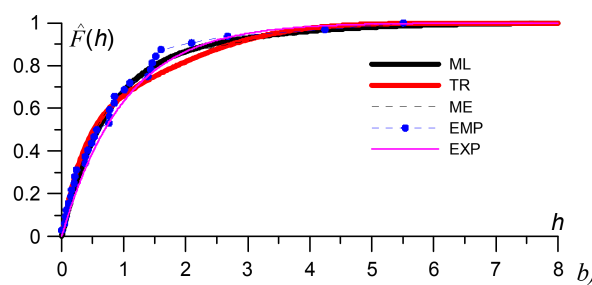


Figure 06SEM MR4: a) estimation of $\hat{\lambda}(h)$ in dimensionless form; b) estimation of $\hat{F}(h)$ in dimensionless form. ML= method of maximum likelihood; TR = threshold method; ME = maximum entropy method; EMP = empirical method; EXP = exponential model

		Δt					
MR4		**5 years**	**10 years**	**20 years**	**30 years**	**50 years**	**100 years**
SA25	t0=18						
ML		0.161054	0.30586	0.502362	0.634005	0.780491	0.921256
TR		0.123765	0.228959	0.375726	0.485153	0.687985	0.949652
ME		0.164582	0.293156	0.48037	0.598669	0.740055	0.898372
SA26	t0=82						
ML		0.096065	0.181464	0.330998	0.447294	0.637769	0.895739
TR		0.142704	0.265269	0.509026	0.674833	0.876424	0.994197
ME		0.083795	0.1601	0.299794	0.417345	0.648214	out range
SA28	t0=5						
ML		0.174731	0.330941	0.550055	0.674371	0.822008	0.939842
TR		0.196914	0.345369	0.517931	0.608422	0.745664	0.947441
ME		0.185425	0.343338	0.553228	0.667558	0.803032	0.923146
SA37	t0=83						
ML		0.095742	0.185693	0.330517	0.447042	0.638091	0.894702
TR		0.161459	0.318902	0.513278	0.679089	0.879156	0.994421
ME		0.083563	0.16426	0.300187	0.418587	0.652739	out range
SA40	t0=240						
ML		0.166269	0.30708	0.53222	0.68576	0.865553	0.988067
TR		Indeterminate	Indeterminate	Indeterminate	Indeterminate	Indeterminate	Indeterminate
ME		out range	out range	out range	out range	out range	out range

SA41	t0=1						
ML		0.187709	0.336656	0.554107	0.687639	0.831979	0.94504
TR		0.223093	0.402558	0.571086	0.658236	0.777032	0.950495
ME		0.204466	0.357768	0.567935	0.690089	0.819918	0.93129
SA56	t0=18						
ML		0.161054	0.30586	0.502362	0.634005	0.780491	0.921256
TR		0.123765	0.228959	0.375726	0.485153	0.687985	0.949652
ME		0.164582	0.293156	0.48037	0.598669	0.740055	0.898372

Table 04P_SEM MR4: Probability of occurrence $\hat{P}_{\Delta t | t_0}$ for different Δt

Macro Region 5 (MR5)

MR5_ interoccurrence times in years							
3.6736149	14.00548	34.78904	55.02297	61.19725	143.7406	194.7283	226.8948
314.05750							
$\mu = 116.45661$ $\quad p = 0.4444$ $\quad \lambda_\infty = 0.0296402$ $\quad \tilde{\lambda}_\infty = 3.4518$							
MR5_ dimensionless interoccurrence times							
0.0315449	0.120263	0.298730	0.472476	0.525494	1.234284	1.672110	1.948321
2.6967770							

Table 06SEM. MR5: interoccurrence time dataset using the interoccurrence time datasets of the SA composing MR5

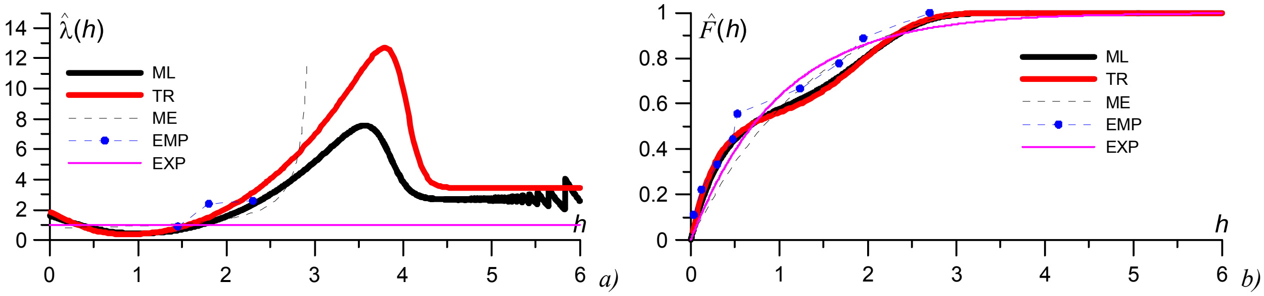


Figure 07SEM MR5: a) estimation of $\hat{\lambda}(h)$ in dimensionless form; b) estimation of $\hat{F}(h)$ in dimensionless form. ML= method of maximum likelihood; TR = threshold method; ME = maximum entropy method; EMP = empirical method; EXP = exponential model.

				Δt			
MR5		**5 years**	**10 years**	**20 years**	**30 years**	**50 years**	**100 years**
SA03	t0=0.0						
ML		0.060341	0.114519	0.226996	0.297595	0.395845	0.543678
TR		0.069412	0.206314	0.303349	0.383509	0.459184	0.592876
ME		0.032007	0.06311	0.137078	0.192546	0.302233	0.521141
SA04	t0=271						
ML		0.095712	0.208365	0.371348	0.528115	0.758347	0.971487
TR		0.103329	0.24665	0.415201	0.557475	0.786432	0.979585
ME		0.07417	0.163838	0.300861	0.445756	0.709225	Out of range
SA05	t0=12						
ML		0.054256	0.114364	0.195249	0.267839	0.34611	0.49762
TR		0.060546	0.113299	0.199351	0.278587	0.349103	0.468856
ME		0.032292	0.071381	0.131072	0.194267	0.303072	0.525802
SA58	t0=54						
ML		0.031337	0.066361	0.114362	0.159076	0.20586	0.365483
TR		0.027057	0.050918	0.090885	0.13024	0.16475	0.310484
ME		0.033714	0.074523	0.136835	0.202811	0.306398	0.548922
SA59	t0=121						
ML		0.017272	0.034532	0.074004	0.115383	0.201969	0.477297
TR		0.014296	0.029031	0.068548	0.103677	0.196822	0.484641
ME		0.037444	0.073828	0.151998	0.225259	0.351431	0.609669

SA75	t0=52						
ML		0.031873	0.060701	0.116209	0.161503	0.228802	0.367283
TR		0.028412	0.053444	0.104218	0.135955	0.191275	0.313431
ME		0.03366	0.066375	0.136645	0.202526	0.315961	0.548164
SA79	t0=69						
ML		0.025185	0.059011	0.093582	0.13225	0.194937	0.354401
TR		0.024778	0.058107	0.092249	0.130573	0.193151	0.354511
ME		0.034355	0.084092	0.13947	0.20672	0.322494	0.559486
SA84	t0=72						
ML		0.023999	0.046021	0.089703	0.127398	0.18985	0.355141
TR		0.018604	0.03543	0.071808	0.097186	0.150983	0.313202
ME		0.034526	0.068076	0.140142	0.207696	0.324019	0.562134
SA89	t0=151						
ML		0.021787	0.050044	0.09788	0.155801	0.276753	0.61443
TR		0.021129	0.043366	0.103919	0.157489	0.292896	0.639001
ME		0.040117	0.089627	0.162857	0.24134	0.376528	0.653209

Table 05P_SEM MR5: Probability of occurrence $\hat{\mathrm{P}}_{\Delta t|\, t_0}$ for different Δt

Macro Region 6 (MR6)

MR6_ interoccurrence times in years							
4.688642	5.881749	13.76984	27.18319	30.71288	56.73634	72.65607	100.3346
122.9360	122.937	161.8029	215.7441				
$\mu = 77.94861$ $p = 0.4167$ $\lambda_\infty = 0.033077$ $\tilde{\lambda}_\infty = 2.5783$							
MR6_ interoccurrence times in dimensionless form							
0.060150	0.075457	0.176653	0.348732	0.394014	0.727869	0.932102	1.287189
1.577142	1.577154	2.075764	2.767774				

Table 07SEM. MR6: interoccurrence time dataset using the interoccurrence time datasets of the SA composing MR6

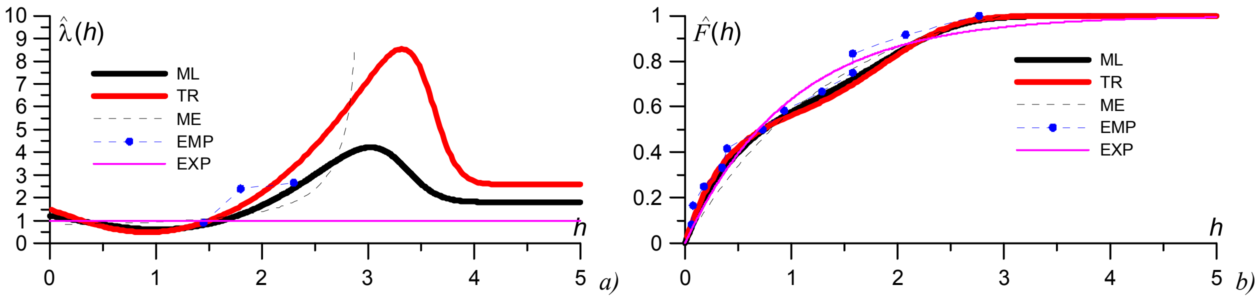


Figure 08SEM MR6: a) estimation of $\hat{\lambda}(h)$ in dimensionless form; b) estimation of $\hat{F}(h)$ in dimensionless form. ML= method of maximum likelihood; TR = threshold method; ME = maximum entropy method; EMP = empirical method; EXP = exponential model

		Δt					
MR6		**5 years**	**10 years**	**20 years**	**30 years**	**50 years**	**100 years**
SA24	t0=40						
ML		0.039451	0.096306	0.175295	0.237261	0.351327	0.627267
TR		0.04145	0.088838	0.147305	0.195926	0.281847	0.580433
ME		0.042291	0.106876	0.204251	0.286431	0.44201	0.721479
SA34	t0=22						
ML		0.056713	0.107703	0.207961	0.277258	0.392574	0.612508
TR		0.058281	0.108403	0.200431	0.259055	0.350627	0.559674
ME		0.049064	0.096063	0.198182	0.277916	0.428859	0.700014
SA38	t0=4						
ML		0.06687	0.136225	0.243931	0.323475	0.445231	0.636549
TR		0.080004	0.148542	0.272828	0.349374	0.456243	0.607427
ME		0.047855	0.101151	0.193301	0.271068	0.413136	0.682776
SA63	t0=92						
ML		0.038474	0.083398	0.16832	0.249702	0.435952	0.845912
TR		0.032162	0.078694	0.150611	0.231663	0.429369	0.867464
ME		0.057749	0.122077	0.233283	0.327149	0.504588	0.823986

Table 06P_SEM MR5: Probability of occurrence $\hat{P}_{\Delta t|t_0}$ for different Δt

Macro Region 7 (MR7)

MR7_ interoccurrence times in years							
0.005613	0.075942	0.507056	1.745634	4.129079	7.242543	8.678101	12.93454
14.55944	15.41725	16.64099	18.33644	25.56771	28.68665	38.34579	39.16575
41.67178	51.84873	54.01703	65.54449	68.24903	81.13173	84.08513	84.10157
125.8909	129.2970	136.4006	156.9056	186.5086			
$\mu = 51.64451$ $\quad p = 0.4138$ $\quad \lambda_\infty = 0.062109$ $\quad \tilde{\lambda}_\infty = 3.2076$							
MR7_ dimensionless interoccurrence times							
0.000109	0.001470	0.009818	0.033801	0.079952	0.140238	0.168035	0.250453
0.281916	0.298526	0.322221	0.355050	0.495070	0.555463	0.742493	0.758370
0.806895	1.003952	1.045937	1.269145	1.321513	1.570962	1.628149	1.628467
2.437638	2.503591	2.641139	3.038179	3.611386			

Table 08SEM. MR7: interoccurrence time dataset using the interoccurrence time datasets of the SA composing MR7

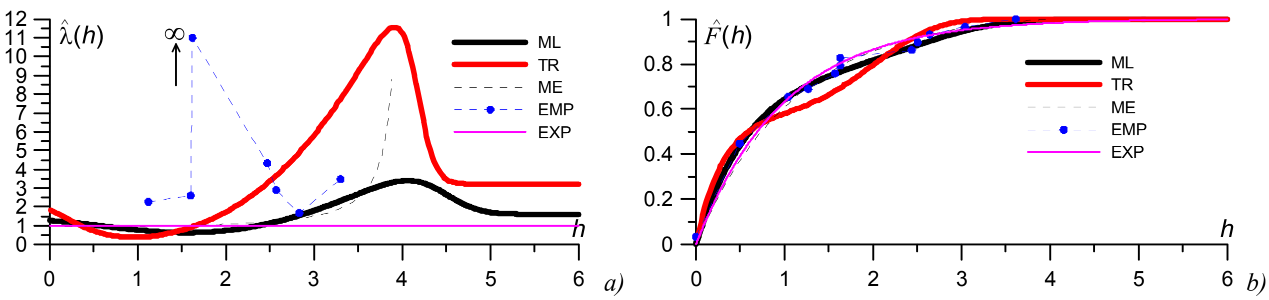


Figure 09SEM MR7: a) estimation of $\hat{\lambda}(h)$ in dimensionless form; b) estimation of $\hat{F}(h)$ in dimensionless form. ML= method of maximum likelihood; TR = threshold method; ME = maximum entropy method; EMP = empirical method; EXP = exponential model

		Δt					
MR7		**5 years**	**10 years**	**20 years**	**30 years**	**50 years**	**100 years**
SA15	t0=89						
ML		0.062719	0.117985	0.239156	0.353902	0.583767	0.946568
TR		0.095297	0.175153	0.380517	0.576199	0.851242	0.99947
ME		0.098628	0.180138	0.339336	0.466234	0.667783	0.947126
SA16	t0=27						
ML		0.094701	0.168263	0.299976	0.394544	0.53231	0.777253
TR		0.06061	0.098543	0.169038	0.225689	0.358858	0.843734
ME		0.089869	0.164127	0.309138	0.424744	0.608344	0.862843
SA19	t0=19						
ML		0.102141	0.181303	0.322229	0.422	0.564376	0.779014
TR		0.082041	0.13195	0.217741	0.274793	0.376999	0.804316
ME		0.08934	0.16317	0.30736	0.422307	0.608728	0.857882
SA53	t0=74						
ML		0.06064	0.117368	0.213178	0.315075	0.511384	0.902229
TR		0.05872	0.123895	0.271931	0.41866	0.715728	0.995848
ME		0.095403	0.182574	0.321133	0.450913	0.645817	0.91599
SA55	t0=95						
ML		0.137188	0.207059	0.435638	0.638186	0.891599	0.99976
TR		0.137188	0.207059	0.435638	0.638186	0.891599	0.99999
ME		0.119388	0.174367	0.345175	0.47423	0.679201	0.96367

SA68	t0=1						
ML		0.106398	0.208072	0.361972	0.481129	0.630591	0.80337
TR		0.250298	0.270214	0.40242	0.484241	0.564588	0.7697
ME		0.079909	0.161435	0.297547	0.41781	0.598406	0.845481
SA80	t0=174						
ML		0.223999	0.391998	0.653394	0.818779	0.950111	0.99312
TR		0.5	0.726027	0.958904	0.993151	1	1
ME		0.198792	0.363214	0.69856	0.939619	Out of range	Out of range

Table 07P_SEM MR7: Probability of occurrence $\hat{\mathrm{P}}_{\Delta t | t_0}$ for different Δt

Macro Region 8 (MR8)

MR8_ interoccurrence times in years							
0.471917	2.732234	9.954695	22.27298	39.89495	47.83341	51.66277	68.88320
69.03186	83.81932	96.21093	113.0211	125.1101	140.4369	193.4059	
$\mu = 70.98282$	$p = 0.400$	$\lambda_\infty = 0.028777$	$\tilde{\lambda}_\infty = 2.0427$				
MR8_ dimensionless interoccurrence times							
0.006648	0.038491	0.140241	0.313780	0.562037	0.673873	0.727821	0.970421
0.972515	1.180839	1.355411	1.592232	1.762541	1.978464	2.724686	

Table 09SEM. MR8 interoccurrence time dataset using the interoccurrence time datasets of the SA composing MR8

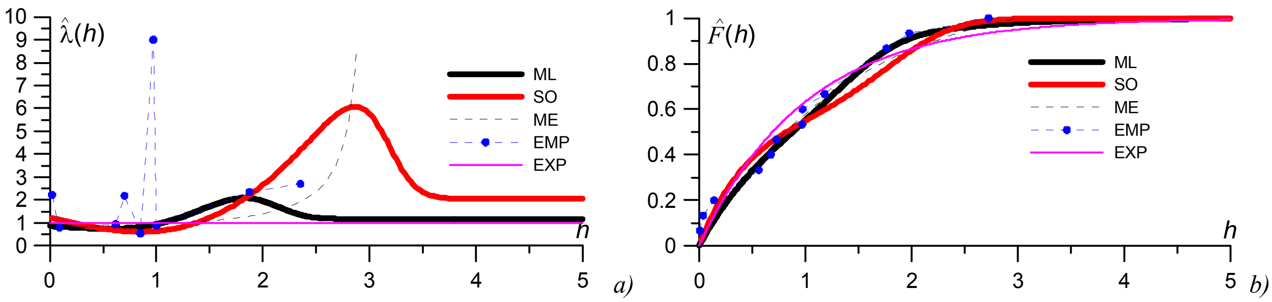


Figure 10SEM MR8: a) estimation of $\hat{\lambda}(h)$ in dimensionless form; b) estimation of $\hat{F}(h)$ in dimensionless form. ML= method of maximum likelihood; TR = threshold method; ME = maximum entropy method; EMP = empirical method; EXP = exponential model

		Δt					
MR8		**5 years**	**10 years**	**20 years**	**30 years**	**50 years**	**100 years**
SA14	t0=0.0						
ML		0.065625	0.118268	0.205584	0.286884	0.422351	0.727319
TR		0.068977	0.144806	0.241345	0.33898	0.454495	0.663497
ME		0.058686	0.115343	0.205756	0.294148	0.446441	0.719792
SA17	t0=88						
ML		0.087958	0.175244	0.34338	0.494549	0.715496	0.902061
TR		0.058539	0.104239	0.200548	0.357906	0.584102	0.961275
ME		0.068271	0.133185	0.253634	0.362604	0.550334	0.887284
SA21	t0=34						
ML		0.043959	0.106281	0.197182	0.254345	0.453463	0.839808
TR		0.043056	0.093954	0.169548	0.245305	0.352983	0.720654
ME		0.050356	0.12193	0.224505	0.310961	0.471959	0.760941
SA35	t0=184						
ML		0.090198	0.151977	0.279681	0.387946	0.558072	0.80648
TR		0.303762	0.477009	0.712959	0.872271	0.942378	0.992104
ME		0.218819	0.375055	0.714159	fuori range	fuori range	fuori range
SA42	t0=24						
ML		0.044983	0.107756	0.196574	0.272604	0.429294	0.807424
TR		0.069899	0.118394	0.2004	0.25935	0.377512	0.682628
ME		0.049424	0.119688	0.220363	0.30523	0.46326	0.746909

Table 08P_SEM MR8: Probability of occurrence $\hat{P}_{\Delta t|t_0}$ for different Δt

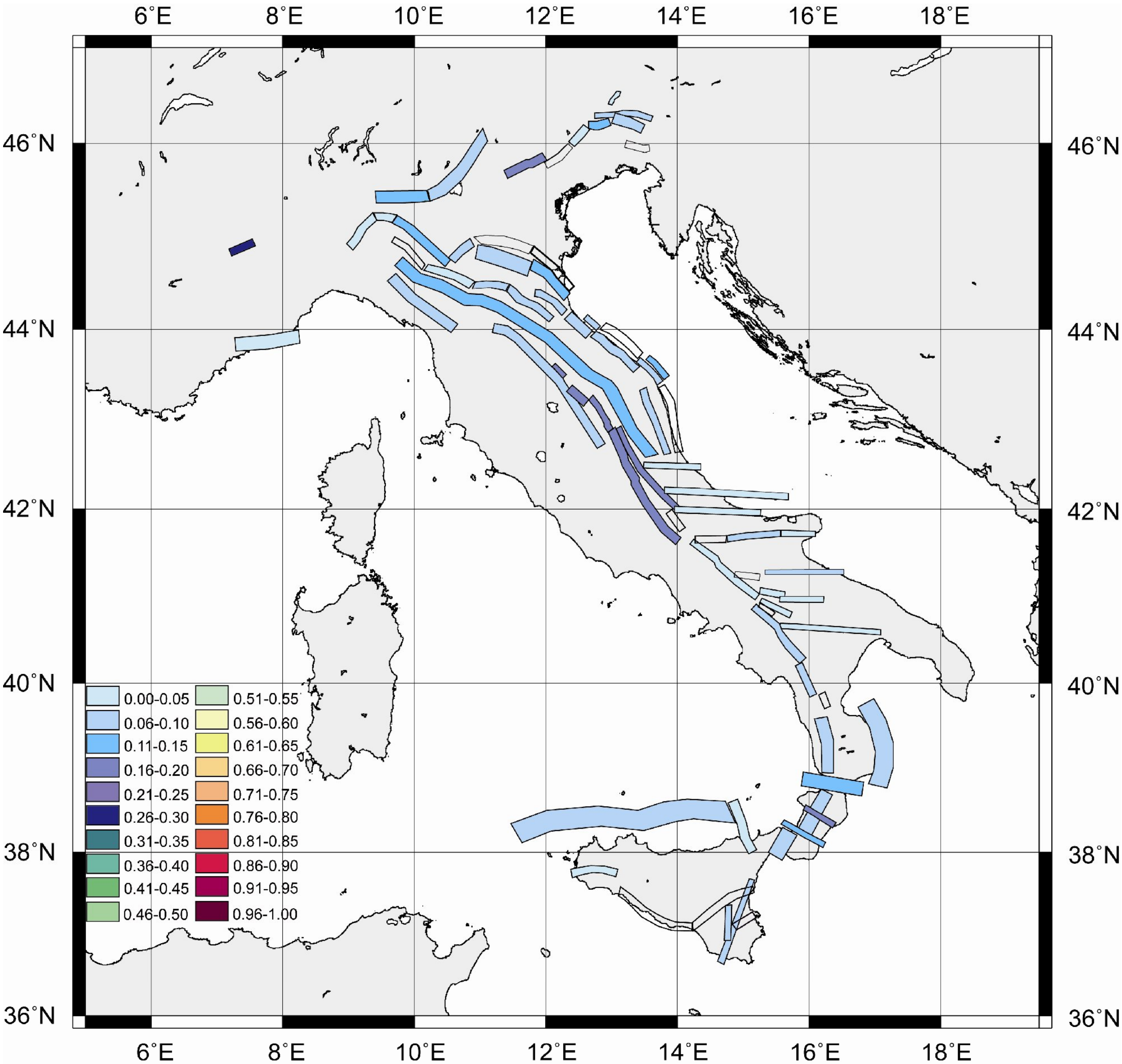


Figure 11SEM Occurrence probability for Δ*t=5years, Method ML*

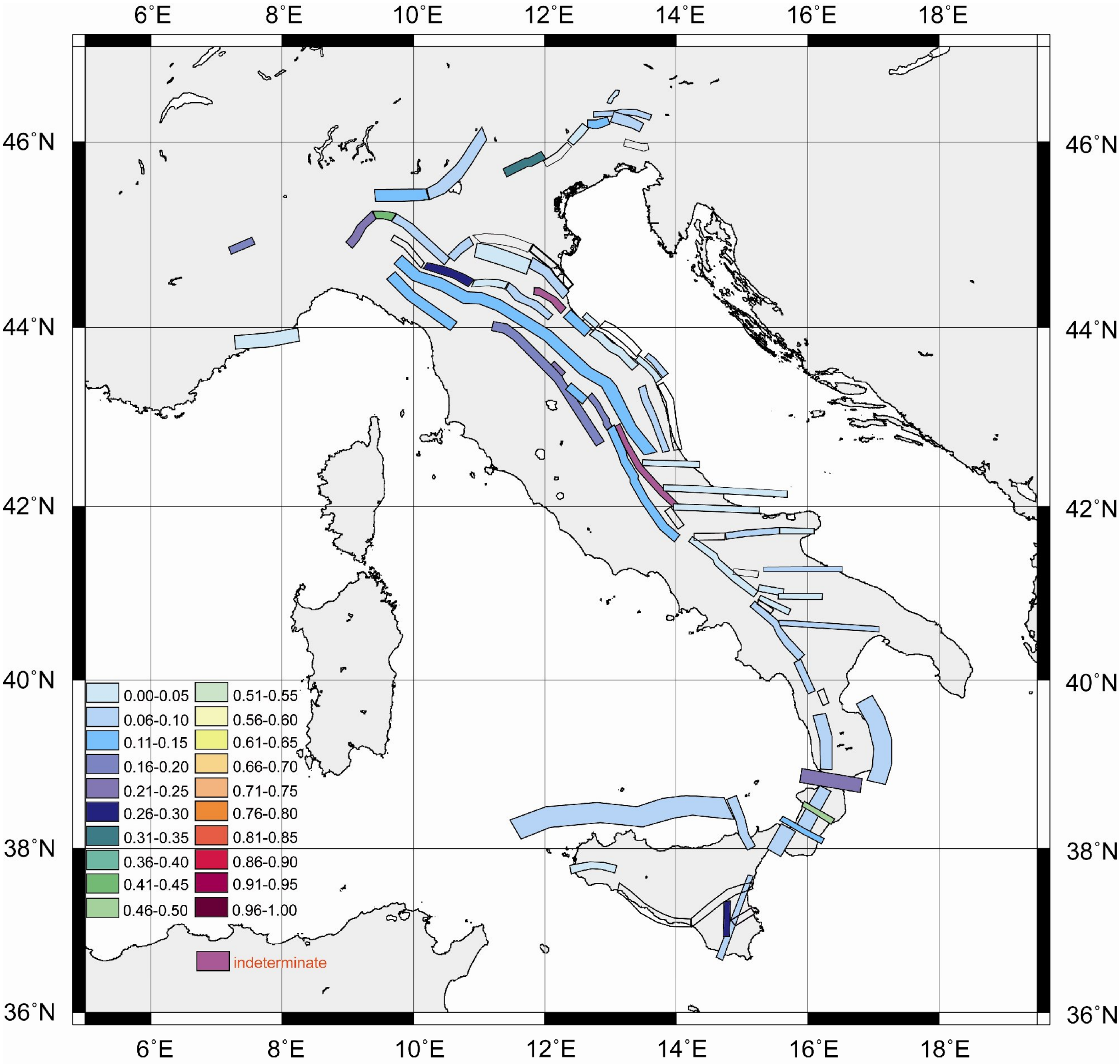


Figure 12SEM Occurrence probability for Δ*t=5years, Method TR*

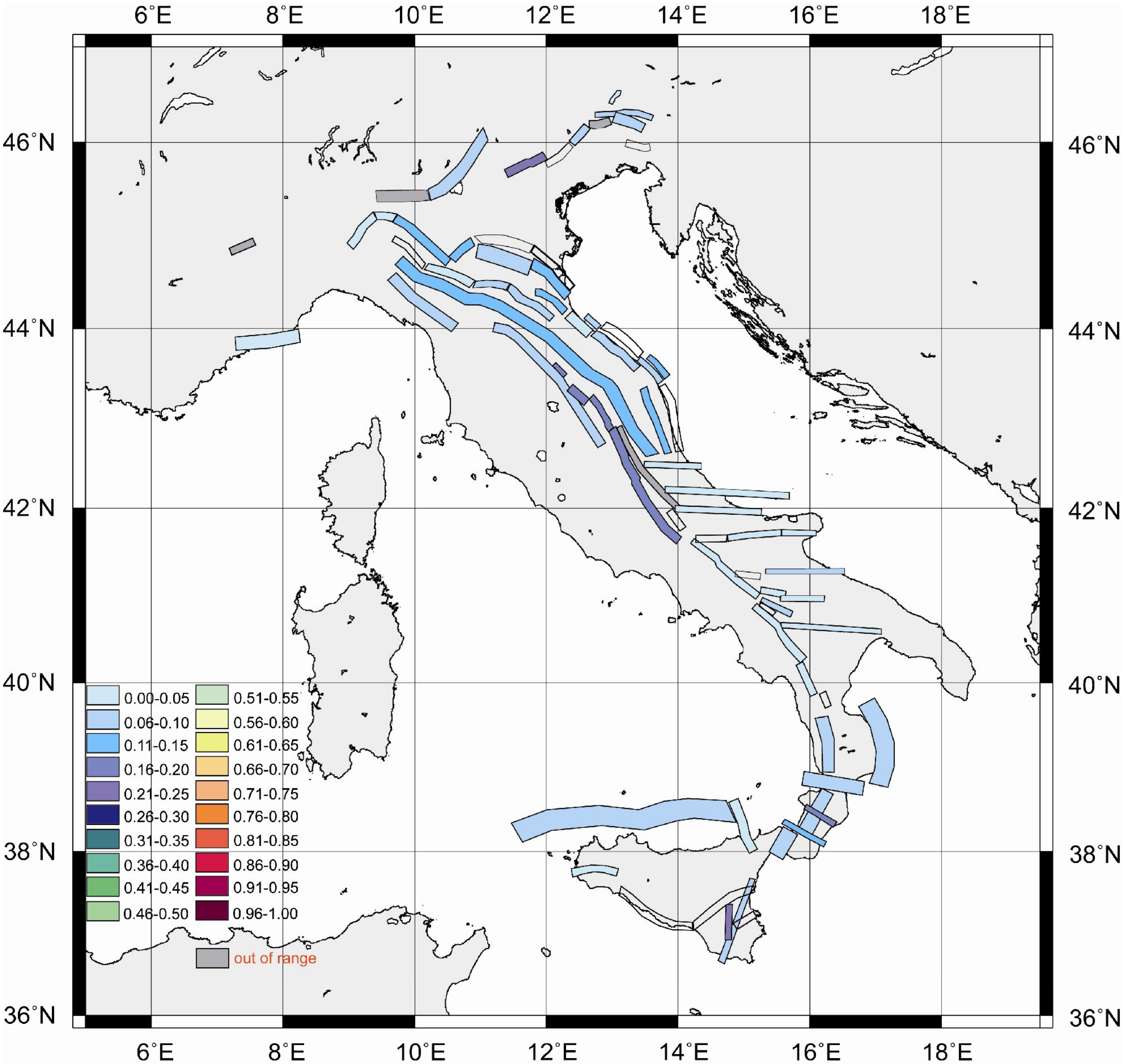


Figure 13SEM Occurrence probability for Δ*t=5years, Method ME*

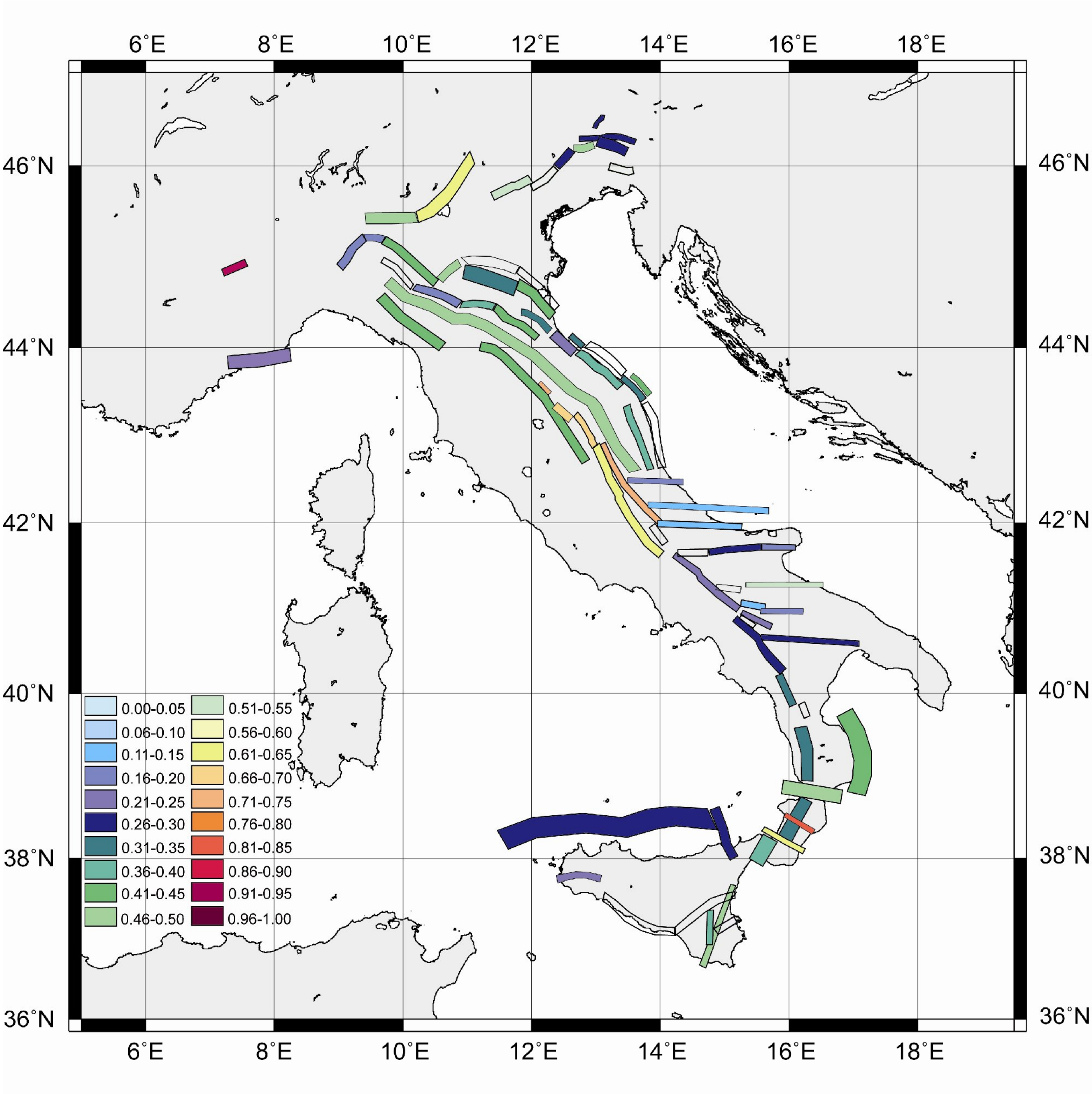


Figure 14SEM Occurrence probability for Δ*t=30years, Method ML*

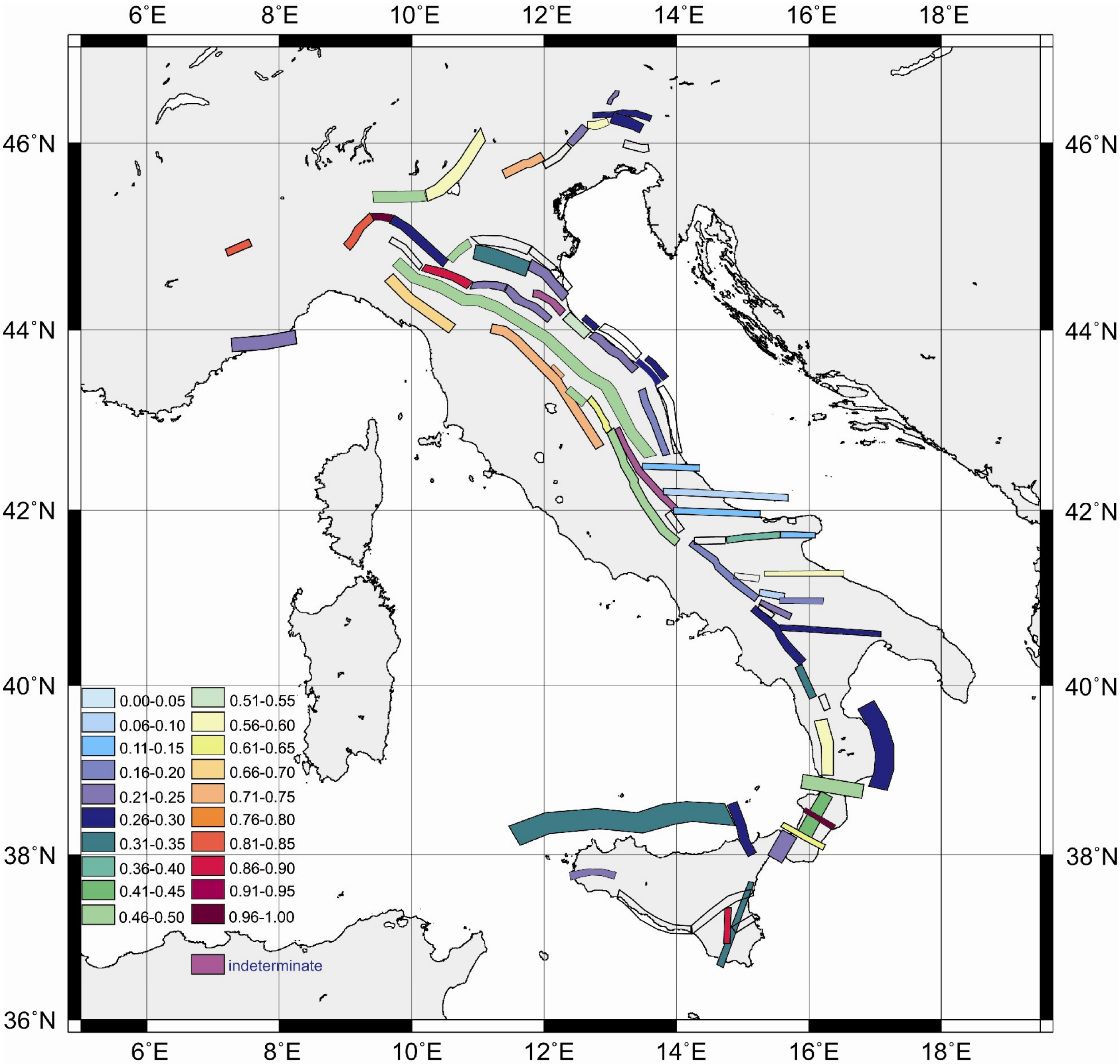


Figure 15SEM Occurrence probability for Δ*t=30years, Method TR*

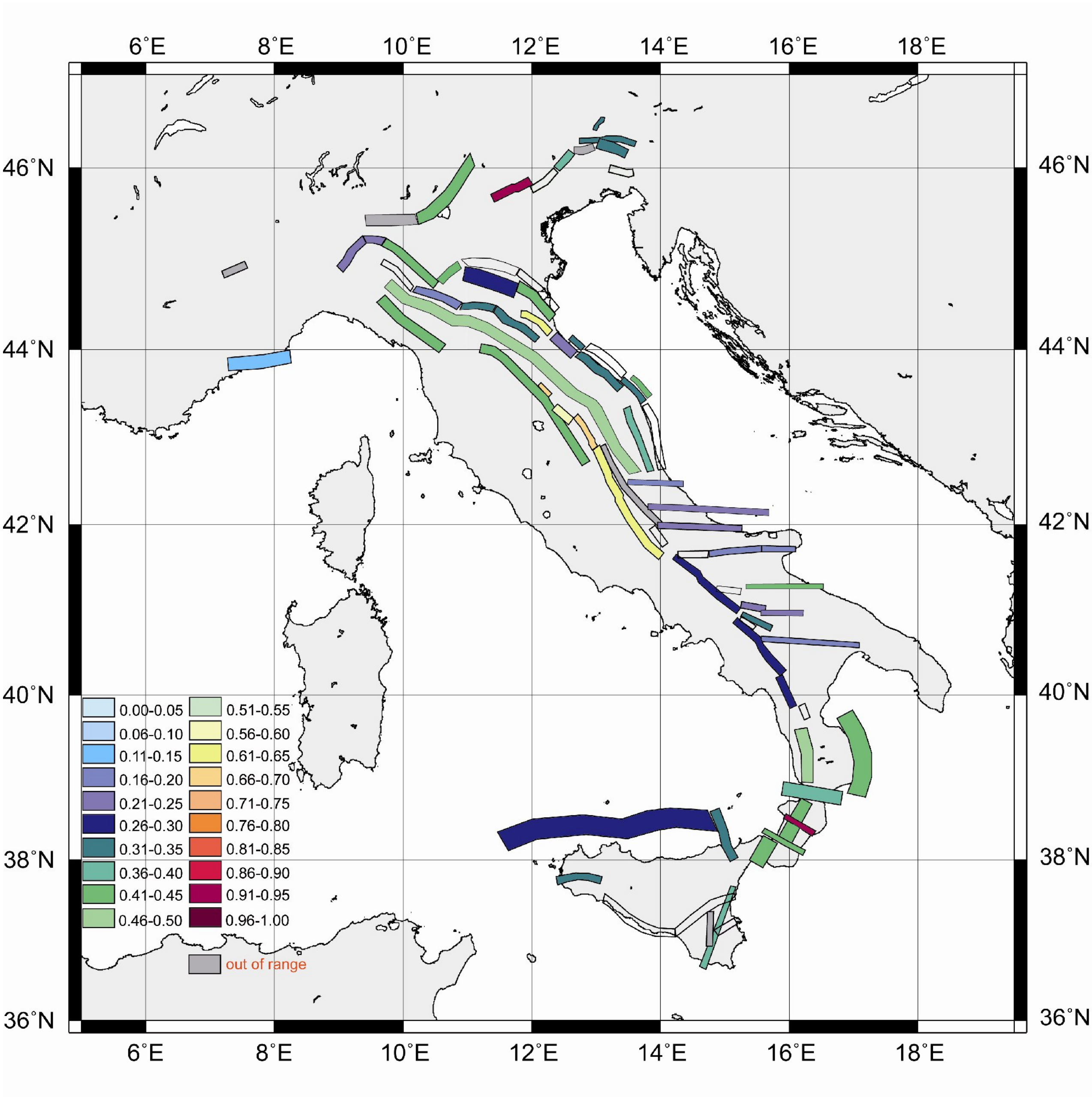


Figure 16SEM Occurrence probability for Δ*t=30years, Method ME*

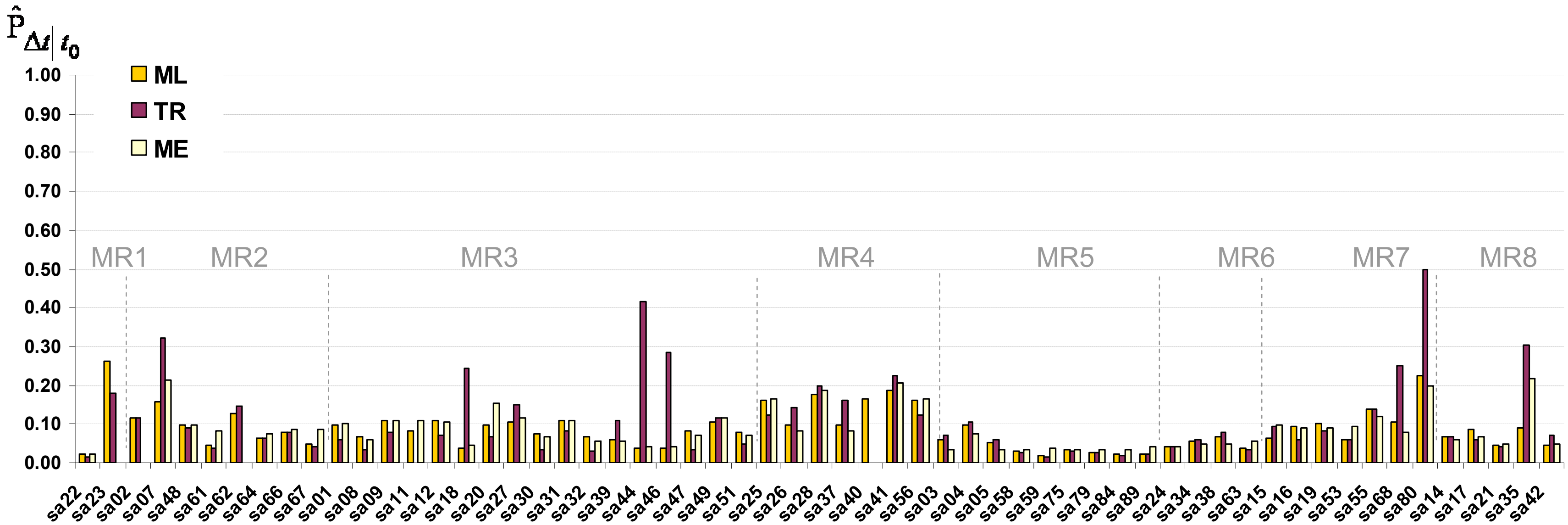


Figure 17SEM Italian Seismogenetic Source Areas (SA): occurrence probability for Δt=5 years, t_0, different for each SA, is the actual elapsed time in the area. Comparison between the ML, TR and ME methods. The vertical dotted lines define the eight MR.

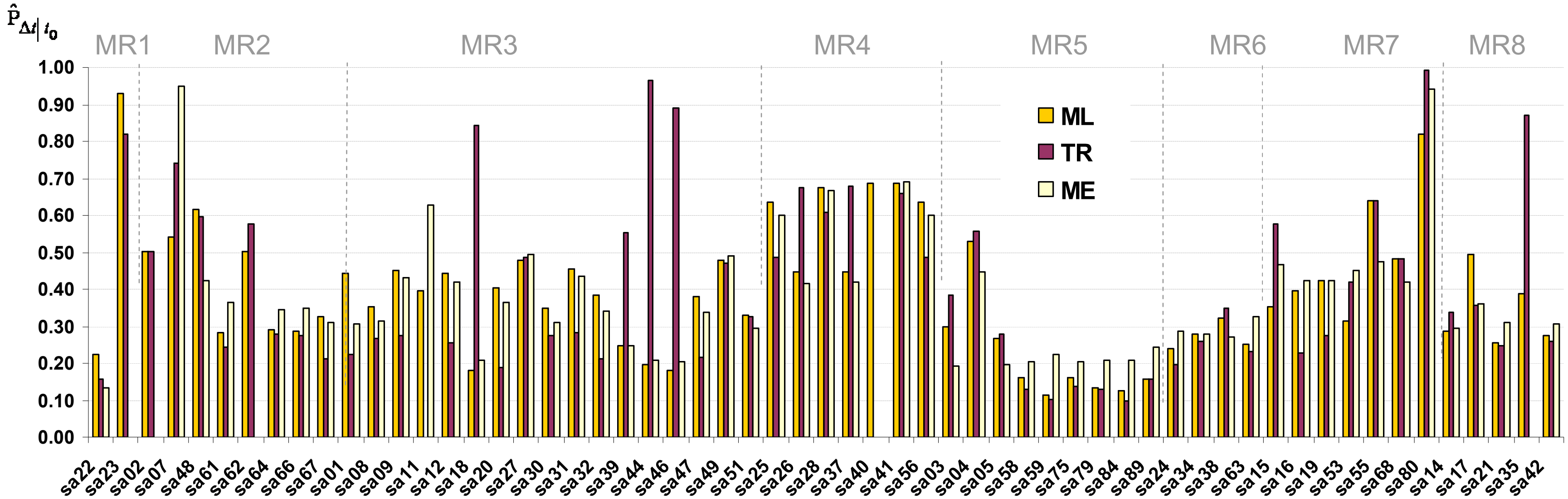


Figure 18SEM Italian Seismogenetic Source Areas (SA): occurrence probability for Δt=30 years, t_0, different for each SA, is the actual elapsed time in the area. Comparison between the ML, TR and ME methods. The vertical dotted lines define the eight MR.