About earthquake forecasting by Markov renewal processes

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Abstract

We propose and validate a new method for the evaluation of seismic hazard. In particular, our aim is to model large earthquakes consistently with the underlying geophysics. Therefore we propose a non-Poisson model, which takes into account occurrence history, improved with some physical constraints. Among the prevalent non-Poisson models, we chose the Markov renewal process, which is expected to be sufficient to capture the main characteristics, maintaining simplicity in analysis. However, due to the introduction of some physical constraint, our process differs significantly from others already presented in literature. A mixture of exponential + Weibull distributions is proposed for the waiting times and their parameters are estimated following the likelihood method. We validated our model, using data of earthquakes of high severity occurred in Turkey during the 20th century. Our results exhibit a good accordance with the real events.

Key Words: *Earthquake prediction, semi Markov processes, renewal processes, mixture distributions.*

AMS 2000 subject classification: 60K20

1. Introduction

During the last decades most countries have developed individual programs of seismic hazard assessment (SHA), aimed at establishing or updating the national seismic codes. Various approaches have been presented depending on different seismic culture and on local culture. However the models available in literature can be broken into a few broad categories. Essentially they can be divided into Poisson and non-Poisson models. Poisson models are widely used in engineering seismic hazard analysis, but they may not be appropriate for large earthquakes in the fault-specific case, because any Poisson process is inherently memory less. Here we propose to model large earthquakes in a way which is more consistent with the underlying physics; therefore we propose a non-Poisson recurrence model which takes into account occurrence history. For this reason, among prevalent non-Poisson models, we chose the Markov renewal processes which keep track of the last event occurring prior or at time t (in particular we refer to Guagenti Grandori et al., 1988, Guagenti Grandori et al., 1990). This kind of renewal process is able to emulate a broad range of temporary behavior of large earthquakes (e.g. Wu et al., 1995), in spite of its lack of complexity. Indeed it is expected to be sufficient to capture the main characteristics which are missing in traditional Poisson models, maintaining simplicity in analysis. Our proposed model is a renewal process where the probability of next event will be conditioned to the last event occurred but, as better explained in the next section, it is not a classical slip predictable model neither a time predictable model (Shimazaki et al., 1980). This means that our model can be considered into the family of the renewal process but it differs from others significantly. Indeed, taking into account the renewal approach introduced by Alvarez (Alvarez, 2005), we improved that model with some physical consideration and with the use of a mixture of two distributions.

In more details, this paper is organized as follows. In Section 2, we define general Markov renewal processes, together with some details. Section 3 specializes those results for Weibull distribution of inter-event times. In this section the state definition is approached; this step highlights that the earthquake sequence can be assumed belonging to a unique state and modeled with a mixture distribution. Then, in Section 4, we fit the proposed models to the Turkey earthquakes dataset and carry out a few tests. In Section 5 the crossing state prediction is evaluated for the two models proposed. Some final remarks are given in Section 6.

2. Markov Renewal Processes

Let us recall the definition and some properties of a Markov renewal process with finite state space.

In what follows $E = \{1, \dots, M\}$ with $M \in \mathbb{N}$; $\mathbf{F} = (F_{ij}(\cdot))_{ij \in E}$ will denote a matrix of distribution functions on $\Re_+$, $\mathbf{P} = (p_{ij})_{ij \in E}$ will denote a transition matrix on E and $\mathbf{a} = (a_1, \dots, a_M)$ a probability distribution on E (i.e. $a_i \geq 0$ and $\sum_{i=1}^{M} a_i = 1$).

Let us consider a two-dimensional stochastic process $(J_n, X_n)_{n \geq 0}$ defined on a complete probability space $(\Omega, \Im, \mathrm{P})$ satisfying (Limnios & Oprisan, 2001):

1. $X_0 = 0$ a.s.
2. $\Pr(J_0 = k) = a_k$ for every $k \in E$
3. $\Pr(J_n = k, X_n \leq x \mid J_0, J_1, X_1, \dots, J_{n-1}, X_{n-1}) = p_{J_{n-1},k} F_{J_{n-1},k}(x)$ a.s. for every $x \in (0, +\infty)$ and $k \in E$.

DEF. *The process* $(J_n, X_n)_{n \geq 0}$ *is called Markov renewal process determined by* (E, $\mathbf{a}$, $\mathbf{P}$, $\mathbf{F}$).

Let us recall some consequences of the definition:

i. $(J_n)_{n \geq 0}$ is a E-valued Markov chain with transition matrix $\mathbf{P}$ and initial distribution $\mathbf{a}$.

ii. for every n>1, X_1, ... , X_n are conditional independent, given $(J_n)_{n\geq 0}$ and

$$\Pr(X_1 \leq x_1, \ldots, X_n \leq x_n \mid J_n, n \geq 0) = \prod_{i=1}^{n} F_{J_{i-1},J_i}(x_i) \qquad \text{a.s.}$$

The Markov chain $(J_n)_{n\geq 0}$ represents the states successively visited and the process $(X_n)_{n\geq 0}$ represents the successive waiting times. In our application the states visited are earthquakes occurred, classified with respect to their severity.

The X_n's are the times between successive earthquakes. Hence, from (ii) one can deduce that, if an earthquake is classified of severity $i\in E$ and the successive earthquake is of severity $j\in E$, the time between the two earthquakes is a positive random variable with distribution F_{ij}.

We will assume that the distribution F_{ij} have a density f_{ij}, for every i,j=1, ..., M.

Let ($j_0, j_1, x_1, \ldots, x_{\tau-1}, j_\tau$) be a realization of the Markov renewal process on the time window [0,T]; τ represents the number of states visited in [0, T] and for the lost event J_τ we have the censored data x_τ> [T − (x_1 +...+ $x_{\tau-1}$)].

Then, the conditional likelihood, given J_0=j_0, is

$$L(j_0) = \left[\prod_{i=0}^{\tau-1} p_{j_i,j_{i+1}}\, f_{j_i,j_{i+1}}(x_i)\, 1_{\tau>0} + 1_{\tau=0}\right] \cdot \sum_{k=1}^{M} p_{j_\tau,k}\left[1 - F_{j_\tau,k}(x_\tau)\right] \tag{1}$$

At first, following Alvarez (2005), we assume the following Weibull density:

$$f_{ij}(x) = \alpha_{ij}\,\rho_{ij}\,(\rho_{ij}x)^{(\alpha_{ij}-1)} \exp\left[-(\rho_{ij}x)^{\alpha_{ij}}\right] \tag{2}$$

with α_{ij} and ρ_{ij} positive parameters. This choice will be discussed in details in Section 3, where also we argue that this assumption can be enlarged.

The maximum likelihood estimates for the parameters p_{ij}, α_{ij}, ρ_{ij} are obtained maximizing the log-likelihood over the window [0,T]. Assuming τ> 0 for the set of data available, the conditional log-likelihood is then given by

$$\begin{aligned}\ell(j_0) = \sum_{i=0}^{\tau-1} \ln(p_{J_i,J_{i+1}}) + \sum_{i=1}^{\tau-1}\left\{\ln\left[\alpha_{J_i,J_{i+1}}\rho_{J_i,J_{i+1}}(\rho_{J_i,J_{i+1}}\,x_i)^{(\alpha_{J_i,J_{i+1}}-1)}\right] - (\rho_{J_i,J_{i+1}}\,x_i)^{(\alpha_{J_i,J_{i+1}})}\right\} + \\ + \ln\left\{\sum_{k=1}^{M} p_{J_\tau,k}\exp\left[-\rho_{J_\tau,k}\,x_\tau\right]^{\alpha_{J_\tau,k}}\right\}\end{aligned} \tag{3}$$

In the second proposed approach, (2) becomes a mixture of two distributions:

$$f_{ij}(x) = \left\{q \cdot b_{ij}\exp[-b_{ij}x]\right\} + \left\{(1-q)\cdot \alpha_{ij}\,\rho_{ij}\,(\rho_{ij}x)^{(\alpha_{ij}-1)}\exp\left[-(\rho_{ij}x)^{\alpha_{ij}}\right]\right\}$$

with b_{ij}, α_{ij} and ρ_{ij} positive parameters respectively of Exponential and Weibull densities, q is the weight given to the first distribution and (1.−q) is its complementary. This choice will be discussed in detail in Section 4.

3. The distribution choice

The choice of distribution is crucial. Obviously, such choice is affected by the physics of the data taken under consideration. In particular, we have chosen a distribution which models a given sequence of earthquakes at best. About this point, we emphasize that whether the sequence of major earthquakes from a given seismic region is Poisson, renewal, clustered or regular is presently a hotly debated issue, with implications on the level of earthquake safety. Nevertheless, as introduced in Section 1, we chose the renewal process approach because we assume that the sequence of events is related to the phases of accumulation and release of energy characterizing a given seismogenetic source. Therefore the renewal process approach is the one which fits better this assumption, according to the assumptions made. The physics of the earthquake generation also suggests that, after a certain elapsed time, the risk of an immediate strong earthquake increases, therefore, according to point 3 in the previous Section, the longer the waiting time x_{ij}, spent by the system in state i, the higher the probability that transition to the next state j will occur in the next dx. In the following we will show that this behavior is modeled by a Weibull distribution with shape parameter greater or equal than 1 that presents an increasing hazard rate function.

Indeed, in order to validate this assumption, geophysical and geological about seismic region analyzed should be known in details. As we had no geophysical data, we resorted to data provided in literature (Alvarez, 2005), referring to earthquakes that occurred in Turkey during the 20th century, according to data provided by Kandilli observatory (Tab. 1), referring to the so called “North Anatolian Fault Zone”.

year	*m*	*d*	*M*	*year*	*m*	*d*	*M*	*year*	*m*	*d*	*M*	*year*	*m*	*d*	*M*	*year*	*m*	*d*	*M*
1903	4	29	**6.7**	1941	5	23	**6.0**	1951	4	8	**5.8**	1965	6	13	**5.7**	1986	6	6	**5.6**
1912	8	9	**7.3**	1941	9	10	**5.9**	1951	8	13	**6.9**	1966	3	7	**5.6**	1992	3	13	**6.8**
1914	10	4	**6.9**	1941	11	12	**5.9**	1952	1	3	**5.8**	1966	8	19	**6.9**	1992	3	15	**5.8**
1924	9	13	**6.8**	1942	11	15	**6.1**	1952	10	22	**5.6**	1967	7	22	**6.8**	1992	11	6	**6.0**
1925	8	7	**5.9**	1942	11	21	**5.5**	1953	3	18	**7.2**	1967	7	26	**5.9**	1995	2	23	**5.8**
1926	10	22	**6.0**	1942	12	20	**7.0**	1953	9	7	**6.0**	1968	9	3	**6.5**	1995	10	1	**6.0**
1928	3	31	**6.5**	1943	6	20	**6.6**	1955	7	16	**6.8**	1970	4	19	**5.8**	1995	12	5	**5.6**
1929	5	18	**6.1**	1943	11	27	**7.2**	1956	2	20	**6.4**	1970	4	23	**5.6**	1996	8	14	**5.6**
1930	5	7	**7.2**	1944	1	2	**7.2**	1957	4	25	**7.1**	1971	5	12	**5.9**	1996	10	9	**6.8**
1933	7	19	**5.7**	1944	6	25	**6.0**	1957	5	26	**7.1**	1971	5	22	**6.8**	1997	1	22	**5.5**
1935	1	4	**6.4**	1944	10	6	**6.8**	1959	4	25	**5.9**	1975	9	6	**6.6**	1998	6	27	**6.3**
1938	4	19	**6.6**	1945	3	20	**6.0**	1961	5	23	**6.3**	1976	11	24	**7.5**	1999	8	17	**7.4**
1939	9	22	**6.6**	1946	2	21	**5.5**	1963	9	18	**6.3**	1983	7	5	**6.1**				
1939	11	21	**5.9**	1946	5	31	**5.9**	1964	1	30	**5.7**	1983	10	30	**6.9**				
1939	12	27	**7.9**	1949	7	23	**6.6**	1964	6	14	**6.0**	1984	9	18	**5.5**				
1940	4	13	**5.6**	1949	8	17	**6.7**	1964	10	6	**7.0**	1986	5	5	**5.9**				

Table 1 Turkish Earthquakes catalogue reported by Alvarez in (Alvarez, 2005). In table each event is identified through: year (year), month (m), day (d) and magnitude (M).

3.1 Data analysis

The application of a Markov renewal process requires the definition of the sates visited by the process of events during its evolution.

With this aim and starting from the data reported in Table 1 and them inter-event times (in years), here reported in Table 2, some data analyses were approached.

Inter-event times							
9.27800	2.15250	9.94130	0.90080	1.20650	1.44330	1.13090	0.97030
3.19840	1.46150	3.28950	1.42310	0.16330	0.09920	0.29820	1.11000
0.29690	0.17140	1.00810	0.01620	0.08030	0.50200	0.43390	0.10250
0.47710	0.28070	0.45680	0.91970	0.27600	3.14440	0.06680	1.64370
0.34550	0.39200	0.79830	0.40820	0.46830	1.85830	0.59580	1.17950
0.08570	1.91430	2.07760	2.31850	0.36840	0.37180	0.31040	0.68690
0.73480	0.44740	0.92510	0.01080	1.10390	1.62820	0.01080	1.05330
0.02700	4.28880	1.21460	6.61670	0.31650	0.88460	1.63290	0.08570
5.76990	0.00540	0.63970	2.29890	0.60460	0.17680	0.69230	0.15250
0.28810	1.42850	1.13900					

Table 2. Turkish Earthquakes catalogue (Alvarez, 2005). Inter-event times expressed in years.

First of all the magnitude frequency has been evaluated and reported in diagram. Figure 1 reports the histogram relating to magnitude of earthquakes; it is clear that there are no earthquakes with magnitude equal to 6.3. Hence it is easy to figure out that earthquakes could be divided into 2 groups.

To investigate the inter-event times behaviour, we plotted inter-event times vs. progressive number of events (they are 76 in all). Figure 2 reports the plot of the inter-event time of each event and from the scatter plot of the data sample we argue that the first and the third values can be considered outliers with the respect to the rest of the data set. The motivation of this behaviour is clear if we check the years of the first four collected data, with the corresponding three first inter-event times; actually we realized that they refer to years at the very beginning of the century and around the First World War and it is very likely that some data were missed in the period from 1903 and 1924; so we decided to disregard such three first sojourn times, since they can be considered unreliable.

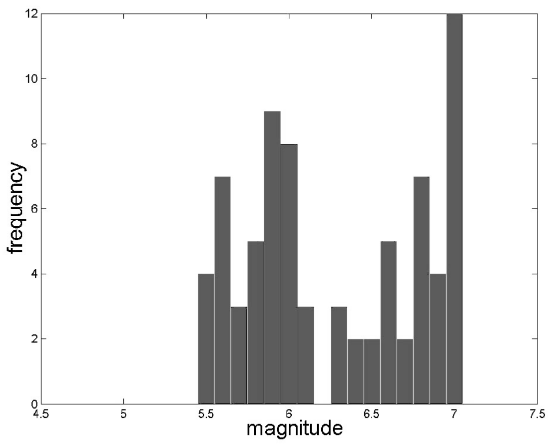

Figure 1 Turkish Earthquakes catalogue (Alvarez, 2005). Histogram of magnitudes.

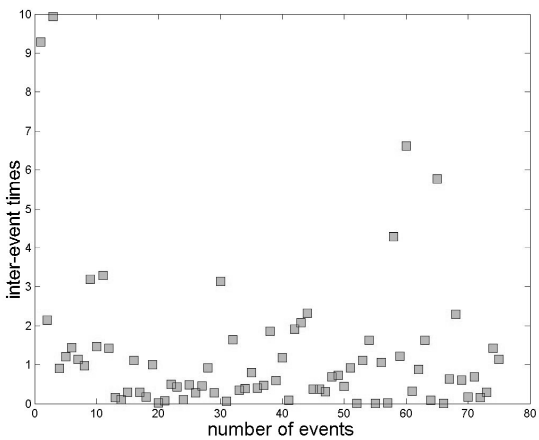

Figure 2 Turkish Earthquakes catalogue (Alvarez, 2005). Plotting of inter-event times event to event.

To investigate the goodness of our choice a boxplot was built with the remaining data. Here below, we report the boxplot for the considered data set, i.e. the original data set without the first three events. Again it is clear that some data can be considered inhomogeneous with respect to the others, but now we do not want to disregard other data, since all the remaining data look significant from a geophysical point of view. Hence we formulate a new conjecture and, by this plot, we motivate the choice of a mixture of two distributions to model the behaviours of the appearing two different data sets, since we want to take both of them into account. Obviously this conjecture has to be validated, and this request will be met in the following.

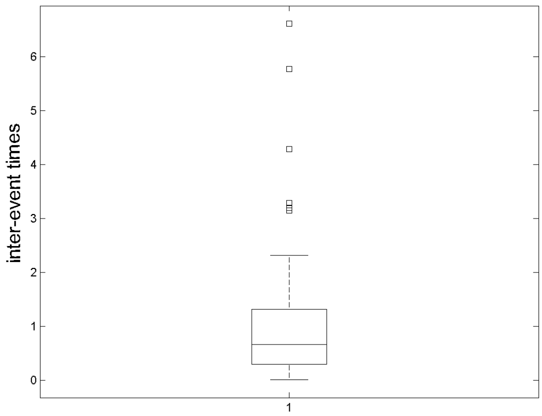

Figure 3 Current inter-event times set

3.2 Some basic assumptions

As underlined in Section 2, Markov renewal process requires the definition of the *states* in which the process evolves. Following the results obtained in 3.1, the ranges of magnitude (in Richter scale) of the two classes are: 5.5-6.3 and greater than 6.3, respectively. The classes coincide with the states of our general Markov renewal process and are characterized by indexes i and j with i, j = 1, 2. This means that only earthquakes with large magnitude are taken under consideration. The time series evidence, based on this dataset, suggests the following conjectures:

- *Earthquakes happen according to a renewal process.*
- *The longer is the waiting time for transition from the state i to the state j the higher is the probability that the transition happens.*

In order to meet these assumptions for the probability, we chose the Weibull distribution which is extremely flexible (Alvarez 2005). To take into account even the geophysical motivations, we before introduced, we imposed a lower bound to parameter α characterizing the Weibull distribution. Indeed, the Weibull distribution hazard rate function behaves in different ways according to different values of shape parameter α. If $\alpha>1$, there is a positive hazard rate increasing to infinity as x increases. If $\alpha<1$ there is a decreasing hazard rate as x increases. If $\alpha=1$ there is a constant hazard rate as x increases. To satisfy the physical hypothesis connected with the accumulation and release of energy, we imposed in each analysis $\alpha\geq1$. As said in previous section the estimation of the shape parameters α and ρ has been done through Eq. (3).

By inspection of the boxplot picture, the double behavior of data appears clearly; however it is difficult to define the states and their amplitude. This leads to think to another model based on the assumption that events belong to a unique state where the evident double behavior can be modeled with a mixture of two distributions. This subject will be better developed in the following Section 4.2.

4. The inter-event times analysis

The Markov renewal process is defined when the properties 1, 2 and 3 in Section 2 are satisfied

Therefore, we followed two different paths:

A) events are divided into two states;

B) events are considered into only one state.

We will present results referring to these two different approaches separately.

4.1 *Case study A)*

In this case we define only two different states:

State 1: events with magnitude $5.5 \le M \le 6.3$

State 2: events with magnitude $M > 6.3$

The transition matrix assumes the form:

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} = \begin{bmatrix} 0.581 & 0.419 \\ 0.586 & 0.414 \end{bmatrix} \tag{4}$$

with $i,j = 1, 2$.

	$f_{11}(x)$	$f_{12}(x)$	$f_{21}(x)$	$f_{22}(x)$
α	1.0946	1.0000	1.0000	1.0000
ρ	1.1686	0.9312	0.7925	0.8337
μ_t	0.8270	1.0739	1.2618	1.1995
cv_t	0.9146	1.0000	1.0000	1.0000
μ_s	0.8302	1.0755	1.2181	1.1756
cv_s	0.8251	1.3206	1.3293	1.1237

Table 3 Weibull densities $f_{ij}(x)$ *modeling the waiting times of transitions i vs. j (with i,j=1,2): distribution parameters* α and ρ, *the estimated distribution mean* μ_t *and coefficient of variation* cv_t, *and samples' mean* μ_s *and* cv_s.

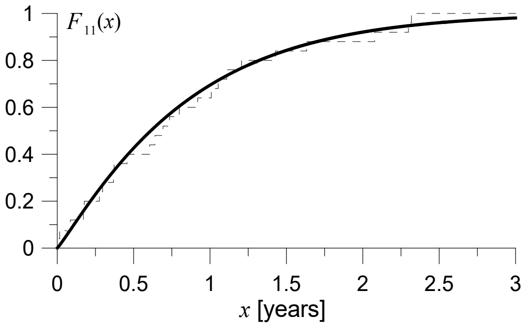

Fig. 4

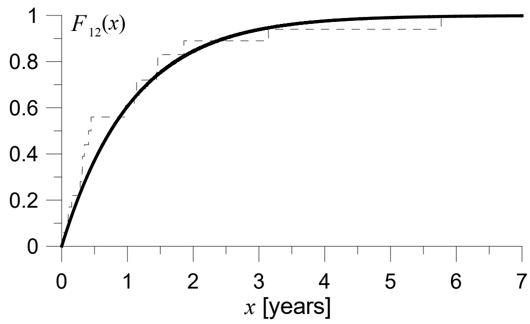

Fig. 5

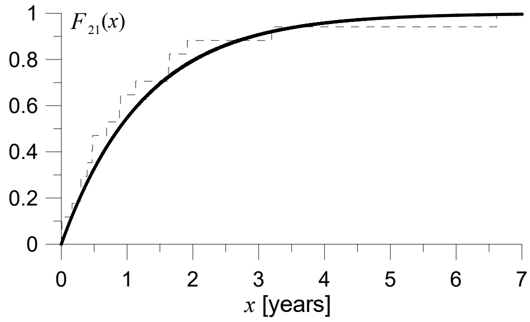

Fig. 6

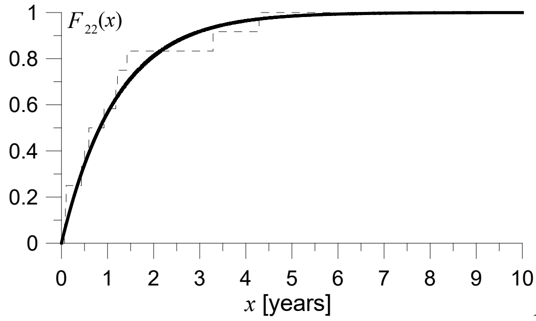

Fig. 7

Figure 4-7 Comparison between empirical and theoretical (Weibull) cumulative distributions $F_{ij}(x)$ with i,j=1,2.

The obtained results put in evidence as a Poisson process could not be completely rejected; in Table 3 the parameter α of the densities $f_{12}(x)$, $f_{21}(x)$ and $f_{22}(x)$, denotes a degeneration of the Weibull distribution into an exponential distribution with a constant hazard rate. However, we have a good agreement between the experimental mean values and theoretical ones as well as between the experimental coefficient of variation and the theoretical one. In Figure 4-7 we report the cumulative distribution of $f_{11}(x)$, $f_{12}(x)$, $f_{21}(x)$ and $f_{22}(x)$ where by dotted line we denote the experimental cumulative distribution and by solid line we denote the theoretical one. Also with the omission of the first 3 inter-events, it is clear that the results obtained in Figure 7 cannot be considered fully satisfying.

By the previous analysis, it is clear that the choice of the states and their length are important and very sensitive, indeed. Different choices lead to different models of the problem that could provide either correct or incorrect forecasting.

The obtained results suggest us to model the inter-event times like a unique set; in other words we are convinced we can use one state only, as we will see in the following.

4.2 ***Case study B)***

We consider only one state neglecting the first three events:

State 1: events with magnitude M≥5.5.

Inter-event times (in years) of events with M≥5.5							
0.0054	0.0108	0.0108	0.0162	0.0270	0.0668	0.0803	0.0857
0.0857	0.0992	0.1025	0.1525	0.1633	0.1714	0.1768	0.2760
0.2807	0.2881	0.2969	0.2982	0.3104	0.3165	0.3455	0.3684
0.3718	0.3920	0.4082	0.4339	0.4474	0.4568	0.4683	0.4771
0.5020	0.5958	0.6046	0.6397	0.6869	0.6923	0.7348	0.7983
0.8846	0.9008	0.9197	0.9251	0.9703	1.0081	1.0533	1.1039
1.1100	1.1309	1.1390	1.1795	1.2065	1.2146	1.4231	1.4285
1.4433	1.4615	1.6282	1.6329	1.6437	1.8583	1.9143	2.0776
2.2989	2.3185	3.1444	3.1984	3.2895	4.2888	5.7699	6.6167

Table 4 Turkish Earthquakes catalogue (Alvarez, 2005). Inter-event times (in years) of events with M≥5.5 *in increasing order.*

If we organize the data of Table 2 in an increasing order per row (see Table 4) the inter-event times present values included in the range 0.005-4.3, but most of them are less than 2, whereas some inter-event times only are greater than 3 and just the last three times are greater than 4; this seems a symptom of a possible double behavior of the inter-event times. To well capture this behavior, the distribution we chose is a mixture of two distributions of the type (Guagenti Grandori et al., 1990):

$$f(x) = qh_1(x) + (1.-q)h_2(x) \qquad (5)$$

where $h_1(x)$ is the density modeling the inter-event times with a small average value, $h_2(x)$ is the density modeling the inter-event times with an high mean value; the weight q is the percentage of inter-event times having the behavior modeled by the density $h_1(x)$ and $(1.-q)$ is its complementary.

As already pointed out, the choice of the distributions is a crucial point. Here the chosen distributions are an exponential for $h_1(x)$ and a Weibull for $h_2(x)$ and Eq. (5) becomes:

$$f(x) = q \cdot \left[b(\exp^{-bx}) \right] + (1.-q) \cdot \left\{ \alpha\rho(\rho x)^{(\alpha-1)} \exp\left[-(\rho x)^{\alpha} \right] \right\} \qquad (6)$$

with b, α and ρ positive distributions parameters.

The hazard rate function will be:

$$\lambda(x) = q \cdot b + (1.-q) \cdot [\alpha\rho(\rho x)^{(a-1)}] \qquad (7)$$

This choice admits an hazard rate function characterized by four different phases: in the first phase the hazard rate function decreases, in the second phase hazard rate function, $\lambda(x)$ is quite stationary, in the third phase $\lambda(x)$ increases and in the fourth phase $\lambda(x)$ decreases and becomes constant with a value close to the value of a Poisson process (Fig. 8); this behavior permits to make prevision also when a long time is already passed since the last event occurred, without emphasizing it with a "catastrophic" prevision of imminent event. This choice has been made because the inter-event times, here reported, are not supported by any other information, such as geophysical or geological data or any kind precursor signs that can justify the prevision of an imminent event after a long seismic silence (Garavaglia et al. 2007).

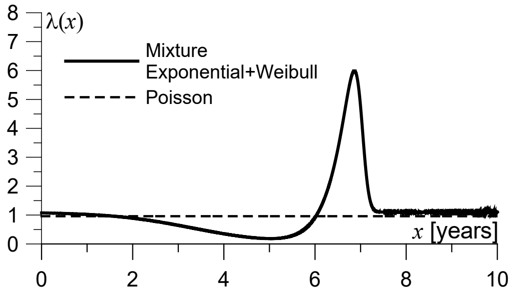

Fig. 8 Comparison between hazard rate functions $\lambda(x)$ of Poisson process and mixture e-w.

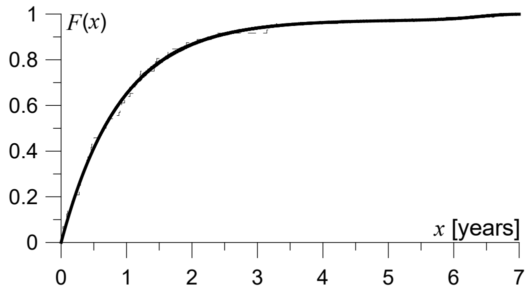

Fig. 9 Comparison between empirical and mixture e-w cumulative distributions $F(x)$.

In Table 5 and 6 the parameters of the mixture e-w and the test of significance are reported. Of course, having just one state in this case study, the transition matrix, **P**, is not necessary.

Parameter of $h_1(x)$	*Parameters of $h_2(t)$*			*Theoretical mean and coefficient of variation*		*Mean and coefficient of variation of the experimental data*		*Mean and coefficient of variation of $h_1(x)$*		*Mean and coefficient of variation of $h_2(x)$*	
b	α	ρ	q	μ_t	cv_t	μ_s	cv_s	μ_1	cv_1	μ_2	cv_2
1.1027	17.9509	0.1559	0.9747	1.0412	1.1773	1.0407	1.1883	0.90683	1.00	6.22552	0.0688

Table 5. e-w mixture parameters.

Statistical tests of significance				
ML	χ^2	*K-S*	*SN**	*α(Kol)*
72.1640	6.6111	0.0486	1.6533	0.9906

Table 6. e-w mixture: some statistical tests of significance

In Figure 9 we report the cumulative distribution of $f(x)$, where dotted line denotes the experimental cumulative distribution and solid line denotes the theoretical one. By Figure 9 it is evident the good accordance between the experimental data and the theoretical curve; such accordance is underlined also by results shown in Table 6, in particular *α(Kol)* that reaches a value very close to 1 (optimal limit).

5. Crossing state prediction

The aim of this work is the prediction of the *state j* of next earthquake being known the *state i* of last event and the time t_0 passed by the last occurred earthquake.

Under these assumptions we have that the probability of the next event can be defined as follow:

$$\Pr(J_{n+1} = j, X_{n+1} \le t_0 + \Delta x \,|\, J_n = i, X_{n+1} > t_0), \qquad i=1,2 \qquad j=1,2 \qquad (8)$$

where, J_n is the state of last earthquake, J_{n+1} is the state of next earthquake, X_{n+1} is the time already passed by the last occurrence until the instant in which the prediction is made, Δx is the discrete time in which the prediction would be obtained.

If the distribution functions $F_{ij}(x)$ are defined, Eq. (8) can assume the following form:

$$\mathrm{P}_{t_0|\Delta x}^{(ij)} = \frac{\left[F_{ij}(t_0+\Delta x) - F_{ij}(t_0) \right] p_{ij}}{\left[1-F_{i1}(t_0)\right] p_{i1} + \left[1-F_{i2}(t_0)\right] p_{i2}} \qquad (9)$$

In *case study B)* where we have just one state, Eq. (9) assumes the form:

$$\mathrm{P}_{t_0|\Delta x} = \frac{\left[F(t_0+\Delta x) - F(t_0) \right]}{\left[1-F(t_0)\right]} \qquad (10)$$

The predictions obtained by the proposed procedure are here presented separately.

5.1 *Case study A)*

Here we refer to the case study A) in pervious Section 4.1. The probability of occurrence of an earthquake of a given class of magnitude, classifiable into state j (j=1, 2), if last occurred event is classifiable into the state i (i=1, 2), is evaluated starting from the instant t_0 immediately after the last event occurred, therefore t_0=0. The results obtained are reported in the following Table 7.

	Last event is in state *i*=1 (5.5 < M ≤ 6.3)		**Last event is in state *i*=2 (M > 6.3)**		**Last event is in state *i*=1 (5.5 < M ≤ 6.3)**		**Last event is in state *i*=2 (M > 6.3)**		**Last event is in state *i*=1 (5.5 < M ≤ 6.3)**		**Last event is in state *i*=2 (M > 6.3)**	
	t_0=0		t_0=0		t_0=1yr		t_0=1yr		t_0=2yr		t_0=2yr	
Δx	$\mathrm{P}_{t_0\|\Delta x}^{(11)}$	$\mathrm{P}_{t_0\|\Delta x}^{(12)}$	$\mathrm{P}_{t_0\|\Delta x}^{(21)}$	$\mathrm{P}_{t_0\|\Delta x}^{(22)}$	$\mathrm{P}_{t_0\|\Delta x}^{(11)}$	$\mathrm{P}_{t_0\|\Delta x}^{(12)}$	$\mathrm{P}_{t_0\|\Delta x}^{(21)}$	$\mathrm{P}_{t_0\|\Delta x}^{(22)}$	$\mathrm{P}_{t_0\|\Delta x}^{(11)}$	$\mathrm{P}_{t_0\|\Delta x}^{(12)}$	$\mathrm{P}_{t_0\|\Delta x}^{(21)}$	$\mathrm{P}_{t_0\|\Delta x}^{(22)}$
1 year	0.404	0.253	0.321	0.238	0.384	0.292	0.326	0.228	0.314	0.354	0.332	0.222
2 years	0.535	0.353	0.466	0.336	0.486	0.407	0.474	0.327	0.392	0.494	0.482	0.320
3 years	0.570	0.393	0.532	0.380	0.511	0.452	0.541	0.371	0.410	0.549	0.550	0.362
4 years	0.579	0.408	0.562	0.399	0.517	0.470	0.571	0.389	0.414	0.570	0.580	0.380
5 years	0.581	0.414	0.575	0.407	0.518	0.477	0.585	0.397	0.415	0.571	0.581	0.380
6 years	0.581	0.417	0.581	0.411	0.518	0.480	0.591	0.401	0.415	0.582	0.601	0.391

Table 7 Probability of occurrence of next event of given magnitude, conditioned to the magnitude of last event occurred, evaluated for different Δx *and different* t_0

5.2 *Case study B)*

Here we refer to the case study B) in pervious Section 4.2. The probability of occurrence of an earthquake with M≥5.5 is evaluated starting from the instant t_0 immediately after the last event occurred, therefore t_0=0 and after one, two and three years from the last event (under the hypothesis that no events were occurred until 1 year, 2 years or 3 years); therefore t_0=1year, t_0=2years and t_0=3years. The results obtained are reported in the following table:

Probability of occurrence of an earthquake of M≥5.5				
Δx	t_0=0	t_0=1 year	t_0=2 years	t_0=3 years
1 year	0.6507443	0.6191927	0.5406810	0.3908641
2 years	0.8667874	0.8250880	0.7202123	0.5252210
3 years	0.9385080	0.8934548	0.7819250	0.6716191
4 years	0.9623233	0.9169554	0.8491684	0.9894753
5 years	0.9705370	0.9425622	0.9951658	0.9976407
6 years	0.9799269	0.9981591	0.9989163	0.9992178

Table 8 Probability of occurrence of next event of magnitude M≥5.5 *evaluated for different* Δx *and different* t_0

5.3 Comparisons and validation

In Table 7 we report the evaluated prediction of the next event of magnitude 5.5 < M≤ 6.3 and the prediction of the next event of magnitude M>6.3, knowing that the last earthquake occurred was an event with magnitude M=7.4 (Tab. 1) and assuming different values for t_0 and Δt.

In Table 8, we present the predictions of an event with magnitude M≥5.5 when the last event was with magnitude M≥5.5 for different t_0 and Δt.

The data reported in Table 7 and 8 cannot be directly compared. It is clear that the prediction of an earthquake of a magnitude M> 5.5 is higher than the prediction of an event of a specify magnitude.

It is evident that *different approaches provide really different predictions*, even though within the same framework of non-Poisson renewal models.

For a first validation of our methods, we have applied them to predict the next event occurred in Turkey after the last event reported in the dataset of Table 1 (i.e. Izmit-Marmara, August 17, 1999, M=7.4). Precisely, the next event was: Adapazari, November, 12, 1999, M=7.3 (http://www.ingv.it/~roma/reti/rms/terremoti/estero/turchia2/turchia.htm). Assuming t_0=0 (immediately after the last event) the prediction has been made both using the first approach proposed in Section 5.1 according to Table 7 (i.e. probability of occurrence of an earthquake with M>6.3 in the three months (Δx=3months) when the last event was an event with M>6.3) and using the second approach proposed in Section 5.2 according to Table 8 (i.e. probability of occurrence of an earthquake with M>5.5 in the next three months (Δx=3months) whatever magnitude characterizes the last event occurred). Results are summarized in the following table.

	Probability next event with M>6.3 if *last event M>6.3*	**Probability of an event with M>5.5**
Δx	t_0=0	t_0=0
3 months	0.078	0.234

Table 9.Comparison between occurrence probability of a rare event of M>6.3 *and a dangerous, but not so rare, event of* M>5.5 *in the next 3 months* (Δx=*3 months*) *after the last event occurred* (t_0=0), *if it was of the same class of magnitude.*

From Table 9, it is evident that the probability of an event with M>5.5 is high and more reliable than the probability connected with the prediction of an event with M>6.3, if the last event had M>6.3. Since earthquakes with M>5.5 really produce disastrous damages and loss of human lives, this prediction is to be preferred.

We have also investigated the probability of occurrence of the event Bingol, May, 01, 2003, M=6.4 (RAI News24_http://www.rainews24.rai.it/Notizia.asp?NewsID=36370) starting from the last event November, 12, 1999, M=7.3 and using this last information into elaborations. This last information is uninfluenced in both the approaches and the shape parameters of mixture distributions remain the same. In Table 10 we have the results concerning the probability of occurrence related to t_0= 0 (immediately after the event of 1999) and for Δt=4year (next event 2003). We see that again the second approach proposed in Section 5.2 provides a very good prediction. Of course, in the future major attention will be paid by the authors in the study of a validation procedure based on a suitable credibility index (Grandori et al. 2007).

	Probability next event with M>6.3 if *last event M>6.3*	**Probability of an event with M>5.5**
Δt	t_0=0	t_0=0
4 years	0.39903	0.9623233

Table 10. Comparison between occurrence probability of a rare event of M>6.3 *and a dangerous, but not so rare, event of* M>5.5 *in the next 4 years* (*Δx=4 years*) *after the last event occurred* (t_0=0), *if it was of the same class of magnitude.*

6. Concluding remarks

A new approach (using a mixture of distributions) is presented to forecast large earthquakes events, together with a classical one; both methods resort to Markov renewal processes, but they differ significantly. We validated our new approach using data from literature referring to earthquakes in Turkey, which allowed us to make interesting comparisons. The reported results show that fitting our renewal process (with a mixture of distributions) to such data is challenging. Even though that earthquake dataset (Alvarez, 2005) is not the best suited to illustrate the methods developed in this paper (since data are there provided without any geophysical information), we have shown that different approaches provide really different predictions. The most reliable predictions are obtained by means of our new approach, which is supposed to take into account some geophysical evidence also. Therefore our results seem to urge more adequate models describing the earthquake occurrence in time and robust hypotheses about the regions where large earthquakes are expected to strike. Indeed, our better method looks very encouraging within this framework.

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