Experimental investigation of amplification, via a mechanical delay-line, in a rainbow-based metamaterial for energy harvesting: Supplementary Material

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To aid insight into the underlying physics of the rainbow-based metamaterial, described in the main text, we give a detailed description of the experimental setup and additional numerical results.

I. SUPPLEMENTARY NOTE 1 - FURTHER EXPERIMENTAL DETAILS

The electro-mechanical system is made of a beam augmented with 30 resonators, which are mounted on the beam through a set of screws, emulating rigid connections. We assume that the tightening of the screws is sufficiently strong to guarantee a linear and frequency-independent behavior of the joint. The beam is also mechanically joined to a *LDS v406* electrodynamic shaker at the left boundary, to provide an out-of-plane input excitation; the beam is placed perpendicularly to the excitation. An accelerometer, *PCB 352C33*, is bonded to the beam to measure the acceleration level provided by the shaker and data is acquired by way of a National Instruments (NI) input module with internal signal conditioning.

The opposite end is attached to an external frame via a set of ropes, to avoid undesired deformations of the beam; the presence of the ropes is assumed not to affect the system dynamics. In addition, an acoustic black hole, whose characteristics are discussed in detail below, is manufactured at the right-hand boundary to minimize any reflection of waves and to emulate absorbing boundaries. The 26^{th} resonator, that is placed at the coordinate at which the wave speed at 2.05 kHz is minimum, is equipped with a harvester, able to convert the mechanical vibrations into electric energy. The geometrical and physical parameters of the harvester are illustrated in the final part of this section.

A Polytec 3D *scanner laser Doppler vibrometer* (SLDV) measures the velocity field along the beam's main dimension, separating out the out-of-plane component. At the same time, the voltage across the harvester's electrodes is acquired through the external Polytec acquisition system. Finally, the excitation signal (provided through a *KEYSIGHT 33500B* waveform generator), synchronously starts with the measurement system and consists in a input function $V(t) = V_0 w(n) \sin(2\pi f_c t)$ with amplitude $V_0 = 5$ V, $w(n)$ is a Hann window, central frequency $f_c = 2.05$ kHz and time duration 30 ms; this results in a spectral content of width $\Delta_f = 0.14$ kHz.

Black hole characteristics

To emulate absorbing boundaries we consider an acoustic black hole similar to Ref.[1–3] and illustrated in Fig. S1(a)-I. Specifically, the beam cross-section is machined to achieve a variable profile for the thickness characterized by the following expression:

$$h(x) = \varepsilon x^m + h_0, \tag{S1}$$

where $m = 2.2$, $h_0 = 2$ mm and $\varepsilon = 1.6$. The addition of a highly dissipative material (*Blu-Tack*) to the variable section provides localized damping during the wave propagation, that allows to avoid undesired reflections at the beam's free edge, therefore emulating absorbing boundary conditions.

We experimentally verified the behavior of the acoustic black hole through the setup illustrated in Fig. S1(a)-II, in which only the host beam is mounted on the optical table. The aforementioned input forcing signal ($f = 2.05$ kHz, $\Delta f = 0.14$ kHz) is imposed at the boundary opposite to the black hole. The experimental dispersion relation $|\dot{w}(\kappa, f)|$ is computed through the *Fourier Transform* (FT) of the velocity field and compared to the analytical dispersion for the A_0 Lamb mode in Fig. S1(b). As expected, the energy content is mainly located on the positive wavenumber domain, whereas only a small amount of energy (almost 50 % of reflected component) is present in the left side.

Geometrical and physical details of the harvester and optimal resistive load

The piezoelectric harvester employed in this work is made of four cantilever beams arranged in a cross-like shape mounted on the 26^{th} resonator by means of a screw. Each of them is made of an aluminum substrate of 25 mm length, 5 mm width and 2 mm thickness, endowed with piezoelectric patch of the same in plane dimensions, with a cross-section thickness of $t = 0.3$ mm and bonded through a conductive epoxy CW2400. To optimize the power extracted during the tests, a passive resistor is selected and connected to each

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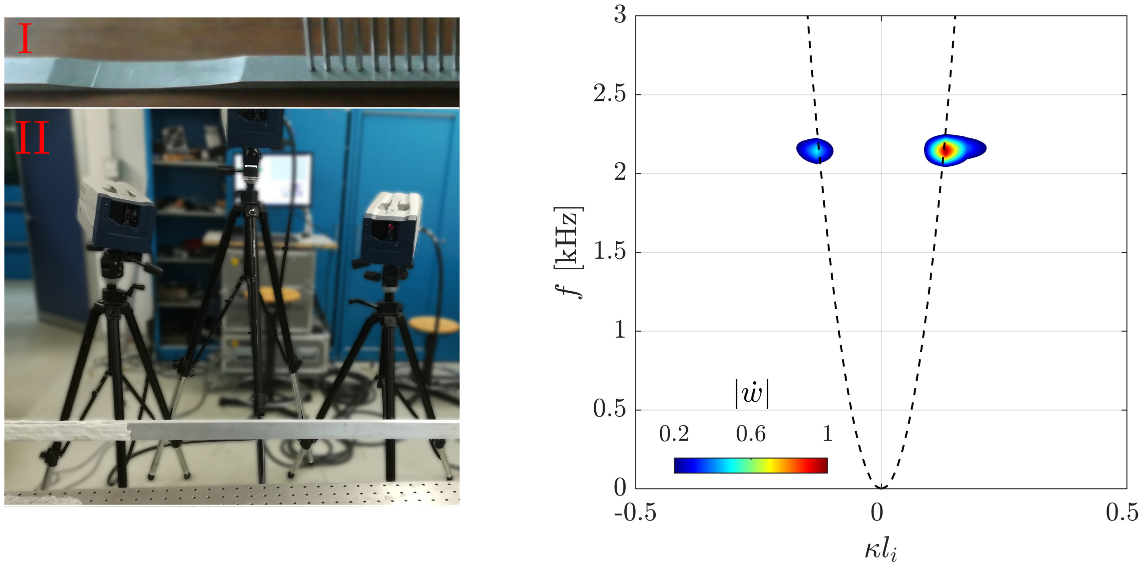
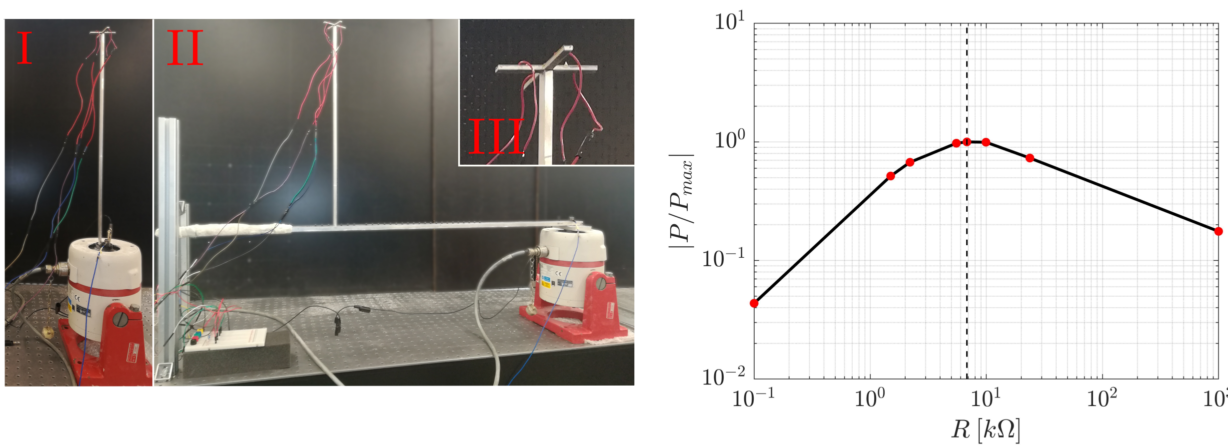


(a) (b)

FIG. S1. (a) Experimental setup for the characterization of the acoustic black hole. The beam is clamped on the left side and the wavefield is measured on the bottom surface using a 3D scanner laser Doppler vibrometer (SLDV). The acoustic black hole is manufactured by gradually reducing the beam thickness of an otherwise constant-thickness beam and through the addition of a highly dissipative material in correspondence of the reduced cross section (I). (b) Experimental dispersion curve illustrating the input and reflected A_0 Lamb modes.



(a) (b)

FIG. S2. (a) Experimental setup adopted for the evaluation of the optimal electric load and associated electric circuit with parallel connection (I). The same load and electric connections are employed in the experimental tests of the beam with attached resonating rod (II) and beam with metamaterial. Zoom-in view of the harvester (III). (b) Representation of the peak power for different resistive loads; the optimum value is at approximately 6.8 $k\Omega$.

piezo-patch based on the experimental characterization of the dynamical response. The analysis is performed through the experimental setup displayed in Fig. S2(a)-I, where the resonator is placed vertically on the shaker. The system is excited using a sweep sine in the neighborhood of the resonance frequency of the cantilever and the rod (which are carefully designed for operating at the same frequency). The voltage drop across the piezoelectric electrodes is measured upon varying the resistance values, whereby the power achieved at 2.05 kHz is displayed in Fig. S2 (b) normalized by the its maximum. The analysis reveals an optimal resistance of 6.8 $k\Omega$, which is in agreement with the lumped model in Ref.[4].

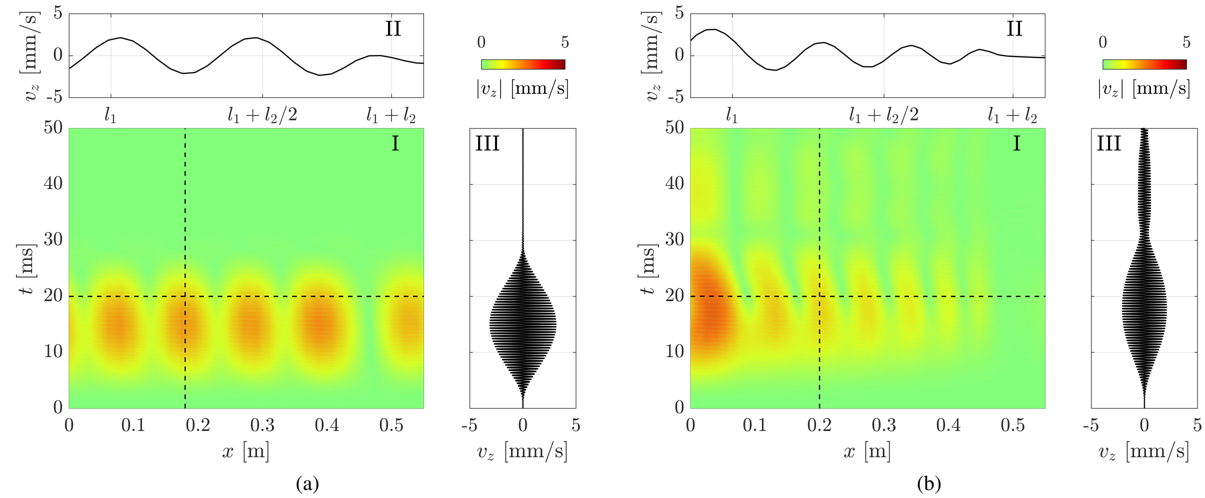


FIG. S3. Numerical velocity field computed for (a) beam with one resonator; (b) beam with graded metamaterial. The horizontal and vertical dashed lines in (I) correspond to the reference coordinate and time instant employed to illustrate the wave behavior in space (II) and time (III), as shown on top and alongside the velocity field.

II. SUPPLEMENTARY NOTE 2 - NUMERICAL METHODS AND ADDITIONAL RESULTS

The numerical model employed in this work is based on a finite element discretization of the system through ABAQUS CAE 2018. In particular, the host beam and the resonators are modeled through full 3D stress quadratic elements (C3D20), while the electromechanical interactions are addressed by linear 3D piezoelectric elements (C3D8E). Electrodes are modelled through a voltage constraint on the piezo-faces, and the electric load is introduced via a user Fortran subroutine, that is able to solve at each time increment a resistive circuit[5]. The analysis is performed using an implicit analysis based on the Hilber-Hughes-Taylor operator, with a constant time increment $dt = 5\ \mu s$. The system is forced using the imposed acceleration that is experimentally measured with the accelerometer placed in correspondence of the shaker, while the black hole is modeled numerically by varying the beam profile and adding a soft material with Rayleigh damping $\alpha = 5000$ rad/s. In addition, we add in the harvester model a Rayleigh damping source, with $\alpha = 19.3$ rad/s and $\beta = 2.93 \times 10^{-6}$ s/rad on the basis of conventional modal damping ratios[6] for aluminum beams with piezoelectric patches.

The dispersion relation is computed using COMSOL multiphysics exploiting periodic boundary conditions applied to the unit cell boundaries, whereby a parametric analysis is performed in order to verify the wavenumber transformation described in Fig. 2(b) and Fig. 4(d) in the manuscript.

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