



Performance of Green Roofs for Rainwater Control

A. Raimondi¹ · G. Becciu¹

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Abstract

Green roofs can be an effective tool for sustainable urban drainage, since they reduce and retain runoff by delaying its peak. Most studies analysing the retention capacity of green roofs are usually referred to a specific place and roof condition and do not consider the possibility that the roof could be partially pre-filled from previous rainfalls at the beginning of the given event. The aim of this paper is to develop an analytical probabilistic approach to evaluate green roof performance for stormwater control in terms of runoff that could be applied for different sites and climate conditions. To this end, the possibility that the green roof retention capacity could not be completely available owing to pre-filling from previous rainfall events has been considered and equations for an optimum green roof design, relating the runoff average return interval to the water retention capacity, have been proposed. The influence of parameters affecting the runoff process has been examined in depth and a case study to test the goodness of fit of the resulting equations has been developed.

Keywords Green roofs · Stormwater control · Probability · Runoff · Retention capacity · Pre-filling

1 Introduction

The urban population of the world has grown rapidly from 751 million in 1950 to 4.2 billion in 2018. Today, 55% of the world's population lives in urban areas, a proportion that is expected to increase to 68% by 2050 (United Nations 2018). The cover of natural surfaces causes many problems as higher temperatures, poorer air quality, increased noise levels, loss of biodiversity and greater runoffs. In particular, the reduction of infiltration into the groundwater system increases the possibility of floods and of surface waters contamination. In this context, the implementation of strategies for a sustainable urban drainage to restore the natural hydrological cycle and to minimize the environmental impact is urgent (Lee 2019; Lee et al. 2019; Li et al. 2017; Wang et al. 2019). Among these, green roofs that do not require additional space beyond

✉ A. Raimondi
anita.raimondi@polimi.it

¹ D.I.C.A., Politecnico di Milano, Milano, Italy

a building's footprint could be an effective tool in densely built urban areas where rooftops constitute from 30 to 50% of impervious surfaces (Carter and Rasmussen 2007). The installation of green roofs involves numerous environmental and economic benefits, as stormwater management, energy conservation, reduction of heat islands, improvement of water quality, protection of biodiversity. Focusing on stormwater management, green roofs allow: the local disposal of runoff; the reduction of runoff volumes, through evapotranspiration from vegetation and exposed surfaces; the delay of runoff, triggered by soil saturation; the reduction and delay of runoff peak rates, for the infiltration of rainwaters into the soil and their temporary storage in the substrate and in the drainage layer; the improvement of stormwater quality for effect of its percolation into the soil. The first green roofs were installed in Germany in the 1970s (Getter et al. 2007) and since then they have spread in all major modern countries where, in some cases, incentive programs to encourage or even impose their installation have been undertaken.

Hydrological performances of green roofs have been investigated by several studies and models in the last decades (Li and Babcock 2014, 2015; Palla et al. 2012; Locatelli et al. 2014; Stovin et al. 2017), but often these studies refer to a specific place or climate (Palermo et al. 2019; Piro et al. 2018; Herrera et al. 2018; Peng and Jim 2015) having a limited perspective in terms of space and time (Hakimdavar et al. 2014; Nawaz et al. 2015). Moreover, especially for full-scale installations, scientific evidence provided is not sufficient yet to demonstrate hydrological benefits of green roofs (Berndtsson 2010; Gnecco et al. 2013; Zhang and Guo 2013). Many authors studied the retention capacity of green roofs, focusing their attention on the relation between storage volume and rainfall depth (Getter et al. 2007; Carter and Rasmussen 2007), antecedent dry period (Chowdhury and Beecham 2012; Voyde et al. 2010), evapotranspiration (Bengtsson 2005).

The aim of this paper is to develop an analytical probabilistic approach to evaluate green roof performances for stormwater control by estimating the runoff probability distribution function. The use of this kind of methodology for the modelling of urban drainage systems was first proposed in the Nineties by Adams and Papa (2000). Once we define the relation that links the output variables to the input variables and to the parameters characterising the examined process, the methodology allows the estimation of the probability distribution functions of output variables starting from the probability distribution functions of the input variables. One of the main advantages of analytical probabilistic approaches is that they combine the simplicity of "design storm" methods with the probabilistic reliability of continuous simulations. Equations resulting from the application of this methodology are generally easy to implement and provide useful information to designers. Moreover, they are not influenced by a specific place, but can be applied to different basins and climates all over the world. In recent years, different applications of the analytical probabilistic approach have been proposed in literature for the modelling of stormwater detention facilities (Raimondi and Becciu 2017; Becciu and Raimondi 2014, 2015); rainwater tanks (Raimondi and Becciu 2011, 2014; Becciu et al. 2018); infiltration trenches (Guo and Gao 2016); permeable pavements (Zhang and Guo 2014a); bioretention systems (Zhang and Guo 2014b), and green roofs (Guo 2016; Guo and Zhang 2014; Zhang and Guo 2013). In these works, the modelling of green roofs, does not take into consideration the possibility that the retention capacity could be partially pre-filled from more than one previous event at the beginning of the considered rainfall; this aspect is not negligible for this kind of systems, since outflows from green roofs are limited to evapotranspiration from vegetation and soil.

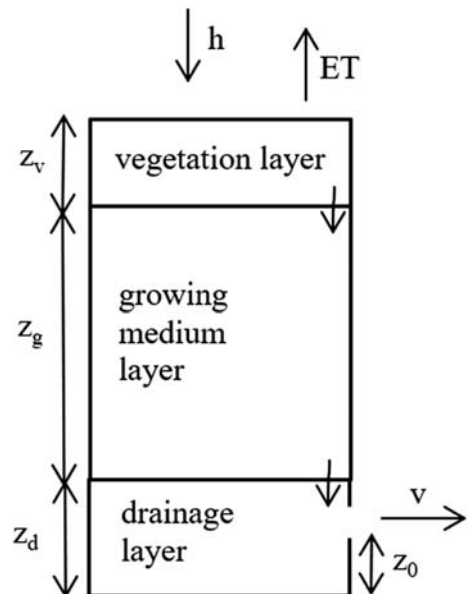
This paper tries to fill the gap considering the possibility that retention volume is not completely available because partially filled by previous rainfall events; in particular, the runoff probability has been calculated considering the possibility of pre-filling from a chain of N rainfall events. Moreover, the proposed method allows for an optimum green roof design since resulting equations relate the maximum retention capacity to the runoff average return interval. The modelling accurately defines the different variables involved in the process, analysing their single effects on retention capacity and runoff. Final equations have been validated comparing the results of their application with those from the continuous simulation of rainfall data recorded at the gauge station of Milano-Monviso.

2 Methodology

Green roofs are engineered multi-layered structures, with a vegetated upper surface, that work in very shallow systems without connections to the natural ground. A typical green roof is composed of: vegetation; growing media, a blend of mineral material enriched with organic material where water is retained and in which vegetation is anchored; filter fabric; drainage layer, generally constituted of plastic profiled elements, that stores water for plants sustainment during dry periods, evacuating excess water in roof drains; root resistant membrane, mechanic protection geotextile. Figure 1 shows the scheme of reference for the modeling of green roofs used in this paper.

The input to the system is rainfall depth, h ; output variables are respectively runoff volume, v and evapotranspiration volume ET . Three different layers have been considered: vegetation layer, of thickness z_v ; growing medium layer, of thickness z_g ; drainage layer of thickness z_d . Volumes must be intended as specific for unit of area. The parameter z_0 represents the height of the overflow threshold into the drainage layer to evacuate excess rainwater when the retention capacity of the roof becomes null.

Fig. 1 Green roof schematization used in the modeling



Rainfall is first intercepted by plants into the vegetation layer and then infiltrated into the growing medium where it is retained, used by vegetation, and released back into the atmosphere through evapotranspiration. The excess is infiltrated into the drainage layer equipped with an overflow to drain heavy rainfalls. The amount of water stored in a green roof w , can be estimated as follows:

$$\left\{ \begin{array}{ll} w_v & 0 < h \leq w_{v,max} \\ w_{v,max} + w_g & w_{v,max} < h \leq w_{v,max} + w_{g,max} \\ w_{v,max} + w_{g,max} + w_d & w_{v,max} + w_{g,max} < h \leq w_{v,max} + w_{g,max} + w_{d,max} \\ w_{v,max} + w_{g,max} + w_{d,max} & h > w_{v,max} + w_{g,max} + w_{d,max} \end{array} \right. \quad (1)$$

The parameters w_v , w_g and w_d represent the volume of water stored respectively into the vegetation layer, the growing medium layer and the drainage layer, while $w_{v,max}$, $w_{g,max}$ and $w_{d,max}$ represent their maximum (always for unit of area). The amount of water stored in a green roof can vary between zero, during dry periods when rainfall depth is null and the volume is not partially pre-filled from previous events, and $w_{max} = w_{v,max} + w_{g,max} + w_{d,max}$ when all the three layers reach their maximum capacity ($0 \leq w \leq w_{max}$). The amount of water volume intercepted by vegetation can be estimated by the following expression:

$$w_v = c_v \cdot w_{v,max} \quad (2)$$

where, c_v is a reduction coefficient ranging between zero and one. The amount of water into the growing medium layer can be calculated as follows:

$$w_g = c_g \cdot w_{g,max} = c_g \cdot \phi_f \cdot z_g = \phi \cdot h_g \quad (3)$$

that is, either as a percentage of the growing medium storage capacity, calculated by means of a reduction coefficient c_g varying between zero and one, or multiplying the growing medium moisture content by the water depth into the layer. In particular, ϕ represents the growing medium moisture content, variable is between zero and the growing medium field capacity ϕ_f and h_g is the water depth into the growing medium layer that can range between zero and the growing medium thickness z_g . The amount of water into the drainage layer can be estimated with the following formula:

$$w_d = c_d \cdot w_{d,max} = c_d \cdot p_d \cdot z_d = p_d \cdot h_d \quad (4)$$

that is either as a percentage of the maximum storage capacity of the drainage layer, calculated by means of a reduction coefficient c_d varying between zero and one, or multiplying the drainage layer porosity p_d by the water depth h_d . In particular, the drainage layer porosity p_d can vary between zero and one, while the water depth into the drainage layer h_d can range between zero and the height of overflow threshold z_0 , obviously lower than the drainage layer thickness z_d .

The evapotranspiration volume, that is the amount of water released to the atmosphere from plants transpiration and soil evaporation can be estimated by the following equation:

$$ET = \begin{cases} ET_P & h \geq ET_P \text{ or } h < ET_P \text{ and } h + \Delta w \geq ET_P \\ h - \Delta w & h < ET_P \text{ and } h - \Delta w < ET_P \end{cases} \quad (5)$$

The actual evapotranspiration ET equals the potential evapotranspiration ET_P , when the rainfall depth is higher than the potential evapotranspiration or when the rainfall depth is lower than the potential evapotranspiration, but the rainfall depth added to the water content already present into the green roof resulting from previous rainfalls Δw exceeds the potential

evapotranspiration. The water content into green roofs resulting from previous rainfalls Δw can vary between zero, if the green roof is completely dry, and w_{max} , if the water content into the green roof is at its maximum, as in the case of two very close heavy rainfall events. In this paper, the actual evapotranspiration has been assumed equal to the potential one. Meteorological variables that most affect runoff from green roofs are rainfall depth h , rainfall duration θ and interevent time d ; in the modeling they have been assumed as independent and exponentially distributed. Numerous studies on different basins concluded that this hypothesis could be considered acceptable to reduce the complexity of the analytical derivation (Adams et al. 1986; Eagleson 1978; Bedient and Huber 1992).

Bacchi et al. (2008) tested that for most Italian basins the Weibull probability distribution function fits the frequency distribution of meteorological input variables better than the exponential probability distribution function; however, its use involves a considerable complication in the equation's integration. Becciu and Raimondi (2014) verified that the double-exponential probability distribution function complies with the frequency distribution of observed data for the main rainfall characteristic parameters; such distribution may be easily integrated but derived expressions are quite complex. Moreover, its application to a case study highlighted the fact that using the double-exponential probability distribution function does not improve so much the accuracy of results and that the bias due to the use of the exponential probability distribution function is negligible when compared to the simplicity of equations integration.

To isolate independent events from a continuous record of rainfalls, a minimum interevent time, the so-called Inter Event Time Definition (IETD) (USEPA 1986), must be defined. If the interevent time between two consecutive rainfall events is smaller than IETD, the two rainfalls are joined into a single event, otherwise they are assumed as independent. The probability distribution function of rainfall depth, rainfall duration and interevent time, considered exponentially distributed, results:

$$f_h = \xi \cdot e^{-\xi \cdot h} \tag{6}$$

$$f_\theta = \lambda \cdot e^{-\lambda \cdot \theta} \tag{7}$$

$$f_d = \psi \cdot e^{-\psi \cdot (d - IETD)} \tag{8}$$

where: $\xi = 1/\mu_h$; $\lambda = 1/\mu_\theta$; $\psi = 1/(\mu_d - IETD)$. The parameters μ_h , μ_θ and μ_d are respectively the mean values of rainfall depth, rainfall duration and interevent time.

To evaluate green roof performance in terms of stormwater control, a chain of rainfall events has been considered (Fig. 2).

With reference to runoff, it generally occurs when the water content at the end of a rainfall event exceeds the green roof retention capacity; the possibility that the retention capacity is not completely available at the beginning of a rainfall event, because it may be partially filled from previous rainfalls has been considered. The water content in a green roof at the end of a generic rainfall event w_i can be expressed by:

$$w_i = \begin{cases} w_{i-1} - Et \cdot d_i + h_i - Et \cdot \theta_i & \text{Condition}_1 \\ h_i - Et \cdot \theta_i & \text{Condition}_2 \\ w_{max} & \text{Condition}_3; \text{Condition}_4 \\ 0 & \text{Otherwise} \end{cases} \tag{9}$$

- Condition*₁ : $w_{i-1}-Et \cdot d_i > 0; 0 < w_{i-1}-Et \cdot d_i + h_i-Et \cdot \theta_i < w_{max}$
- Condition*₂ : $w_{i-1}-Et \cdot d_i \leq 0; 0 < h_i-Et \cdot \theta_i < w_{max}$
- Condition*₃ : $w_{i-1}-Et \cdot d_i \leq 0; h_i-Et \cdot \theta_i \geq w_{max}$
- Condition*₄ : $w_{i-1}-Et \cdot d_i > 0; w_{i-1}-Et \cdot d_i + h_i-Et \cdot \theta_i \geq w_{max}$

for $i = 1, \dots, N$ where N is the number of given rainfall events.

The water content for $i = 0$, that is w_0 , results:

$$w_0 = \begin{cases} h_0-Et \cdot \theta_0 & 0 < h_i-Et \cdot \theta_i < w_{max} \\ w_{max} & h_i-Et \cdot \theta_i \geq w_{max} \\ 0 & h_i-Et \cdot \theta_i \leq 0 \end{cases} \tag{10}$$

With reference to eq. (9): *Condition*₁ expresses the case when there is pre-filling resulting from previous rainfall events at the beginning of the given event and the given event does not produce runoff; *Condition*₂ expresses the case when there is no pre-filling resulting from previous rainfall events at the beginning of the given event and the given event does not produce runoff; *Condition*₃ expresses the case when there is no pre-filling resulting from previous rainfall events at the beginning of the given event and the given event produces runoff; *Condition*₄ expresses the case when there is pre-filling resulting from previous rainfall events at the end of the given event and the given event produces runoff. The variable Et in eqs. (9) and (10) represents the evapotranspiration rate. The runoff at the end of a generic event v_i can be calculated by the following expression:

$$v_i = \begin{cases} w_{i-1}-Et \cdot d_i + h_i-Et \cdot \theta_i-w_{max} & \textit{Condition}_1 \\ h_i-Et \cdot \theta_i-w_{max} & \textit{Condition}_2; \textit{Condition}_3 \\ w_{max}-Et \cdot d_i + h_i-Et \cdot \theta_i-w_{max} & \textit{Condition}_4 \\ 0 & \textit{Otherwise} \end{cases} \tag{11}$$

- Condition*₁ : $w_{i-1} \leq w_{max}; w_{i-1} > Et \cdot d_i; w_{i-1}-Et \cdot d_i + h_i-Et \cdot \theta_i > w_{max}$
- Condition*₂ : $w_{i-1} \leq w_{max}; w_{i-1} \leq Et \cdot d_i; h_i-Et \cdot \theta_i > w_{max}$
- Condition*₃ : $w_{i-1} > w_{max}; w_{max} \leq Et \cdot d_i; h_i-Et \cdot \theta_i > w_{max}$
- Condition*₄ : $w_{i-1} > w_{max}; w_{max} > Et \cdot d_i; w_{max}-Et \cdot d_i + h_i-Et \cdot \theta_i > w_{max}$

for $i = 1, \dots, N$ where N is the number of given rainfall events.

The runoff for $i = 0$, that is v_0 , results:

$$v_0 = \begin{cases} h_0-Et \cdot \theta_0-w_{max} & h_i-Et \cdot \theta_i > w_{max} \\ 0 & \textit{Otherwise} \end{cases} \tag{12}$$

With reference to the eq. (11): *Condition*₁ expresses the case when there is no runoff at the end of event $i - 1$, there is pre-filling from event $i - 1$ at the beginning of event i and there is runoff from the green roof at the end of event i ; *Condition*₂ expresses the case when there is no runoff at the end of event $i - 1$, there is no pre-filling from event $i - 1$ at the beginning of event i and there is runoff from the green roof at the end of event i ; *Condition*₃ expresses the case when there is runoff at the end of event $i - 1$, there is no pre-filling from event $i - 1$ at the beginning of event i and there is runoff from the green roof at the end of event i ; *Condition*₄ expresses the case when there is runoff at the end of event $i - 1$, there is pre-filling from event $i - 1$ at the beginning of event i and there is runoff from the green roof at the end of event i . The probability that the runoff volume exceeds a given threshold \bar{v} has been estimated, setting

$h = h_i = h_{i+1}$, $\theta = \theta_i = \theta_{i+1}$, $d = d_i = d_{i+1}$ in eq. (11); this leads to the deletion of Condition₂. Raimondi and Becciu (2014) maintained that to consider two chained rainfall events may be acceptable only for a long IETD and high outflow rates; for rainwater detention facilities with low outflow rates (e.g., infiltration basins, green roofs..) or when strict limitations on discharges in the downstream drainage system are imposed, the Authors suggested to assume three chained rainfall events. For green roofs, outflows rates are very low since they are only due to evapotranspiration, so it is reasonable to consider a chain of N rainfall events in the estimation of the runoff probability. In the calculation two different conditions have been recognized: maximum emptying time, which is the time needed to empty the retention capacity when it is full, lower and higher than minimum interevent time $IETD$. For Case 1, the pre-filling from previous rainfalls at the beginning of the given event has been excluded and the full storage capacity has been considered available; for Case 2, the possibility that the retention volume is partially filled from previous rainfalls has been analyzed. Moreover, a threshold runoff volume \bar{v} has been used.

Case 1: $w_{max}/Et \leq IETD$:

$$P_v = P(v > \bar{v}) = \int_{h=w_{max}+\bar{v}+Et-\theta}^{\infty} f_h \cdot dh \int_{\theta=0}^{\infty} f_{\theta} \cdot d\theta = \gamma \cdot e^{-\xi \cdot (w_{max}+\bar{v})} \tag{13}$$

whit: $\gamma = \frac{\lambda}{\lambda + Et \cdot \xi}$

Case 2: $w_{max}/Et > IETD$:

$$\begin{aligned} P_v = P(v > \bar{v}) &= \int_{\theta=0}^{\infty} f_{\theta} \cdot d\theta \int_{d=IETD}^{\infty} f_d \cdot dd \int_{h=w_{max}+\bar{v}+Et-\theta}^{\infty} f_h \cdot dh \\ &+ \sum_{i=2}^N \left[\int_{\theta=0}^{\infty} f_{\theta} \cdot d\theta \int_{d=IETD}^{\frac{w_{max}+\bar{v}}{Et}} f_d \cdot dd \int_{h=\frac{w_{max}+\bar{v}+(i-2) \cdot Et \cdot d}{i} + Et - \theta}^{\frac{w_{max}+\bar{v}+(i-2) \cdot Et \cdot d}{i} + Et - \theta} f_h \cdot dh \right] = \\ &= \gamma \cdot \left\{ e^{-\xi \cdot (w_{max}+\bar{v})} + \psi \cdot \sum_{i=2}^N \left[-(i-1) \cdot \beta_i \cdot e^{-\xi \cdot Et \cdot IETD \cdot \left(\frac{i-2}{i}\right) - \frac{\xi}{i-1} \cdot (w_{max}+\bar{v})} \cdot \beta_i^* \cdot e^{-\xi \cdot \left[Et \cdot IETD \cdot (i-1) + (w_{max}+\bar{v}) \right]} \cdot Et \cdot \beta_i \cdot \beta_i^* \cdot e^{\psi \cdot IETD \cdot (w_{max}+\bar{v})} \cdot \left(\frac{\psi}{Et} + \xi\right) \right] \right\} \tag{14} \end{aligned}$$

Where: $\beta_i = \frac{1}{\xi \cdot Et \cdot (i-2)} + \psi \cdot (i-1)$; $\beta_i^* = -\frac{1}{i \cdot \psi + (i-1) \cdot \xi \cdot Et}$.

Equation (14), that considers the possibility of pre-filling resulting from previous rainfall events, depends on the expected values of rainfall depth, rainfall duration, interevent time, as well as on the evapotranspiration rate, interevent time definition, maximum retention capacity, runoff threshold and the number of chained events. If the possibility of pre-filling resulting from previous events is excluded, then the whole retention capacity is available at the beginning of the given rainfall event (eq. 13), in this case the runoff probability distribution function only depends on the expected values of rainfall depth and rainfall duration,

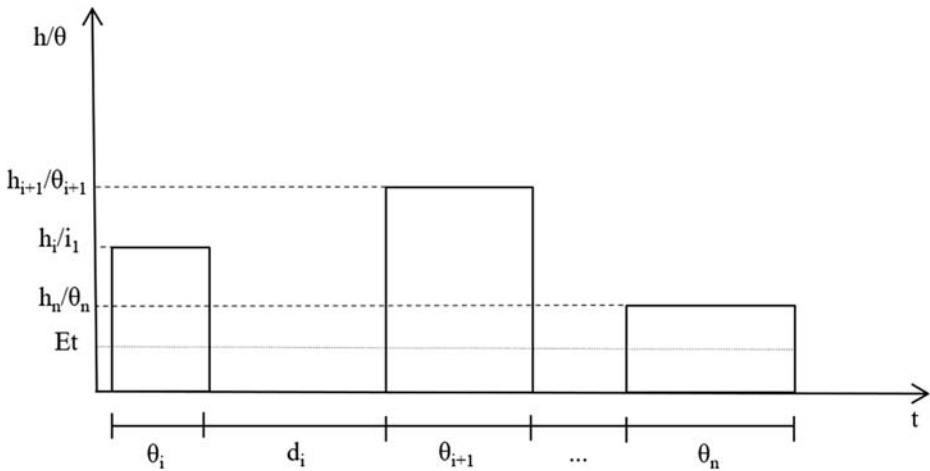


Fig. 2 Chain of rainfall events

evapotranspiration rate, maximum retention capacity and runoff threshold. Two characteristic variables to analyze green roof performance for stormwater control are the water retention capacity w_{max} and the evapotranspiration rate Et ; in the following pages, eqs. (13) and (14) have been studied with respect to these variables. If the water retention capacity tends to infinite, the probability of runoff from green roofs obviously tends to zero since the storage volume is unlimited (for $w_{max} \rightarrow \infty$, it results $P_v \rightarrow 0$); for the limit case in which the water retention capacity tends to zero, a single rainfall event is considered (eq. 13) and the runoff probability depends on the runoff threshold, the evapotranspiration rate and the expected values of rainfall depth and rainfall duration (for $w_{max} \rightarrow 0$, it results $P_v \rightarrow \gamma \cdot e^{-\xi \cdot \bar{v}}$). If the evapotranspiration rate tends to infinite, the runoff probability obviously tends to zero, since no rainwater is stored into the green roof (for $Et \rightarrow \infty$, it results $P_v \rightarrow 0$); if the evaporation rate tends to zero, the runoff probability is a function of the maximum retention capacity, the runoff threshold, the mean rainfall depth and the number of given events: that is for $Et \rightarrow 0$, it results in $P_v \rightarrow e^{-\xi \cdot (w_{max} + \bar{v})} + \sum_{i=2}^N \left[-e^{-\frac{\xi}{i-1} \cdot (\bar{v} + w_{max})} + e^{-\frac{\xi}{i} \cdot (\bar{v} + w_{max})} \right]$.

3 Application

Resulting formulas have been tested using the rainfall series recorded from Milano-Monviso rain gauge station during a period of twenty years. An $IETD = 10$ [hours] has been selected identifying $N = 979$ independent rainfall events. Mean μ , standard deviation σ and a coefficient of variation V of the three hydrologic parameters used in the modeling rainfall depth h , rainfall duration θ and interevent time d have been reported in Table 1.

The hypothesis of characteristic rainfall variables as exponentially distributed, while well suited for rainfall duration, is not perfectly respected for rainfall depth and interevent time, however the bias on results due to its use can be considered negligible, as discussed and tested by Becciu and Raimondi (2014). Table 2 contains the correlation coefficients with the three hydrological parameters.

Table 1 Mean, standard deviation and coefficient of variation of hydrologic parameters

	h [mm]	θ [hour]	d [hour]
μ [mm]	18,49	14,37	172,81
σ [mm]	21,33	14,81	223,89
V [-]	1,15	1,03	1,30

Interevent time results only weakly correlated to the other two variables, while the correlation between rainfall depth and duration is not negligible. However, to overcome the correlation with rainfall variables, copula functions have been recently introduced in the hydrologic research in order to broaden the multivariate inference capability (Abdollahi et al. 2019); for simplicity they have not been considered in this work. The runoff probability has been estimated by varying the maximum retention capacity w_{max} ; it has been calculated summing up the maximum retention capacity of the three layers composing the green roof (vegetation layer, growing medium and drainage layer): $w_{max} = w_{v,max} + w_{g,max} + w_{d,max}$. The maximum retention capacity of the vegetation layer is generally of a few millimeters. The maximum retention capacity of the growing medium layer can be estimated considering both the case of extensive green roofs covered with grass (with a thickness of a few centimeters) and the case of intensive green roofs covered with shrubbery and small trees (their thickness could be of some hundreds of centimeters). Considering a soil moisture content being variable between 0 and 100 [%], the maximum retention capacity of the growing medium layer can vary between 0 and 1000 [mm]. The maximum retention capacity of the drainage layer usually varies from 0 to 150 [mm], therefore the maximum retention capacity of the whole roof can vary between 0 and 1250 [mm]. In the calculation an extensive green roof has been considered and the maximum retention capacity underwent a variation between 0 and 200 [mm]. Different studies in literature tried to define the value of the evapotranspiration rate on green roofs: Lazzarin et al. (2005) estimated that evapotranspiration rates range from 0,69 to 6–9 [mm day] with typical values of 1–6 [mm day] using a Penman-Monteith model; Wolf and Lundholm (2008) found evapotranspiration rates varying between 0 to 5 [mm/day] analysing wet, intermediate, and dry conditions; Voyde et al. (2010) averaged evapotranspiration rates of about 2 [mm day] for the seven days following saturation. In the calculation two different values of evapotranspiration rates have been considered: $Et = 0,125$ [mm/hour] and $Et = 0,25$ [mm/hour], respectively corresponding to daily values of $Et = 3$ [mm/day] and $Et = 6$ [mm/day]. The runoff probability estimated by the application of eqs. (13) and (14) have been compared with the runoff frequency calculated by the continuous simulation of recorded data. For both cases, the runoff threshold \bar{v} has been set equal to zero. Figure 3 shows results for $Et = 0,125$ [mm/hour]; in this case the runoff probability estimated from the application of eqs. (13) and (14) adequately fits the runoff frequency calculated by the continuous simulation of recorded data, if four chained rainfall events are considered. If only two or three chained events are considered in eqs. (13) and (14), formulas underestimate the possibility of runoff from green roofs; this underestimation is due to the fact that for low outflow rates, the retention volume results partially pre-filled from previous rainfalls and it does not completely empty during interevent time.

Figure 4 reports results for $Et = 0, 25$ [mm/hour]; in this case the runoff probability estimated from the application of eqs. (13) and (14) fits the runoff frequency calculated by the continuous simulation of recorded data, if two chained rainfall events are considered. If three or four chained events are considered in eqs. (13) and (14), formulas overestimate the

Table 2 Correlation coefficients among hydrologic parameters

$\rho_{\theta,d}$ [-]	0,11
$\rho_{h,d}$ [-]	0,11
$\rho_{\theta,h}$ [-]	0,62

possibility of runoff from green roofs; when increasing outflow rate, the possibility that the retention volume results partially pre-filled from more than one previous rainfall events quickly decreases.

Equation (13) and (14) enable to obtain full stormwater control, since they relate the average return interval of runoff to the maximum retention capacity. For example, for an average return interval equal to 10 [years], it results: $w_{max} = 65$ [mm] for $Et = 0,125$ [mm/hour] and $w_{max} = 50$ [mm] for $Et = 0,25$ [mm/hour].

4 Conclusions

Results from the application of final equations for the estimation of runoff probability to a case study and their comparison with results obtained from the continuous simulation of recorded data have shown the efficiency of the proposed method. To consider more than two chained events in the estimation of the runoff probability is a very effective tool in the analysis of green roofs characterized by low outflow rates: in this case the possibility that the retention volume is partially pre-filled from previous events cannot be neglected, since it strongly influences the runoff probability. Moreover, the proposed method allows an optimum design of green roofs for stormwater control, since the retention capacity for different average return intervals can be estimated from the runoff probability distribution function. The key point that the Authors aim to develop in future studies is the estimation of the number of chained rainfall events that characterize runoff from green roofs, considering the variation of the evapotranspiration rate, and the runoff threshold.

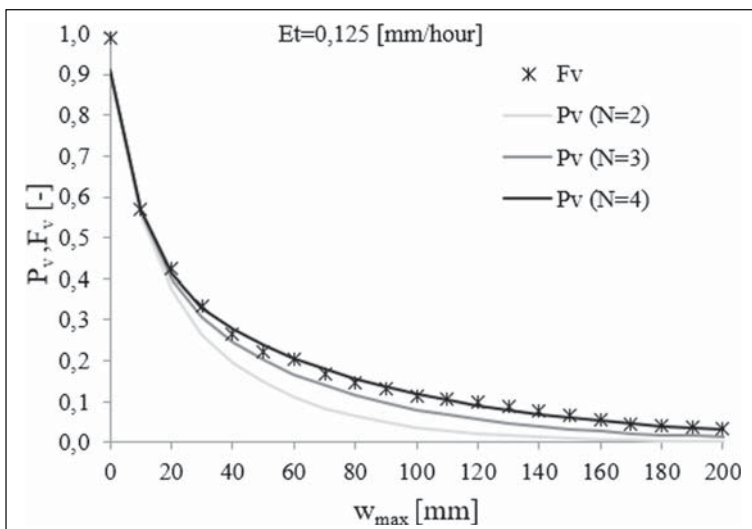


Fig. 3 Runoff probability and frequency for $Et = 0,125$ [mm/hour] and $N = 2 - 3 - 4$

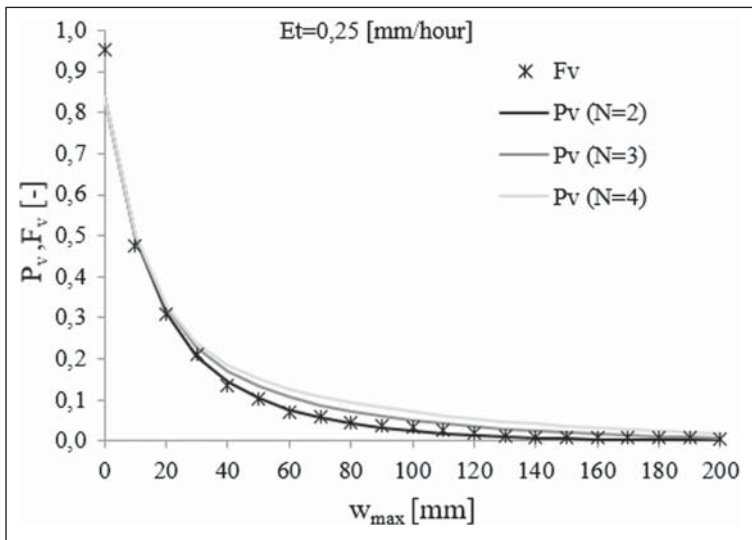


Fig. 4 Runoff probability and frequency for $E_t = 0, 25$ [mm/hour] and $N = 2 - 3 - 4$

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Availability of Data and Materials Authors agree with data transparency and undertake to provide any required data and material.

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Compliance with Ethical Standards

Conflict of Interest The authors declare that they have no conflict of interest.

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References

- Abdollahi S, Akhoond-Ali AM, Mirabbasi R, Adamowski JF (2019) Probabilistic event based rainfall-runoff modeling using copula functions. *Water Resour Manag* 33:3799–3814
- Adams BJ, Fraser HG, Howard CDD, Hanafy MS (1986) Meteorological data analysis for drainage system design. *J Environ Eng* 112(5):827–848
- Adams BJ, Papa F (2000) *Urban Stormwater management planning with analytical probabilistic models*. New York John Wiley & Sons
- Bacchi B, Balistocchi M, Grossi G (2008) Proposal of a semi-probabilistic approach for storage facility design. *Urban Water J* 5(3):195–208. <https://doi.org/10.1080/15730620801980723>
- Becciu G, Raimondi A (2014) Probabilistic analysis of spills from stormwater detention facilities. *WIT Trans Built Environ* 139:159–170. <https://doi.org/10.2495/UW140141>
- Becciu G, Raimondi A (2015) Probabilistic modeling of the efficiency of a stormwater detention facility. *Int J Sustainable Dev Plann* 10(6):795–805. <https://doi.org/10.2495/sdp-v10-n6-795-805>
- Becciu G, Raimondi A, Dresti C (2018) Semi-probabilistic design of rainwater tanks: a case study in northern Italy. *Urban Water J* 15(3):192–199. <https://doi.org/10.1080/1573062X.2016.1148177>
- Bedient PB, Huber WC (1992) *Hydrology and floodplain analysis*, 2nd edn. Addison Wesley, New York
- Bengtsson L (2005) Peak flows from thin sedum-moss roof. *Hydrol Res* 36(3):269–280. <https://doi.org/10.2166/nh.2005.0020>
- Berndtsson JC (2010) Green roof performance towards management of runoff water quantity and quality: a review. *Ecol Eng* 36:351–360. <https://doi.org/10.1016/j.ecoleng.2009.12.014>
- Carter TL, Rasmussen TC (2007) Hydrologic behavior of vegetated roofs. *J Am Water Resour Assoc* 42:1261–1274. <https://doi.org/10.1111/j.1752-1688.2006.tb05299.x>
- Chowdhury RK, Beecham S (2012) Characterization of rainfall spells for urban water management. *Int J Climatol* 33:959–967. <https://doi.org/10.1002/joc.3482>
- Eagleson PS (1978) Climate, soil and vegetation. The distribution of annual precipitation derived from observed storm sequences. *Water Resour Res* 14(5):713–721. <https://doi.org/10.1029/WR014i005p00713>
- Getter KL, Rowe DB, Andresen JA (2007) Quantifying the effects of slope on extensive green roof stormwater retention. *Ecol Eng* 31(4):225–231. <https://doi.org/10.1016/j.ecoleng.2007.06.004>
- Gnecco I, Palla A, Lanza LG, La Barbera P (2013) The role of green roofs as a source/sink of pollutants in storm water outflows. *Water Resour Manag* 27:4715–4730
- Guo Y (2016) Stochastic analysis of hydrologic operation of green roofs. *J Hydrol Eng* 21(7):04016016. [https://doi.org/10.1061/\(ASCE\)HE.1943-5584.0001371](https://doi.org/10.1061/(ASCE)HE.1943-5584.0001371)
- Guo Y, Gao T (2016) Analytical equations for estimating the total runoff reduction efficiency of infiltration trenches. *J Sustainable Water Built Environ* 2(3):06016001. <https://doi.org/10.1061/JSWBAY.0000809>
- Guo Y, Zhang S (2014) Runoff reduction capabilities and irrigation requirements of green roofs. *Water Resour Manag* 28:1363–1378. <https://doi.org/10.1007/s11269-014-0555-9>
- Hakimdarav R, Culligan PJ, Finazzi M, Barontini S, Ranzi R (2014) Scale dynamics of extensive green roofs: quantifying the effect of drainage area and rainfall characteristics on observed and modeled green roof hydrologic performance. *Ecol Eng* 73:494–508. <https://doi.org/10.1016/j.ecoleng.2014.09.080>
- Herrera J, Flamant G, Gironás J, Vera S, Bonilla CA, Bustamante W, Suárez F (2018) Using a hydrological model to simulate the performance and estimate the runoff coefficient of green roofs in semiarid climates. *Water* 10(2):198. <https://doi.org/10.3390/w10020198>
- Lazzarin RM, Castellotti F, Busato F (2005) Experimental measurements and numerical modeling of a green roof. *Energy Build* 37(12):1260–1267. <https://doi.org/10.1016/j.enbuild.2005.02.001>
- Lee EH, Choi YH, Kim JH (2019) Real-time integrated operation for urban streams with centralized and decentralized reservoirs to improve system resilience. *Water* 11(1):69. <https://doi.org/10.3390/w11010069>
- Lee EH (2019) Advanced operating technique for centralized and decentralized reservoirs based on flood forecasting to increase system resilience in urban watersheds. *Water* 11:1533. <https://doi.org/10.3390/w11081533>
- Li J, Deng C, Li Y, Li Y, Song J (2017) Comprehensive benefit evaluation system for low-impact development of urban Stormwater management measures. *Water Resour Manag* 33:4745–4758
- Li Y, Babcock RW Jr (2014) Green roof hydrologic performance and modeling: a review. *Water Sci Technol* 69(4):727–738. <https://doi.org/10.2166/wst.2013.770>
- Li Y, Babcock RW Jr (2015) Modeling hydrologic performance of a green roof system with HYDRUS-2D. *J Environ Eng* 141(11):04015036. [https://doi.org/10.1061/\(ASCE\)EE.1943-7870.0000976](https://doi.org/10.1061/(ASCE)EE.1943-7870.0000976)
- Locatelli L, Mark O, Mikkelsen PS, Ambjerg-Nielsen K, Bergen JM, Binning PJ (2014) Modelling of green roof hydrological performance for urban drainage applications. *J Hydrol* 519:3237–3248. <https://doi.org/10.1016/j.jhydrol.2014.10.030>

- Nawaz R, McDonald A, Postoyko S (2015) Hydrological performance of a full-scale extensive green roof located in a temperate climate. *Ecol Eng* 82:66–80. <https://doi.org/10.1016/j.ecoleng.2014.11.061>
- Palermo A, Turco M, Principato F, Piro P (2019) Hydrological effectiveness of an extensive green roof in a Mediterranean climate. *Water* 11(7):1378. <https://doi.org/10.3390/w11071378>
- Palla A, Gnecco I, Lanza LG (2012) Compared performance of a conceptual and a mechanistic hydrologic model of a green roof. *Hydrol Process* 26:73–84. <https://doi.org/10.1002/hyp.8112>
- Peng LL, Jim CY (2015) Seasonal and diurnal thermal performance of a subtropical extensive green roof: the impacts of background weather parameters. *Sustainability* 7(8):11098–11113. <https://doi.org/10.3390/su70811098>
- Piro P, Carbone M, De Simone M, Maiolo M, Bevilacqua P, Arcuri N (2018) Energy and hydraulic performance of a vegetated roof in sub-Mediterranean climate. *Sustainability* 10(10):3473. <https://doi.org/10.3390/su10103473>
- Raimondi A, Becciu G (2011) An analytical probabilistic approach to size cisterns and storage units in green buildings. *Proc CCWI 2011 Exeter 5-7 September 2011 volume 1* (ISBN: 0953914089;978-095391408-1)
- Raimondi A, Becciu G (2014) Probabilistic modeling of rainwater tanks. *Procedia Eng* 89:1493–1499. <https://doi.org/10.1016/j.proeng.2014.11.437>
- Raimondi A, Becciu G (2017) On the efficiency of stormwater detention tanks in pollutant removal. *Int J Sus Dev Plann* 12:144–154. <https://doi.org/10.2495/SDP-V12-N1-144-154>
- Stovin V, Vesuviano G, De-Ville S (2017) Defining green roof detention performance. *Urban Water J* 14(6): 574–588. <https://doi.org/10.1080/1573062X.2015.1049279>
- United Nations (2018) *World Urbanization Prospects, The 2018 Revision*
- U.S. Environmental Protection Agency (1986) *Methodology for analysis of detention basins for control of urban runoff quality* 440(5):87–001. Washington, DC: EPA
- Voyde E, Fassman E, Simcock R (2010) Hydrology of an extensive living roof under sub-tropical climate conditions in Auckland, New Zealand. *J Hydrol* 394(3–4):384–395. <https://doi.org/10.1016/j.jhydrol.2010.09.013>
- Wang J, Zhang S, Guo Y (2019) Analyzing the impact of impervious area disconnection on urban runoff control using an analytical probabilistic model. *Water Resour Manag* 33:1753–1768
- Wolf D, Lundholm JT (2008) Water uptake in green roof microcosm: effects of plant species and water availability. *Ecol Eng* 33(2):179–186. <https://doi.org/10.1016/j.ecoleng.2008.02.008>
- Zhang S, Guo Y (2013) Analytical probabilistic model for evaluating the hydrologic performance of green roofs. *J Hydrol Eng* 18(1):19–28. [https://doi.org/10.1061/\(ASCE\)HE.1943-5584.0000593](https://doi.org/10.1061/(ASCE)HE.1943-5584.0000593)
- Zhang S, Guo Y (2014a) An analytical equation for evaluating the stormwater volume control performance of permeable pavement systems. *J Irr Drain Eng* 141(4):06014004. [https://doi.org/10.1061/\(ASCE\)IR.1943-4774.0000810](https://doi.org/10.1061/(ASCE)IR.1943-4774.0000810)
- Zhang S, Guo Y (2014b) Stormwater capture efficiency of bioretention systems. *Water Resour Manag* 28(1): 149–168. <https://doi.org/10.1007/s11269-013-0477-y>