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Settlement analysis of the masonry umbrella vault of the Masegra Castle

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Abstract **– In this work, the first results on the settlement analysis of the masonry umbrella vault located in the Masegra Castle (Sondrio, Italy) are presented. An umbrella vault is a particular type of cross vault in which the high number of sails is disposed according to an umbrella-shaped configuration. For geometry reasons, these vaults presents typically a good load-bearing capacity when subjected to vertical and horizontal loads. Therefore, the most critical issue is given by settlements of the supporting walls. A geometrical model based on NURBS surfaces has been derived starting from a detailed point cloud. A recently published method based on an adaptive kinematic limit analysis is adopted to find the cracks configuration deriving from a given base settlement.**

I. INTRODUCTION

The modern theory of masonry vaults derives mainly from Heyman’s studies [1], in which masonry has been idealized as no-tension material (i.e. null tensile strength, crushing and sliding failures avoided). This theory opened access to the study of masonry arches through limit analysis techniques [2]. The three-dimensional extension of the statics of the masonry arches allowed to extend the same methodology to the study of the most spread typologies of masonry vaults [3].

One of the most complex typologies of masonry vaults consists of stellar vaults. Stellar vaults are a specific kind of Gothic ribbed vault, that can be considered as an extension of the traditional cross vault with more complex ribbed systems and a higher number of sails [4]. The complex geometry of stellar vaults results in a mechanical behavior quite difficult to predict, which usually requires advanced numerical techniques to be studied [5,6].

A masonry umbrella vault can be considered as a particularly complex type of stellar vault, or cross vault, where sails are disposed around the center of the vault resulting in an umbrella shape on the horizontal projection. The umbrella vault of the Masegra Castle is described by a total of eight sails, with four half-cross vaults added at the corner, supported by four walls and resulting in a quadrangular horizontal plan whose side is 4 m length (see pictures in Fig. 1(a)). Four half-cross vaults are localized at the corners. Some in-situ investigations revealed that one of the four walls supporting the vault lacks a foundation system. As a confirmation of this, the cracks typical of vertical settlement phenomena have been observed in the wall and also in the umbrella vault (Fig. 1(b)).

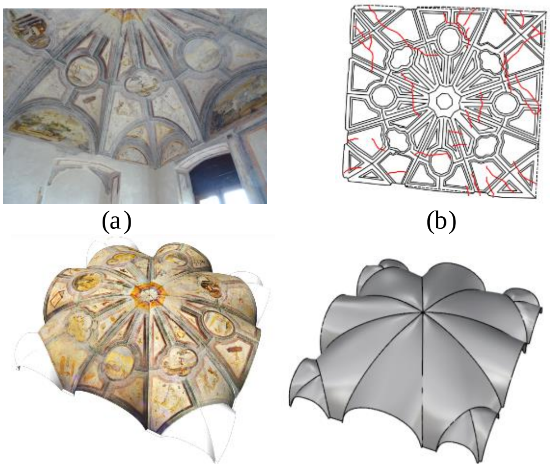
A geometrical survey was carried out by means of photogrammetric and laser scanner techniques, and a dense point cloud was acquired [7]. Starting from here, a 3D model of the umbrella vault based on NURBS surfaces has been derived, see Fig. 1(c). NURBS (Non-Uniform Rational B-Spline, [8]) surfaces are parametric surfaces commonly adopted within CAD and BIM environments that allow representing in exact way and with low computational efforts complex curved geometries. The NURBS modeling techniques has proven to be very suited for historical masonry constructions [9]. Therefore, a novel limit analysis procedure based on the utilization of NURBS surfaces has been recently presented specifically for historical masonry vaults [10].

The geometry of this vault, characterized by a low height-to-span ratio, makes its vulnerability to horizontal actions negligible, differently from most of the masonry buildings [11,12]. Therefore, in this work, the attention is focused on the structural response to the settlement induced by one of the supporting walls.

Previous studies about curved structures analyzed under settlements can be found in [13–16]. In a recent paper published by some of the Authors, it has been exhaustively exposed that the problem of settlement in masonry structures discretized with rigid elements can be treated as a unilateral contact problem [17]. The structural response to a given differential settlement is a discontinuous displacement field where jump of displacements (i.e. cracks) between adjacent rigid elements occur to maximize the work of external loads or to minimize the work of the reaction forces for the applied settlement. In this work, the vault is discretized few triangular NURBS rigid elements, with internal dissipation allowed at interfaces only. Then, the mesh is iteratively adjusted until the maximum of the external work is reached. A similar application to masonry walls and façades can be found in [18].

II. NURBS ADAPTIVE KINEMATIC LIMIT ANALYSIS

The method here presented is an extension to the settlement problem of the adaptive NURBS-based limit analysis procedure firstly published in [10] and successively applied to several types of curved masonry structures (see for instance stellar vaults [6], historical domes [19], and masonry buildings including vaulted elements [20,21]).


(a) (b)

(c)

Fig. 1. (a) The masonry umbrella vault of Masegra Castle, (b) observed crack pattern, and (c) obtained NURBS model.

The masonry vault is discretized by using a reduced number of NURBS rigid elements. Differently by Finite Element, the use of NURBS elements allows to maintain unvaried the real geometry of the vault even by using discretization composed of very few elements. By using NURBS properties, the subdivision is conducted directly within the parametric domain of the NURBS surface. In this way, each element is a trimmed NURBS surface obtained from the initial one, and the overall geometry remains not affected by the discretization (for a detailed dissertation on NURBS, we refer to [8]).

A kinematic limit analysis formulation is defined. Each element is idealized as a rigid block whose kinematic is described through 6 degrees of freedom in the three-dimensional Euclidean space. In order to correctly represent the behavior of vaults, a rigid-plastic behavior is assigned at interfaces between adjacent elements. The rigid plastic behavior is preferred to the traditional no-tension material to reproduce even crushing and sliding failures, which cannot be excluded a priori for lowered arches and vaults. Therefore, a three-dimensional failure surface, describing a Mohr-Coulomb failure criterion enriched with a tension cut-off and a linear cap in compression, is applied (see Fig. 2). In this way, interfaces represent the possible fracture lines, able to reproduce both flexural and sliding failures, on which depends the discontinuous displacement field.

Given an initial discretization into few rigid blocks, the discontinuous displacement field can be derived by solving the following linear programming problem:

$$\min\{-\mathbf{q}^{\mathbf{T}}\mathbf{d}+\mathbf{D_F u}\}$$

$$\text{subjected to } \begin{cases} [\mathbf{A}+\mathbf{B}]\mathbf{d} - \mathbf{A_F u} = \mathbf{0} \\ \mathbf{Cd} = \hat{\mathbf{u}}_{\mathbf{0}} \end{cases}$$

where $\mathbf{d}$ is the vector containing the unknown centroids displacements of each rigid block, $\mathbf{u}$ is the vector containing the unknown displacement jumps at interfaces, $\mathbf{q}^{\mathbf{T}}\mathbf{d}$ is the external work, $\mathbf{D_F u}$ is the internal dissipated work, $\mathbf{A}$, $\mathbf{B}$, $\mathbf{A_F}$ and $\mathbf{C}$ are matrixes representing kinematic compatibility constraints and boundary conditions, and finally $\hat{\mathbf{u}}_{\mathbf{0}}$ is the vector of known displacements (i.e. the settlement) at the boundaries. Let us observe that, in order to formulate dissipated works and kinematic constraints through linear equations, the hypothesis of small displacements must be adopted.

As a consequence of the rough discretization adopted, the solution of the linear programming problem depends on the assumed position of possible fracture lines. In order to find the absolute minimum of the objective function, i.e. to obtain the real position of fracture lines deriving from an occurred settlement, a mesh adaptation procedure is applied. The mesh adjustment is here conducted through a meta-heuristic algorithm. A Genetic Algorithm [22] is here adopted, even if it has been recently observed that other meta-heuristic procedures can be followed [23].

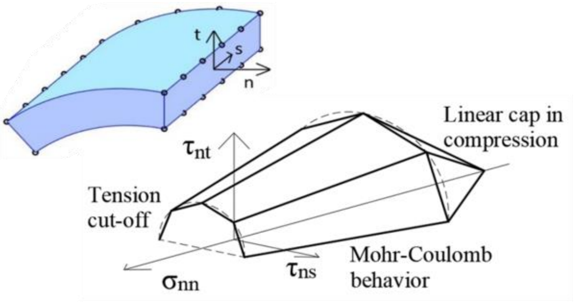


Fig. 2. NURBS curved element and 3D failure surface assigned to interfaces.

III. ANALYSES AND RESULTS

A mesh composed of triangular curved elements is adopted. The initial subdivision of the singular sail (see Fig. 3) has been decided considering the statics of cross vaults [3]. In order to take into account the typical arch-behavior of cross vaults' free edges, the external boundaries of each sail have been subdivided through three nodes. The internal disposition of interfaces reproduces the Sabouret-cracks commonly observed on these vaulted elements. In the mesh adaptation scheme, each node is moved along the current edge or within the sail: as a result, the mesh adjustment of the single sail is managed through 7 parameters, each one defining the position of the node in the parametric space (movements of each node are depicted in Fig. 3). Provided that boundaries between different sails have to be subdivided according to the same node, the mesh adaptation is performed by using a total of 84 parameters.

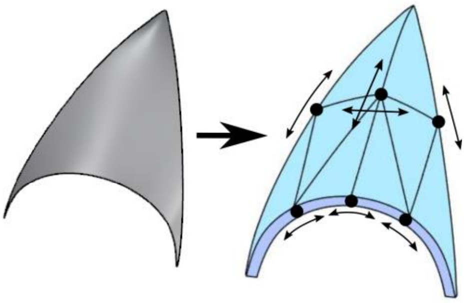


Fig. 3. Mesh by triangular curved elements and adjustment scheme on the single sail.

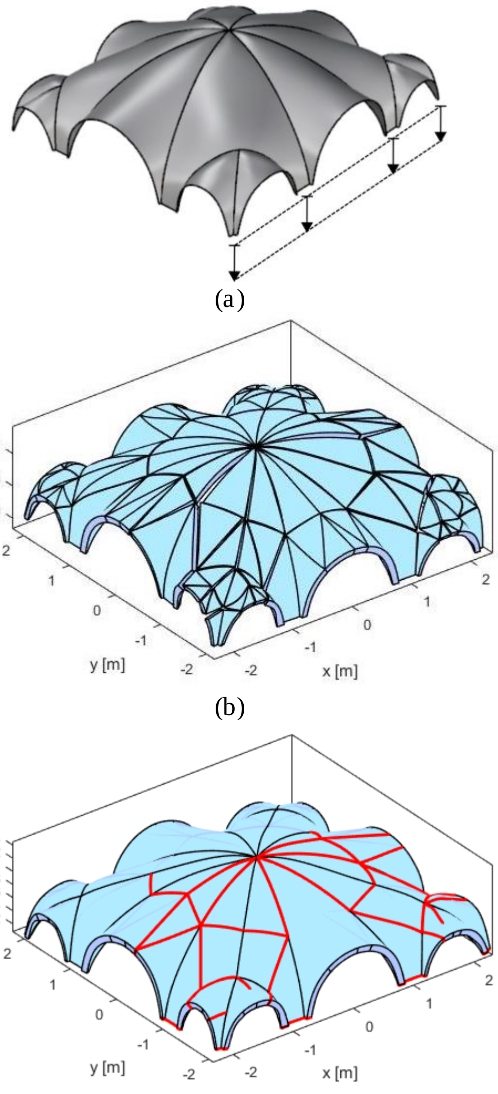
The lack of a foundation system under one of the supporting walls and the observed crack pattern make quite realistic the hypothesis of vertical settlements occurred along the corresponding perimeter line. By observing cracks on this wall, it is not clear of the settlement is uniform or localized in the middle of the perimeter line. Moreover, the presence of a wide opening on one of the adjacent supporting walls suggests the possibility that vertical settlements occurred according to a complex configuration distributed along more perimeter sides. Therefore, the first analysis is conducted considering a uniform vertical settlement along one perimeter side. However, further configurations will be analyzed in upgoing papers.

Without further information, the vault thickness has been supposed to be equal to 5 cm. The assigned Mohr-Coulomb failure surface has been derived by using the following resistance parameter: null value of tensile stress, compressive stress equal to 2.6 MPa, cohesion equal to 0.1 MPa, and a friction angle of 27°. Finally, the specific weight has been assumed equal to 18 kN/m^3. The presence of backfill at the extrados of the vault is quite uncertain, thus no additional vertical load has been applied in the following analysis.

A schematization of the applied differential settlement is shown in Fig. 4(a). The value of the vertical displacement applied is equal to 1 cm. The results obtained are depicted in Fig. 4(b, c). For sake of clarity, the deformed structure (Fig. 4(b)) has been depicted by scaling displacements according to a factor equal to 50. In Fig. 4(c) the main cracks which open because of the applied settlements are shown.

It can be observed that most of the cracks obtained are localized on the corners. The external boundaries of the corner cross vaults show the typical behavior of the arch subjected to differential vertical settlements. No cracks occur on the sail located between these two corners, since this portion moves as a rigid body.

Good correspondence is observed between cracks obtained through the presented procedure and the pattern observed in-situ. However, more diffused fractures can be noted in the real crack pattern (Fig. 1(b)). Therefore, considering also the presence of big openings in some of the supporting perimeter walls, it can be hypothesized that the real settlement occurred is characterized by a more complex shape.



(a)

(b)

(c)

Fig. 4. Settlement of one supporting wall: (a) schematization of the applied settlement, (b) deformed structure (displacement scaled by 50), and (c)

localization of the highest relative displacements.

IV. CONCLUSIONS

The first analysis of the masonry umbrella vault of the Masegra Castle (Sondrio, Italy) subjected to differential settlement has been presented. The vault has been studied by using a novel adaptive NURBS kinematic limit analysis procedure. Starting by an initial discretization into few curved rigid elements, the discontinuous displacement field deriving from an applied settlement is derived through a simple linear programming formulation. Then, the correct position of cracks is derived through an automatic mesh adaptation scheme governed by a Genetic Algorithm. The procedure has allowed to obtain reliable results in a fast and efficient way. In particular, the use of NURBS allowed to deal with the real geometry of such a complex masonry vault by maintaining low the computational effort and without introducing simplifications. In future research, different settlement configurations will be investigated. Moreover, some new automatic procedures aimed at finding the settlement shape from which derives an observed crack pattern will be proposed.

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