Statistical control of complex geometries, with application to Additive Manufacturing

Controllo statistico di geometrie complesse, con applicazioni all'Additive Manufacturing

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Abstract New production processes, as Additive Manufacturing (AM), allow the production of objects and shapes characterized by a growing complexity, especially when compared with those normally manufactured in traditional production processes. It is thus necessary to develop methods which can be applied to the study of the variability of a dataset whose elements are manufactured realizations of the same nominal model, which can carry a very high degree of complexity. In the present work, we propose a method allowing modeling a wide variety of possibly local geometric deviations of the manufactured object with respect to the nominal model. A way of identifying and monitoring these kind of deviations, based on Principal Component Analysis in Hilbert spaces, is proposed as well. The proposed method is tested on a real datased of items produced via AM.

Abstract Lo sviluppo di nuovi metodi di produzione, quali l'Additive Manufacturing (AM), rende realizzabili forme geometriche sempre più articolate e molto più complesse di quelle normalmente realizzate in processi produttivi più tradizionali. Si rende dunque necessario lo sviluppo di metodi statistici applicabili all'esplorazione della variabilità di dataset in cui ogni osservazione è una realizzazione effettiva-

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mente prodotta di un modello nominale, che può presentare un elevato grado di complessità. Nel presente lavoro viene proposto un metodo che consente, in linea di principio, di modellare un insieme molto ampio di deviazioni geometriche, anche locali, degli oggetti prodotti rispetto al modello nominale, e di rilevare successivamente tali deviazioni via Analisi delle Componenti Principali in spazi di Hilbert. Tale metodo viene altresì testato su un dataset di oggetti reali prodotti via AM.

Key words: Statistical Process Control, Object Oriented Statistics, Compositional Data Analysis, Functional Data

1 Introduction

Additive Manufacturing processes are becoming more and more important, since they are facing a continuous technological improvement, and they allow the realization of a wide variety of shapes and geometries. The production of complex shapes (e.g, objects with a complete asimmetry, or with an internal lattice structure) constitutes an interesting challenge for Statistical Process Control. The necessity of adequate methods, suitable to the statistical control of this kind of objects, is moreover driven by the fact that the industrial sectors in which AM presents the greatest potentialities are the aerospace and the biomedical sectors ([10]), both areas in which the identification of production errors is foundamental.
After the manufacturing phase, data usually come as reconstructed shapes (e.g., via tomography), in the form of a point cloud, possibly associated to a triangulated mesh. Point clouds are a class of data whose statistical process monitoring has been studied by several authors ([6], [7], [11], [12]). In all cited works, points in the reconstructed point cloud were associated with their deviation from the nominal geometry (usually a CAD model), hence producing a *deviation map*. However, the deviation map generated by the distances of the points in the reconstructed point cloud is usually different from the deviation map generated by the distances of the points in the nominal point cloud from the reconstructed object. Different information are, in general, carried by the corresponding deviation maps, as follows from the formal definition of Hausdorff distance between subsets of a metric space, which was firstly introduced, in a completely different context, by Hausdorff in [1]. In [15] we propose to summarize all the information content about the differences between an object and its corresponding model using two *deviation maps*. These maps are then used in a monitoring method based on a functional Principal Component Analysis for compositional data (SFPCA, [8]), allowing to exploit this information content completely. In this communication, performances of the method shall be showcased on a dataset of manufactured objects, produced via Additive Manufacturing at the Department of Mechanical Engineering at Politecnico di Milano. These objects will be briefly introduced in the next section.

2 Test Dataset: a motivating example

The dataset on which all the analyses are carried out consists of a sample of 16 meshes, resulting from the tomography of plastic objects produced via Additive Manufacturing at the Department of Mechanical Engineering of Politecnico di Milano. These objects are trabecular egg-shaped shells, and they constitute a quite realistic example of the geometric complexity that can be achieved by Additive Manufacturing processes. We show the nominal model in Fig. 1.

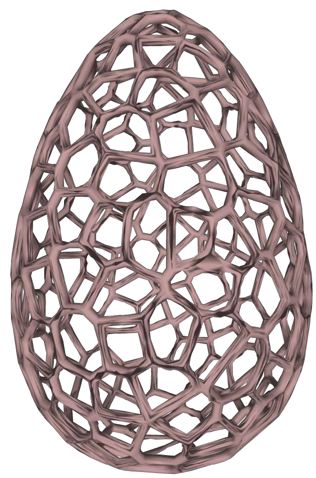

Fig. 1 Nominal CAD model for the produced trabecular structures.

Among the sixteen produced items, we have objects with no evident defects (In Control), as well as items affected by different kinds of geometrical deviations (Out of Control). In Fig. 2 we show two realizations of defective elements, on the left an item affected by irregularities on the geometrical structure, on the right a case in which a struct is missing.

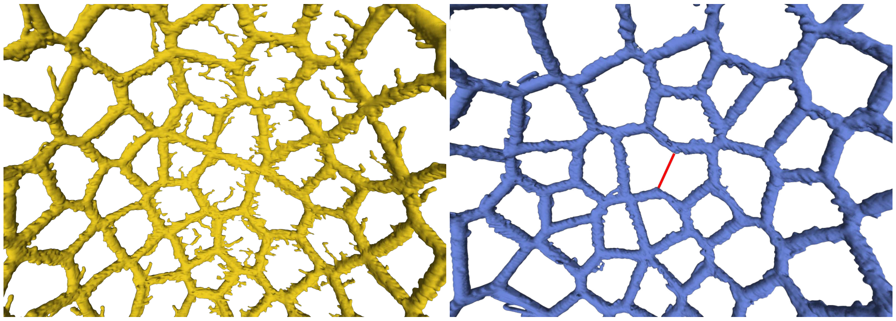

Fig. 2 Different kind of geometric deviations in the manufactured objects

It is worth noting that these geometrical deviations are an example of the complementariety of the two deviation maps mentioned in Section 1. Indeed, while the local irregularities can be detected if one measures the distance of points of the manufactured object from the nominal model, a missing struct is visible only when considering distance of points of the nominal model from the manufactured object.

3 Methodology: a density representation of Hausdorff maps

The general problem of analyzing the differences between two meshes or point clouds can be efficiently stated by referring to the definition of Hausdorff distance, given below (see [2] for a deeper insight).

Definition 1. Let X, Y be two closed, bounded, non-empty subsets of a metric space (U, d). Their Hausdorff distance is defined as

$$d_H(X,Y) := \max \left\{ \sup_{x \in X} \inf_{y \in Y} d(x,y), \sup_{y \in Y} \inf_{x \in X} d(x,y) \right\} \tag{1}$$

This definition implicitly introduces two maps, which are

$$d_X : Y \to \mathbb{R}^+, \, d_X(y) := \inf_{x \in X} d(x,y) \tag{2}$$

and

$$d_Y : X \to \mathbb{R}^+, \, d_Y(x) := \inf_{y \in Y} d(x,y) \tag{3}$$

which will be called, as already done, *deviation maps*. A key idea underlying the method we propose in [15] is that both deviation maps should be analyzed to retrieve all possible differences between a reconstructed geometry and the nominal one. In the case of a dataset of point clouds, these maps are easily computed, since they are represented by finite, discrete sets. From each map we estimate a density obtained by dropping the spatial information carried by the data. These are f_X and f_Y, representing the PDFs of the values of d_X and d_Y respectively (or of a monotone transformation of these). The dataset of PDFs can be analyzed on the common support of the densities through SFPCA ([8]), which extends Functional Principal Component Analysis ([3]) to probability density functions. The natural space in which PCA of density functions can be coherently performed has been defined by ([4]), and laterly studied in other works ([5], [9]). It is the space $B^2(a,b)$, defined as the set (of equivalence classes) of positive functions with square-integrable logarithms

$$B^2(a,b) := \{f > 0 \text{ s.t. } \log f \in L^2(a,b)\} \tag{4}$$

where the equivalence relation is defined as

$$f_1 = f_2 \iff \exists c > 0 \text{ s.t. } f_1 = c f_2 \text{ a.e..} \tag{5}$$

In $B^2(a,b)$, operations and inner product are defined as

$$f_1 \oplus f_2 = f_1 f_2, \tag{6}$$

$$\alpha \odot f_1 = f_1^{\alpha}, \alpha \in \mathbb{R} \tag{7}$$

$$\langle f_1, f_2 \rangle_{B^2} = \frac{1}{2(b-a)} \iint \log \frac{f_1(x)}{f_1(y)} \log \frac{f_2(x)}{f_2(y)}\, dx\, dy. \tag{8}$$

This space is a useful extension of the Aitchison geometry (see [13] for a complete reference), in which PCA can be applied in the form of SFPCA ([8]), and it can be practily applied relying on the centered log-ratio transformation, as in [14]. Dimensionality reduction via SFPCA allows reducing the monitoring problem to a multivariate one based on the scores along the first K principal components. In [15], we build a control chart scheme based on the scores vectors, which allows detecting out of control conditions for the produced objects.

4 Results and conclusion

SFPCA, based on a geometric structure coherent with the features of probability density functions, provides interpretation tools which are quite powerful when coupled with the geometric iterpretation of the introduced deviation maps. This dimensional reduction technique provides a very good compromise between spatial simplification (all spatial information is dropped) and loss of information (the functional approach allow to performs data analysis directly on density functions). The control chart scheme based on SFPCA proves to be very effective in detecting out-of-control conditions deriving from either widespread or localized defects. Indeed, the method performs well also in presence of local defects affecting just a small part of the whole point cloud, exhibiting then good detection power.

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