Nexus 2014
Ph.D. Day
presentations

June 9-12
Ankara, Turkey

Nexus 2014

Relationship between Architecture and Mathematics

MIDDLE EAST TECHNICAL UNIVERSITY
DEPARTMENT OF ARCHITECTURE

Nexus 2014 : Relationships Between Architecture and Mathematics

Ph.D. Day Presentations

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Relationships Between Architecture and Mathematics

Ph.D. Day Presentations

Edited by Arzu Gönenç Sorguç

Proceedings of Nexus 2014 : Relationships Between Architecture and Mathematics Conference, Ph.D. Day Presentation Session

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Foreword

The year of 1956 could be regarded as a significant demarcation point as far as the contemporary history of Turkey's higher education has been considered; Middle East Technical University (METU) akin to those of prestigious examples that of the United States was introduced to Turkey's cultural landscape as part of the nation's a-century-long-yearning for a full-fledge modernization. Originally planned as a UN project, academically structured by the University of Pennsylvania and carefully exercised by the Turkish government METU was in fact regarded as a new model in higher education and became fully operational in the following years with an increasing attempt to make it "a center of excellence" in higher education, research and societal endeavors. Since then METU has already proven itself not solely as a leading institution in its region but also as a globally recognized and internationally acclaimed academic environment – after 60 years of its establishment METU can now be regarded as a success story as it has long been listed as one of the top institutions in many of the ranking systems world-wide. In this short history of METU, Department of Architecture as one of the founding departments of the original establishment of the post-war years had a significant role in research and education as it radically changed the nation's course of architectural culture. Along with those of Francophone and Germanic school systems all established in Istanbul were forced to challenge a new model that of truly American, which had its unique curricula, pedagogy and an un-orthodox environment, all translated from the Bauhaus model via the East Coast. As part of its new curricula the school's philosophy was simple: not completely fine arts or sole technical talent architecture is rather a multi-faceted discipline that constantly requires analytical thinking, individual rationale, social responsibility and the critical thought. Of the related fields and disciplines, mathematics was thought to be important because it was believed that as the core science of all the pure logic of computation could pave the way through which the said qualities would be achieved at all levels. It is no coincidence that today METU Department of Architecture receives the top entries of all high school students who take the central university exam annually: e.g. in 2013, of 1.8 million entries the department accepted only 72 students ranked within top 8000 with the best mathematic scores. It is now a nationally accepted fact that METU Department of Architecture enrolls the best students in each year with superb competences in mathematics and the related basic sciences. We believe this is a good indication that METU's founding principles as well as the department's prime philosophy on analytical thinking, rationale, social research and the critical thought still remains, making a strong tradition of its own. In this respect, I believe that the upcoming conference of NEXUS at METU is not an utterly new enterprise but rather a remainder of the school's long-lasting tradition that fosters mathematics and architecture not separate domains but rather an interrelated sphere, which constantly enhances a unique co-existence, exclusive in nature.

About Middle East Technical University

Established in 1956 Middle East Technical University (METU) is one of the most renowed universities in Turkey. According to the recent rankings of the Times Higher Education, METU is ranked in the top 100 universities by reputation in the world. In addition METU is the only Turkish university which is included among the "Highest Group" in the international university rankings by QS, Webometrics, HEEACT, URAP and Leiden. The campus of METU is located about 5 km's away from the Ankara city center along the Ankara-Eskişehir highway. METU currently has about 26,500 students of which %30 are enrolled to graduate programs. METU hosts over 1,700 international students from nearly 94 countries studying toward myriad of academic degrees. The campus has dormitory capacity for approximately 6,000 students who benefit from a shopping area, banks, post office, many restaurants and cafes and also a wide variety of sports facilities. METU campus also hosts the biggest technology park (The METUTECH) in Turkey. About 3,750 people most of who are R&D personnel are employed in about 250 firms that operate at METUTECH.

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Nexus 2014: Relationships Between Architecture and Mathematics

9-12 June 2014

The 10th international, interdisciplinary Nexus conference for architecture and mathematics will take place 9-12 June 2014 in Ankara, Turkey, hosted by the Department of Architecture of Middle East Technical University.

This is the tenth conference in the Nexus series, following those in 1996 (Fucecchio, Florence, Italy), 1998 (Mantua, Italy), 2000 (Ferrara, Italy), 2002 (Óbidos, Portugal), 2004 (Mexico City), 2006 (Genoa, Italy), 2008 (San Diego, USA), 2010 (Porto, Portugal) and 2012 (Milan, Italy). The papers from the conference will be published in future issues of the Nexus Network Journal.

The Nexus conferences are dedicated to explorations of the relationships between architecture and mathematics, through a broad panorama of topics. In the past, these topics have included: symmetry in architecture, projective and descriptive geometry, soap bubbles and minimum surfaces, systems of proportions, geometry and urban design, the development of structural forms, the use of arithmetical, geometrical, and harmonic means, calculations of domes and arches, linear algebra and geometric forms, music theory and architecture, fractals in architecture, etc. Presentations have also included discussions of the work of individual architects, such as Alberti, Palladio, Frank Lloyd Wright; historical periods, such as Roman, Incan and Renaissance; the application of particular branches of mathematics to architectural design, such as geometry, topology and algebra.

link : http://nexus2014.org/

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Ph.D. Day Poster Presentations

The Order Complex: Generating Triply Periodic Minimal Surface Structures

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Keywords: Algorithms, Generative Software, Minimal Surfaces, Patterns

A minimal surface is a geometry concept which refers to a surface with zero mean curvature that has the property of having the smallest area within a given boundary. Studies on minimal surfaces was traced back 250 years ago (1744) with Euler as the forerunner, whose research focused on the rotation surface with minimal area. Since then the research of minimal surfaces has been active for several hundred years. In 1760, Lagrange derived the equation minimal surfaces satisfy. The well-known Plateau (1855-90) problem is the existence problem of constructing a piece of surface that interpolates the given boundary curve and has minimal area. This problem, though raised by Lagrange in 1760, was named after Plateau, who created several special cases experimenting with soap films and wire frames. In 1856 Schwarz discovered and investigated the first triply periodic minimal surfaces (TPMS), a new classes of surfaces, obtained by a series of rotational symmetries. The next development in TPMS did not take place until the 1960s, when Schoen investigated for NASA whether surfaces of this type might be of use as space structures, and found more than a dozen new example.

TPMS have a large amount of different configurations, reaching very high levels of complexity by forming infinite repetitive crystalline structures. Furthermore they divides space into two disjoint but intertwining phases that are simultaneously continuous. This topological feature of bicontinuity is rare in two dimensions and is therefore virtually unique to three dimensions (Torquato 2002).

TPMS configurations have been found in many natural phenomena and offer a great attraction to many disciplines. Reasons for the common interest lie in the deep problems which open up during closer investigation of their properties, and others in the widespread applications in different areas of scientific research. Some interesting features are:

- A natural geometric stiffness

- They enable the optimal use of materials

- They configure stable and resistant structures

- It has come to light that TPMS allow the optimization for simultaneous transport of heat and electricity (Torquato et al . 2005, 2006)

In recent time, great progress was made by using new generative software that have enabled researchers to enlarge the family of minimal surfaces as well as to confirm old ideas, to see complex minimal surfaces, to alter them and to check their properties. Thanks to these tools have been realized architecture inspired from minimal surfaces that embodied the unity of economy and beauty. In art world we see plenty of sculpture works playing on minimal surfaces. Scientists and engineers have anticipated the nanotechnology applications of minimal surfaces in the areas of molecular engineering and materials science.

This paper is focused on both the form-finding and the fabrication related to the geometric properties of TPMS. The aim is understand how the translation from the virtual three-dimensional space to the built artifact could be embodied into a computational process which would also solve the issues within the fabrication framework.

Generative software like Grasshopper is capable of simulating these geometries and controlling their properties through the employment of algorithm. Usually this method has a dialogue with a great number of data (elements) and operations run by the flow which connects the different algorithms, where an algorithm is a process which allows to calculate a result from input data through a logic and finite sequence of elementary instructions. The criterion followed by an algorithm is linear: it concerns input data, the elaboration of them and an output result. It's possible to develop a deterministic and traceable processes that have the advantage of more control: as they are highly determinable, their output is predictable and can therefore be easily refined through subsequent adjustments. The knowledge of geometry plays a major role in application of this process, a deep understanding of geometrical relationships is becoming the key for parametrical optimization and associative modeling techniques.

From the point of view of the generative process, the study involves a form-finding strategy considering the properties of infinitely periodic minimal surfaces.

The first step is to create a minimal surface module, within a basic kaleidoscopic cell (usually a cube consisting of tetrahedra and prism) characteristic to the specific potential geometric configuration, starting from implicit equation that describes it. Subsequently the surface could be reflected and form the triply periodic minimal surface which will be adapted to the desired morphology. The created digital model, which can reach a considerable level of complexity, could be analyzed via computer simulations. First results, that will be further investigated in research's next stage, seem to show interesting requirements like artifacts that have novel mechanical, flow properties, and low weight.

Up to now, these objects were extremely difficult to produce, but advances in tridimensional printing techniques have significantly improved the control over the whole design of solid and porous structures. The working principle of of these technologies is based on spatially controlled solidification of a liquid photo-polymerisable resin. Using a computer-controlled laser beam or digital light projection, and a computer-driven support platform, a 3d object can be constructed in a layer-by-layer fashion. Structural parameters such as porosity and pore size, and even gradients thereof, can be freely varied to obtain a new breed of products that possess innovative multifunctional characteristics.

Triply Periodic Minimal surfaces

Minimal surfaces are defined within the language of differential geometry as surfaces of zero mean curvature. This means they are equally convex and concave at all points and their form is therefore saddle–like, or hyperbolic. They are called minimal because given a fixed boundary curve the area of a “minimal surface” is extremal with respect to other surfaces with the same boundary
Three–periodic minimal surfaces have three lattice vectors, i.e., they are invariant under translation along three independent directions. They are also called triply–periodic or infinite periodic minimal surfaces, abbreviated to TPMS and IPMS, respectively. Numerous examples are now known with cubic, tetragonal, rhombohedral, and orthorhombic symmetries
The symmetries of a TPMS allow the surface to be constructed from a single asymmetric surface patch, which extends to the entire surface under the action of the symmetry group. The most important local symmetries of minimal surfaces are euclidean reflections (in mirror planes) and two–fold rotations.

P surface

D surface

Gyroid

Neovius surface

Double-Gyroid

Kaleidoscopic Cells

Prisms

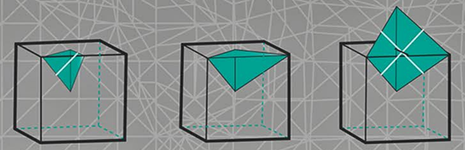
Rectangular Parallelepiped
A rectangular box, shown in its maximally symmetric form of a cube.

Equilateral Prism
A prism based on an equilateral triangle.

Isosceles Prism
A prism based on a 45-45-90 degree triangle.

30-60-90 Prism
A prism based on a 30-60-90 degree triangle.

Tetrahedra



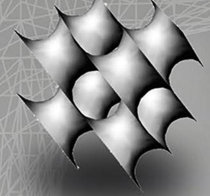
Quadrirectangular Tetrahedron
his tetrahedron is shown as 1/48 of a cube; it is the fundamental region for the full symmetry group of the cube. There is one possible C2 axis, shown in white. The name quadrirectangular refers to the fact that each of the four faces has a right angle.

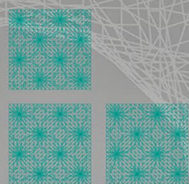
Trirectangular Tetrahedron
This tetrahedron is shown as 1/24 of a cube. There are no possible C2 axes.

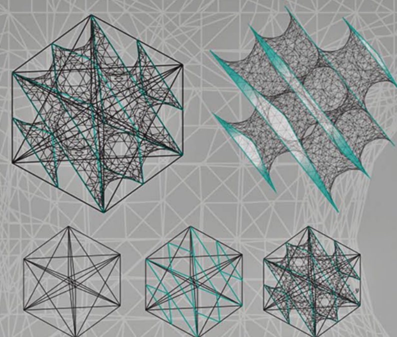
Tetragonal Disphenoid
This tetrahedron can be viewed as two trirectangular tetrahedra stacked up. There are three possible C2 axes, shown in white and brown.

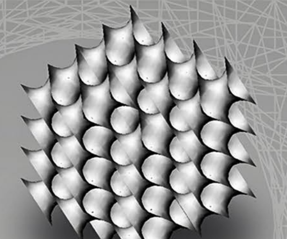
Many ***triply periodic minimal surfaces*** can best be understood and constructed in terms of fundamental regions bounded by mirror symmetry planes.
There are ***two classes*** of kaleidoscopic cells: the prisms and the tetrahedra.
A prism in the general sense is a plane polygon extended at right angles in the third dimension.
A tetrahedron is a polyhedron with four flat faces.

Diamond







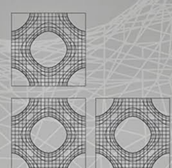


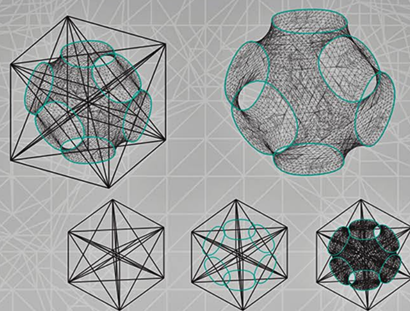
D surface

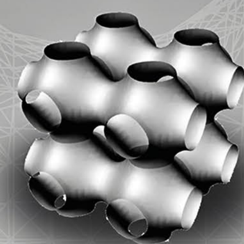
The P surface was first described and parametrised by the 19th century analyst, Hermann Amandus Schwarz in 1865, though his predecessors, Riemann and Weierstrass also contributed to the problem.
The P surface is balanced, with a pair of identical sub-volumes on either side of the surface. The labyrinth graph of each sub-volume is the 6-coordinated jungle gym pattern, the graph connecting nearest vertices of the primitive cubic lattice.

f(x)= sin(x) * sin(y) * sin(z) + sin(x) * cos(y) * cos(z) + cos(x) * sin(y) * cos(z) + cos(x) * cos(y) * sin(z)

Schwartz P





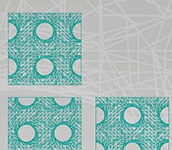


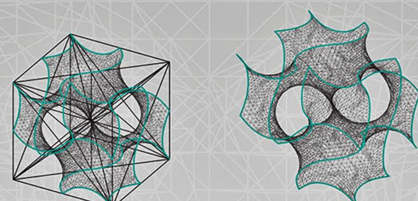
D surface

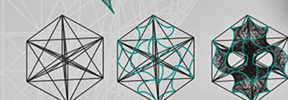
P surface
The discovery of the D surface is —like the P— due to Schwarz, with input from his teachers Weierstrass and Riemann. Schwarz realised that a patch bounded by a tetrahedral polygon could be globally extended in 3D euclidean space via rotations to form an infinite crystalline surface without self-intersection; thus the D surface was the first explicitly known TPMS. The face-centred cell of the oriented space group is why the D surface is also called the F surface. The name diamond surface comes from its labyrinth graphs: these are a pair of interwoven networks each with the bonding structure of diamond.

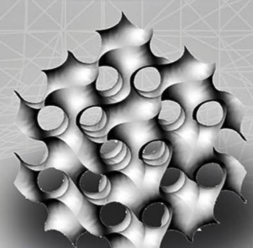
f(x)= cos(x) + cos(z) + cos(y)

Gyroid





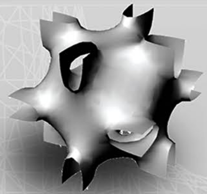


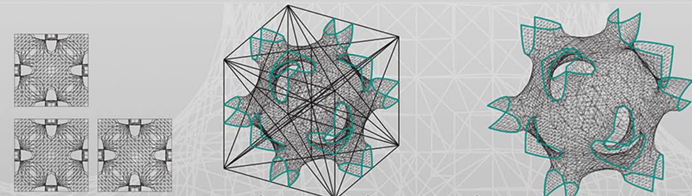


Gyroid

The G surface or gyroid is a relative newcomer to the stable: it was discovered experimentally by Alan Schoen in the 1960's. It is a remarkable structure, mathematically subtle and not readily amenable to the parametrisations used by Schwarz to derive the simpler P and D examples. It is (so far) the most ubiquitous TPMS found in physical systems, most likely due to its combination of local homogeneity (like the P and D surfaces) and global homogeneity.
Like the P and D surfaces, the gyroid has 2D intrinsic symmetry, however, none of the hyperbolic mirror lines map to 3D euclidean symmetries, so that it is entirely free of straight lines and mirror planes. The latter space group is chiral and implies that a single subvolume of the bicontinuous structure is chiral. As it lacks straight lines, the gyroid is not balanced. However, the pair of subvolumes are related to each other by a 3D inversion symmetry (through any flat point on the surface). So while one labyrinth of the spatial partition induced by the gyroid is right-handed and the other is left-handed, the G surface itself is not chiral.

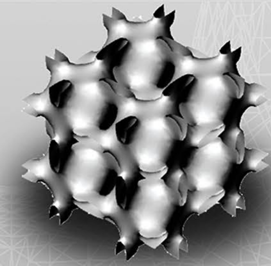
f(x)= cos(x)*sin(y)+cos(y)*sin(z)+cos(z)*sin(x)





Neoviuos surface

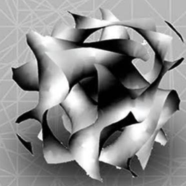
the Neovius surface is a triply periodic minimal surface originally discovered by Finnish mathematician Edwin Rudolf Neovius. The surface divides the space into two infinite non-equivalent labyrinths. In Schoen's categorisation it is called the C(P) surface, since it is the "complement" of the Schwarz P surface. It can be extended with further handles, converging towards the expanded regular octahedron



Neovious surface

f(x)= 3*(cos(x) + cos(y) + cos(z)) + 4*cos(x) * cos(y) * cos(z)

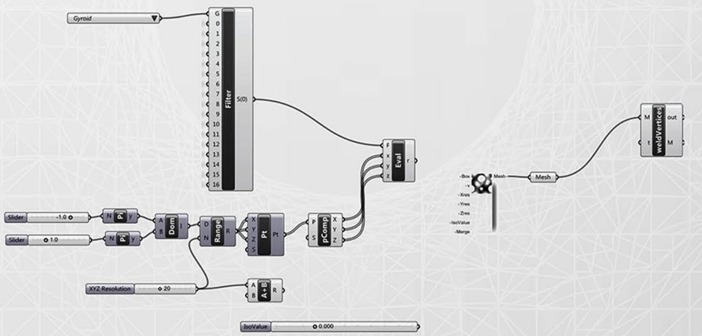
Double-Gyroid

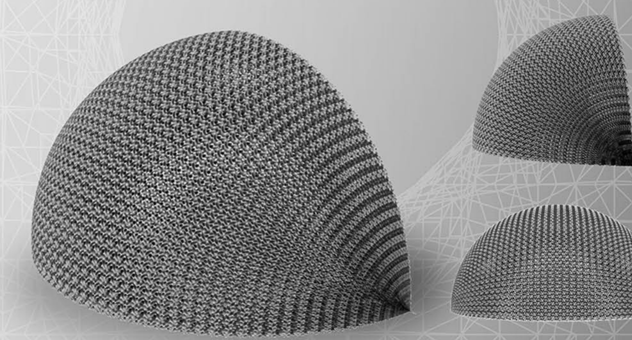


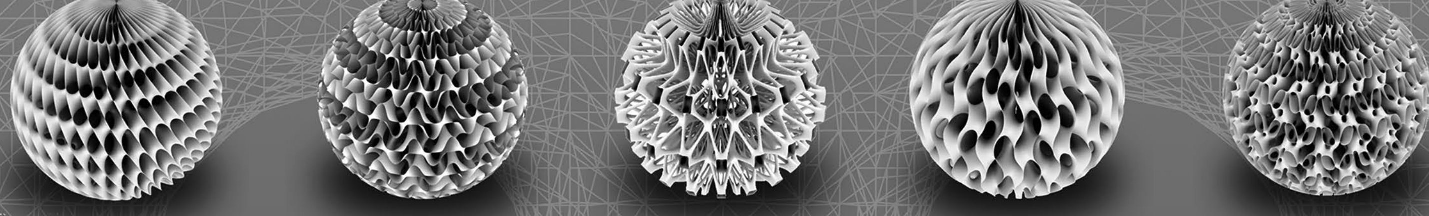
Double-Gyroid

In 1986, Thomas et al. were the first to discover the double-gyroid IMDS inmultiarm star diblock copolymers of poly(isoprene) and poly(styrene), but incorrectly identified the new morphology, lying between the lamellar and cylindrical phase, as the double diamond Eight years later, in 1994, two groups independently characterized the IMDS observed in a low molecular diblock copolymer melt and in a diblock copolymer blend correctly as DG.

f(x)= 0.49*(cos(2*x + y + z - pi) + cos(2*x - y + z - pi) + cos(- 2*x + y - z - pi) + cos(- 2*x - y - z - pi) + cos(x + 2*y + z - pi) + cos(x + 2*y - z - pi) + cos(- x - 2*y + z - pi) + cos(- x - 2*y - z - pi) + cos(x + y + 2*z - pi) + cos(- x + y + 2*z - pi) + cos(x - y - 2*z - pi) + cos(- x - y - 2*z - pi) + cos(- 2*x + y + z) + cos(2*x + y - z) + cos(- 2*x - y + z) + cos(2*x - y - z) + cos(- x + 2*y + z) + cos(x - 2*y + z) + cos(- x + 2*y - z) + cos(x - 2*y - z) + cos(x - y + 2*z) + cos(x + y - 2*z) + cos(- x - y + 2*z) + cos(- x + y - 2*z)) + 0.27*(cos(- 2*x + 2*y - pi) + cos(2*x - 2*y - pi) + cos(2*x + 2*y - pi) + cos(- 2*x - 2*y - pi) + cos(- 2*y + 2*z - pi) + cos(2*y - 2*z - pi) + cos(2*y + 2*z - pi) + cos(- 2*y - 2*z - pi) + cos(- 2*z + 2*x - pi) + cos(2*z - 2*x - pi) + cos(2*z + 2*x - pi) + cos(- 2*z - 2*x - pi)) - 0.69







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