Noise in a FM-converted Self-Mixing Interferometer

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Abstract— **We measure the noise of the cFM (converted Frequency-Modulation) signal in a self-mixing interferometer (SMI) and confirm the theoretical prediction of a dependence of rms noise from $(1+C)^{-1}$, where *C* is the Acket feedback factor, up to a high value (i.e., *C*= 30) in the region III of Tkach and Chraplyvy feedback diagram. We verify also the minor dependence of noise from the phase factor 2*ks*+atan *α*. Last, we show that the cFM SMI easily attains an unprecedented limit of small-signal detection, with amplitudes down to 0.25 pm/√Hz.**

Keywords— Optical interferometers, noise measurements, optical feedback, measurements, semiconductor laser diodes.

I. INTRODUCTION

The self-mixing interferometer (SMI) is a novel configuration for the measurements of optical phase-shifts of a remote moving target, like those generated by a displacement, a minute vibration, or a tilt of the target surface.

It is based on the interaction, inside the laser cavity, of the usually very small field returning from the target and the pre-existing unperturbed field oscillating in the cavity. As a result of interaction, the perturbed field emitted by the laser carries both AM (amplitude modulation) and FM (frequency modulation) whose modulation indexes are proportional to *cos(2ks)* and *sin(2ks)*, respectively, where $2ks=\phi$ is the optical phase suffered on the propagation path to the target at distance *s*, and $k=2\pi/\lambda$ is the wave vector. Operation can be on untreated (diffusing) surface up to several meters distance (or, with returns down to 10^{-8} in power) thanks to the mechanism of coherent detection.

Several review papers have been published to illustrate the mechanism of operation of the SMI [1,2] as well as the many applications of the SMI, proposed or developed, in the fields of kinematic measurements [3-6], mechanical engineering [4-7], biomedicine [8], physical parameters [9,10], consumer [1], etc., just to quote a few.

Although SMI has been demonstrated, and can actually be made out of virtually any type of laser, provided it is single mode, all the above applications have been preferably developed with inexpensive and compact LD (laser diode) which provides the AM component as a signal superposed to the CW power emitted by the laser and readily detected by the monitor photodiode normally included in the laser package. So, with as little as $10 laser and an oscilloscope you may start playing with a SMI detecting μm-vibrations picked up around the lab.

On the other hand, while the AM component *cos 2ks* is readily available, the *sin 2ks* of the FM cannot be easily detected and is normally unused in a SMI, and this is unfortunate because a single interferometric signal has ambiguity for 2ks>2π (and two orthogonal signal will solve the problem) and even more, because the FM signal has much better SNR than the AM, so it can potentially achieve a few orders of magnitude better sensitivity to small signals (roughly speaking, from the nanometer of AM to the picometer of FM).

Recently, the deadlock of FM has been overcome [11-13] thanks to the idea of using a sharp edge optical filter, like either: - an acetylene cell like in [11], or a Mach-Zehnder [12] or any another optical interferometer [13] - to convert the FM into a AM with the impressive result that signal amplitude increases by a factor roughly 10^3 and SNR by a factor about 10^2 [11], thus better approaching the quantum limit of detection [14], contrary to the AM which suffers of having small useful signal over a large dc bias that carries a large shot noise.

The principle of operation of interferometer filters is described in Ref. [13] together with a comparison of different interferometer configurations. With a Mach-Zehnder fiber Interferometer (MZI), we have then demonstrated a compact setup [12] with 120-pm detectable minimum amplitude respect to the 6.6-nm currently achieved by the AM-based SMI. In addition, the analysis of the cFM (converted frequency modulated) SMI signal has been carried out in [13], quantifying the substantial improvement respect to the AM and predicting a (1+C) dependence of the SNR, where C is the feedback factor [1,13].

In this paper, we carry out an experimental study and check the validity of the theoretical results of [13]. Also, with an improved setup, we achieve a record minimum (at SNR=1) detectable signal of 0.25 pm/√Hz.

II. SIGNAL AND NOISE OF THE CONVERTED FM COMPONENT

The theoretical analysis of the cFM SMI signal amplitude and noise has been developed in [13] starting from the Lang and Kobayashi equations [see Ref.1] for the laser operating under feedback conditions. We will not repeat the treatment of [13] here, and limit ourselves for the reader's sake, to report the main results of amplitude and noise of the cFM and AM signals for our discussion and for the comparison to the experimental results.

IA-Signal amplitude

The E_p and $\Delta\omega_p$ of AM and FM amplitudes are found respectively as [13]:

$$\Delta E_p = E_0 \kappa C, \quad \text{and} \quad \Delta\omega_p = C/\tau_{ext}, \qquad (1)$$

where E_0 is the (unperturbed) field amplitude, $\kappa=(1+\alpha^2)^{-1/2}\tau_p/\tau_{ext}$ is a coupling factor with α = linewidth enhancement factor, τ_p =photon lifetime in the cavity, and τ_{ext} =2ns/c = round trip time of external cavity to external target at

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distance s; C is the feedback factor explicitly given by $C=(1+\alpha^2)^{1/2}K(\tau_{ext}/\tau_{in})$, with $K=\eta_s(1-r_2^2)(r_3/r_2)$ = fraction of field coupled into the oscillating mode, in terms of (field) reflectivity of the mirrors, and mode superposition factor η_s; last, $\tau_{in}=2nL_{in}/c$ = round trip time inside the laser cavity.

As the detected power at the output of the laser is $P_0 = E_0^2$ in unperturbed conditions, the AM power signal developed by ΔE_p is [13]:

$$P_{AM} = 2\, E_0\, \Delta E_p = 2\, P_0\, \kappa\, C \qquad (2)$$

About the signal carrying the FM modulation, we pass it through a steep filter of frequency slope $S_F = dP_{out}/df$ so that the output photocurrent is given by the power $P_{out}=E^2$ multiplied by the filter slope S_F and by the (peak) frequency deviation $\nu_p = \omega_p/2\pi$. Thus, the converted cFM signal is written as [13]:

$$P_{cFM} = P_{out}\, S_F\, \omega_p/ 2\pi = P_{out}\, S_F\, C/ 2\pi\tau_{ext} \qquad (3)$$

where we have allowed P_{out} be eventually different from P_0.

Making the ratio of Eqs 2 and 3, we get the cFM-to-AM conversion gain as [13]:

$$G_{cFM\text{-}AM} = [S_F\, (1+\alpha^2)^{1/2}/ 4\pi\tau_p\,]\, (P_{out}/ P_0) \qquad (4)$$

which has already been checked and verified experimentally [13].

I B - Noise of AM and cFM signals

For the AM signal, the fluctuation is due to the shot noise of the (large) unperturbed component E_0, and has a value [10]:

$$p_{nAM} = (2h\nu P_0 F_n B)^{1/2} \qquad (5)$$

where F_n is for the excess noise factor respect to the quantum limit [13], and B the bandwidth of measurement.

Taking the ratio of Eq.2 for the signal and of Eq.5 for the noise and rearranging, we get [13]:

$$SNR_{AM} = 2\, \eta_s\, K(\tau_p/\tau_{in})\, [P_0/ 2h\nu\, F_n\, B]^{1/2} \qquad (6)$$

About the noise of the FM channel, we start from the Schawlow-Townes linewidth $\Delta\nu_0= (1+\alpha^2)\, h\nu(P_{out}8\pi\tau_p^2)^{-1}$ [15] and calculate the frequency noise by using a result of Laurent [16] saying that the noise spectral density $S(\Omega)$ of an oscillator is 2 times the linewidth $\Delta\Omega_0 = 2\pi\,\Delta\nu_0$. Inserting in the previous expressions and integrating over the measurement bandwidth B, we get the frequency noise rms of the laser, as [13]:

$$\Delta\nu_{n1} = [\Delta\nu_0\, B / \pi]^{1/2} = (1+\alpha^2)^{1/2}[h\nu\, B/2P_{out}]^{1/2}(2\pi\,\tau_p)^{-1} \qquad (7)$$

Linewidth given by Eq.7 is for the unperturbed conditions, but under optical feedback $\Delta\nu_{n1}$ may be either narrowed or broadened depending on the phase ϕ=2ks of the returning field, whereas the amplitude (AM signal) is unaffected. This is the behavior observed in the region II of the well-known Tkach and Chraplyvy feedback diagram [17, 18]. In addition, Petermann [19,20] has studied the dependence of linewidth to increasing C factor (or feedback strength) in the region III and up at the boundary to the coherence collapse of region IV where the laser enters in new high-dynamic level phenomena of multi-periodicity, bifurcations and chaos [20,21], initially depending on phase ϕ with some jumps into chaos and back, and then filling all the interval $\phi=0\text{-}2\pi$ at increasing C [21,22]. Except for the jumps, the perturbed linewidth $\Delta\nu_{pert}$ is found to be reduced, respect to the unperturbed linewidth $\Delta\nu_0$, of the factor

$$\Delta\nu_{pert}/\Delta\nu_0 = [1 + C \sin(2ks + \operatorname{atan}\alpha)\,]^{-2} \quad (8)$$

Eq.8 tells us that the narrowing reaches the maximum value $(1+C)^{-2}$ when 2ks + atan $\alpha=\pi/2$, whereas the maximum gain is at 2ks = 0, for which the reduction factor is $(1+\kappa\tau/\tau_{in})^{-2}$ [23-25]. However, the laser will spontaneously lock at the minimum linewidth, at $C>1$ where a mode hopping occurs on the external cavity modes [1,21], therefore the factor $(1+C)^{-2}$ is the naturally occurring decrease of linewidth $\Delta\nu_0$ and, in view of Eq.(7), the decrease of frequency rms noise $\Delta\nu_{n1}$ will be a factor $(1+C)^{-1}$. So, we may write the rms noise of the cFM channel as:

$$\Delta\nu_{n1} = (1+\alpha^2)^{1/2}[h\nu\, B/2P_{out}]^{1/2}\,[2\pi\,\tau_p\,(1+C)]^{-1} \qquad (9)$$

In the following Sections we carry out experiments to check the validity of Eq. (9).

III. Experimental Setup

There are many choices for the steep-response filter needed for the conversion of the FM signal. Choice of the optical interferometers, of branched or in-line configurations, and of free-space or fiber propagation have been discussed in [13] after the exemplary experiment carried out in [12] with a filter made of an all-fiber Mach-Zehnder Interferometer.

In this work, we use a *free-space* version of the Mach-Zehnder interferometer (MZI) to alleviate alignment and efficiency problems (see Fig.1).

The laser diode is a commercial DFB laser for optical communication, model WSLD-1550-020m emitting 20 mW at 1550 nm. The laser had a threshold current of 10-mA and was biased at 20-mA. [Virtually any other laser diode, including Fabry-Perot type, could have been used equally well, provided the emission is a single longitudinal mode].

A beam sampler, made by a glass flat with anti-reflection coating on one surface is used to deviate about 4% of the emitted power to the MZI filter, the frequency-selective element converting the frequency modulation into an amplitude modulation. The MZI is made up by two 50% beam-splitters and two mirrors on a path with ΔL = 45 cm imbalance. The laser beam sent to the MZI is collimated, and at the output a lens focuses it to photodiode PD.

The (power) transmission T of the MZI is a periodic function of laser frequency f:

$$T(f) = ½ [1+ cos (2\pi f\Delta L/c)] \qquad (10)$$

The filter is periodic in frequency of the so called FSR (free spectral range), given by:

$$FSR= c/\Delta L \qquad (10A)$$

and has a maximum sensitivity of response $S_{F(MZ)}$ at :

$$S_{F(MZ)} = dT/df_{\,(max)} = \pi \Delta L/c \qquad (11)$$

In our setup, with 45-cm $=\Delta L$ imbalance, we get a maximum relative sensitivity of $S_{F(MZ)} = 4.7\cdot10^{-9}$ Hz^{-1} and a FSR = 660 MHz. The maximum sensitivity is obtained when the MZI is in quadrature (i.e., midway of the cosine transmission function). To tune the interferometer in quadrature, one mirror of the MZI is mounted on a piezo-actuator (Fig.1), and operation is governed by a proportional controller working in closed loop. The error signal is given by the difference between the signal of the external photodiode (FM) and a fixed dc reference (set point). The set point corresponds to half of the maximum power. The bandwidth of the electronic loop is about 500 Hz, large enough for compensating slow ΔL variations due to thermal drift of laser wavelength and to mechanical positioning errors and microphonics. The main part of the emitted power (96%) is focused on a loudspeaker (the target) placed at a distance of L=55 cm (if not differently specified), realizing the usual self-mixing interferometer.

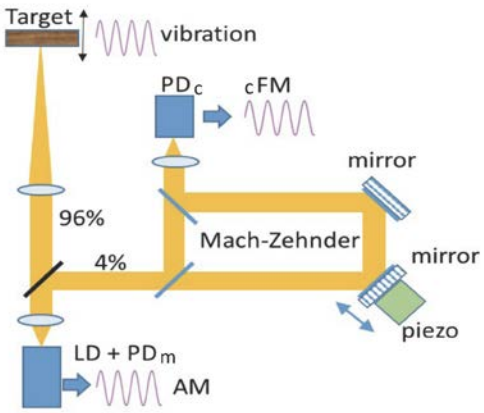

Fig 1. Schematic of the SMI with FM conversion. The Mach-Zehnder interferometr is used as a filter, and is locked at half fringe by the piezo actuator, so that the slope of the transfer function is a maximum ($4.7\cdot10^{-9}$ Hz^{-1}) for a 45-cm imbalance of the two arms). Phodiode PDc detects the converted FM signal (cFM), whereas the monitor photodiode PDm ot the laser detecs the usual AM signal of the SMI.

The FM signal from the external photodiode is acquired by a trans-impedance op-amp with 125-MHz bandwidth and a R_F=40-kΩ feedback resistance, is then sampled by a digital oscilloscope (DO) The bandwidth of the DO is limited to 20 MHz, in order to cancel residual oscillation of the electronics at high frequency. The frequency response of the entire receiver is flat up to 20 MHz.

The AM signal is acquired by a trans-impedance op-amp with 28 kHz bandwidth, and a feedback resistance R_F =47-kΩ. The AM signal is useful to check that the SMI signal when it becomes larger than the MZI dynamic range, given by the free spectral range (FSR), when working at high C.

For example, Fig.2 shows an SMI signal at C=30, a value at the boundary of the coherence collapse for this laser [17,18,20]. In this case, the AM signal shows the regular SMI waveform (Fig.2 middle) with 14 minute fringes, barely visible over the upper and lower plateau of each semiperiod. In contrast, the cFM channel shows an additional number of fringes, (7 per semiperiod, Fig.2 bottom), because in this high-C experiment the signal frequency deviation has become so large (about 5 GHz) that each time it crosses the spectral range FSR=660 MHz a new additional fringe is generated. Of course, this case is exemplary of the waveforms encountered in the experiments, whereas in the normal use of the SMI we will keep the frequency swing of the signal smaller than the FSR. Considering that the peak-to-peak amplitude of the cFM signal is $V_{PP} \cong$2-V (Fig.2), we can calculate the sensitivity of our MZI filter as $S_{MZ}=S_{MZr}\cdot V_{PP}$ $=9.4\cdot10^{-9}$ V/Hz.

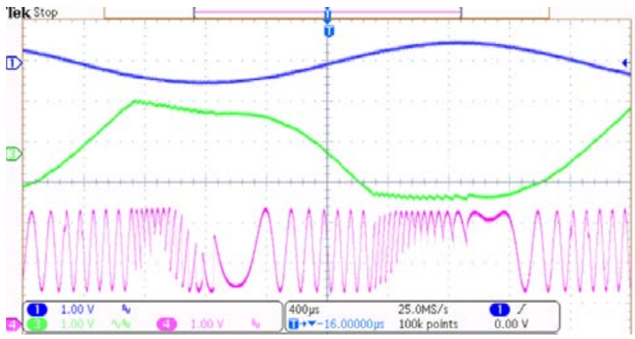

Fig 2. Exemplary SMI signal acquired for C=30 at a distance L=15-cm with loudspeaker peak-to-peak amplitude of vibration ΔL=11 μm. Upper trace: driving signal. Middle trace: AM SMI signal. Lower trace: converted FM SMI signal.

IV. Measurement Results

The aim of our measurements on FM-SMI is to verify the theoretical dependence of the noise from the feedback level, expressed by the C value. In order to adjust the feedback, we used an optical attenuator inserted in the propagation path to the target. As a first step, we let the target vibrate (at about 25 Hz) and acquire both the AM and cFM signals. From the waveform we have evaluated the C value, as described in [10, 14, 25]. For example, Figs.3 the acquisitions are for C=0.7 and 14 respectively.

Important to note, at the large value C=14 of Fig.3 and C=30 of Fig.2, as well at all the other large-C cases, we didn't observe any fringe loss due to mode hopping on the external cavity modes (ECM) [1,17,18,20,26], and this may look surprising because it is well known from literature [1,17,26] that the two-stable mode operation is limited to the range $1<C<4.6$, the level of feedback normally used in SMI measuring applications, while going to $C>4.6$ we open the way to five ECM modes, and at $C>7.8$ to seven modes, etc., [1,27,28]. Inside these ranges, mode hopping with fringe loss is well known to occur, rather erratically and in most diode lasers (see e.g. Fig.5 of [1] and Fig.7.6 of [26]). Though we carefully watched for the appearance of fringe losses in all the C= 5 to 30 experiments, lasting several months, we have never seen it and can safely conclude that, at least for our DFB laser, regular operation is possible without fringe loss up to C =20...30, just at the boundary of the coherence collapse [1, 17, 19-22] where the laser starts to emit spikes and the spectrum has a large 1/f component. Explaining this unexpected (and welcome for applications) behavior is out of the scope of the present paper and will be the subject of a forthcoming work.

About the noise floor, we switched off the loudspeaker and acquired signals after a suitable change of the oscilloscope vertical scale, in order to make quantization noise negligible. The noise spectrum was evaluated for all the feedback level, including the absence of target, together with the noise floor of the photodetector. Fig.4 shows some of the average spectra, measured as a function of the frequency and for some values of the feedback level C. Average is because spectra have a small dependence on phase ϕ=2ks, of about 1-dB for $-\pi/2< \phi<\pi/2$ as shown later (see Fig.7).

Incidentally, the data confirm that the cFM-SMI is correctly designed, because the cFM noise is always dominant with respect to photodetector noise.

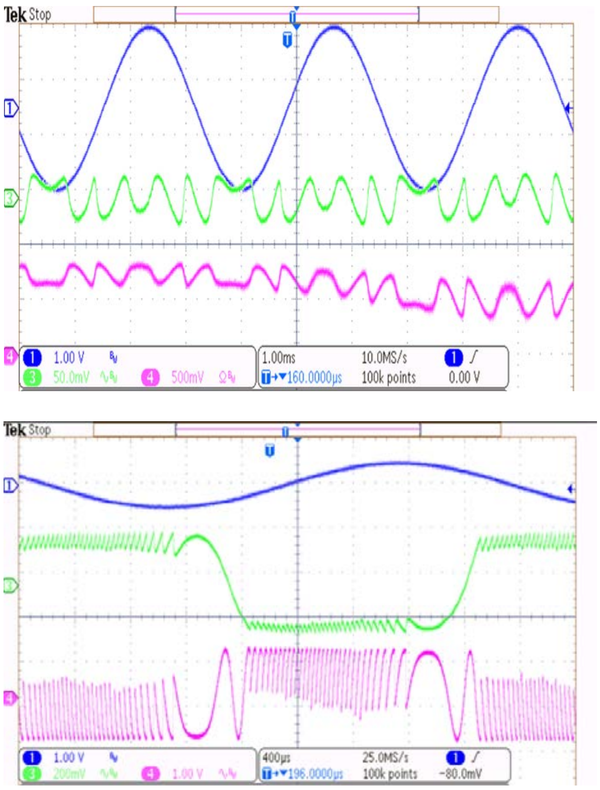

Fig 3. Exemplary waveforms of SMI signals acquired for: C=0.7 and loudspeaker peak-to-peak amplitude of vibration ΔL=2-µm (upper panel) and for C=14, ΔL=24-µm (lower panel). In both panels distance is L=55-cm, and top trace is the loudspeaker driving signal, middle the AM SMI signal and bottom the converted cFM SMI signal.

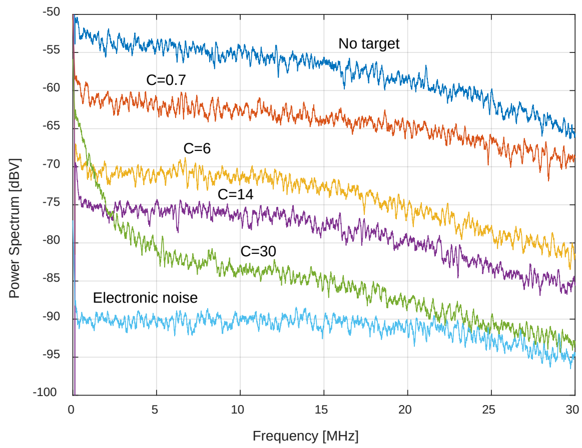

Fig.4 The noise spectra of the cFM signal, as s function of frequency, and for some values of C, the optical feedback level. Respect to the case without the target, noise is decreasing at increasing feedback, until at about C=30 coherence collapse (revealed by the low-frequency increase) starts to set in.

As expected from theory, the cFM noise spectrum is about white (the cutoff at 20 MHz is given by the oscilloscope), and the noise level *decreases* with increasing feedback level (curves for C=0.7 to 15 in Fig.4). Then, reaching out to about C≈25, the laser diode enters the coherence collapse regime (14,15,17), and a low-frequency excess component (or 1/f) becomes evident (see Fig.4 for C=30). In the time-domain, the cFM signal at C=30 looks like a series of small pulses superposed to the 2ks signal, as shown in Fig.5: upper trace is the noise floor for C=30, compared with lower trace for C=14.

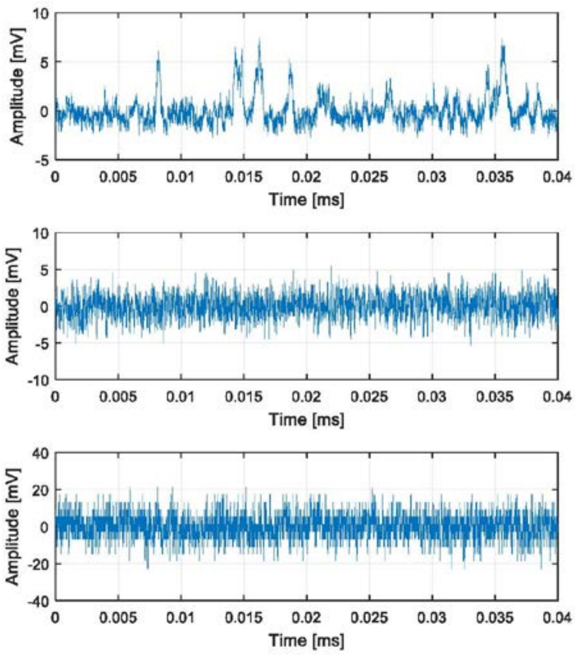

Fig.5 Noise floor acquisitions. The top trace for C=30 exhibits a small baseline noise but is at the boundary of coherence collapse, as revealed by the spikes. Middle trace is for C=14 and has larger baseline noise. Bottom trace is for C=1.7 and exhibits a still larger noise (note the scale change).

Considering that the fringe amplitude is higher than 1 V, these pulses (of amplitude lower than 10 mV) were not visible in Fig.2. Even if coherence collapse is starting, the laser linewidth is still narrowing, as demonstrated by the noise floor reduction for frequency larger than 10 MHz. Table I reports the rms voltages measured in the time domain, as a function of *C* value. These values are obtained with a two-poles low-pass filter at 20 MHz. Due to the low-frequency signal accompanying the coherence collapse, at large values of *C* the rms voltage in table I and Fig. 6 are derived from the value of spectral density at 10 MHz (Fig.4).

Table I

C	0	0.7	1.7	2.3	3	6	8	14	25	30
Noise level rms (mV)	35	21	13	11	9.6	5.1	4	2.8	1.7	1.4

Fig.6 shows the noise levels of Table I expressed as noise-equivalent displacement (NED) as a function of the *C* value. NED is calculated considering the noise level in Table I as the integral of a white spectral density, filtered at 20 MHz by two poles. For converting the values in equivalent displacement, the SMI signal for $C > 1$ is considered linear inside a single fringe. The fringe amplitude is ~1 V (Fig.3) and corresponds to $\lambda/2=775$ nm of displacement, therefore the FM-SMI sensitivity for small-displacements is $S_{SMI} \cong 1.3$ V/µm.
Dots in Fig.6 are measurement data, while continuous line is the theoretical behavior of FM-SMI NED for laser diode linewidth equal to $\Delta\nu_0 = 3.5$ MHz, a reasonable estimation for our DFB laser. Using Eqs.9 and 11 we can write:

$$NED_{THEORY} = \sigma_{FM} S_{MZ}/S_{SMI} = \sqrt{\frac{\Delta\nu}{\pi}} S_{MZ}/S_{SMI} =$$

$$= \sqrt{\frac{\Delta\nu_0}{\pi}} S_{MZ}/[S_{SMI}(1+C)] \qquad (12)$$

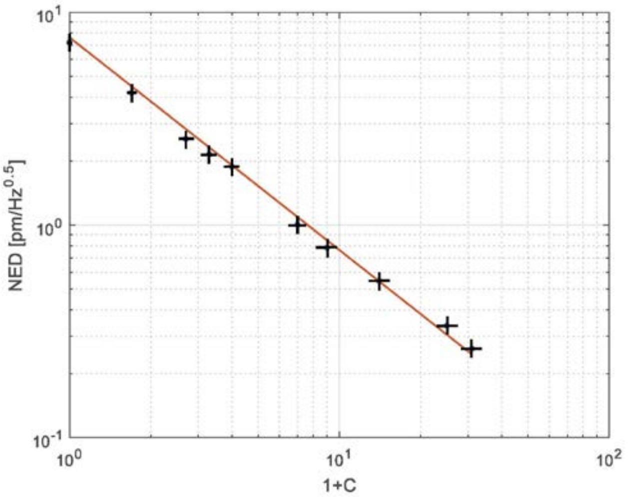

Fig 6. Noise equivalent displacement as a function of the optical feedback factor 1+*C*. Crosses are experimental data, line is the theoretical curve for laser linewidth $\Delta\nu_0 = 3.5$ MHz.

The agreement of theory and experimental results in Fig.6 is really good, considering just the laser linewidth dependence on *C* value. Actually, Petermann's theory of laser feedback [19,20] predicts a laser linewidth dependence also on the phase *2ks* of optical back injection (Eq.8). It should influence the noise floor of FM SMI, given a stable back-injection level. For the experimental measurement of the noise level as a function of SMI phase, the loudspeaker was driven at low frequency (20 Hz) with an amplitude corresponding to almost one interferometric fringe (about 750 nm). Fig.7 shows the acquisition, for $C \cong 1$, evaluated by acquiring the signals with a larger vibration at higher frequency (Fig. 8). The number of samples of the whole acquisition of Fig.8 is 10^7, and the spectrum evaluation was made on a window of 10^6 points, moving at different phase positions.

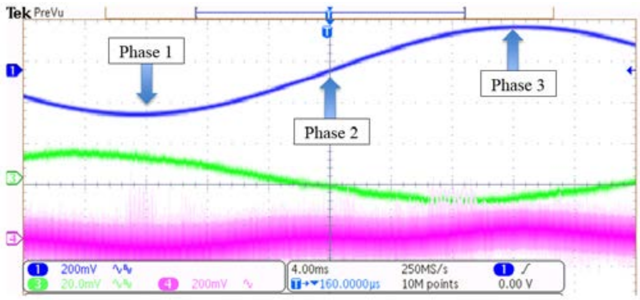

Fig. 7. Acquisitions for measuring noise level as a function of SMI phase. Top trace: loudspeaker driving signal, middle trace: AM SMI signal, bottom trace: cFM SMI signal.

The SMI phase is measured on the driving signal, because the AM signal is high-pass filtered, and at low frequency (20 Hz) the electronic feedback loop is acting on the cFM signal, keeping the Mach-Zehnder interferometer in quadrature, therefore compensating the modulation. It is worth to note that the present cFM layout was not designed for low-frequency vibrations. At 20 Hz, instead, the loudspeaker movement is in phase with the electric driving signal, because its resonance frequency is about 150 Hz. the motor speed in round per minute (rpm) during time, for seven sloshing levels (phases a, b, c, d, e, f, and g). Phase a and g are steady state conditions when there is no fluctuation on the surface. In phase d, the speed reaches to its maximum while it is minimum during phase b.

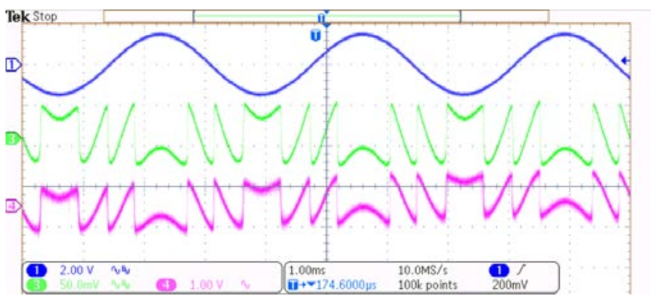

Fig. 8. Signals acquired for *C*=0.7. Upper trace: loudspeaker driving signal. Middle trace: AM SMI signal. Lower trace: cFM SMI signal.

The experimental data show a deterministic behavior of FM noise level as a function of the SMI phase, but its maximum variation is limited to about 1 dB. Fig.9 shows the power spectra of the noise level for three positions in phase, corresponding to the arrows depicted in Fig.7. Even though the noise dependence on SMI phase is evident, its contribution is negligible with respect to the variation as a function of optical feedback level, 1 dB against about 30 dB, the reason why the SMI phase was not reported in the measurements of Fig. 5. These last were all made at the center of SMI fringes, i.e. at about the same phase.

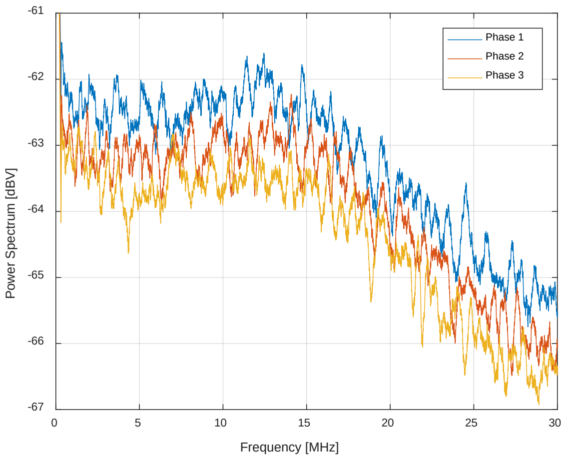

Fig. 9. Noise spectra as a function of the optical feedback phase for *C*=1. With reference to Fig.7, top trace corresponds to phase 1, middle trace to phase 2 and bottom trace to phase 3.

V. THE CFM SMI AS A VIBROMETER

The results reported in previous Sections are interesting because the measurements we have performed confirm the theory of SMI measurements at moderate-to-high level of feedback, and this was the aim of our present work. Yet the work also brings about a record of performances for state-of-the-art vibrometers, both for improved bandwidth - with a cutoff frequency of 20 MHz, and minimum detectable signal. Looking at Fig.6, we have obtained at *C*=30, the unprecedented NED (noise equivalent displacement) of 0.25 pm/√Hz for a target at 55 cm distance. In contrast, typical NED of AM-based vibrometers are in the range 100..200 pm/√Hz [3]. Fig.10 reports an exemplary high frequency performance of the cFM SMI of Fig.1, the pickup of the piezo response to a step of excitation with only 1 nm of amplitude, revealing the 100-kHz resonance of the target. The acquisition was made after 128 averages with bandwidth 20 MHz, and shows a NED equal to about 200 pm, corresponding to about 0.5 pm/√ Hz.

VI. CONCLUSION

We have checked the theory of noise of converted-frequency self-mixing signals, and found very good agreement up to incipient coherence collapse, at the large value of C=30 where usually it was generally accepted that operation of the SMI is not possible.

We have also confirmed the findings of our previous works about the superior performance of the cFM SMI respect to the traditional AM-based SMI, and gave a practical example of performance down and below the pm/√Hz on a large record bandwidth (tens of MHz).

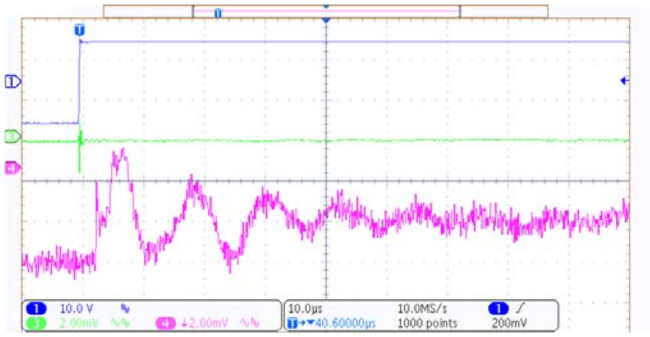

Fig.10 Excitation voltage of a piezo-ceramic actuator (top) supplying a 1-nm step displacement, and corresponding displacement response detected by the cFM SMI setup of Fig.1, with the target placed at 55 cm distance and untreated diffusive surface. Bandwidth of acquisition is 20 MHz, 128 averages, *C*=15. Noise is read as about 200 pm rms corresponding to 0.5 pm/√Hz.

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