

## Collision avoidance algorithms for Space Traffic Management applications

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### Abstract

Establishing cost-effective Space Traffic Management (STM) systems presents diverse challenges, from technical and operational aspects, to regulatory, policy and legislation. On the technical part, new types of missions and more objects in orbit will increase collisional activities and require advances in Space Situational Awareness. To deal with the growing amount of data, fast and efficient algorithms are a must. Furthermore, autonomous on-orbit collision avoidance capabilities would reduce the workload on satellite operators and improve safety. Also, the analysis of an avoidance manoeuvre should not be limited to the two objects involved but the general effect on the environment. To tackle some of these issues, this work reviews recent advances in collision avoidance manoeuvre modelling, analysis, and optimisation. The focus is put on computational efficiency and suitability for autonomous on-board applications. To achieve these goals analytical and semi-analytical models are used. Both impulsive and low-thrust propulsion systems are considered, as well as maximum deviation and minimum collision probability manoeuvres. The models can also be applied to uncertainties propagation. The algorithms are tested in different practical scenarios and traded-off for computational performance, accuracy, and reliability. Based on the results, their applicability for future STM systems is discussed.

**Keywords:** Collision avoidance manoeuvre, Space Traffic Management, Space Situational Awareness, Analytical methods, Semi-analytical methods

### Nomenclature

$a$	Semimajor axis, km
$a_t$	Continuous thrust acceleration, m/s <sup>2</sup>
$e$	Eccentricity
$f$	True anomaly, deg or rad
$i$	Inclination, deg or rad
$M$	Mean anomaly, deg or rad
$T$	Period of the spacecraft, s
$\alpha$	Vector of Keplerian elements $[a, e, i, \Omega, \omega, M]$
$\delta\alpha$	Orbit modification due to the manoeuvre
$\Delta t_{\text{CAM}}$	Duration of the low-thrust manoeuvre, s
$\Delta t$	Lead time of the manoeuvre, s
$\omega$	Argument of perigee, deg or rad
$\Omega$	Right ascension of ascending node, deg or rad

### Acronyms/Abbreviations

CA	Close approach
CAM	Collision avoidance manoeuvre
SSA	Space Situational Awareness
STM	Space Traffic Management
TLE	Two-line elements

## 1. Introduction

Commercial space utilisation finds itself in a defining moment. Space-based assets are experiencing an accelerated growth due to technological enablers, such as cheaper and more flexible satellite platforms and more

affordable launch opportunities, as well as high-societal-impact applications, such as communications, navigation, or Earth observation. Like air transport in the 40s, this blooming of the so-called New Space should be nurtured through adequate international cooperation efforts analogous to those which ultimately led to current air traffic management systems.

Establishing a cost-effective network of Space Traffic Management (STM) systems faces diverse challenges, from technical and operational aspects, to regulatory, policy and legislation [1]. Such a multifaceted issue requires an equally multifaceted response. From the regulatory and legislative point of view, it is essential to introduce internationally agreed and enforced regulations limiting the accumulation of debris in orbit. The most significant developments in this area are the space debris mitigation guidelines from the Inter-Agency Space Debris Coordination Committee and the guidelines for long-term sustainability of outer space defined by the UN Committee on the Peaceful Uses of Outer Space [2]. Another key policy aspect is to promote the collaboration between space-faring nations, particularly regarding Space Situational Awareness (SSA) data sharing.

Focusing on the technical aspects, new types of missions and platforms and more objects in orbit will increase collisional activities and require advances in SSA capabilities, space surveillance, and tracking. To

deal with the growing amount of data, fast and efficient algorithms for the prediction, assessment, and avoidance of conjunctions are a must. Furthermore, autonomous on-orbit collision avoidance capabilities would reduce the workload on satellite operators and improve safety. Finally, a more congested scenario calls for global approaches, where the analysis of an avoidance manoeuvre is not limited to the two objects involved but considers the general effect on the environment.

To tackle some of these issues, this work presents recent advances and new developments in CAM modelling, analysis, and optimisation [3][4][5][6][7]. A modular framework is adopted [4], composed of three blocks: 1) orbit modification due to the CAM, 2) corresponding deviations at the Close Approach (CA), and 3) analysis of the outcomes through their projection in the b-plane and the change in collision probability. Analytical [3][5] and semi-analytical [6] models are leveraged to obtain fast and accurate expressions for the orbit modification. Both impulsive and low-thrust propulsion systems are considered, as well as maximum deviation and minimum collision probability optimal manoeuvres. Furthermore, the models can also be applied to uncertainties propagation. The algorithms are tested in different practical scenarios and traded-off for computational performance, accuracy, and reliability. Based on the results, their applicability for future STM systems is discussed, putting the focus on computational efficiency, suitability for autonomous on-board applications, and synergies with artificial intelligence.

The rest of the manuscript is organized as follows. First the problem is defined, introducing the nominal CA and detailing the assumptions for the computation of collision probabilities. Next, the analytical and semi-analytical CAM models are presented. For brevity, mathematical formulations and detailed descriptions are omitted as they can be found in previous works. Then, the performance of the algorithms is assessed through numerical test cases, and their applicability to future STM systems is discussed. Finally, conclusions are drawn and future steps are outlined.

## 2. Nominal close approach definition

Let us consider a generic CA between two Earth-orbiting objects, only one of which may manoeuvre to avoid a possible collision. For simplicity, the manoeuvring and non-manoevring objects will be referred to as spacecraft and debris, respectively, and the predicted CA is assumed to be a direct impact (zero miss distance). Let  $\alpha = [a, e, i, \Omega, \omega, M]$  be the nominal Keplerian elements of the spacecraft orbit, where  $a$  is the semimajor axis,  $e$  is the eccentricity,  $i$  is the inclination,  $\Omega$  is the right ascension of the ascending node,  $\omega$  is the argument of perigee, and  $M$  is the mean anomaly. The CAM may be impulsive or continuous low-thrust. Two figures of merit are used to evaluate and design the CAM:

miss distance and collision probability. To compute the collision probability, short-term encounters are assumed [8] (i.e. conjunctions with high relative speed). Also, it is assumed that the computation of the covariance matrices of spacecraft and debris are statistically independent. Under these assumptions, the truncated series method proposed by Chan [8] can be used to compute the collision probability, reducing the original problem where both bodies have a covariance matrix and a spherical envelope to an equivalent one where the debris has no envelope and a combined covariance matrix resulting from the sum of the covariance matrices of each object, and the spacecraft has no covariance and a combined spherical envelope with radius equal to the sum of the radii of the envelopes of each object.

## 3. CAM models

To analyse this problem, a set of analytical and semi-analytical models were proposed [3][5][6]. They follow a modular framework with three main blocks, dealing with: orbit modification due to the CAM, deviations in position and velocity at the CA, and analysis and interpretation of the results. A detailed description of the structure and capabilities of these tools together with their mathematical formulation is provided in [4][7]. In the following, we just provide a brief overview of their characteristics.

Orbit modifications are expressed as a change  $\delta\alpha$  of the nominal Keplerian elements of the spacecraft's orbit. For impulsive CAMs, orbit modifications are modelled analytically from Gauss' planetary equations [3][4][5], following an approach analogous to that of [9]. The model is then extended to include the change in Keplerian elements due to a change in position at the initial time, which allows one to analytically propagate covariance matrices for the unperturbed case. For low-thrust CAMs, a semi-analytical model for constant, tangential thrust is presented in [6][7], based on a single-average of the equations of motion over the eccentric anomaly. Analytical expressions in terms of elliptic integrals of the first and second kind are reached for the mean evolution of all Keplerian elements except  $M$ , for which a numerical integration is required [10]. Furthermore, the short-term periodic evolutions of  $a$  and  $e$ , needed for the accurate computation of  $M$ , are approximated through a sinusoidal expression with coefficients numerically fitted over a single revolution of the spacecraft. It follows that this approach requires numerically propagating one orbit of the spacecraft for the fitting of the short-term coefficients, and the numerical integration of the time law (i.e. the differential equation for  $M$ ) over the whole duration of the CAM. The model proposed in [6][7] uses three parameters to model the CAM: the low-thrust acceleration magnitude  $a_t$ , the duration of the low-thrust CAM  $\Delta t_{\text{CAM}}$ , and the anticipation of the manoeuvre  $\Delta t$ .

At a second step, orbit modifications are mapped into displacements at the nominal CA using linearised relative motion equations [11]. The linearised approach gives good accuracy if the displacements are small compared to the nominal orbit, which is the case for practical CAMs [3]. The model includes both the change in position, needed for CAM characterisation, and the change in velocity, needed for the evaluation of uncertainties evolution.

Finally, the displacement at the CA is projected into the b-plane of the nominal CA to provide physical insight and simplify CAM analysis and design. This allows to separate total displacement into displacements inside the b-plane, which contribute to increasing the miss distance, and perpendicular to it, which only affect to the new time of the CA. Furthermore, inside the b-plane we can distinguish between displacement along the so-called time axis, driven by changes in phasing, and the geometry axis, related to the geometrical modification of the orbit.

### 3.1 Applications

The impulsive CAM model can be used to: 1) evaluate the outcome for a given manoeuvre, 2) design maximum displacement or minimum collision probability optimum manoeuvres. Both optimum CAMs leverage the linear structure of the analytical model to reduce the optimal control problem to an eigenproblem like the one proposed by Conway [12] for asteroids.

In the low-thrust case, the semi-analytical model describes the CAM for a given CA based on three parameters:  $a_t$ ,  $\Delta t_{CAM}$ , and  $\Delta t$ . Although a numerical propagation and fitting is required for the short-term periodic evolution of  $a$  and  $e$  the resulting coefficients only depend on  $a_t$ . Nevertheless, in all cases the numerical integration of  $M$  is required, due to the high influence of phasing in the deflection. The test cases in [4] show computational speed gains of nearly 70% compared to the full numerical integration of the equations of motion, with small errors for typical values of  $a_t$ . It is also possible to design optimal low-thrust CAMs by combining this model with a parametric optimizer.

Finally, the models have also been applied to the propagation of uncertainties. For the unperturbed case, the extended impulsive CAM model provides fully analytical state transition matrices allowing to propagate the covariance matrix. For the perturbed case, a single-averaged numerical propagator [13] is used to numerically evaluate the covariances evolution. A comparison between the unperturbed and perturbed cases is shown in [4]. The perturbed model includes  $J_2$ , atmospheric drag, and solar radiation pressure, and the test cases consider several values for the area-to-mass ratio and account for uncertainties not only in the position and velocity but also in the area-to-mass and the drag and reflectivity coefficients. The results show that the

differences in the evolution of uncertainties in the b-plane between the perturbed and unperturbed cases are negligible for practical values of area-to-mass ratio, justifying the use of the analytical model as a first approximation. Note that this conclusion only applies to the validity of the analytical state transition matrix for the propagation of uncertainties, not to the state transition matrix itself. In other words, the nominal deviations due to the perturbations will be missing in the analytical model, but their effect in uncertainty growth will be small.

## 4. Numerical test cases

Several numerical results are now presented to illustrate the performance of the algorithms introduced in the previous section. More detailed numerical analyses can be found in references [3][4][5][6][7]. Particularly, a detailed sensitivity analysis over the conjunction geometry for the impulsive CAM is performed in [3], together with a numerical quantification of the errors of the analytical model. Moreover, some of the test cases in [3] are specifically chosen to illustrate the effect of uncertainty growth in time and the evolution of dynamics in the b-plane. A quantification of the error for the low-thrust semi-analytical model can be found in [4][6].

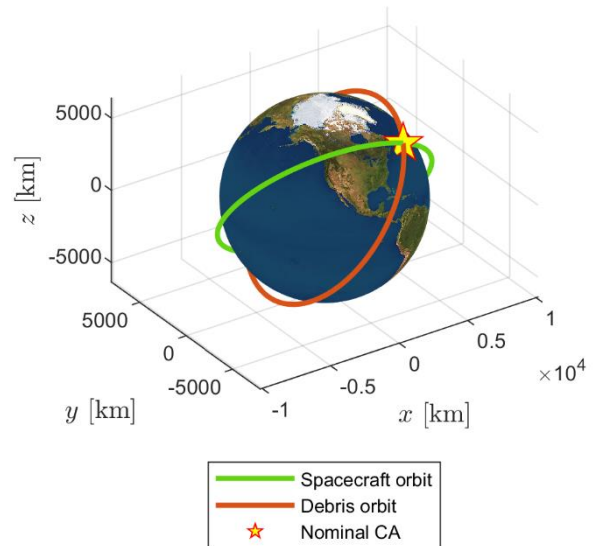


Fig. 1. Orbit representation of the nominal CA.

For this work, a nominal CA is constructed from data corresponding to the Starlink constellation being deployed by SpaceX. This choice is made because Starlink is expected to be the largest constellation ever put in orbit.

Table 1 shows the Keplerian elements for object Starlink-24 (NORAD catalogue ID 44238), computed from two-line elements (TLEs) retrieved from Space-

Track<sup>\*</sup> with epoch 2020-09-13, 01:36:48. The orbital period of this spacecraft is  $T = 95.147$  min.

Table 1. Nominal orbital elements of object Starlink-24 at epoch 2020-09-13, 01:36:48, from Space-Track.

Object	NORAD ID	Epoch	$a$ [km]	$e$ [-]	$i$ [deg]	$\Omega$ [deg]	$\omega$ [deg]	$M$ [deg]
Starlink-24	44238	2020-09-13 01:36:48	6903.824	1.673E-4	52.997	171.294	70.632	289.486

Table 2. Orbital elements of spacecraft and debris at the nominal CA.

Object	$a$ [km]	$e$ [-]	$i$ [deg]	$\Omega$ [deg]	$\omega$ [deg]	$f$ [deg]
Starlink-24	6903.824	1.673E-4	52.997	171.294	70.632	50.410
Debris	6904.614	2.236E-4	52.997	261.294	67.666	351.292

For the debris, it is assumed that another Starlink satellite with the same nominal  $a$ ,  $e$ ,  $i$ , and  $\omega$  and a right ascension of the ascending node 90 deg higher has failed and ended up having a direct-impact CA with Starlink-24. The nominal elements of both objects at the time of CA are reported in Table 2. Regarding the uncertainties, the covariance matrices in position and velocity of both

objects are assumed to be known at manoeuvre time and propagated up to the CA using the analytical state transition matrix. In all cases, the initial value of the covariance in the transversal–normal–out-of-plane frame is:

$$\mathbf{C}^{\text{TNH}} = \begin{bmatrix} 3.0\text{E-}1 & -3.7\text{E-}3 & 4.7\text{E-}3 & -4.6\text{E-}7 & 3.0\text{E-}4 & -3.3\text{E-}7 \\ -3.7\text{E-}3 & 1.1\text{E-}2 & -4.0\text{E-}5 & 1.1\text{E-}5 & -4.1\text{E-}6 & 4.0\text{E-}7 \\ 4.7\text{E-}3 & -4.0\text{E-}5 & 1.9\text{E-}2 & -1.2\text{E-}7 & 7.2\text{E-}6 & 1.2\text{E-}6 \\ -4.6\text{E-}7 & 1.1\text{E-}5 & -1.2\text{E-}7 & 1.2\text{E-}8 & -7.7\text{E-}10 & -2.1\text{E-}10 \\ 3.0\text{E-}4 & -4.1\text{E-}6 & 7.2\text{E-}6 & -7.7\text{E-}10 & 3.1\text{E-}7 & -2.1\text{E-}10 \\ -3.3\text{E-}7 & 4.0\text{E-}7 & 1.2\text{E-}6 & -2.1\text{E-}10 & -2.1\text{E-}10 & 2.2\text{E-}8 \end{bmatrix}$$

with values in km and km/s. The combined envelope radius is 20 m. The collision probability for the nominal CA is  $2.081 \cdot 10^{-3}$ .

Fig. 2 shows the results for the maximum miss distance and minimum collision probability CAMs for increasing values of lead time  $\Delta t$  up to 3 periods of the spacecraft  $T$ , and a fixed manoeuvre magnitude of  $\delta v = 0.1$  m/s. For this nominal CA the differences between the maximum miss distance and minimum collision probability CAMs are very small. This contrast with the results in [3][5], where significant differences could be found between both strategies during the first periods. The reason lies in the geometry of the conjunction and the way each covariance matrix is projected in the nominal b-plane of the CA. For the smooth encounter proposed in this example the combined covariance

evolves slowly in time, reducing the differences between both approaches.

Short lead times may provide more insight from a physical point of view, but their practical applicability to operations is limited due to the high risk associated to last-minute manoeuvres. Fig. 3 presents the results for  $\Delta t$  up to  $50T$ , which corresponds to a maximum lead time of 79.29 hours (i.e. over three days). The maximum miss distance shows a linear long-term evolution, as it is not affected by the uncertainties. This is not the case for the collision probability: as we consider longer  $\Delta t$  the uncertainties at the CA also grow, limiting the reduction in collision probability. Note that these results have been obtained assuming that the covariances are known only at the time of the manoeuvre. In practical scenarios, the covariance is normally provided at the predicted CA

\* <https://www.space-track.org/>

through conjunction data messages/conjunction summary messages, and it is updated as the time of the CA approaches.

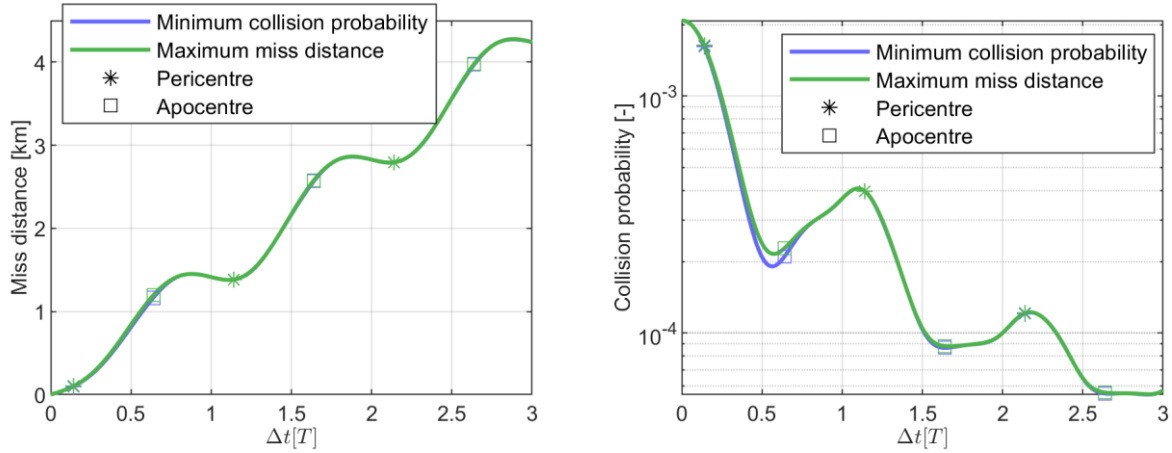


Fig. 2. Miss distance and collision probability for the nominal CA, for the maximum displacement and minimum collision probability impulsive CAMs with fixed magnitude 0.1 m/s and lead times up to 3 periods of the spacecraft.

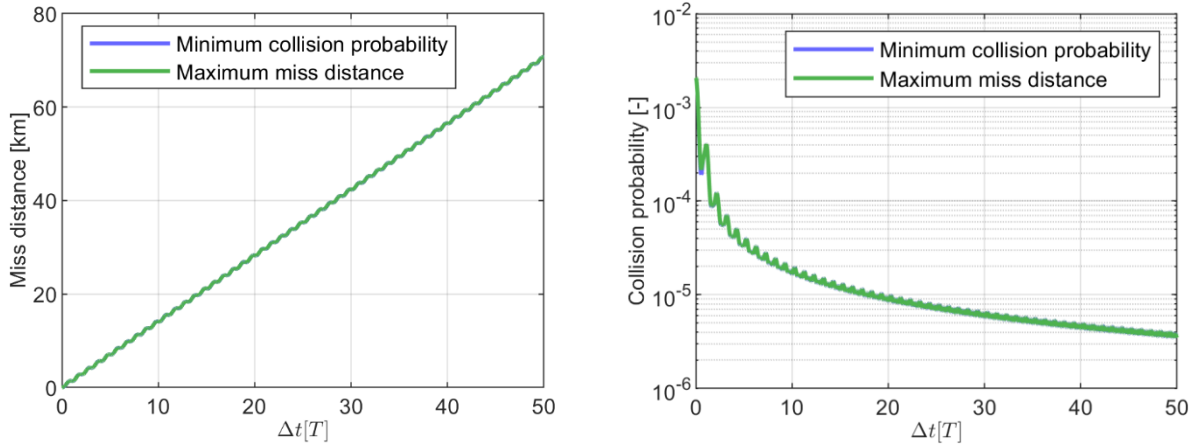


Fig. 3. Miss distance and collision probability for the nominal CA, for the maximum displacement and minimum collision probability impulsive CAMs with fixed magnitude 0.1 m/s and lead times up to 50  $T$ .

An example of low-thrust CAM is depicted in Fig. 4, for the same CA defined in Table 2. The CAM is modelled as a tangential-thrust arc with constant thrust acceleration  $a_t$  lasting  $\Delta t_{CAM}$ , followed by a coasting arc until the nominal CA of duration  $\Delta t_f$ . Therefore, the low-thrust CAM is characterized by the three parameters  $a_t$ ,  $\Delta t_{CAM}$ , and  $\Delta t_f$ . The plot on the left represents the miss distance versus  $\Delta t_{CAM}$  and  $\Delta t_f$ , for a fixed thrust acceleration  $a_t = 10^{-5} \text{ m/s}^2$  (this would correspond to a 1 mN thrust for a 100 kg spacecraft). It is observed that both  $\Delta t_{CAM}$  and  $\Delta t_f$  have a significant impact in the miss

distance, but only  $\Delta t_{CAM}$  will affect fuel consumption. In other words, it is possible to reduce CAM fuel requirements by increasing the lead time. Moreover, thrusting continuously during several periods is not practically feasible due to power supply restrictions. The plot on the right shows the evolution of the miss distance with  $a_t$  and  $\Delta t_f$  for a fixed value of  $\Delta t_{CAM} = 0.4 T$  (that is, the thrusting arc covers 40% of the orbit). Significant displacements can be achieved even for very small thrust accelerations by assigning a sufficiently long lead time.



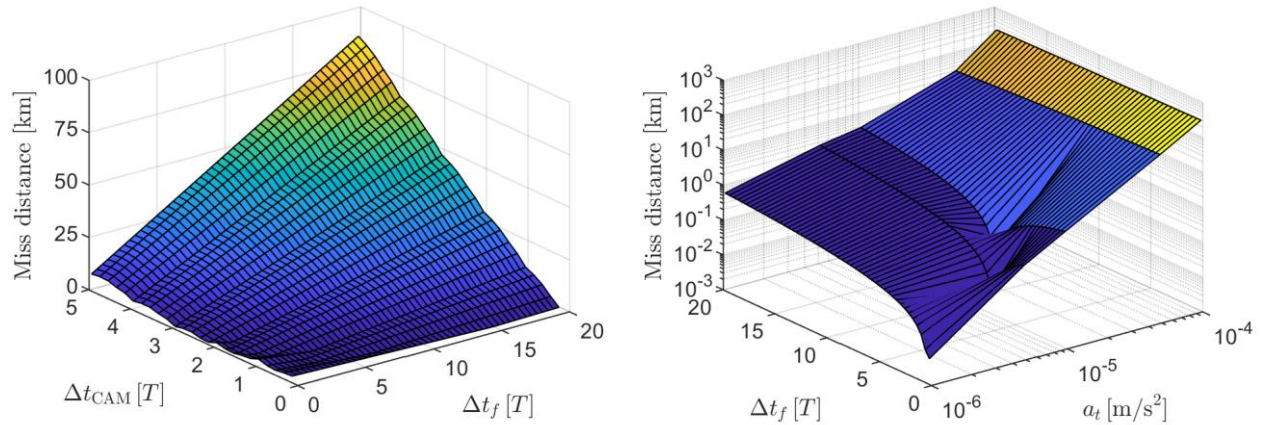


Fig. 4. Miss distance evolution for the nominal CA, for the low-thrust tangential CAM parametrized through thrust acceleration  $a_t$ , thrust arc duration  $\Delta t_{CAM}$ , and CAM lead time  $\Delta t_f$ .

## 5. Space Traffic Management applications

The models presented in Section 2 are characterized by their computational efficiency and the relative simplicity of their algorithmic implementation. This makes them suitable to address several of the challenges faced by future STM systems.

First, the amount of data to be processed is steadily growing due to the increase in launch traffic and improvements in SSA capabilities that allow to track smaller debris. Furthermore, the higher number of objects will not only increase the number of predicted CAs, but also the possibility of a CAM affecting neighbouring objects. In this scenario, the highly-efficient impulsive CAM algorithm allows to perform sensitivity analyses over several design parameters and update them as more accurate data about the predicted conjunction becomes available.

A different approach to tackle the increasing complexity of the space domain is the use of artificial intelligence. In a recent work Vasile et al. [14] proposed an artificial intelligence system to assist ground operators in the design and implementation of CAMs. They identified as a key challenge the lack of publicly available datasets of past CAMs that could be used for the training of the artificial intelligence, and addressed it by generating datasets for real and virtual scenarios from TLE information. They modelled the impulsive CAMs using the approach proposed by Vasile and Colombo [9] for asteroid deflection, which is also the basis for the impulsive CAM model used in this work and related references [3][4][5][7]. The key advantages of our model are that it allows for the analytical design of minimum collision probability CAMs, not only maximum miss distance ones, and for the evaluation of the state

transition matrices, which can be leveraged for the construction of the datasets.

Aside from applications to ground operations, there is also significant interest in providing satellites with autonomous collision avoidance capabilities. The models proposed in [3][4][5][6][7] meet the fundamental requirement that the algorithms have to be lightweight in order to fit within on-board computers limited power. They also perform very well for last-minute manoeuvres, which have limited practical interest for current operations but could be of interest for automatic CAMs in scenarios where the CA was not identified in time or limitations in satellite-ground communications prevent the upload of manoeuvre commands in time. However, these applications will also require to regularly upload to the satellite relevant information about nearby objects or even equip them with sensors to perform tracking of potentially offending objects. Sensor design falls out of the scope of this work, but a system similar to the Automatic Dependent Surveillance - Broadcast (ADS-B) used in airplanes could be considered.

There are still some open challenges regarding the application of the algorithms presented in Section 2 for STM. Hardware considerations regarding the on-board use of the impulsive CAM algorithms have yet to be included. Another hurdle is that the current model for low-thrust CAMs is only semi-analytical and requires numerical integrations. These numerical integrations are the most time-consuming parts of the algorithm and hinder potential on-board applications. Several approaches are being studied to reduce the need for numerical integrations. One is to replace the numerical fitting of the short-term periodics of  $a$  and  $e$  with analytical expressions. Another is to resort to other

perturbations methods such as the method of multiple scales, recently applied by Gonzalo and Bombardelli [15] to obtain fully analytical approximate solutions for the constant radial thrust problem. As previously noted, the modular structure of the framework allows to replace one of the blocks, in this case the model for the orbit modification, without affecting the others.

## 6. Conclusions and future work

This work has reviewed the current development status of a family of algorithms for the efficient modelling of CAMs between spacecraft and debris, and their application to future STM activities. These algorithms leverage analytical and semi-analytical techniques to model the orbit modification due to a generic manoeuvre in terms of its Keplerian elements, and then map it to changes in position and velocity at the nominal CA through a linearized relative motion model. The results are interpreted in terms of the miss distance in the b-plane and the collision probability for given uncertainties of spacecraft and debris. The model for the impulsive CAM is fully analytical, and it also allows for the computation of state transition matrices which can be used for the quantification of the time evolution of uncertainties. On the other hand, the model for low-thrust CAMs is semi-analytical, requiring a small number of numerical integrations. Several numerical test cases have been presented, highlighting the computational efficiency of the methods.

The remarkable computational performance and relatively simple mathematical structure of these models make them suitable to tackle a number of open challenges for future STM activities, such as: 1) efficiently analysing increasingly large sets of data, 2) generating sample dataset for training of artificial intelligence algorithms, or 3) developing lightweight algorithms suitable for on-board applications.

Based on the current status of the CAM analysis framework and the identified STM applications, two key items to be tackled in future works have been individuated:

- Introduce hardware considerations regarding the on-board applications for the impulsive CAM algorithm.
- Advance the semi-analytical models for the low-thrust CAM to reduce or eliminate the need for numerical integrations. The numerical integrations are the most time-consuming part of the algorithms and hinder on-board applications.

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