Non-linear vibrations of variable-stiffness plates: Fast analysis using semi-analytical methods

C.A. Yan[1], R. Vescovini[1], and E.L. Jansen[2]

[1]*Dipartimento di Scienze e Tecnologie Aerospaziali*

Politecnico di Milano, Milano, Italy

[2] *School of Engineering and Applied Science*

Rotterdam University of Applied Sciences, Rotterdam, Netherlands

Abstract

Aeronautical structures can benefit from the adoption of variable-stiffness (VS) configurations, and recent advances in fiber-steering and tow placement render these configuration more and more appealing. In the past, several studies have clearly shown that improved static and dynamic responses can be achieved with respect to classical quasi-isotropic configurations. Buckling and post-buckling responses were investigated by Weaver and co-workers [1, 2], while free-vibration and buckling are analyzed in Ref. [3] to mention a few. As regards nonlinear vibrations, many studies are available for isotropic and straight fiber composites [4, 5], while the field of variable-stiffness configurations is still relatively unexplored [6, 7]. However, nonlinear vibrations and, to a more general extent, dynamic stability (e.g. dynamic buckling, flutter) are important topics, both practically and theoretically, to be considered for the design of modern aerostructures. Presented is a formulation for the analysis of thin-plates layered with an arbitrary number of plies with non-uniform properties, i.e. variable-stiffness plates (VSP). The approach is developed in the context of a mixed variational framework, where the unknowns are the Airy stress function and the out-of-plane displacements. The Ritz method is employed to perform the spatial discretization: the continuous (in space) problem is so translated into a discrete one, characterized by a relatively small number number of degrees of freedom. Thus, a complex, time-consuming, non-linear problem can be solved with relative ease. This is particularly important to allow tailoring opportunities offered by variable-stiffness to be assessed and explored. With this aim in mind, a number of strategies is discussed to handle time-dependency. The first one refers to direct-time integration of the governing equation in modal coordinates. A second possibility relies upon an iterative procedure based on the Harmonic Balance Method (HBM), which is employed for evaluating the nonlinear forced steady- state response and the nonlinear free vibrations. Another approach refers to the method of averaging and is based on an assumptions regarding the time-dependence of amplitude and phase of the problem, where an averaging procedure is applied to eliminate time-dependency, i.e. by replacing the vibration amplitudes and phases with their average values over the time period. In addition, a perturbation approach – this method being essentially an extension of Koiter's approach to the case of vibrations – is proposed as a further efficient alternative for improving the time for the analysis. In this case the nonlinear problem is converted into a sequence of linear problem whose solution can be performed in fractions of a second. The proposed tools are computationally effective inasmuch the nonlinear analysis can be performed in seconds or fractions of seconds. For this reason, they are believed particularly suitable for gathering insight into the nonlinear vibration response of variable-stiffness configurations, a relatively new field demanding further investigation.

References

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