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A seminar on quantum
measurement of gravity for
geodesists and geophysicists

– Monograph –

March 18, 2019

Springer

Preface

During the last 30 years a great advancement in low energy physics, particularly for interactions of atoms with the electromagnetic field, has been achieved. Quoting the Nobel Prize talk of C. Cohen-Tannoudji, we can say that the development of electronics and laser techniques has allowed to implement a fine manipulation of atoms with photons. In this way, following the theory already worked out in the 50ies, physicists have learnt how to cool a sample of atoms at the level of the μk and, nowadays, even in the range of the nk .

A wealth of important applications has sprung out from this ability of manipulating large samples ($N \sim 10^7$) of cold atoms; among them, we mention only the improvement of atomic clocks, the creation of atomic gyroscopes and that of atomic gravity meters. This last item is obviously of great interest for geodesists and geophysicists, particularly for the potential applications to space geodesy.

More than one year ago my coauthor, Federica Migliaccio, asked me to look into the matter and give a seminar to a group of scientists of the Italian geodetic/geophysical community. After some discussion we soon realized that the initiative could be carried out following one of two completely different approaches: either to provide a “description” of the phenomena that permit the quantum measurement of gravity, or to “explain” the physical principles that are at the basis of what we could call with R.P. Feynman a theoretical experiment.

In fact, many geodesists and geophysicist do not have the scientific background to understand the quantum process leading to the measurement of g , so we decided to follow the second approach, also in view of the many different future applications of such matters in our disciplines; be it enough to mention the direct measurement of gravity potential differences by the observation of the different beat of clocks.

True is that one can always claim that there is some magic box producing numbers (measurements) and we should only be interested in how to use them for our purposes, however we firmly believe that loosing completely the contact with the science behind the measurement can lead us in the long run to a dead-end street.

In this way this text was born, conceived as a road that leads the reader from classical physics (mechanics and electromagnetism, considered as a common scientific background), to the basics of quantum mechanics, and finally to understand

the dynamics of a bunch of atoms falling in the gravity field, while interacting with suitably resonant laser beams.

Of course this is not a general text on theoretical physics, but it explains only those concepts that are necessary to understand atom manipulation by photons. Among these however there are the principles of quantum mechanics, presented in a special chapter, following my own views, from the period of my studies in physics.

We notice as well that there are basically two types of measurements of g , one based on stimulated Raman transition, and the other on so-called Bloch oscillations, although a new generation of instruments based on Bose–Einstein condensation is currently under study.

Among them, we have chosen to present the first one, which can be easily worked out analytically. Not to be mentioned however, the principles presented in the text can provide a good starting point to understand also the others.

The reader will realise that in the References most textbooks are in Italian. This is because they are the books on which I studied at my university time, yet you will realise too that most of them are famous books from the founders of quantum mechanics and, though mentioned in an Italian translation, they say the same things as the original text.

Place, month year

Fernando Sansò

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