Optimal Feeder Routing in Urban Distribution Networks Planning with Layout Constraints and Losses

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Abstract—**We address the problem of optimally re-routing the feeders of urban distribution network in Milano, Italy, which presents some peculiarities and significant design challenges. Milano has two separate medium-voltage (MV) distribution networks, previously operated by two different utilities, which grew up independently and incoordinately. This results in a system layout which is inefficient, redundant, and difficult to manage due to different operating procedures. The current utility UNARETI, which is in charge of the overall distribution system, aims at optimally integrating the two MV distribution networks and moving to a new specific layout that offers advantages from the perspectives of reliability and flexibility. We present a mixed-integer programming (MIP) approach for the design of a new network configuration satisfying the so-called 2-step ladder layout required by the planner. The model accounts for the main electrical constraints such as power flow equations, thermal limits of high-voltage (HV)/MV substation transformers, line thermal limits, and the maximum number of customers per feeder. Real power losses are taken into account via a quadratic formulation and a piecewise linear approximation. Computational tests on a small-scale system and on a part of the Milano distribution network are reported.**

Index Terms—**Mixed-integer programming (MIP), graph theory, power system planning, urban distribution network, feeder routing, optimization.**

I. INTRODUCTION

IN this paper, we describe the work on re-planning the medium-voltage (MV) urban distribution network in Milano, Italy, carried out within a long-term collaboration with UNARETI, the distribution system operator (DSO) of Milano. Before the deregulation in 1999, Milano had two separate distribution networks operated by two companies, which grew up independently and incoordinately. Recently, the two companies merged into UNARETI and today the main goal is to optimally integrate the two distribution networks and move to a specific new layout called 2-step ladder layout at the same time, which is selected and required by UNARETI planners.

The distribution network planning has attracted important research efforts from both university and utility companies. Reviews and surveys of the historical developments on the field are available in [1]-[5]. The main goal of distribution network planning is to determine the substation location, size and service area, number of feeders and their routes. From a mathematical point of view, distribution network planning is a nonlinear combinatorial optimization problem involving a large number of continuous and discrete variables. According to [1], the studies on power distribution planning (PDP) mainly differ in terms of objectives, design variables, voltage level (MV, low-voltage (LV)), problem types (greenfield planning, expansion), load models, planning periods (single-stage, multi-stage), constraints and optimization methods (numerical, heuristic). Regarding numerical methods, a mixed-integer linear programming (MILP) model is developed in [6], using DC power flow in combination with a linear disjunctive model. In [7], an evaluation of the model in [6] is carried out, which compares sequential single-stage and multi-stage planning with DG and investment constraints. An example of an MILP model with the computation of reliability indexes is presented in [8], while [9] presents a PDP model to determine the trade-off between the minimum cost and higher reliability.

In [10], an optimization model is formulated for the simultaneous planning of primary (i.e., MV) and secondary (i.e., LV) distribution networks and applied to a real residential grid of 75 buses. A multi-stage multi-objective PDP model for distribution substation siting, sizing and timing is presented in [11] and a new method for the spatial PDP considering variant environmental factors in the feeder routing formulation has been proposed in [12]. In [12], the derived mixed-integer nonlinear programming (MINLP) formulation is transformed into an MILP problem by integer algebra techniques.

A direct solution method to the optimal feeder routing

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problem of radial distribution systems is proposed in [13], which allows to reduce the complexity of the problem without compromising the optimality of the solution. In [14], the application of graph theory to decompose a dynamic PDP into a number of static PDP problems results in a method that improves the quality of the multi-stage PDP process. Reference [15] proposes an algorithm for the optimal feeder routing problem using the dynamic programming technique and geographical information systems (GIS) facilities. The effectiveness of the algorithm is also illustrated for a "real-world" study case. Reference [16] presents an algorithm to find the optimal distribution substation placement and sizing by using the particle swamp optimization (PSO) and optimal feeder routing by using modified minimum spanning tree (MST) .

Moreover, among the PDP literature, some works are specifically related to urban distribution networks. For instance, [17] proposes a software tool for urban MV distribution network planning to determine the optimum network configuration. Similarly, [18] uses an automatic network routing algorithm considering different switching and feeder backup scenarios in urban distribution networks. Reference [19] presents a method to re-design distribution networks based on GIS and a cascading of MILP approach. In [20], an evolutionary algorithm is presented to identify the optimal configuration of a real MV urban distribution network. Reference [21] proposes a method for improving the power supply capability based on structural optimization. In [22], an adaptive simulated annealing genetic algorithm (ASAGA) combined with traveling salesman problem (TSP) path optimization is used for urban open-loop MV distribution network planning. Dijsktra algorithm and TSP are used for feeder routing in [23], which deals with the optimal expansion planning of the urban distribution network of Grenoble in France.

Specifically, regarding losses, a multi-criterion algorithm is applied in [24] to reduce the system power losses. The algorithm is based on network reconfiguration. In [25], it is demonstrated that by means of MILP formulation, the network reconfiguration in a practical LV Indian utility distribution feeder reduces energy loss. A branch exchange technique is used in [26] for distribution network planning using loss reduction approach. The final results have been checked on a real case to demonstrate the feasibility of the proposed method.

A conceptual framework of resilience domain and its measurement approaches, especially a thorough conceptual framework of resilience as a subcategory of vulnerability in electric power systems is presented in [27]. Moreover, the paper presents conceptual difference between resilience and other indices of power system assessment such as reliability, risk and security.

As stated in [1], the majority of distribution utilities, like UNARETI, still rely on heuristic planning processes and empirical rules. The collaboration between researchers and utility planners leads to the development of models and methods that are suited to real-word PDP problems and meet the demands of utilities.

Adopting a single-stage network expansion approach, we present mixed-integer programming (MIP) formulations to determine the optimum feeder routing of the primary MV distribution network taking into account not only the layout and electrical constraints but also power losses, which are crucial from the perspective of application. The objective is to minimize the sum of capital costs, i.e., installation of new lines, and operation costs, i.e., distribution network losses. In order to find a trade-off between solution accuracy and computation time, a quadratic formulation with power losses and a piecewise linear approximation are considered. Thus, the main contribution of the paper is to express and convert the traditional planning approaches in a mathematical model in order to make them more efficient and effective.

A simplified preliminary version of the model without power losses is summarized in [28], while a modified version considering electric power cable joints is summarized in [29].

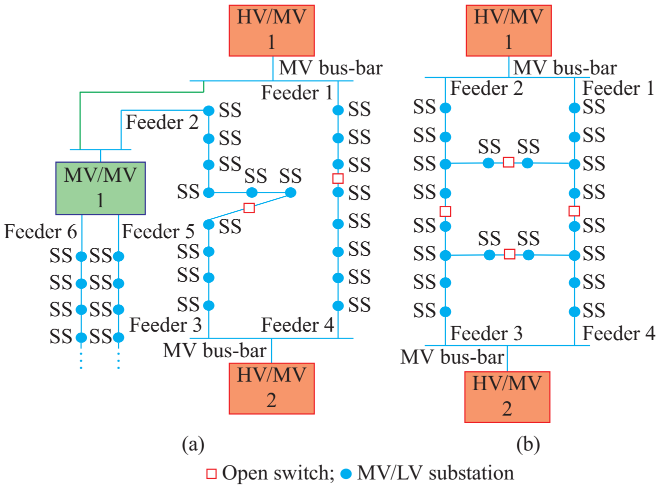
The remainder of this paper is organized as follows. In Section II, the Milano urban distribution network and the particular 2-step ladder layout requirement are described, along with the advantages and disadvantages. In Section III, modeling issues related to the layout and power losses are presented. The detailed MIP models for feeder routing with layout constraints and losses are described in Section IV. Numerical results are reported in Section V and conclusions are given in Section VI.

II. MV URBAN DISTRIBUTION NETWORK IN MILANO

The urban distribution network in Milano is spread over a metropolitan area of about 190 km^2, serving about 1.3 million people. The MV network is supplied by 11 HV/MV substations, connected to the national transmission system either at 220 kV or 132 kV, and 14 MV/MV substations directly supplied from the aforementioned HV/MV substations. In terms of extension, the MV network is about 4000 km, mostly consists of underground cables at 23 kV, 15 kV, 9 kV and 6.4 kV. The LV network consists of about 3000 km and it is supplied by about 6000 MV/LV substations. The distribution network counts 890000 LV customers and 1750 MV customers. The peak power demand in 2018 was 1672 MW out of a total installed power of 2805 MW.

In general, as for urban distribution network, voltage drop is not a concern for the MV network in Milano. MV feeders are too short and costumers are sited relatively close to the feeding MV/LV substation to cause substantial voltage drops [30], [31]. In this case, capacity limits dominate the design. Therefore, reactive power and voltage drop can be neglected and DC power flow is formulated as explained in Section IV.

In terms of layout, Fig. 1 shows the current layout and the new 2-step ladder layout between two adjacent HV/MV substations. One of the 14 MV/MV substations is directly supplied from HV/MV substations, which feeds local demand, as shown in the green box in Fig. 1. Those substations are considered as an extension of an HV/MV bus-bar in areas with difficulty in building an HV/MV substation due to space constraints. The loss of one of the dedicated power cables (in green) results in the loss of all the feeders arising from the MV/MV substation. This design philosophy leads to high losses, complicated cables protection system and limitation in cables capacity for contingency margin purpose. In Fig. 1, SS represents secondary substation, i.e., MV/LV substation.



(a) (b)

Fig. 1. Current and new 2-step ladder layouts. (a) Current layout. (b) New 2-step ladder layout.

Moreover, the distribution network in Milano shows some unnecessary and inefficient redundancy. Figure 2 shows a part of the distribution network in Milano in the area of HV/MV substation GADIO and VENEZIA. Feeders are colored based on the feeding substation. Different colors in the same service area highlight the network with redundancy layout to be solved.

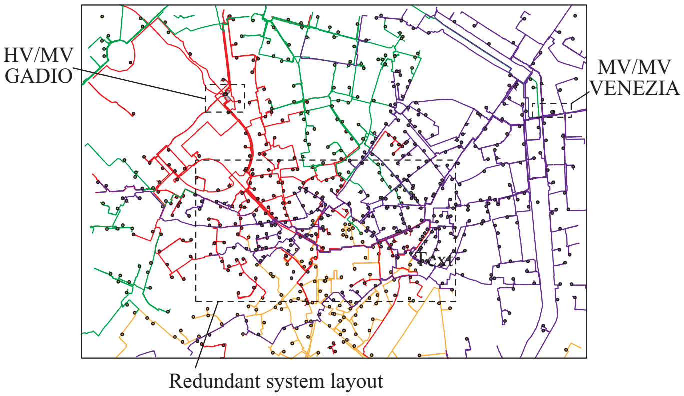


Fig. 2. View of distribution network in Milano.

The aim of the proposed methodology is to plan future upgrading of the distribution network in order to get a completely new optimized network layout.

A new layout called 2-step ladder layout is proposed in Fig. 3. The red squares are the MV switches installed at the beginning of each feeder for protection purposes, and the green circles represent secondary substations. For the sake of clarity, only the four MV switches related to a single 2-step ladder layout are depicted.

The 2-step ladder layout consists of four MV feeders interconnected with each other. The system, which is operated radially, has four tie points which can be switched to connect feeders to alternative sources in case of outage. This kind of layout is suitable for densely populated areas such as the city of Milano, where the load density requires many connections, and therefore, the number of feeders does not increase substantially [32]-[35].

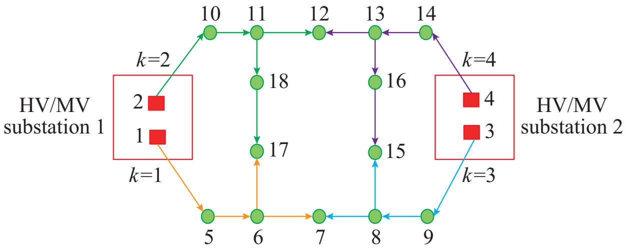


Fig. 3. Example of a 2-step ladder layout.

This type of layout, normally used by UNARETI planners, has advantages in terms of line capacity and voltage drop during normal operation as well as in $N-1$ contingency. Moreover, a layout standardization makes the network easier to be operated, reduces operation mistakes, and improves the scheduling of maintenance and repairing.

III. Modeling Issues

In order to model the 2-step ladder shape using mathematical programming approach, a directed graph $D=(V,E)$ is considered, containing the set of nodes V (corresponding to HV/MV and MV/LV substations) and the set of edges E (corresponding to the network connections). The goal is to determine a collection of additional edges to be activated so that the resulting network has a 2-step ladder layout and achieves a minimum cost.

As depicted in Fig. 3, the graph of the distribution network consists of HV/MV substations (square nodes) and MV/LV substations (circle nodes). Moreover, we define four types of paths identified by k, which are represented in different colors. In particular, each source node such as node 1 has only one edge connected, and each tie node such as node 15 has two connections belonging to two different paths. Moreover, there are nodes like node 14 with two connections belonging to the same path, and nodes like 8 called T-node with three incident edges, one coming and two leaving.

Unlike tie nodes, source nodes are known in our case, which will be selected and provided as an output of the optimization process.

In the optimization, the following assumptions are considered.

1) Single-stage optimization approach only yields the final picture of the distribution network layout.

2) The distribution network has to be shaped with 2-step ladder layouts.

3) Balance equations are approximated by DC power flow in order to keep the model linear, but losses are considered as a posteriori. This will be presented in Section IV.

4) Substation transformers and feeders have to be loaded within their capabilities and operation constraints.

5) Constraints on the maximum number of customers per feeder are included in order to reduce the impact of interrup-

tions in case of maintenance or fault of network elements. The utility company has to deal with a reward/penalty framework in terms of distribution network performance, which is positively affected by reducing the maximum number of customers per feeder.

In dense urban areas, distribution systems are dominated by limitations in terms of power capacity rather than voltage drop limitations. Therefore, voltage drop constraints are not included in the model. Considering the high load density, distributed generation is not an issue for the distribution network in Milano. Moreover, for the selection and size of conductor, according to UNARETI policies, a single standard cable type is considered suitable for the maintenance and repairing strategies.

Since the electrical transmission and distribution losses account for most of the power losses in the entire power system, and the largest amounts of these losses occur in MV and LV distribution lines, the feeder routing optimization should include power loss minimization.

The optimization model receives topological and electrical data as input such as HV/MV and MV/LV substations, existing connections, LV customers, MV/LV power demand, line data and so forth. The expected output is the optimal 2-step ladder layout which minimizes fixed costs related to the installation of new lines, and variable costs related to power losses, as shown in Fig. 4.

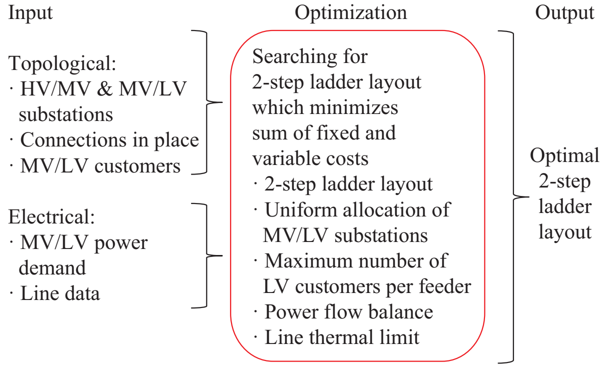


Fig. 4. Outline of optimization process.

IV. MIP FORMULATIONS WITH LAYOUT CONSTRAINTS AND POWER LOSSES

In this section, we present MIP formulations aiming at minimizing the sum of the installation costs of the new edges and the costs deriving from power losses, while taking into account the above-mentioned layout and electrical constraints. Real power losses are modeled either with a quadratic objective function or with a piecewise linear approximation.

Figure 5 shows variables and parameters used in the mathematical formulation. On one hand, for each node i, the following binary variables are defined: x_i^k (which is 1 if node i belongs to path k, 0 otherwise), y_i^k (which is 1 if node i is the T-node for path k, 0 otherwise), v_i^t (which is 1 if node i is one of the tie nodes, 0 otherwise), w_i^k (which is 1 if node i is supplied from path k, 0 otherwise). On the other hand, for each edge (i,j), the following variables are defined: binary variables z_{ij}^k (which is 1 if connection (i,j) belongs to path k, 0 otherwise), continuous variables f_{ij}^k (which is the power flow of connection (i,j) if (i,j) belongs to path k, 0 otherwise) and continuous variables Pl_{ij}^k (which represents real losses of connection (i,j) if (i,j) belongs to path k, 0 otherwise). l_{ij} is the length; r_{ij} is the resistance; U_{ij} is the thermal limit of connection (i,j); p_i is the local demand of node i; and NLV_i is the number of LV customers supplied from node i.

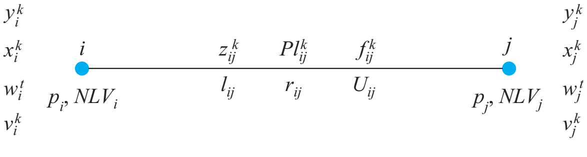


Fig. 5. Definition of variables and inputs.

In the objective function (1), the first term corresponds to the fixed costs (due to the new edges), and the second one corresponds to the costs of real power losses along all paths.

$$\min\left(\sum_{(i,j)\in E} c_{ij}^{in} l_{ij} \sum_{k=1}^{4} z_{ij}^k + \sum_{(i,j)\in A} \alpha N_{yr} N_{heq} c_{ij}^{l} \sum_{k=1}^{4} Pl_{ij}^k\right) \tag{1}$$

where c_{ij}^{in} is the installation cost of edge (i,j); α is the net present value coefficient of future real losses; N_{yr} is the number of years considered for real losses; N_{heq} is the number of equivalent peak power hours; and c_{ij}^{l} is the cost of losses of edge (i,j).

To take into account the existing edges, a fixed cost $c_{ij}^{in}=0$ is considered for connections already in place.

Some layout constraints used to shape the 2-step ladder layout, and some electrical constraints to model physical phenomena, complete the mathematical formulation.

The layout constraints can be categorized based on the type of node considered, i.e., HV/MV substation node, MV/LV substation node, T-node and tie node. Constraints (2) and (3) are for HV/MV substation nodes and allow a single edge to leave each source node, where s^k is the source node of path k; and K is the set of paths of the graph $\{1, 2, 3, 4\}$. Constraints (4)-(11) are for MV/LV substations. Each MV/LV substation must have two edges, except for tie nodes, which have one edge connected per path and $v_i^t=1$. T-nodes have three edges connected and $y_i^k=1$.

$$\sum_{j:(i,j)\in E} z_{ij}^k = 1 \quad i=s^k, k\in K \tag{2}$$

$$\sum_{j:(i,j)\in E} z_{ji}^k = 0 \quad i=s^k, k\in K \tag{3}$$

$$\sum_{j:(i,j)\in E} z_{ij}^1 + \sum_{j:(j,i)\in E} z_{ji}^1 = 2x_i^1 + y_i^1 - v_i^1 - v_i^3 \tag{4}$$

$$\sum_{j:(i,j)\in E} z_{ij}^2 + \sum_{j:(j,i)\in E} z_{ji}^2 = 2x_i^2 + y_i^2 - v_i^2 - v_i^1 \tag{5}$$

$$\sum_{j:(i,j)\in E} z_{ij}^3 + \sum_{j:(j,i)\in E} z_{ji}^3 = 2x_i^3 + y_i^3 - v_i^3 - v_i^4 \tag{6}$$

$$\sum_{j:(i,j)\in E} z_{ij}^4 + \sum_{j:(j,i)\in E} z_{ji}^4 = 2x_i^4 + y_i^4 - v_i^4 - v_i^2 \tag{7}$$

$$\sum_{j:(i,j)\in E} z_{ij}^1 - \sum_{j:(j,i)\in E} z_{ji}^1 = y_i^1 - v_i^1 - v_i^3 \tag{8}$$

$$\sum_{j:(i,j)\in E} z_{ij}^2 - \sum_{j:(j,i)\in E} z_{ji}^2 = y_i^2 - v_i^2 - v_i^1 \tag{9}$$

$$\sum_{j:(i,j)\in E} z_{ij}^3 - \sum_{j:(j,i)\in E} z_{ji}^3 = y_i^3 - v_i^3 - v_i^4 \tag{10}$$

$$\sum_{j:(i,j)\in E} z_{ij}^4 - \sum_{j:(j,i)\in E} z_{ji}^4 = y_i^4 - v_i^4 - v_i^2 \tag{11}$$

For example, for node 15 in Fig. 3, $v_{15}^3 = 1$, $x_{15}^3 = 1$, and $x_{15}^4 = 1$, which identify it as a tie node for paths 3 and 4. For node 11, $y_{11}^2 = 1$, and $x_{11}^2 = 1$, which identify it as a T-node for path 2.

Constraints (12)-(20) link the variables x_i^k, y_i^k and v_i^t. As shown in Fig. 3, each node must belong to only one path except for tie nodes which belong to two different paths (constraint (12)). Any path has only one T-node (constraint (13)) as well as only one node is a tie node for each couple of paths (constraint (14)). Moreover, (15) states that nodes can not be T-node and tie node at the same time.

$$\sum_{k\in K} x_i^k = 1 + \sum_{t\in T} v_i^t \quad i \in V\backslash S \tag{12}$$

$$\sum_{i\in V\backslash S} y_i^k = 1 \quad k \in K \tag{13}$$

$$\sum_{i\in V\backslash S} v_i^t = 1 \quad t \in T \tag{14}$$

$$\sum_{t\in T} v_i^t + \sum_{k\in K} y_i^k \le 1 \quad i \in V\backslash S \tag{15}$$

$$y_i^k \le x_i^k \quad i \in V\backslash S, k \in K \tag{16}$$

$$\begin{cases} v_i^1 \le x_i^1 \\ v_i^1 \le x_i^2 \end{cases} \quad i \in V\backslash S \tag{17}$$

$$\begin{cases} v_i^2 \le x_i^2 \\ v_i^2 \le x_i^4 \end{cases} \quad i \in V\backslash S \tag{18}$$

$$\begin{cases} v_i^3 \le x_i^3 \\ v_i^3 \le x_i^1 \end{cases} \quad i \in V\backslash S \tag{19}$$

$$\begin{cases} v_i^4 \le x_i^4 \\ v_i^4 \le x_i^3 \end{cases} \quad i \in V\backslash S \tag{20}$$

where S is the set of source nodes; and T is the set of tie nodes of the graph.

Constraints (16) - (20) complete the layout modeling. A node i can have $y_i^k = 1$ if and only if $x_i^k = 1$ (constraints (16)). For example, in the case of paths 1 and 2, the four nodes selected as tie nodes must have $v_i^1 = 1$ and the corresponding variables x_i^1 and x_i^2 equal to 1 (constraints (17)-(20)). Moreover, to avoid double edges between nodes, constraints (21) is added to the model.

$$z_{ij}^k + z_{ji}^k \le x_i^k \quad (i,j) \in E, k \in K \tag{21}$$

Finally, constraint (22) is also included to guarantee a uniform allocation of MV/LV substations among the four possible paths.

$$\sum_{i\in V\backslash S} x_i^k \ge |V\backslash S|/4 \quad k \in K \tag{22}$$

where $|\cdot|$ is used to obtain the number of elements of the set. The electrical constraints are necessary to guarantee the feasibility of network operation. Constraint (23) represents the power balance equations at MV/LV substations while constraints (24)-(27) represent the power balance at the HV/MV substation level. The lines thermal limit is enforced by constraint (28), and (29) presents the maximum number of LV customers per feeder.

$$\sum_{j:(i,j)\in E} f_{ji}^k - \sum_{j:(j,i)\in E} f_{ij}^k = p_i w_i^k \quad i \in V\backslash S, k \in K \tag{23}$$

$$\sum_{j:(i,j)\in E} f_{ij}^1 - \sum_{j:(j,i)\in E} f_{ji}^1 = \sum_{i\in V\backslash S} p_i w_i^1 \quad i = s^k \tag{24}$$

$$\sum_{j:(i,j)\in E} f_{ij}^2 - \sum_{j:(j,i)\in E} f_{ji}^2 = \sum_{i\in V\backslash S} p_i w_i^2 \quad i = s^k \tag{25}$$

$$\sum_{j:(i,j)\in E} f_{ij}^3 - \sum_{j:(j,i)\in E} f_{ji}^3 = \sum_{i\in V\backslash S} p_i w_i^3 \quad i = s^k \tag{26}$$

$$\sum_{j:(i,j)\in E} f_{ij}^4 - \sum_{j:(j,i)\in E} f_{ji}^4 = \sum_{i\in V\backslash S} p_i w_i^4 \quad i = s^k \tag{27}$$

$$f_{ij}^k < U_{ij} z_{ij}^k \quad (i,j) \in E, k \in K \tag{28}$$

$$\sum_{i\in V} NLV_i \cdot w_i^k \le N_{\max} \quad k \in K \tag{29}$$

where $N_{\max}$ is the maximum allowed number of LV customers per feeder.

Constraints (30)-(33) are used to ensure the radial system operation. In fact, combining w_i^k with x_i^k and v_i^t make it possible to impose the radiality operation of the 2-step ladder layout.

$$w_i^k \le x_i^k \quad i \in V\backslash S, k \in K \tag{30}$$

$$w_i^k \ge x_i^k - \sum_{t\in T} v_i^t \quad i \in V\backslash S, k \in K \tag{31}$$

$$\sum_{k\in K} w_i^k \ge \sum_{t\in T} v_i^t \quad i \in V\backslash S \tag{32}$$

$$\sum_{k\in K} w_i^k \le 1 \quad i \in V\backslash S \tag{33}$$

A. Quadratic Formulation

Real power losses are crucial from the perspective of application. In general, the power loss Pl_{ij}^k in a branch is given by the quadratic function $Pl_{ij}^k = 3R_{ij}\left(I_{ij}^k\right)^2$, where R_{ij} and I_{ij}^k are the resistance and current of branch (i,j), respectively.

Define r_{ij} as the resistance of branch (i,j), V_n as the nominal voltage, and $\cos\phi_n$ as the nominal power factor, respectively. The quadratic objective function and linear constraints (QOF-LC) formulation then directly minimizes the following objective function subject to the above linear constraints (2)-(33):

$$\sum_{(i,j)\in E} c_{ij}^{in} l_{ij} \sum_{k=1}^{4} z_{ij}^k + \sum_{(i,j)\in E} \alpha N_{yr} N_{heq} c_{ij}^l r_{ij} l_{ij} \sum_{k=1}^{4} \left(\frac{f_{ij}^k}{V_n}\cos\phi_n\right)^2 \tag{34}$$

Clearly, (34) is quadratic in the variable f_{ij}^k. This is equivalent to the minimization of the linear objective function (1) while adding the following quadratic constraints:

$$Pl_{ij}^k = r_{ij} l_{ij} \left(\frac{f_{ij}^k}{V_n} \cos\phi_n \right)^2 \quad (i,j) \in E, k \in K \tag{35}$$

The resulting MIP formulation is referred to as linear objective function and quadratic constraints (LOF-QC) formulation.

B. Piecewise Linear Approximations

In order to obtain an MILP formulation, we consider piecewise linear approximations of the convex objective function corresponding to real losses in terms of power flows [36], [37]. Given the cable parameters, the real losses are calculated as $Pl = 3RP_{epf}^2\left(1+\tan^2\phi\right)/V^2$, where R is the cable resistance; P_{epf} is the line power flow; ϕ is the nominal line power factor; and V is the line voltage.

A piecewise linear objective function and linear constraints (PLOF-LC) approximation with three pieces is shown in Fig. 6. This can be achieved by minimizing the objective function (1) subject to:

$$Pl_{ij}^k \geq m_1 f_{ij}^k + q_1 \quad (i,j) \in E, k \in K \tag{36}$$

$$Pl_{ij}^k \geq m_2 f_{ij}^k + q_2 \quad (i,j) \in E, k \in K \tag{37}$$

$$Pl_{ij}^k \geq m_3 f_{ij}^k + q_3 \quad (i,j) \in E, k \in K \tag{38}$$

where m_i is the slope; and q_i is the y-intercept of the straight line i.

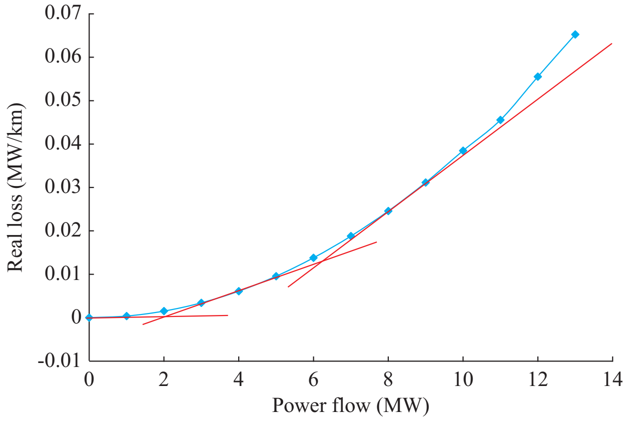


Fig. 6. Piecewise linearization with three pieces.

Clearly, the larger the number of pieces is, the more accurately real losses are approximated. For comparison purposes, a piecewise linear objective function and linear constraints (LOF-LC) approximation is also considered. The linear approximation is the tangent at the point of 8 MW.

V. Numerical Results

In this Section, we present results on both a test network and a real power system. The different approaches discussed above are implemented in general algebraic modelling system (GAMS). CPLEX 12.0 is used for solving linear or quadratic MIP formulations. All simulations are performed on a PC with Intel Core i5-7300 3.50 GHz CPU with 16.0 GB memory, running 64-bit Windows 10. A CPU time limit of 60000 s for each instance is set.

A. Results on a Test Network

Tests have been carried out on the 20-node test network shown in Fig. 7. The input data of the optimization model are given in Table I. Some of them are from UNARETI experience that characterize the distribution network in Milano.

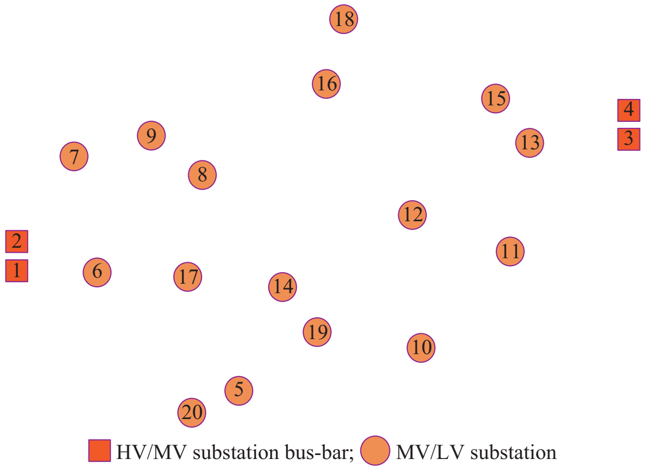


Fig. 7. 20-node test network.

TABLE I
INPUT DATA

Parameter	Value
Installation cost c_{ij}^{in}	90 k€/km
Cost of losses c_{ij}^{l}	77 €/MWh
Number of equivalent hours N_{heq}	3000 hour/year
Time horizon considered for losses N_{yr}	20
Net present value coefficient α	0.8
Line resistance r_{ij}	0.164 Ω/km
Thermal limit of connection (i,j) U_{ij}	9.75 MW
Nominal voltage V_n	23 kV
Nominal power factor $\cos\phi$	0.95
Maximum number of LV customers per feeder $N_{\max}$	4000

The time horizon N_{yr} is set to be 20 years, which is the usual life time of underground cables. The resistance and thermal limit are for a cable Al 3×185 mm^2, which is the standard cable installed by UNARETI.

Table II shows the results for the proposed formulations, where the following figures are reported: real losses, overall network extension, capital costs and costs of losses, total costs, CPU time and number of nodes in the branch and bound (B&B) tree. The benchmark for comparison is the quadratic formulation (QOF-LC) since it is more accurate.

The model with LOF-LC overestimates the losses, and results in a higher corresponding cost. The PLOF-LC shows a little underestimation of losses, which implies a lower cost of losses in the objective function. The LOF-QC turns out to be harder to solve than QOF-LC and runs out of memory, which makes the results different from QOF-LC.

The optimal layouts (solutions) obtained with the three formulations are shown in Fig. 8, where different colors highlight the 4 different feeders of the 2-step ladder layouts. Since the layouts yielded by QOF-LC and PLOF-LC coincide, the piecewise linearization works well in Fig. 8(a), as it gives the optimal configuration more than 200 times faster. For LOF-LC, due to the overestimation of losses, the optimal layout is different and has a higher total capital cost. The dashed lines represent open switches, which make the feeder system radial. This is a useful output of the optimization process. Considering the computation time, the formulation of PLOF-LC turns out to be the best one since it provides the optimal layout in less than 100 seconds.

TABLE II
COMPARISON OF RESULTS

Case	*Pl* (kW)	*l* (km)	Installation cost (k€)	Loss cost (k€)	Total cost (k€)	CPU time (s)	No. of B&B nodes
QOF-LC	18.82	4.30	387	253	641	16589	434581
LOF-QC	25.89^{*}	4.36	392	267	659	16140	1400000
LOF-LC	59.40	4.44	399	268	667	4	691
PLOF-LC	15.85	4.30	387	214	601	80	8825

Note: * represents out of memory.

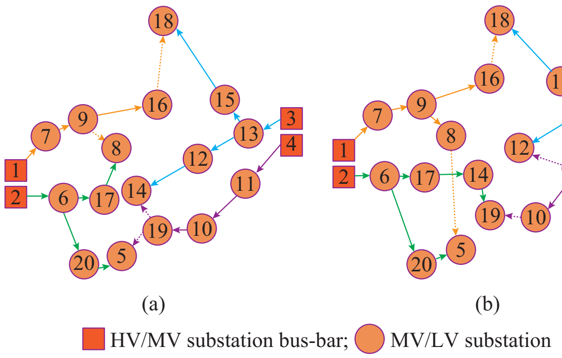


Fig. 8. 20-node network. (a) QOF-LC and PLOF-LC. (b) LOF-LC.

B. Results on a Real-world Subnetwork

The best formulation (PLOF-LC) is tested on a real-world subnetwork made of 70 nodes (66 MV/LV substations and 4 HV/MV substation bus bars) isolated from the whole distribution network. The locations and connections of HV/MV and MV/LV substations already in place are depicted in Fig. 9, where the numbers are the names of the substations. PV1 and PV2 are the two source nodes (substation bus bars) of HV/MV substation Porta Volta, while GA1 and GA2 are the two source nodes (substation bus bars) of HV/MV substation GADIO.

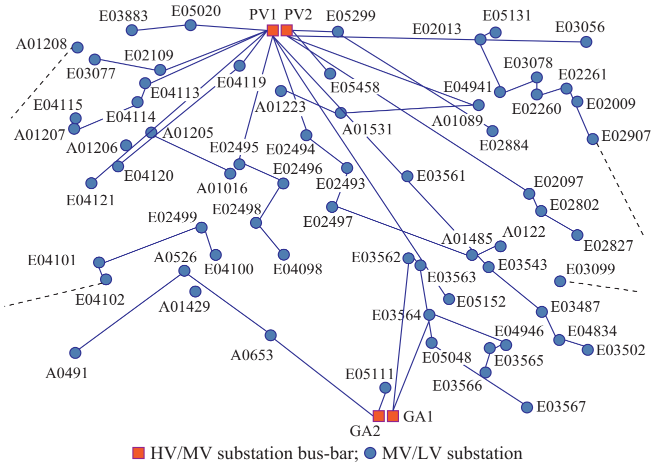


Fig. 9. Topological input data of 2-step ladder layout for a real-world subnetwork.

The blue lines are the existing connections. The dashed black lines represent connections which start in the subnetwork but end outside of this set. These connections are not considered as candidate here. The computation time limit is set to 100000 CPU seconds.

Figure 10 shows the result of optimal layout considering the network already in place. The four feeders of the 2-step ladder have different colors. The red ovals show the lines opened on one side, which guarantees the radiality of the distribution network.

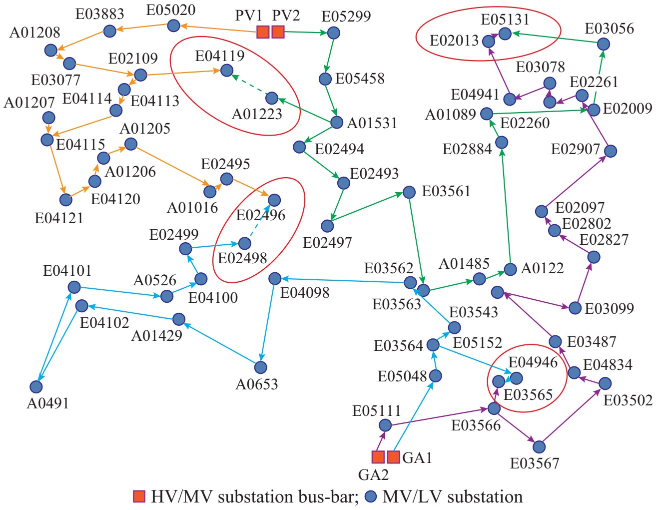


Fig. 10. Optimal 2-step ladder layout for a real-world subnetwork.

Figure 11 clarifies new lines installed and lines already in place kept even in the 2-step ladder optimal layout.

Table III shows the feeder results. All the feeders are quite balanced, especially if we compare the couple from PV1 and PV2 with those from GA1 and GA2, in terms of number of MV/LV substations connected, LV customers bounded to 4000 per feeder, length of feeders, and MV customers.

According to Table IV, the network extension can be reduced by about 30%. Starting from an initial network extension of 16.1 km, the algorithm removes 12.1 km lines, which are now unnecessary in the new 2-step ladder layout.

4 km lines are kept and 7.4 km of new lines are proposed to be installed down to a new network extension of 11.4 km. Considering the investment cost related to new proposed lines of around 665 k€, and a saving of about 780 k€ due to the reduciton of losses and faults, a payback period of 17 years is obtained.

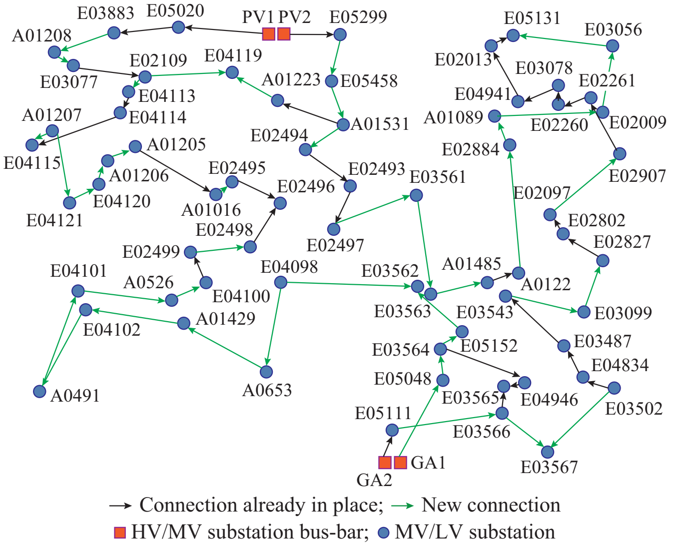


Fig. 11. Existing and new connections of 2-step ladder layout for a real-world subnetwork.

TABLE III
FEEDER RRESULTS

Feeder	l (km)	Power supplied (MW)	No. of MV/LV substations	No. of LV customers	No. of MV customers	Loss (kW)
PV1	2.2	4.5	17	3844	4	4.63
PV2	3.0	3.8	16	2625	4	3.80
GA1	3.3	3.3	15	2257	2	2.97
GA2	2.9	5.1	18	2861	7	9.15
Total	11.4	16.8	66	11587	17	20.55

TABLE IV
LENGTH RESULTS

Item	Value
Initial network extension	16.1 km
Lines removed	12.1 km
Lines kept	4 km
New lines installed	7.4 km
Extension of new network layout	11.4 km
Extension change	30%
Installation cost	665 k€

Finally, Fig. 12 shows a representation of the optimization output using Google Earth map.

VI. CONCLUSION

The paper presents MIP models to optimally integrate the two MV distribution networks in Milano to achieve the required 2-step ladder layout.

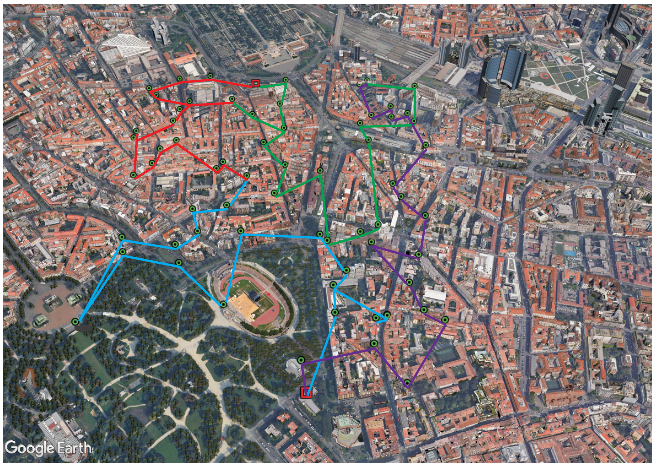


Fig. 12. Optimal 2-step ladder layout for a real-world subnetwork on Google Earth map.

The cable routes are kept, installed, or removed so as to minimize the capital costs and losses. The objective function and constraints are firstly explained. Moreover, advantages and disadvantages of four different model formulations are reported and verified on a test network. Since the piecewise linear approximation with three pieces leads to a good trade-off between network layout quality and computation time, it has been applied on a real subnetwork. Simulation results show a significant reduction of the distribution network extension using the proposed approach, which potentially increases the reliability of the network.

We are now working to improve the proposed model using a dynamic approach to take into account interactions between variables over time. Moreover, we are considering to adopt a robust approach to deal with traditional uncertainties related to long-term planning of distribution network expansion.

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