First experiences of creativity in mathematics for a primary school in Italy

Federico Malucelli
DEIB – Politecnico di Milano
Piazza L. da Vinci 32
Milan – Italy
federico.malucelli@polimi.it

Matteo Fantinati
Scuola Primaria Bombonati
Ist. Comprensivo 5
Ferrara – Italy
fantinatimatteo@libero.it

Abstract:
In the early years of school, pupils have a very positive approach towards any new topic. Very often, the first contact with mathematics in the Italian school risks giving a limited and somehow negative impression, generating long lasting effects. In this paper we report on a series of experiences carried out in a second-year class of a primary school, that allowed us to introduce new concepts and approaches through games and puzzles. The aim was to stimulate the creativity and empower pupils. We have used simple materials and applied the *Puzzle Based Learning* method. Despite the limited time devoted to this experience, the outcomes have been extremely positive.

1. Introduction

Early primary school children generally tackle any topic with great enthusiasm and motivation. Catching their attention and actively involve them in the learning process is relatively easy. Generally, in the first classes of primary school, the first contact with mathematics is about numbers and arithmetic. This is undoubtedly important in order to give pupils the basic tools to juggle with more advanced concepts that they will meet later in their school career. However, this activity takes up most of the time. Beyond what are the "National Curriculum Guidelines" of the Italian Ministry, it must be agreed that the equivalence that is likely to be established between mathematics and arithmetic can be misleading, generating a dry and uncreative idea of the discipline.

We often ask ourselves trivial questions: why do the first approaches to mathematics concern basic operations and "tables" to be learned by heart? The same first approaches to Italian and music, just to mention two very engaging subjects, but no less creative than mathematics, rarely concern grammar analysis and solfeggio.

Why is it that when we look at a drawing of a child, where we leave her free to express her creativity, we find it beautiful and original, but we do not give the possibility to do the same thing in mathematics?

The impression and feelings you get from the first approaches remain for a long time. As argued in [11], the combination of performing repetitive tasks, learning by heart and mechanical repetition of concepts creates the preconditions to inhibit any creative momentum, not only in mathematics.

The risk, therefore, is that children will grow up with a distorted idea of mathematics with repercussions that can be decisive for their school career and, in the longer term, for their future, such as possible university choices and job opportunities.
In this context, the key issue, even in primary school pupils, is to try to stimulate creativity through curiosity and to give pupils the opportunity for "inclusive" learning, where they can play an active role in identifying and exploring knowledge [11]. Some teachers may have the impression that they will not be able to cope with such a task. With our experience, where we propose simple and affordable activities, we want to demonstrate the opposite.
With this article we do not want to make a theoretical-pedagogical contribution to the teaching of mathematics in primary school. Rather we want to illustrate a small project (less than 10 hours in total) carried out in the school year 2016-17 in a second primary class in Italy. The objectives of the project were mainly to open a small window on an alternative vision of mathematics and computer science with the emphasis on the creative and intuitive component of the two subjects, with the sole aim of stimulating pupils' curiosity and desire to discover.
According to the psychologist Wertheimer [19], the key objective of school activity is to foster the so-called "productive thinking" in students, i.e. to orient pupils to solve new and unusual problems in order to stimulate the intuitive component. Our aim was precisely this: to propose activities that would bring out the children's intuition and ability to deal with new problems, the so-called Köhler insight [13, 5], i.e. to activate the cognitive mechanisms that allow to solve situations never faced before.
According to Emma Castelnuovo [3], the observation, the use of materials and objects and their manipulation are the main tools to trigger the process of intuition and creation. To pursue this goal we have used storytelling [6], we have used simple materials such as ropes, cards, flashlights, coat hangers [2], and above all we have made children move around in open spaces or in the gym, even if sometimes the risk has been to create chaotic situations.

Even if the pupils were very young, we used a *participationist* approach [18] being inspired by the methods of Puzzle Based Learning [14]. We took advantage of the playful component of the activities, using as much as possible materials, tools taken from everyday life, offering the pupils games and group work.

In the following we will briefly go back to the basic concepts of Puzzle Based Learning, and then briefly describe the activities carried out, highlighting each time the purpose of the meeting. A brief report on the results achieved will conclude the article.

2. *Puzzle Based Learning*

Puzzle Based Learning [14] is a didactic approach that aims to develop reasoning skills, perseverance and motivation in dealing with problems, thus providing the basic foundations of problem-solving techniques. In this context, the teacher only guides the activities, leaving pupils free to tackle the problem and experiment with methods of solution, or getting them back on track if necessary.

The role of the teacher is not to "teach" how to solve problems, but rather to propose stimulating games for everyone, to follow the students' reasoning, to arrive with them at the solution. He must be ready to accept methods of problem solving different from what he has thought. The teacher must therefore be ready to get involved and not feel uncomfortable in front of possible obstacles. By adopting a sporting similarity, the teacher assumes, during Puzzle Based Learning, the role of the coach, whose motivational skills must be predominant compared to purely technical ones.

In this way students are called upon to look for new problem-solving strategies, to choose between different options, to make decisions and act flexibly, to acquire and interpret information, to identify links and relationships [10].

Teachers only need to encourage learning by discovery, i.e. learning based on the experience of the learners through doing and working, cooperative and collaborative learning that can enhance student motivation. Teachers play the role of motivator - stimulator, a kind of tutor who uses learners' skills to build higher level skills ("scaffolding") [15].

3. Experience in class

The class in which we carried out the activity is composed of 26 pupils, some of them very lively and others difficult to involve in structured activities. From the very beginning the pupils understood the spirit of the activities, also because they appreciated the playful component.

We report briefly the experiences made during the 6 meetings lasting about an hour and a half, held during the curricular timetable, briefly listing the activities and their purpose.

Meeting 1: Let's break the ice and discover the topology

The first meeting had the aim of dropping any reverential fear towards the external teacher and immediately bring the pupils into the spirit of Puzzle Based Learning by taking an active role. Given the age of the children and their spontaneity, this did not require a great deal of effort, more attention required instead to make them play according to the established rules. We proposed a very addictive physical puzzle, already experienced previously in an upper secondary school class [9]. The pupils were divided into groups of 3 and in each one a volunteer was designated who wore a T-shirt and to whom the wrists were tied with a rope of about 50 centimetres.

The challenge was to take off the shirt and put it back on again so that it would be upside down with the label on the outside and on the back. The task of the volunteer's assistants was to help him/her with the limited movements of the rope and to suggest the actions to be taken to arrive at the solution. The teacher only stimulated the observation: what happens to the shirt once it has been pulled off? It hangs like when you hang the laundry, someone observed. Is it straight or is it upside down? If we put it on again by doing the reverse operation, what happens? How can we reverse it before putting it on again? After a few tries, all the groups came up with the solution.

Then we proposed to everyone to try to design a T-shirt using paper ribbons about twenty centimetres long and 3 wide, glue and scissors, obtaining a ring, with an "inside" and an "outside" that can be coloured differently. We then invited the children with another paper ribbon of the same size to build an "innovative T-shirt", thus another ring, in which it was not possible to distinguish the "inside" from the "outside", that is a Moebius ring. The property was verified by drawing with a pencil a longitudinal line inside the closed ribbon. We then pushed them to experiment what happens to the two types of rings by cutting them with scissors longitudinally on the centre, or remaining one cm from the edge, discovering something magical. We concluded the meeting by showing the class of artwork images where the Moebius ring is represented, as for example many works by Escher, and a short video with the funny story of "Wind and Mr. ug"[1].

[1] https://www.youtube.com/watch?v=4mdEsouIXGM

Meeting 2: ***The oldest optimisation problem in history***

Optimisation problems are simple enough to grasp, after all, optimisation is inherent in human nature, and in nature more generally. The purpose of this meeting was to let the children touch the spirit of optimisation. Emma Castelnuovo herself proposed in her textbooks for middle school, courageously but with good reasons, the first rudiments of linear programming. Experiences of optimisation problems faced in a high school are reported in [8,17]. The importance of optimisation in high school and its place in programmes in Italy and abroad is discussed in [4].

We started by telling the legend of Queen Dido, when she arrived with her ships on what is now the Tunisian coast and asked the locals if they could give her a piece of land as big as an ox's skin. Dido, however, had the skin of the ox cut to form a very long rope and used it to enclose the largest possible piece of land and in this way founded the city of Carthage.

We went to the gym to play Dido. The class was divided into 3 groups of about 8 children and each group was equipped with a closed loop rope 8 meters long. Each group pretended to be the crew of a Dido ship that needed space to sleep. Since the class obviously did not yet have the concept of an area, we used the children lying on the ground as a unit of measure. Initially we limited ourselves to making rectangular shapes, discovering that in the long and narrow rectangles only a couple children could lay, in the square there were 4. If instead we left the rope free to assume the most useful shape, we discovered that the circle could accommodate the full crew, even if not so comfortably. So, we asked them to guess what would happen if we joined two ropes and therefore had a perimeter twice as wide as before, and everyone was unbalanced in saying that if one rope could accommodate 8 children, two joined ropes would accommodate twice as many. To their amazement, however, they verified that with two strings joined together they could accommodate the whole class, and also very comfortably.

A secondary aim of the game was also to introduce the concepts of perimeter and area and to observe how the two measures are related.

Meeting 3: ***Ariadne, Theseus and the Maze***

The purpose of this meeting was to bring the children into contact with the first coding elements.

We started with the story of Theseus and the Minotaur locked in the labyrinth and explained the stratagem used by Ariadne to allow Theseus to

escape. This was the cue to play with the labyrinths. We distributed a small labyrinth represented on a squared sheet of paper and asked to find the exit from a fairly central point. We also invited the children to pay attention to the sequence of the movements made: number of steps forward, right turns, left turns. We then proposed the same labyrinth in a larger scale on a large sheet of wrapping paper, where the squares had a side of 20 cm. We opened a challenge to groups to those who could get a Beebot[2] out of the labyrinth, planning its movements. The group were asked to test their Beebot programme, passing the turn to the subsequent group at the first fail.
As a last activity, we pointed out to the children how to get out of a labyrinth by seeing it from above is not so difficult, while Theseus could not have the vision from above. So, we invited the children to think about what Theseus could do in the labyrinth. Since we did not have a real labyrinth in which to immerse ourselves, on the type of hedge or corn plant mazes that can be found in various locations, we gave them a labyrinth enclosed in a shoebox and made of appropriately glued "walls" of drink straws. On the floor of the box we put a sheet of glossy paper. The exit of the labyrinth was opened on the edge of the box. We then covered the box with a cardboard in which we drilled a hole where we inserted a whiteboard marker. This helped us in reconstructing the path made once the lid was removed, and we could easily erase the trace to make another attempt. The children in turn tried to get out of the labyrinth experimenting various strategies. We verified that if we always kept the right, or always the left, when hitting a wall, we could eventually reach the exit. Though it was not always the shortest trajectory.

Meeting 4: ***The invention of chess and binary numbers***
The purpose of this fourth meeting was twofold. On the one hand, we wanted to make children reflect on the concept of exponential growth and how it goes beyond intuition. On the other hand, we wanted to introduce binary numbering and how it is used in computers.

We started with the story of Lahur Sessa, the inventor of the game of chess, who as a reward asked to be given a grain of wheat for the first square of the chessboard, 2 for the second, 4 for the third, 8 for the fourth, and so on, doubling the quantity each time until the sixty-fourth square. The king who had accepted the request lightly, even believing it to be modest, as the

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[2] www.bee-bot.us

number of squares and grains of wheat increased, realized that the request for a reward was exorbitant and impossible to satisfy. We repeated the experiment in the courtyard of the school using small stones. But we had to give up after the first row of squares. So, we tried to think of the thickness of a sheet of paper that is folded in half repeatedly. We realize that with relatively few iterations we could reach incredible thicknesses, enabling us to reach Mt. Everest or even the Moon.
Then, we built some cards: on each one we represented as many dots as the first powers of 2: 1, 2, 4, 8 and 16. We placed the cards on a magnetic board in descending order of number of dots from left to right. We invited the children to use the cards to compose numbers by adding the dots together, taking each one at most once, and we found that each number from 0 to 31 can be obtained in a unique way. Afterwards, we tried to represent the choices of the cards by putting a 1 under the chosen card and 0 vice versa. This "secret" code, at this point without the cards, was enough to decipher the numbers. Instead of the cards, we put flashlights, hanging on hooks, where the lit torch is to be interpreted as a 1 and the torch off as a 0. It was a fairly simple and involving game to get the numbers from the configuration of the lights and vice versa. We concluded by saying how the memory inside computers or mobile phones works on the principle of the lights that we used in our game, although of course there are many more.

Meeting 5: ***Let's guess, weigh and sort***
The aim of this meeting was to introduce the idea of efficiency in the computation, i.e. to make pupils think about how many operations are needed to reach an objective and how to use the information to minimize this number. Thus, we introduce the basis of computational complexity.
We started the meeting by playing a guessing game. We prepared a deck of eight cards. The back of the cards had written different random numbers between 1 and 100. The game consisted in guessing whether a particular number was present in the deck or not. We gave volunteers 8 candies to use as money, for each attempt they had to "pay" one candy. We immediately noticed that if you were lucky, you would only spend 1 and if you were unlucky you would need 8. We then moved on to the case where the cards were sorted and the children developed quite quickly a strategy that took advantage of the information by using a binary search with which they were able to find the card in at most 4 attempts, in the worst case.
Pointing out the importance of having the objects sorted, we carried out the next activity. We presented to the class 8 objects (a bag of rice, a bag of

pasta, a bag of flour, a bag with carrots, a bag with oranges, a bag with apples and a bag with bread) of unknown weight and we asked ourselves the problem of putting them in order by weight, from the lightest to the heaviest. Before proceeding with the sorting, however, we had to think about how to compare them. After hearing the various proposals and analysing the material available, we built a 2-arm scale using a skirt hanger hanging from a fairly sturdy and thin stick placed between two chairs. We then proceeded to compare the objects, making comparisons between all pairs of objects. We then thought about whether some comparisons could be avoided, taking advantage of the transitive property of the sorting.
Therefore, we proposed to use a comparison strategy (known in computer literature as "sorting network" [7]) drawn on a large sheet of parcel paper lying on the ground. The sorting network has in input the items to be compared and it represent the pairwise comparisons to be made. The sorted pair is the input of subsequent pairwise comparisons. The overall number of comparisons is minimum. The sorting network was used to sort a group of 6 children in height. Our sorting network remained in the classroom and it was used any time the children had to sort objects.

Meeting 6: ***The Problems of a Mayor: Games with graphs***
The purpose of the last meeting was to address decision-making problems, such as determining the minimum spanning tree or a graph colouring. See [12] for an introduction to these problems and to other graph optimisation problems. At first glance they might seem complex problems and far from the interests of 7-8 year old children, but actually seen within a gaming context they become simple and intuitive.
The secondary purpose of the meeting was also to try to educate children to make shared decisions.
Before we started, we drew with the chalk in the schoolyard some undirected graphs with about ten nodes, so that there would be no downtime during the activity. The graph to be used in for the spanning tree problems had one value associated with every arc.
The common context for the two games was the city one. In the first game we imagined a city that has suffered a flood making it necessary to connect the islands that have formed (the graph nodes) through the construction of bridges (the arcs) so that from any island you can reach any other island using paths that involve crossing even more than one bridge. The cost of reconstruction of each bridge is given by the number we put on the arcs.

The goal is to minimize the overall cost. The abstract problem is known as the minimum spanning tree.
In the second game, instead, we pretended to be in a city where the inhabitants are very quarrelsome and want to colour the houses (the nodes of the graph) so that each inhabitant from his window cannot see other houses with the same colour as his own. The arcs of the graph represent the line of sight. In order to limit expenses, we want to use as few colours as possible.
The children were organized in groups, one per graph, and in each group were identified: a mayor (the one who makes the decisions after consulting the citizens), an engineer (the one who with a coloured plaster builds bridges between islands), a chief accountant and a helper (who take into account the expenses by making the sum) and the citizens, one per island, who require the construction of bridges and who try to move around the city once the bridges are present.
In the second problem, since simpler accounting is required, we only have the mayor and the citizens, who personally colour the houses with coloured chalks, after the mayor has assigned them a colour. In both cases, after some heated discussion, the solution was found.

4. Outcomes and discussion

The first aim of these meetings was to experiment with a creative and original approach to mathematics. Pupils were asked to act personally, to use knowledge (assimilation of information), skills (applying knowledge and using know-how to solve problems) and competences (ability to use personal, social, methodological skills and abilities) in their possession.
The learner no longer passively receives information, but becomes the protagonist of the learning process himself. In addition to stimulating the cognitive process of individual learners, “Learning Together” and “Learning by doing” strategies were implemented [1] thanks to which an attempt was made to develop a spirit of initiative, of shared participation, which would favour the acquisition of key citizenship skills. In particular, it has been tried to actively involve the pupils in the proposed activities by bringing their personal contribution.
Collaboration and participation have become indispensable tools to organize one’s own learning and acquire new study skills, but also to understand and discuss the different points of view of classmates.

A second objective of the experiment was to try to bring primary school children into contact with advanced concepts of mathematics and computer science, such as algorithmics, computational complexity, topology and optimisation, which are generally dealt with at university level, although in greater depth. Counting on the fact that we used a playful approach and that children, unlike adults, do not show any reverential fear, the experience was very positive and it was exciting to see how 7-8 year old children came to make their discoveries with great naturalness.

A posteriori, we can say that we met these two objectives. This series of meetings had a positive feedback on the class group. The pupils joyfully welcomed the presence of an external teacher and immediately showed an interested approach to the new activities presented. It should be remembered that the children's attention during this age group is relatively low (about one hour), and it is therefore essential to structure the activities proposed in such a way as to favour concentration, variety of activities and physical movement, so as not to overburden the lesson. The playful aspect of learning, originality and novelty in teaching methods are fundamental components that attract the student and condition his/her interest and participation.

It should also be taken into account that every child reacts subjectively to new activities in which he or she is involved: he or she may show enthusiasm, willingness to do and participate, exuberance or, on the contrary, may be disinterested or restless, depending on the character of each one. It is the teachers' task here to encourage the participation and involvement of all pupils in the class, on the one hand by containing those who are a little too "excited", and on the other by stimulating those who are more shy and reserved.

During our activities there have certainly been some chaotic moments. Remember that the class was very numerous (26 pupils), the pupils are almost always euphoric towards new activities that go beyond traditional teaching and demonstrate this with a more active behaviour. However, this is also partly due to the fact that each pupil is directed to take advantage of their previous knowledge and experiences and, consequently, want to share them with teachers and their classmates.

From an objective point of view, the effects of the activities of this experience have been tangible. The class, which was previously considered difficult to manage and with elements with inclusion problems, came out more cohesive and motivated. The results in the mathematics tests following the experiment were overall discrete with some excellence. We can therefore say that the activity motivated the students to commit themselves to the discipline and led to general improvements in the profit of the individual students.

There has been also a third implicit objective that involved the teacher learning. With these activities the school teacher experimented creative teaching techniques [16] and new concepts. This experience has been much more effective than any instructional course that could be organized on the topic.

5. Conclusions

In this article, we reported on a new and ambitious experiment whose aim was getting very young pupils in contact with the creative face of mathematics in an early stage of their career.
This workshop has been an excellent opportunity to provide students with a different and original approach to mathematics and to make them understand that mathematics is present in many activities we face every day, even if we are not always aware of its presence.
The proposed activities were intended to strengthen the ability to frame and tackle certain mathematical problems that arise from a child's life, rather than from textbooks. This involved obtaining relevant information, developing strategies, verifying them in the field and experiencing the results. The latter was facilitated by the use of materials. It was also intended to give a point of view of computer science that frees it from the link with the hardware, while linking it with algorithmic thinking and systematization of the instructions needed to achieve a goal or a solution.
From a behavioural point of view, these activities have been designed to stimulate divergent thinking and critical thinking, as well as the involvement and inclusion of all pupils by highlighting their skills.
This type of experience is not confined to mathematics, nut can be integrated with other disciplines, such as Italian, art or theatre. In this way, the experience can gain strength and be even more effective, as highlighted in [11], where it is pointed out how the connection with art strengthens creative and problem-solving skills.

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Federico Malucelli MS and PhD in Computer Science at the University of Pisa. He is full professor of Operations Research at the Politecnico di Milano. He has been teaching Operations Research and Optimization courses at all levels in several Universities and Schools, in the last 30 years. He applies Operations Research for solving real problems in transportation, telecommunications, logistics and energy management. He has published more than 100 contributions on the subject and some articles on teaching optimization and active learning.

Matteo Fantinati MS in “Geography and territorial processes” at the University of Bologna. He got a postgraduate degree in “GIS (Geographical Information System) and remote sensing” at the University of Venezia. He taught scientific subjects for ten years in different primary schools of Ferrara. Nowadays he teaches General Geography and Touristic Geography in a high school. He also cooperates with other schools to develop digital teaching.