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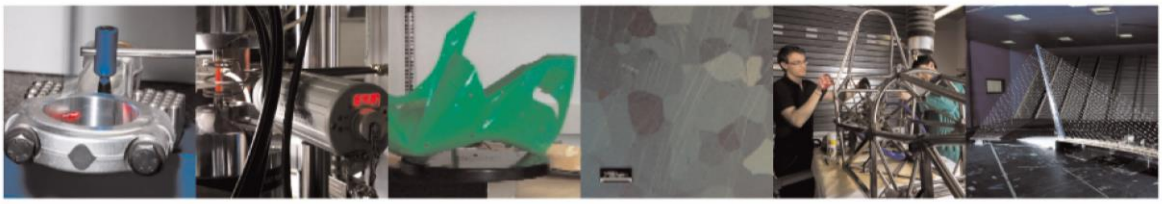

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Matta, Andrea; Simone, Francesca

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DIPARTIMENTO DI MECCANICA ■ POLITECNICO DI MILANO
via G. La Masa, 1 ■ 20156 Milano ■ EMAIL (PEC): pecmecc@cert.polimi.it
http://www.mecc.polimi.it
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Analysis of Two–Machine Lines with Finite Buffer, Operation–Dependent and Time–Dependent Failure Modes

Andrea Matta

Institute of Industrial Engineering, School of Mechanical Engineering
Shanghai Jiao Tong University,
800 Dong Chuan Road, 200240, Shanghai - China.
(email: matta@sjtu.edu.cn)

Francesca Simone

ME Dimensional Strategy and Management, Chrysler Group LLC,
Auburn Hills, MI, USA.
(email: fs656@chrysler.com)

Abstract

An analytical model for evaluating the throughput of two–machine lines, characterized by intermediate buffer with finite capacity, deterministic processing times and multiple failure modes for each machine is presented in this paper. Both operation–dependent failure and time–dependent failure are captured in a unique model as extension of the existing literature that was dealing with either one of them. Each machine has two failure modes, one is operation-dependent and the other is time-dependent. Time to failure and time to repair are assumed to be geometrically distributed. The presented method calculates the steady-state probabilities of the manufacturing system with a computational effort that depends only on the number of failure modes and not on the buffer capacity. A performance comparison of the proposed model with existing techniques is also reported, the aim is to show the error introduced by an analytical model that considers the operation–dependent failure mode as approximation of the time–dependent one.

Keywords: transfer line, markov chain, time–dependent failures, operation–dependent failures

1 Introduction

Research in automation is continuously providing new technologies implemented in modern production systems. The presence of electronics such as PLCs (Programmable Logic Controller), control systems, data acquisition systems etc. is increasing more and more at all levels (component, station, system). A characteristic of these devices is their failure distribution that, in general, does not depend on the workload of the machine but only on the time elapsed from the occurrence of the last failure event. These failures are known in literature as time–dependent failures (TDFs).

In order to derive accurate system performance estimations, it becomes important to explicitly consider the nature that rules the disruption of electronic devices. Actually, manufacturing lines are affected not only by operation-dependent failures (ODFs) (Dallery and Gershwin, 1992), but also by TDFs. ODFs are failure types occurring only if the machine is processing a part, for instance a tool breakage. TDFs are failure types occurring whether or not the machine is processing a part. Despite the fact that this situation is quite common in practice, the existing analytical methods for performance evaluation tend to keep separated the treatment based on the ODF hypothesis rather than the TDF one.

Approximate analytical methods have been developed to evaluate the performance of production systems (Dallery and Gershwin, 1992; Gershwin, 1994; Altiok, 1997; Buzacott and Shantikumar, 1993; Li and Meerkov, 2008; Papadopoulos et al., 2009). Li et al. (2006) provide rather a complete taxonomy of two–machine line models for throughput analysis. They present and study different analytical models, classified by synchronous and asynchronous lines with ODF and TDF modes, with the main objective of providing useful observations and suggestions about how these models are applicable to different systems and what are the major differences among them. Mourani et al. (2007) address the impact of the ODF and the TDF models on the performance of long lines with intermediate finite buffers. According to their results, replacing in the line an ODF machine by a TDF machine leads to a smaller throughput, if the times between failures of the related machine are exponentially distributed. However, this result should be valid also for other distributions because the machine reliability decreases due to the reduction of the time between failures that derives from the TDF assumption. Their results also point out that the differences in modeling between TDF and ODF increase as the system buffer capacity decreases. Thus, ODF machines can be accurately modeled as TDF only in presence of large buffers, otherwise a significant underestimation of the system throughput will occur. Therefore, in case of transfer lines with small buffer capacities between machines (e.g., automotive plants), the ODF modes should be explictly considered in a performance evaluation model together with the TDF ones due to the impact they have on the system throughput.

The analytical modeling of TDF modes is much simpler than ODF modes because it is not necessary to track the time in which machine is operational. This is also a benefit when failure distributions are fitted. Indeed, machine log data do not have to be depurated by the machine inactivity times (i.e., blocking and starvation) in case of TDFs. As far as the two–machine line models based on TDF hypothesis, the literature includes only models characterized by a single failure mode. Xie (2002) considers two–machine lines with continuous material flow and TDF modes. He studies second order properties of the system throughput and developes a gradient estimator for sample–path optmimization. Colledani et al. (2009) consider a simpler version model of ODF and TDF co-existing in the same machine, for instance the possibility that a joint failure ODF and TDF mode is repaired in one time period is not considered. On the other hand, a vast literature concerns machines in which the nature that rules their failure modes is considered operation–dependent (Gershwin, 1994; Tolio et al., 2002; Tan and Gershwin, 2009; Colledani and Tolio, 2011; Mhada et al., 2014). Therefore, in the existing literature, analytical two–machine line models consider machines in which the nature that rules machine failure modes is always considered either operation-dependent or time-dependent.

Machines simultaneously characterized by TDF and ODF modes have never been addressed and represent the subject of this work. The modeling of such machines is not straightforwad because of the possibility of multiple failure occurrence. Indeed, a TDF may occur when the machine is failed in an ODF mode. This increases the complexity of modeling analytically the system by changing the internal equations of the two–machine line (Gershwin, 1994). In the work of (Tan and Gershwin, 2011) a general two–machine line is presented to model different cases. Among these, the model could be properly used to represent either ODF or TDF multiple failures in the same machine. However, the model is defined in the continuous–material continuous–time case and, to the best of our knowledge, no similar model has been proposed in the discrete–material discrete–time case.

This paper proposes an exact method to calculate the performance of a two–machine line with machines characterized by multiple failure modes. Specifically, the machines can fail in two different disruption types, TDF and ODF. Each failure mode could aggregately represent the several failure modes of the same type that affect the machine behaviour. The analytical model presented in this work can be used as building block in decomposition algorithms for estimating the performance of long lines with machines affected by ODF and TDF (Gershwin, 1987; Dallery et al., 1988; Tolio and Matta, 1998). Additionally, in this paper we show the impact of the TDF assumption on the estimation accuracy of the major performance indicators of a transfer line. In particular, we are interested to understand when the approximation, caused by the use of models in which ODFs and TDFs are separately considered, can be considered negligible or not.

The paper is organized as follows. Section 2 presents the model assumptions and the mathematical notation used. In section 3 the proposed solution technique to calculate the system performance is described. Section 4 is dedicated to the numerical analysis of the proposed model. In particular the shape of the error committed if a TDF mode is modeled as an ODF one is investigated. Section 5 concludes the paper.

2 System modeled

The two–machine one–buffer system is described in this section in terms of assumptions, dynamics and performance measures.

2.1 Assumptions and notation

We consider a two–machine line with an intermediate buffer between. The first and second machines are called the upstream and downstream machines, respectively, and they are denoted with M^u and M^d. The buffer has a limited capacity equal to N.

Time and flow of material in the system are considered discrete. Each part enters the system from the first machine, then goes to the buffer, then to the second machine and then exits the system. The downstream machine takes pieces from the upstream buffer according with the FCFS (First Come First Served) rule. Transportation of parts among buffers and machines is considered negligible respect to machining operation time. Processing times are deterministic and equal for each machine of the line. By convention, processing times are scaled to the time unit, failure and repair events occur at the beginning of the time unit, and changes in the buffer level take place at the end of time unit. Work pieces are not destroyed or rejected at any stage in the line. Also, partly processed work pieces are not added in the line.

A machine is starved if the upstream buffer is empty. A machine is blocked if the downstream buffer is full. It is assumed that the first machine M^u is never starved and the last machine M^d is never blocked. Blocking before service (BBS) convention is considered in the model (Dallery and Gershwin, 1992). Machines are considered unreliable and can fail in two failure modes. One of them is ODF and the other TDF. This means that a machine, if it is operational, can fail in any of the two ways. If a machine is not operational because it is in either an ODF status or it is starved or blocked, a TDF can still occur. Independently from the type, all the times to failure and the times to repair are assumed to be geometrically distributed.

The dynamics of the upstream machine is now described in detail. At the beginning of a time unit, if the upstream machine M^u is operational it can fail in mode u_i with probability p_i^u while attempting to perform an operation, where $i = 1, 2$ with $i = 1$ if the failure is ODF and $i = 2$ if the failure is TDF. If the upstream machine is in the blocking state or ODF state, it can fail only in TDF mode with probability p_3^u. If the machine fails in mode u_i, it can be repaired during a time unit with probability r_i^u. If the upstream machine is failed in ODF mode, at the beginning of the time unit a TDF may occur. If the machine is failed in both of the modes, it can be completely repaired with probability r_3^u coming back to its operative state. An alternative to leave the double failure mode is to repair one failure out of the two; this may happen with probabilities r_4^u and r_5^u for TDF and ODF, respectively.

The dynamics of the downstream machine is now described in detail. At the beginning of a time unit, if the downstream machine M^d is operational it can fail in mode d_i with probability p_i^d while attempting to perform an operation, where $i = 1, 2$ with $i = 1$ if the failure is ODF and $i = 2$ if the failure is TDF. If the downstream machine is in the starvation state or ODF state, it can fail only in the TDF mode with probability p_3^d. If the machine fails in mode d_i, it can be repaired during a time unit with probability r_i^d. If the downstream machine is failed in ODF mode, at the beginning of the time unit a TDF may occur. If the machine is failed in both of the modes, it can be completely repaired with probability r_3^d coming back to its operative state. An alternative to leave the double failure mode is to repair one failure out of the two; this may happen with probabilities r_4^d and r_5^d for TDF and ODF, respectively.

2.2 State representation

The dynamic behavior of the system is modeled with a discrete–time discrete–state Markov chain. The behavior of the upstream machine is graphically illustrated in Figure 1, whereas that of the downstream machine is similar. The state of the whole system is represented by the vector (n, α^u, α^d) where $n = 0, \dots, N$ is the level of the buffer, α^u and α^d refer to the state in which the machines M^u and M^d can be respectively:

$$\alpha^u = \begin{cases} 1 & \text{if} \quad M^u \quad is \quad Up \\ u_1 & \text{if} \quad M^u \quad is \quad Down \quad in \quad ODF \\ u_2 & \text{if} \quad M^u \quad is \quad Down \quad in \quad TDF \\ u_3 & \text{if} \quad M^u \quad is \quad Down \quad in \quad ODF\&TDF \end{cases}$$

$$\alpha^d = \begin{cases} 1 & \text{if} \quad M^d \quad is \quad Up \\ d_1 & \text{if} \quad M^d \quad is \quad Down \quad in \quad ODF \\ d_2 & \text{if} \quad M^d \quad is \quad Down \quad in \quad TDF \\ d_3 & \text{if} \quad M^d \quad is \quad Down \quad in \quad ODF\&TDF \end{cases}$$

The total number of the system states is $8(N+1)$. It is straightforward to demonstrate that the Markov chain is ergodic, thus there exists a limited and unique value $\pi(n, \alpha^u, \alpha^d)$ for the steady state probability of the system of being in state (n, α^u, α^d).

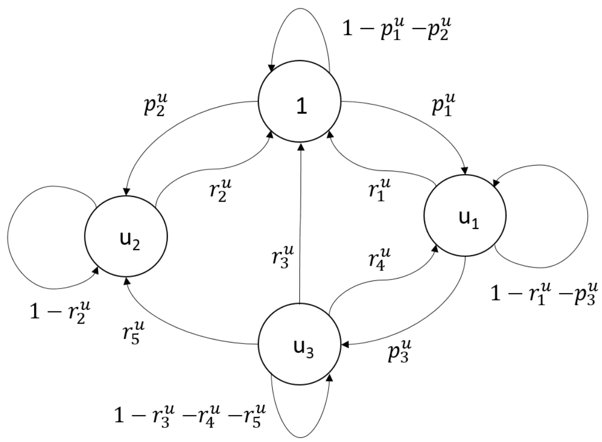

Figure 1: State–transition diagram of the upstream machine

2.3 Performance measures

Assuming to know the steady state probability values for each state of the Markov chain, it is possible to calculate the performance measures of the system under analysis.

The most important performance measure is the overall system efficiency, denoted with E, that provides a synthesis on how the resources are utilized to produce a piece. This measure is calculated by considering all the states that are productive, i.e. those states in which the system releases a finished part from the downstream machine:

$$E = E^d = \sum_{n=1}^{N} \left[\pi(n,1,1) + \sum_{i=1}^{3} \pi(n,u_i,1) \right]$$

where E^d is the probability that M^d is operational and not starved. E^u, the probability that M^u is operational and not blocked, is calculated as follows:

$$E^u = \sum_{n=0}^{N-1} \left[\pi(n,1,1) + \sum_{j=1}^{3} \pi(n,1,d_j) \right]$$

Due to the law of flow conservation (Gershwin, 1994), it is possible to write:

$$E = E^u = E^d$$

The expected number of parts that are present in the buffer is calculated as:

$$\bar{n} = \sum_{n=1}^{N} n \left[\pi(n,1,1) + \sum_{i=1}^{3} \pi(n,u_i,1) + \sum_{j=1}^{3} \pi(n,1,d_j) + \sum_{i=1}^{3} \sum_{j=1}^{3} \pi(n,u_i,d_j) \right]$$

The probabilities of starvation and blocking are calculated as follows:

$$Ps = \sum_{i=1}^{3} \pi(0, u_i, 1) \qquad Pb = \sum_{j=1}^{3} \pi(N, 1, d_j)$$

3 Solution technique

The computational effort related to the numerical calculation of the steady state probabilities of the Markov chain representing the dynamic of the two–machine line system depends on the buffer capacity and on the sum of the number of upstream and downstream states. In this paper, a solution that does not depend on the buffer capacity is presented. The proposed solution is an extension of the method originally presented in (Berman and Gershwin, 1981; Gershwin, 1994).

3.1 Internal states

All the possible states in which the system can be are classified in function of the buffer level. The threes subsets created are named *internal states* (denoted with I), *lower boundary states* (denoted with LB) and *upper boundary states* (denoted with UB). These subsets are defined as follows:

$$(n, \alpha^u, \alpha^d) = \begin{cases} LB & \text{if} \quad 0 \leq n < 2 \\ I & \text{if} \quad 2 \leq n \leq N-2 \\ UB & \text{if} \quad N-2 < n \leq N \end{cases}$$

We first consider the internal states to define the equations which explain the dynamic behavior of this subset. The internal equations express the dynamics of the system in visiting the internal states in which $2 \leq n \leq N-2$. For each level of the buffer, there are nine combinations that consider all the possible states in which the system can be. For these states, we can write the following node equations. The first equation regards the state in which both machines are operational:

$$\pi(n,1,1) = \left[\pi(n,1,1)(1-P^U) + \sum_{i=1}^{3} \pi(n,u_i,1) r_i^u\right](1-P^D) + \\ +(1-P^U)\sum_{j=1}^{3} \pi(n,1,d_j) r_j^d + \sum_{i=1}^{3}\sum_{j=1}^{3} \pi(n,u_i,d_j) r_i^u r_j^d \tag{1}$$

where the following coefficients have been defined for clarity of exposition: $P^U = p_1^u + p_2^u$ and $P^D = p_1^d + p_2^d$. The second set of equations describes the states in which the first machine is down and the second is operational. Here, only the equation related to the state $(n, u_1, 1)$ is shown, the other two equations are reported in Appendix:

$$\pi(n,u_1,1) = \sum_{j=1}^{3} \left[\pi(n+1,u_1,d_j)(1-r_1^u-p_3^u) + \pi(n+1,u_3,d_j) r_4^u + \pi(n+1,1,d_j) p_1^u\right] r_j^d + \\ + \left[\pi(n+1,u_1,1)(1-r_1^u-p_3^u) + \pi(n+1,u_3,1) r_4^u + \pi(n+1,1,1) p_1^u\right](1-P^D) \tag{2}$$

The third set of equations describes the states in which the first machine is operational and the second is down. Here, only the equation related to the state $(n,1,d_1)$ is shown, the other two equations are reported in Appendix:

$$\pi(n,1,d_1) = \sum_{i=1}^{3} \left[\pi(n-1,u_i,d_1)(1-r_1^d-p_3^d)+\pi(n-1,u_i,d_3)r_4^d+\pi(n-1,u_i,1)p_1^d\right] r_i^u + \quad (3)$$
$$+\left[\pi(n-1,1,d_1)(1-r_1^d-p_3^d)+\pi(n-1,1,d_3)r_4^d+\pi(n-1,1,1)p_1^d\right](1-P^U)$$

The fourth set of equations describes the states in which both machines are down. Here, only the equation related to the state (n,u_1,d_1) is shown, the other five equations are reported in Appendix:

$$\pi(n,u_1,d_1) = [\pi(n,u_1,d_1)(1-r_1^u-p_3^u)+\pi(n,u_3,d_1)r_4^u+\pi(n,1,d_1)p_1^u](1-r_1^d-p_3^d)+ \quad (4)$$
$$+[\pi(n,u_1,d_3)(1-r_1^u-p_3^u)+\pi(n,u_3,d_3)r_4^u+\pi(n,1,d_3)p_1^u]r_4^d+$$
$$+[\pi(n,u_1,1)(1-r_1^u-p_3^u)+\pi(n,u_3,1)r_4^u+\pi(n,1,1)p_1^u]p_1^d$$

Following the Gershwin's approach (Gershwin, 1994), a guess regarding the structure of the unknown steady state probabilities is introduced as extension of that one proposed in (Tolio et al., 2002):

$$\pi(n,1,1) = X^n \quad (5)$$
$$\pi(n,u_i,1) = X^n U_i \quad (6)$$
$$\pi(n,1,d_j) = X^n D_j \quad (7)$$
$$\pi(n,u_i,d_j) = X^n U_i D_j \quad (8)$$

where X, U_i and D_j are unknown parameters.

Substituting the guess defined in (5)–(8), and factorizing according to upstream and downstream terms, we can write the internal equations in a different form.

From equation (1) it is obtained:

$$\pi(n,1,1): 1 = \left[1-P^U+r_1^u\,U_1+r_2^u\,U_2+r_3^u\,U_3\right]\left[1-P^D+r_1^d\,D_1+r_2^d\,D_2+r_3^d\,D_3\right] \quad (9)$$

From equations (2),(37) and (38) it is obtained:

$$\pi(n,u_1,1): U_1 = X\left[(1-P^{U_1})U_1+r_4^u\,U_3+p_1^u\right]\left(1-P^D+\sum_{j=1}^{3} r_j^d D_j\right) \quad (10)$$

$$\pi(n,u_2,1): U_2 = X\left[(1-r_2^u)\,U_2+r_5^u\,U_3+p_2^u\right]\left(1-P^D+\sum_{j=1}^{3} r_j^d D_j\right) \quad (11)$$

$$\pi(n,u_3,1): U_3 = X\left[(1-P^{U_3})U_3+p_3^u\,U_1\right]\left(1-P^D+\sum_{j=1}^{3} r_j^d D_j\right) \quad (12)$$

where the following coefficients have been defined for clarity of exposition: $P^{U_1}=r_1^u+p_3^u$ and $P^{U_3}=r_3^u+r_4^u+r_5^u$.

From equations (3),(39) and (40) it is obtained:

$$\pi(n,1,d_1) : XD_1 = \left(1 - P^U + \sum_{i=1}^{3} r_i^u U_i\right) \left[(1 - P^{D_1})D_1 + r_4^d\, D_3 + p_1^d\right] \tag{13}$$

$$\pi(n,1,d_2) : XD_2 = \left(1 - P^U + \sum_{i=1}^{3} r_i^u U_i\right) \left[(1 - r_2^d)D_2 + r_5^d\, D_3 + p_2^d\right] \tag{14}$$

$$\pi(n,1,d_3) : XD_3 = \left(1 - P^U + \sum_{i=1}^{3} r_i^u U_i\right) \left[(1 - P^{D_3})D_3 + p_3^d\, D_1\right] \tag{15}$$

where the following coefficients have been defined for clarity of exposition: $P^{D_1} = r_1^d + p_3^d$ and $P^{D_3} = r_3^d + r_4^d + r_5^d$.

From equations (4),(41)–(48) it is obtained:

$$\pi(n,u_1,d_1) : U_1D_1 = \left[(1 - P^{U_1})\, U_1 + r_4^u\, U_3 + p_1^u\right] \left[(1 - P^{D_1})D_1 + r_4^d\, D_3 + p_1^d\right]$$

$$\pi(n,u_1,d_2) : U_1D_2 = \left[(1 - P^{U_1})\, U_1 + r_4^u\, U_3 + p_1^u\right] \left[(1 - r_2^d)D_2 + r_5^d\, D_3 + p_2^d\right]$$

$$\pi(n,u_1,d_3) : U_1D_3 = \left[(1 - P^{U_1})\, U_1 + r_4^u\, U_3 + p_1^u\right] \left[(1 - P^{D_3})D_3 + p_3^d\, D_1\right]$$

$$\pi(n,u_2,d_1) : U_2D_1 = [(1 - r_2^u)\, U_2 + r_5^u\, U_3 + p_2^u] \left[(1 - P^{D_1})D_1 + r_4^d\, D_3 + p_1^d\right]$$

$$\pi(n,u_2,d_2) : U_2D_2 = [(1 - r_2^u)\, U_2 + r_5^u\, U_3 + p_2^u] \left[(1 - r_2^d)D_2 + r_5^d\, D_3 + p_2^d\right]$$

$$\pi(n,u_2,d_3) : U_2D_3 = [(1 - r_2^u]\, U_2 + r_5^u\, U_3 + p_2^u) \left[(1 - P^{D_3})D_3 + p_3^d\, D_1\right]$$

$$\pi(n,u_3,d_1) : U_3D_1 = \left[\left(1 - P^{U_3}\right)\, U_3 + p_3^u\, U_1\right] \left[(1 - P^{D_1})D_1 + r_4^d\, D_3 + p_1^d\right]$$

$$\pi(n,u_3,d_2) : U_3D_2 = \left[\left(1 - P^{U_3}\right)\, U_3 + p_3^u\, U_1\right] \left[(1 - r_2^d)D_2 + r_5^d\, D_3 + p_2^d\right]$$

$$\pi(n,u_3,d_3) : U_3D_3 = \left[\left(1 - P^{U_3}\right)\, U_3 + p_3^u\, U_1\right] \left[(1 - P^{D_3})D_3 + p_3^d\, D_1\right]$$

It is possible to notice that a model with TDF modes is more general than a model with ODF modes. Indeed, if we set $p_3^u = 0$ and $p_3^d = 0$ the states with u_3 and d_3 do not exist anymore, the terms U_3 and D_3 disappear from the internal equations and coefficients become $P^{U_1} = r_1^u$ and $P^{D_1} = r_1^d$. Indeed, in this particular case the internal equations are the same as in (Tolio et al., 2002).

If we divide every equation by its left member, it is always possible to find a product between two factors, one constituted by only upstream machine parameters and one constituted by only downstream machine parameters. In particular, we can observe that every product has to be equal to 1. This means that it is possible to pose one factor equal to k and the other one equal to $1/k$ as shown in the following equations:

$$\frac{\left(1 - P^{U_1}\right)\, U_1 + r_4^u\, U_3 + p_1^u}{U_1} = k \tag{16}$$

$$\frac{(1 - r_2^u)\, U_2 + r_5^u\, U_3 + p_2^u}{U_2} = k \tag{17}$$

$$\frac{\left(1 - P^{U_3}\right)\, U_3 + p_3^u\, U_1}{U_3} = k \tag{18}$$

for the upstream machine and the following for the downstream machine:

$$\left(1 - P^{D_1}\right) D_1 + r_4^d\, D_3 + p_1^d = \frac{1}{k} \tag{19}$$

$$\left(1 - r_2^d\right) D_2 + r_5^d\, D_3 + p_2^d = \frac{1}{k} \tag{20}$$

$$\left(1 - P^{D_3}\right) D_3 + p_3^d\, D_1 = \frac{1}{k} \tag{21}$$

This implies the following equations for the upstream parameters:

$$U_1 = \frac{p_1^u\,\left(k - 1 + P^{U_3}\right)}{\left(k + P^{U_1} - 1\right)\left(k - 1 + P^{U_3}\right) - r_4^u\, p_3^u} \tag{22}$$

$$U_2 = \frac{p_2^u}{k - 1 + r_2^u} + \frac{r_5^u\, p_1^u\, p_3^u}{\left(k - 1 + r_2^u\right)\left[\left(k + P^{U_1} - 1\right)\left(k - 1 + P^{U_3}\right) - r_4^u\, p_3^u\right]} \tag{23}$$

$$U_3 = \frac{p_1^u\, p_3^u}{\left(k + P^{U_1} - 1\right)\left(k - 1 + P^{U_3}\right) - r_4^u\, p_3^u} \tag{24}$$

and for the downstream parameters we obtain:

$$D_1 = \frac{p_1^d\,\left(k^{-1} - 1 + P^{D_3}\right)}{\left(k^{-1} + P^{D_1} - 1\right)\left(k^{-1} - 1 + P^{D_3}\right) - r_4^d\, p_3^d} \tag{25}$$

$$D_2 = \frac{p_2^d}{k^{-1} - 1 + r_2^d} + \frac{r_5^d\, p_1^d\, p_3^d}{\left(k^{-1} - 1 + r_2^d\right)\left[\left(k^{-1} + P^{D_1} - 1\right)\left(k^{-1} - 1 + P^{D_3}\right) - r_4^d\, p_3^d\right]} \tag{26}$$

$$D_3 = \frac{p_1^d\, p_3^d}{\left(k^{-1} + P^{D_1} - 1\right)\left(k^{-1} - 1 + P^{D_3}\right) - r_4^d\, p_3^d} \tag{27}$$

and finally:

$$X = k^{-1}\left(1 - P^U + r_1^u\, U_1 + r_2^u\, U_2 + r_3^u\, U_3\right) \tag{28}$$

We are now able to express every unknown variable in function of k. For this reason, if we substitute the equations from (22) to (27) in (9) we obtain a polynomial in k:

$$1 = \left[1 - P^U + \sum_{i=1}^{3} r_i^u U_i\right]\left[1 - P^D + \sum_{j=1}^{3} r_j^d D_j\right]$$

The solution of this polynomial provides the $R = 6$ roots that can be used to calculate the corresponding values of $U_{i,r}$, $D_{i,r}$ and X_r (with $r = 1, ..., R$) by means of equations (22)–(28). In particular, we define K_r (with $r = 1, ..., 6$) to be the r^{th} root of the polynomial. For each root, we have a corresponding value of $U_{i,r}$, $D_{j,r}$ and X_r that satisfy the internal equations. The next step is to verify if it is possible to find a linear combination of these solutions that also satisfies the boundary equations and the normalization equations.

3.2 Boundary states

The model assumptions imply that certain boundary states are transient, i.e. they cannot be reached from any state except from possible other transient states. Transient states have zero steady–state probability. In the proposed model, the transient states are less than those identified in (Tolio et al., 2002), due to the possibility of the machines to fail during blocking and starvation. The transient states are:

- $(0, 1, 1)$ and $(0, 1, d_j)$ because they cannot be reached from any other state;
- $(0, u_i, d_3)$ because they can be reached only from theirselves or $(0, 1, d_1)$, $(0, 1, d_3)$, or $(0, u_i, d_1)$, $(0, u_1, d_3)$;
- $(1, 1, d_1)$ because they can be reached only from $(0, u_i, d_1)$ or $(0, 1, d_j)$;
- $(1, 1, d_3)$ because they can be reached only from $(0, u_i, d_1)$, $(0, u_i, d_3)$ or $(0, 1, d_1)$, $(0, 1, d_3)$.

Similarly, the states $(N, 1, 1)$, $(N, u_i, 1)$, (N, u_3, d_j), $(N-1, u_1, 1)$ and $(N-1, u_3, 1)$ are transient. For the remaining boundary states (i.e. those which are not transient), we write in the following only those equations that represent how the TDF impacts on the boundary. All the other equations are as in (Gershwin, 1994; Tolio et al., 2002).

As far as the lower boundary states, we can write:

$$\pi(0, u_1, d_2) = (1 - P^{U_1})(1 - r_2^d)\pi(0, u_1, d_2) + (1 - P^{U_1})p_2^d\pi(0, u_1, 1) + \\ + r_4^u(1 - r_2^d)\pi(0, u_3, d_2) + r_4^u p_2^d \pi(0, u_3, 1) \quad (29)$$

$$\pi(0, u_2, d_2) = (1 - r_2^u)(1 - r_2^d)\pi(0, u_2, d_2) + (1 - r_2^u)p_2^d\pi(0, u_2, 1) + \\ + r_5^u(1 - r_2^d)\pi(0, u_3, d_2) + r_5^u p_2^d \pi(0, u_3, 1) \quad (30)$$

$$\pi(0, u_3, d_2) = (1 - P^{U_3})(1 - r_2^d)\pi(0, u_3, d_2) + (1 - P^{U_3})p_2^d\pi(0, u_3, 1) + \\ + p_3^u(1 - r_2^d)\pi(0, u_1, d_2) + p_3^u p_2^d \pi(0, u_1, 1) \quad (31)$$

$$\pi(1, 1, d_2) = (1 - r_2^d)\left(r_1^u\pi(0, u_1, d_2) + r_2^u\pi(0, u_2, d_2) + r_3^u\pi(0, u_3, d_2)\right) + \\ + r_1^u p_2^d\pi(0, u_1, 1) + r_2^u p_2^d\pi(0, u_2, 1) + r_3^u p_2^d \pi(0, u_3, 1) \quad (32)$$

Equation (32) can be expressed using (29),(30) and (31), which represent the case in which M^d is starved, i.e. the machine is not producing parts, but it may fail in a TDF mode.

As far as the upper boundary states, we can write:

$$\pi(N, u_2, d_1) = (1 - r_2^u)(1 - P^{D_1})\pi(N, u_2, d_1) + p_2^u(1 - P^{U_1})\pi(N, 1, d_1) + \\ + (1 - r_2^u)r_4^d\pi(N, u_2, d_3) + p_2^u r_4^d \pi(N, 1, d_3) \quad (33)$$

$$\pi(N, u_2, d_2) = (1 - r_2^u)(1 - r_2^d)\pi(N, u_2, d_2) + p_2^u(1 - r_2^d)\pi(N, 1, d_2) + \\ + (1 - r_2^u)r_5^d\pi(N, u_2, d_3) + p_2^u r_5^d \pi(N, 1, d_3) \quad (34)$$

$$\pi(N, u_2, d_3) = (1 - r_2^u)(1 - P^{D_3})\pi(N, u_2, d_3) + p_2^u(1 - P^{D_3})\pi(N, 1, d_3) + \\ + (1 - r_2^u)p_3^d\pi(N, u_2, d_1) + p_2^u p_3^d \pi(N, 1, d_1) \quad (35)$$

$$\pi(N-1, u_2, 1) = (1 - r_2^u)\Big(r_1^d\pi(N, u_2, d_1) + r_2^u\pi(N, u_2, d_2) + r_3^u\pi(N, u_2, d_3)\Big) + \\ + p_2^u r_1^d\pi(N, 1, d_1) + p_2^u r_2^d\pi(N, 1, d_2) + p_2^u r_3^d \pi(N, 1, d_3) \quad (36)$$

Equation (36) can be expressed using (33),(34) and (35), which represent the case in which M^u is blocked, but it may fail in a TDF mode.

If the guess introduced for the internal states is correct, there must be a linear combination of the R different solutions found for the internal equations which satisfies the boundary equations and the normalization equation. The steady–state probabilities of the internal states are assumed to follow the form:

$$\pi(n,1,1) = \sum_{r=1}^{6} C_r X_r^n$$
$$\pi(n,u_i,1) = \sum_{r=1}^{6} C_r X_r^n U_{i,r}$$
$$\pi(n,1,d_j) = \sum_{r=1}^{6} C_r X_r^n D_{j,r}$$
$$\pi(n,u_i,d_j) = \sum_{r=1}^{6} C_r X_r^n U_{i,r} D_{j,r}$$

The value of the constants C_r can be found by solving a linear system defined by five lower and upper boundary equations and the normalization equation. The solution expresses the values of the steady–state probabilities of the boundary states and the value of the constants that jointly satisfy internal, boundary and normalization equation. Once all the state probabilities have been calculated, it is possible to evaluate the performance measures of the two–machine line using the expressions defined in section 2.3.

4 Numerical comparison

The possibility of considering both ODF and TDF for each machine leads to a better evaluation of the expected throughput, which in our system corresponds to the overall system efficiency E. Since ODF modes are generally assumed for machines either in practice or in academia (Gershwin, 1994; Dallery and Gershwin, 1992), we compare the performance of the developed model with the one proposed by Tolio et al. (Tolio et al., 2002) that consider multiple ODF modes. The goal is to try quantifying the error we could commit if the multiple ODF model was improperly used to study a two–machine line with machines having ODF and TDF modes.

The comparison is carried out using the machine data reported on Table 1. The buffer capacity is varied from 4 to 30. The probability of failing in a TDF mode is varied from 0.05 to 0.25 to consider different cases. The percentage error committed using the model in (Tolio et al., 2002) is calculated as:

$$\%error = \frac{E_{pm} - E_{cm}}{E_{pm}} \cdot 100$$

where E_{pm} and E_{cm} are the mean throughputs estimated by the proposed model and the comparison model (Tolio et al., 2002), respectively. Notice that E_{pm} is exact and for this reason it is used as benchmark.

Parameter	p_1^u	p_3^u	r_1^u	r_2^u	r_3^u	r_4^u	r_5^u	p_1^d	p_3^d	r_1^d	r_2^d	r_3^u	r_4^u	r_5^u
Value	0.15	p_2^u	0.7	0.75	0	0.75	0.7	0.1	p_2^d	0.65	0.6	0	0.6	0.65

Table 1: Probabilities of failure in ODF mode and probabilities of repair in ODF and TDF modes; the probabilities of failure in TDF mode (i.e., p_2^u and p_2^d) are varied in the numerical experiment keeping the equality $p_2^u = p_2^d$.

In Figure 2, we first present a typical concave trend of the average throughput as a function of the buffer capacity for different values of the probability of failure in the TDF mode. As the buffer capacity increases the mean throughput also increases, asymptotically reaching the maximum production rate achievable by the system. As the probability of failing in a TDF mode increases, the average throughput decreases. This behaviour is aligned with that of systems with only ODF or TDF modes. This is not surprisingly being such models are specific cases of the proposed ODF & TDF model.

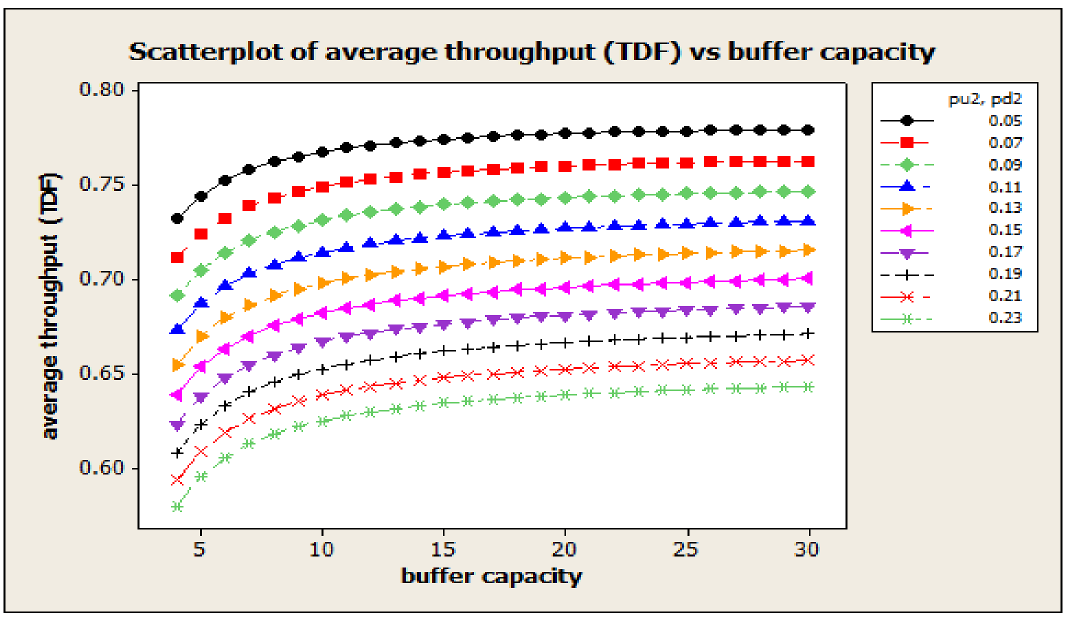

Figure 2: Avarage Throughput as a function of buffer capacity and TDF mode probability

Figure 3 shows the percentage error between the two models as a function of the buffer capacity and for different values of the parameters of the TDF mode. In particular, we can observe that for low buffer capacities and for increasing probabilities of failing in the TDF mode, the approximation introduced by the comparison model is relevant being close to 2.5 %. Consider that the system under analysis is a two–machine line, for longer lines this error can be much larger. This behavior is due to fact that, when the buffer capacities are low and the probabilities of failure are high, the impact of blocking and starvation phenomena on the performance of the system is considerable.

In Figure 4 we present the relationship between the percentage error and the probability of the system to be idle, i.e. when the system is starved (because M^u is failed) or blocked (because M^d is failed). If this probability increases, also the difference between the two

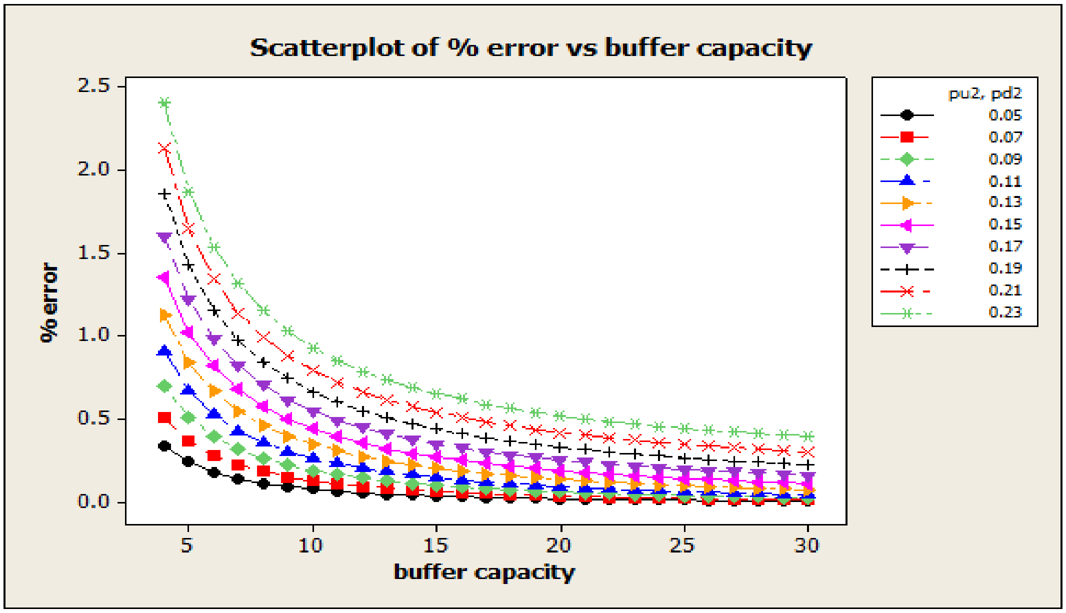

Figure 3: Percentage error as a function of buffer capacity and TDF mode probability

models increases, because TDFs may occur even if the system is idle. Therefore, the more the idle time has an impact on the real system, the more the models will provide different estimations. In Figure 5, it is also possible to observe that the percentage error assumes almost a linear form if compared to the expected number of TDF mode occurrences in a time unit. This behavior is due to the fact that the percentage error jointly depends on both the probability of being idle and the probability of failure.

On the whole, it is possible to infer that, on the basis of the carried out analysis, the approximation introduced by the model that consider only ODF leads, in the worst case considered, to an error around 2.5 percent. This error may be larger for long lines.

5 Conclusion

In this paper an analytical model to exactly evaluate the performance of two–machine lines with deterministic processing times, finite buffer capacity and two failure modes for each machine, one operation–dependent and the other one time–dependent, has been presented. The innovative aspect of the proposed model lies in considering, at the same time, these two failure modes. Beside the basics of the complete mathematical treatment developed, the paper provides also some helpful comparisons with another remarkable model for throughput analysis.

A future work may comprise the adoption of the developed building block for evaluating the performance of longer transfer lines with decomposition techniques, for which existing analytical models consider only either ODF or TDF. An extension to consider multiple ODF and multiple TDF is also straightforward based on our results.

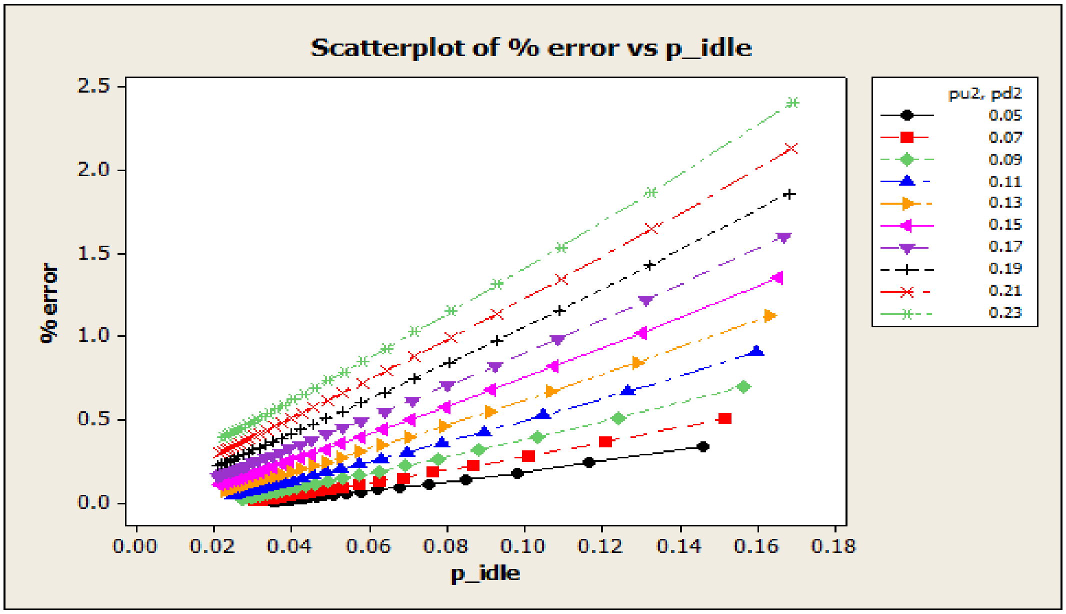

Figure 4: Percentage error as a function of idle time and TDF mode probability

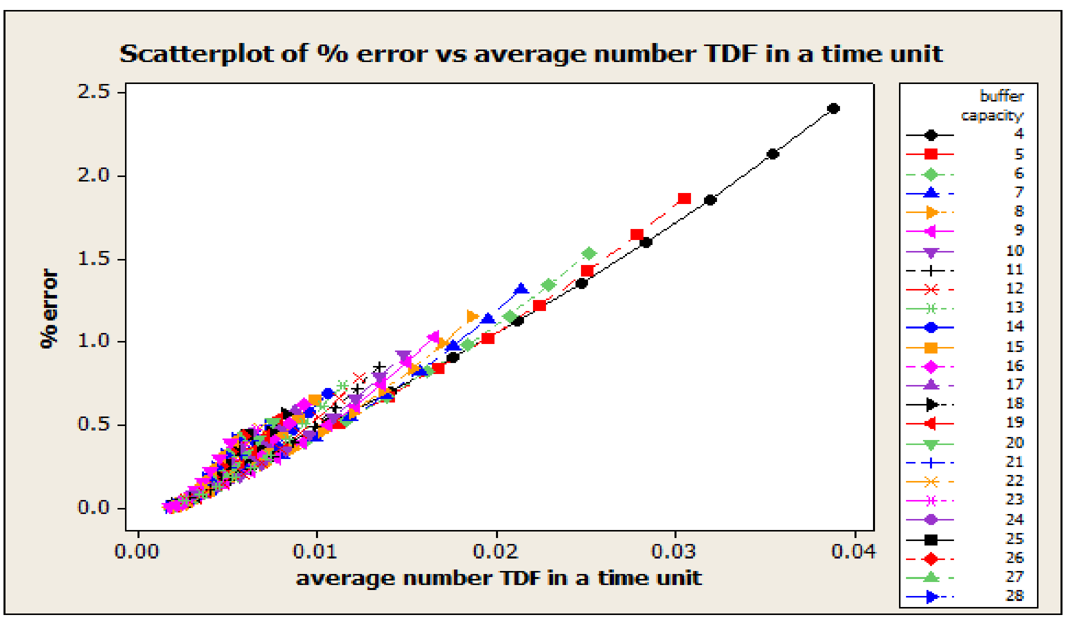

Figure 5: Percentage error as a function of the average number of TDF mode occurences in a time unit

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A Appendix: internal equations

The first equation (1) regards the state in which both machines are operational $(n, 1, 1)$.

The second set of equations describes the states in which the first machine is down and the second is operational:

$$\pi(n,u_2,1) = \sum_{j=1}^{3} \left[\pi(n+1,u_2,d_j)(1-r_2^u) + \pi(n+1,u_3,d_j)r_5^u + \pi(n+1,1,d_j)p_2^u\right] r_j^d + \tag{37}$$
$$+ \left[\pi(n+1,u_2,1)(1-r_2^u) + \pi(n+1,u_3,1)r_5^u + \pi(n+1,1,1)p_2^u\right](1-P^D)$$

$$\pi(n,u_3,1) = \sum_{j=1}^{3} \left[\pi(n+1,u_3,d_j)(1-r_3^u-r_4^u-r_5^u) + \pi(n+1,u_1,d_j)p_3^u\right] r_j^d + \tag{38}$$
$$+ \left[\pi(n+1,u_1,1)p_3^u + \pi(n+1,u_3,1)(1-r_3^u-r_4^u-r_5^u)\right](1-P^D)$$

plus equation (2).

The third set of equations describes the states in which the first machine is operational and the second is down:

$$\pi(n,1,d_2) = \sum_{i=1}^{3} \left[\pi(n-1,u_i,d_2)(1-r_2^d) + \pi(n-1,u_i,d_3)r_5^d + \pi(n-1,u_i,1)p_2^d\right] r_i^u + \tag{39}$$
$$+ \left[\pi(n-1,1,d_2)(1-r_2^d) + \pi(n-1,1,d_3)r_5^d + \pi(n-1,1,1)p_2^d\right](1-P^U)$$

$$\pi(n,1,d_3) = \sum_{i=1}^{3} \left[\pi(n-1,u_i,d_3)(1-r_3^d-r_4^d-r_5^d) + \pi(n-1,u_i,d_1)p_3^d\right] r_i^u + \tag{40}$$
$$+ \left[\pi(n-1,1,d_1)p_3^d + \pi(n-1,1,d_3)(1-r_3^d-r_4^d-r_5^d)\right](1-P^U)$$

plus equation (3).

The fourth set of equations describes the states in which both machines are down, with the upstream being failed in ODF mode:

$$\begin{aligned}
\pi(n,u_1,d_2) = {} & \left[\pi(n,u_1,d_2)(1-r_1^u-p_3^u)+\pi(n,u_3,d_2)r_4^u+\pi(n,1,d_2)p_1^u\right](1-r_2^d)+ \\
& +\left[\pi(n,u_1,d_3)(1-r_1^u-p_3^u)+\pi(n,u_3,d_3)r_4^u+\pi(n,1,d_3)p_1^u\right]r_5^d+ \\
& +\left[\pi(n,1,1)p_1^u+\pi(n,u_1,1)(1-r_1^u-p_3^u)+\pi(n,u_3,1)r_4^u\right]p_2^d
\end{aligned} \tag{41}$$

$$\begin{aligned}
\pi(n,u_1,d_3) = {} & \left[\pi(n,u_1,d_3)(1-r_1^u-p_3^u)+\pi(n,u_3,d_3)r_4^u+\pi(n,1,d_3)p_1^u\right](1-r_3^d-r_4^d-r_5^d)+ \\
& +\left[\pi(n,u_1,d_1)(1-r_1^u-p_3^u)+\pi(n,u_3,d_1)r_4^u+\pi(n,1,d_1)p_1^u\right]p_3^d
\end{aligned} \tag{42}$$

plus equation (4).

The following set of equations describes the states in which both machines are down, with the upstream being failed in TDF mode:

$$\begin{aligned}
\pi(n,u_2,d_1) = {} & \left[\pi(n,u_2,d_1)(1-r_2^u)+\pi(n,u_3,d_1)r_5^u+\pi(n,1,d_1)p_2^u\right](1-r_1^d-p_3^d)+ \\
& +\left[\pi(n,u_2,d_3)(1-r_2^u)+\pi(n,u_3,d_3)r_5^u+\pi(n,1,d_3)p_2^u\right]r_4^d+ \\
& +\left[\pi(n,u_2,1)(1-r_2^u)+\pi(n,u_3,1)r_5^u+\pi(n,1,1)p_2^u\right]p_1^d
\end{aligned} \tag{43}$$

$$\begin{aligned}
\pi(n,u_2,d_2) = {} & \left[\pi(n,u_2,d_2)(1-r_2^u)+\pi(n,u_3,d_2)r_5^u+\pi(n,1,d_2)p_2^u\right](1-r_2^d)+ \\
& +\left[\pi(n,u_2,d_3)(1-r_2^u)+\pi(n,u_3,d_3)r_5^u+\pi(n,1,d_3)p_2^u\right]r_5^d+ \\
& +\left[\pi(n,u_2,1)(1-r_2^u)+\pi(n,u_3,1)r_5^u+\pi(n,1,1)p_2^u\right]p_2^d
\end{aligned} \tag{44}$$

$$\begin{aligned}
\pi(n,u_2,d_3) = {} & \left[\pi(n,u_2,d_3)(1-r_2^u)+\pi(n,u_3,d_3)r_5^u+\pi(n,1,d_3)p_2^u\right](1-r_3^d-r_4^d-r_5^d)+ \\
& +\left[\pi(n,u_2,d_1)(1-r_2^u)+\pi(n,u_3,d_1)r_5^u+\pi(n,1,d_1)p_2^u\right]p_3^d
\end{aligned} \tag{45}$$

The following set of equations describes the states in which both machines are down, with the upstream being failed in $ODF\&TDF$ mode:

$$\begin{aligned}
\pi(n,u_3,d_1) = {} & \left[\pi(n,u_3,d_1)(1-r_3^u-r_4^u-r_5^u)+\pi(n,u_1,d_1)p_3^u\right](1-r_1^d-p_3^d)+ \\
& +\left[\pi(n,u_3,d_3)(1-r_3^u-r_4^u-r_5^u)+\pi(n,u_1,d_3)p_3^u\right]r_4^d+ \\
& +\left[\pi(n,u_3,1)(1-r_3^u-r_4^u-r_5^u)+\pi(n,u_1,1)p_3^u\right]p_1^d
\end{aligned} \tag{46}$$

$$\begin{aligned}
\pi(n,u_3,d_2) = {} & \left[\pi(n,u_3,d_2)(1-r_3^u-r_4^u-r_5^u)+\pi(n,u_1,d_2)p_3^u\right](1-r_2^d)+ \\
& +\left[\pi(n,u_3,d_3)(1-r_3^u-r_4^u-r_5^u)+\pi(n,u_1,d_3)p_3^u\right]r_5^d+ \\
& +\left[\pi(n,u_3,1)(1-r_3^u-r_4^u-r_5^u)+\pi(n,u_1,1)p_3^u\right]p_2^d
\end{aligned} \tag{47}$$

$$\begin{aligned}
\pi(n,u_3,d_3) = {} & \left[\pi(n,u_3,d_3)(1-r_3^u-r_4^u-r_5^u)+\pi(n,u_1,d_3)p_3^u\right](1-r_3^d-r_4^d-r_5^d)+ \\
& +\left[\pi(n,u_2,d_1)(1-r_2^u)+\pi(n,u_3,d_1)r_5^u+\pi(n,1,d_1)p_1^u\right]p_3^d
\end{aligned} \tag{48}$$