

Resilience-based component importance measures for critical infrastructure network systems

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Abstract - In this paper, we propose two metrics, i.e. the optimal repair time and the resilience reduction worth, to measure the criticality of the components of a network system from the perspective of their contribution to system resilience. Specifically, the two metrics quantify (i) the priority with which a failed component should be repaired and re-installed into the network, and (ii) the potential loss in the optimal system resilience due to a time delay in the recovery of a failed component, respectively. Given the stochastic nature of disruptive events on infrastructure networks, a Monte Carlo-based method is proposed to generate probability distributions of the two metrics for all the components of the network; then, a stochastic ranking approach based on the Copeland's pairwise aggregation is used to rank components importance. Numerical results are obtained for the IEEE 30 Bus test network and a comparison is made with three classical centrality measures.

Index Terms - Critical Infrastructure, system resilience, component importance measures, system recovery, stochastic ranking

I. INTRODUCTION

Complexity of critical infrastructures (CIs), such as power grids, the Internet, transportation networks, and so forth, is increasing. Disruptive events, whether they are malevolent attacks, natural disasters, or human-caused accidents, can have significant impacts on these real world complex networks composed of numerous interconnected functional and structural elements.

Justifiably, then, critical infrastructure protection (CIP) has become a priority for all nations [1]. The focus has been traditionally placed on physical protection and asset hardening [2]-[5]. However, in recent years, lessons learned from some catastrophic accidents have pushed part of the focus on the concept of

“resilience” [6], [7]. The outcomes of the 2005 World Conference on Disaster Reduction (WCDR) witness the significance of introducing the term “resilience” into the disaster discourse, giving birth to a new culture of disaster response [8]. Consequently, government policy has also evolved to encourage efforts that would allow assets to continue operating at some level, or quickly return to full operation after the occurrence of disruptive events [9].

“Resilience” comes from the Latin word “resilio” that literary means “to leap back” and denotes a system attribute characterized by the ability to recover from challenges or disruptive events. The Merriam-Webster dictionary defines resilience as “the ability to recover from or adjust easily to misfortune or change.” In this view, systems should not only be reliable, i.e. having an acceptably low failure probability, but also resilient, i.e. having the ability to optimally recover from disruptions of the nominal operating conditions [10], [11].

In this context, the present paper addresses the issue of quantifying the *importance* of components in contributing to the *resilience* of a critical infrastructure. Component importance measures (CIMs) have been thoroughly studied in the field of reliability theory and risk analysis. Various analytical and empirical CIMs have been proposed in the literature, e.g. Birnbaum [12], Fussell-Vesely [13], Reliability Achievement/Reduction Worth [14], [15], Barlow–Proschan [45], Natvig [46], and their extensions [16]-[20], [34], [47], [48]. CIMs have been shown valuable in establishing direction and prioritization of actions related to an upgrading effort (e.g., *reliability* improvement) in system design, or in suggesting the most efficient way to operate and maintain system status. However, none of the existing classical CIMs based on the reliability concept are directly applicable to the post-disaster phase, since there is no scope to exhibit reliability after the occurrence of system failure.

The role that a component plays in a network system has been measured by various so-called centrality measures, looking from the point of view of the complex interaction and communication flow in the network [21], [22]. Classical topological centrality measures are the degree centrality [23], [24], the closeness centrality [24]-[26], the betweenness centrality [24], and the information centrality [27]. They specifically rely on topological information to qualify the importance of a network component. Additionally, Freeman et al. [28] proposed a *flow betweenness* centrality measure based on the idea of maximum network flow; Newman [29] suggested a *random walk betweenness* measure that counts essentially all paths between vertices and which makes no assumptions of optimality; Jenelius et al. [30] proposed several vulnerability-based importance measures for transportation networks; Hines and Blumsack [31] introduced an “electrical centrality” measure for electrical networks by taking into account the electrical topology of the network; Zio and Piccinelli [32] provided a randomized flow model-based centrality measure specifically for electrical networks; Zio and Sansavini [33] introduced component criticality measures from the cascade failure process point of view, for general network systems.

Nevertheless, none of these analyses takes into account the dynamics of system recovery from the effects of a disruptive event.

Resilience-based metrics of component criticality with respect to their influence on the overall resilience of the system (i.e., on the system's ability to quickly recover from a disruptive event) can be helpful for preparing an efficient component repair checklist in the event of system failure [34]. Natvig et al. introduced a dual extended Natvig measure for repairable systems: according to this measure, the components that are considered important are those whose repair reduces significantly the expected time of residence of the system in the worst states [34]. Hence, this dual Natvig measure is a resilience measure for multistate components in a multistate system. A dual extension of the Barlow-Prochan measure has also been suggested for multistate repairable systems, based on the probability that the repair of the i -th component is the cause of a system state improvement, given that this has occurred [48].

Recently, Barker et al. [35] introduced two resilience-based network component importance metrics. Although the resilience definition, which the importance metrics rely on, actually embraces the temporal dimension of system recovery, it can be considered "memoryless": in the sense that the system resilience metric $R_F(t_r|e_j)$ at a given time t_r does not take into account the information about the restoration behavior before time t_r . Thus, it may happen that different restoration curves with different levels of favorability have the same value of resilience $R_F(t_r|e_j)$. Besides, the two metrics introduced in [35] seek to quantify the effect that the *disruption*, rather than the *recovery (behavior)*, of an individual component has on the system resilience, implying that the resilience improvement is achieved by actions related to system protection and fortification efforts in system design, although they have also been applied to compare network recovery strategies [36].

In this study, a new definition of system resilience and a resilience optimization framework are firstly presented, based on which we then introduce two network components importance measures, namely, the optimal repair time and the resilience reduction worth, useful for prioritizing restoration activities. The two measures quantify (i) the priority with which a failed component should be repaired and re-installed into the network and (ii) the potential loss in the optimal system resilience due to a time delay in the recovery of a failed component, respectively. A stochastic ranking technique, based on the Copeland's pairwise aggregation [37], is applied to rank the components criticalities.

As a case study, the IEEE 30 Bus test network is considered: the criticalities of the components computed by the proposed indicators are compared to those produced by three classical measures of betweenness centrality [28], [29], [38].

It is noted that the differences of the resilience-based CIMs proposed in the present paper with respect to that in [35] mainly fall into the following two aspects: (i) the concept and definition of system resilience

which the CIMs rely on takes into account the cumulative restored system functionality; (ii) the focus of the proposed resilience-based CIMs is to quantify the effect that the recovery, rather than the disruption, of individual components has on the global system resilience, for most effective system (recovery) operation.

The remainder of the paper is organized as follows. Section II provides the general framework of the study, recalling the definition of system resilience and the resilience optimization model. In Section III, two measures of component criticality for system resilience, and a simulation methodology for their calculation and ordering are presented. Section IV illustrates the calculation of the proposed metrics on the IEEE 30 Bus test network: the obtained components rankings are compared to those produced by classical betweenness centrality measures. Concluding remarks are drawn in Section V.

II. METHODOLOGICAL BACKGROUND: SYSTEM RESILIENCE DEFINITION AND OPTIMIZATION FOR INFRASTRUCTURE NETWORK SYSTEMS

This section provides the definition of system resilience and the resilience optimization framework, which serve as methodological background for the resilience-based component importance measures that will be discussed in Section III.

A. System Resilience Definition

As illustrated in Fig. 1, a quantifiable and time-dependent system performance function (also called system level delivery function or figure-of-merit) $F(t)$ is the basis for the assessment of system resilience [8], [35], [36]. It has a nominal value $F(t_0)$ under nominal operating conditions. The system operates at this level until suffering a disruptive event at time t_e . The disruption generally deteriorates system performance to some level $F(t_d)$ at time t_d . Then, recovery action is started, affecting and improving system performance until it achieves, at a later time t_r , a targeted level of performance $F(t_r)$ that could be the same, close to, or better than original system performance $F(t_0)$, for which recovery is considered completed. The dotted curve $TF(t)$ in Fig. 1 denotes the targeted system performance if not affected by disruption, which is generally evolving due to the dynamic nature of service demand and system upgrading; in this study, it is assumed to be equal to $F(t_0)$ and remain invariant, for simplicity of illustration. Besides, it is noted that various strategies exist for recovery activities, and system performance is ultimately a function of recovery decisions. The period of $t_d \leq t \leq t_r$ is generally considered as the recovery time [8].

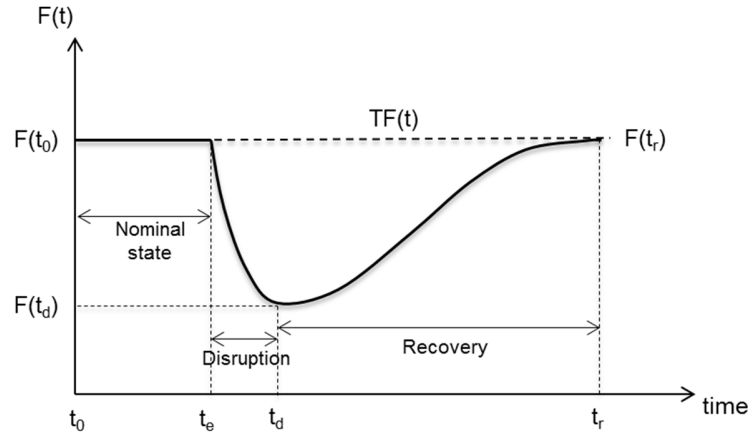


Fig. 1. Generic system performance transition curve under the occurrence of disruption.

Let $R(t)$ be the resilience of a system at time t ($t \geq t_d$). In its basic form, $R(t)$ describes the cumulative system functionality that has been restored at time t , normalized by the expected cumulative system functionality supposing that the system has not been affected by disruption during this time period (Eq. (1) below): graphically, $R(t)$ is quantified by the ratio of the area with diagonal stripes S_1 to the area of the shaded part S_2 , as shown in Fig. 2.

$$R(t) = \frac{\int_{t_d}^t [F(\tau) - F(t_d)] d\tau}{\int_{t_d}^t [TF(\tau) - F(t_d)] d\tau}, t \geq t_d \quad (1)$$

Note that the formulation in Eq. (1) focuses mainly on the recoverability dimension of resilience and $R(t)$ is in the range of $[0, 1]$. $R(t) = 0$ when $F(t) = F(t_d)$, which means that a system has not recovered from its disrupted state (i.e., there has been no “resilience” action); $R(t) = 1$ when $F(t) = TF(t)$, which corresponds to the ideal case where a system recovers to its target state immediately after disruption. This resilience quantification is consistent with the original meaning of the concept of resilience and is capable of measuring at the same time the magnitude and rapidity of system recovery action. More importantly, this definition of system resilience is not memoryless since it considers cumulatively the system functionality restored, differently from [35].

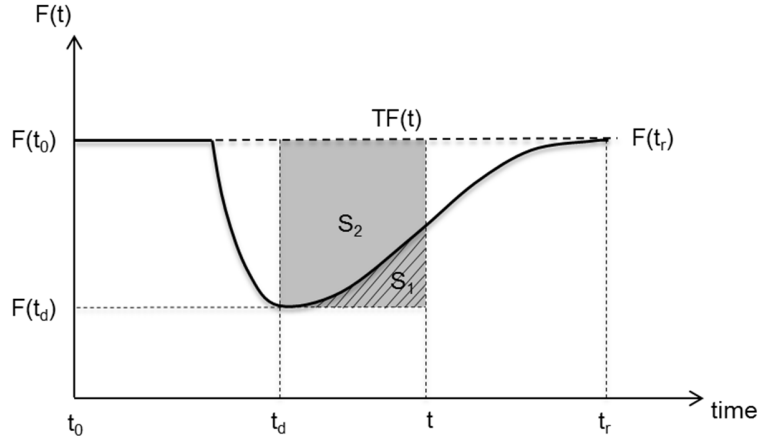


Fig. 2. Conceptual illustration of the proposed resilience measurement.

B. System Resilience Optimization

A disruptive event could impact one or more components of an infrastructure network system. In the case of multiple component failures, a systemic recovery action should be undertaken with the order of failed components to repair such that system resilience is maximal, i.e., to achieve optimal (restored) cumulative system functionality over the recovery time considered.

A variety of frameworks of optimization for post-disaster recovery of an infrastructure network system can be designed, focusing on different aspects of the restoration strategy, e.g., the order of repair of the damaged components, where and how to allocate repair resources and so forth. This study focuses on the significance of the recovery of a component with respect to the resilience of the system. Consequently, the optimization is designed to find the optimal order of repair of the set of failed components with the objective of achieving maximum system resilience over the restoration time horizon.

The mathematical model for the resilience optimization concerns a network $G(V, E)$ comprising a set of nodes V connected by a set of links or arcs E . The network nodes are distinguished in supply nodes V_S , transshipment nodes V_T and demand nodes V_D ($V_S \cup V_T \cup V_D = V$). Each arc $ij \in E$ has an associated capacity $(ij) \in \mathbf{R}_0^+$, each supply node $i \in V_S$ has a supply capacity per time unit $P_i^S \in \mathbf{R}_0^+$ and each demand node $j \in V_D$ has a demand $P_j^D \in \mathbf{R}_0^+$ per time unit. Network flow is delivered from supply nodes to demand nodes respecting the flow capacities of the links and supply/demand capacities of the nodes. The performance of the network is evaluated by determining the maximum amount of flow that can be received by the demand nodes. Formally, the system performance function is defined as:

$$F(t) = \sum_{j \in V_D} f_j(t) \quad (2),$$

where $f_j(t)$ represents the amount of flow received by demand node j at time t .

Disruptions happen and create damages to nodes and/or links in the network, which is modeled as removal of a subset of arcs, $E' \subset E$, from the network.¹ The arcs in set E' are viewed as non-operational immediately after the disruption. System performance $F(t)$ achieves its minimum value at this time, which is seen as the initial stage of system restoration that we focus on (we set $t = 0$ at this time for computational convenience, i.e. $F_{min} = F(0)$).

The recovery optimization framework aims at identifying the subset of links in E' to repair and the order, in which the links should be repaired so as to achieve maximum system resilience over the restoration horizon $T \in \mathbf{Z}^+$. In this study, link repairs are assumed to be discrete tasks and only a single arc can be repaired at a given time period. Thus, discrete time periods $t = 1, \dots, T$ are considered, hereafter, instead of the continuous one in the resilience definition of Eq. (1). Besides, we do not model in detail the mechanisms and procedures according to which a single failed component is repaired; rather, the focus is on the identification of the exact (optimal) time when the disrupted arcs should be brought back online and, thus, on the temporal sequence of the restoration actions on all the failed components. Obviously, the inclusion of possibly different repair times for different components may produce a different component ranking. However, this would be automatically accounted for in the procedure by the inclusion of proper “hard” constraints and, thus, it would not impair the applicability and generality of the approach. By combining Eqs. (1) and (2), system resilience to be maximized at time T is given by

$$R(T) = \frac{\sum_{t=1}^{t=T} [\sum_{j \in V_D} f_j(t) - F_{min}]}{T \cdot (\sum_{j \in V_D} P_j^D - F_{min})} \quad (3),$$

where $\sum_{j \in V_D} P_j^D = TF(t)$ is the target system performance. The variables of the resilience optimization problem include: (i) continuous variables $f_{ij}(t) \in \mathbf{R}_0^+$, $ij \in E$ and $t = 1, \dots, T$, that denote the flow moving from node i to node j through link ij at time unit t ; (ii) continuous variables $f_j(t) \in \mathbf{R}_0^+$, $j \in V_D$, that represent the amount of flow received by demand node j at time unit t and (iii) binary state variables $s_{ij}(t)$, $ij \in E$ and $t = 1, \dots, T$, such that $s_{ij}(t) = 1$ if arc ij is operational and $s_{ij}(t) = 0$ if arc ij is not operational at time unit t . We are interested in optimizing the resilience over the whole restoration process: thus, the timespan T is the total recovery time, defined as the period necessary to restore the system functionality to the same level as the original system. Consequently, the formulation of the resilience optimization problem is as follows:

$$\max \frac{\sum_{t=1}^{t=T} [\sum_{j \in V_D} f_j(t) - F_{min}]}{T \cdot (\sum_{j \in V_D} P_j^D - F_{min})} \quad (4)$$

Subject to

¹ For nodes, they can be converted to equivalent arcs by introducing additional arcs and nodes into the network, i.e. by ‘splitting’ a node into two nodes and an arc.

$$\sum_{(i,j) \in E} f_{ij}(t) - \sum_{(j,i) \in E} f_{ji}(t) \leq P_i^S \quad \forall i \in V_S, t = \{1, \dots, T\} \quad (5)$$

$$\sum_{(i,j) \in E} f_{ij}(t) - \sum_{(j,i) \in E} f_{ji}(t) = 0 \quad \forall i \in V_T, t = \{1, \dots, T\} \quad (6)$$

$$\sum_{(i,j) \in E} f_{ij}(t) - \sum_{(j,i) \in E} f_{ji}(t) = -f_j(t) \quad \forall i \in V_D, t = \{1, \dots, T\} \quad (7)$$

$$0 \leq f_j(t) \leq P_j^D \quad \forall i \in V_D, t = \{1, \dots, T\} \quad (8)$$

$$0 \leq f_{ij}(t) \leq s_{ij}(t)P(ij) \quad \forall ij \in E, t = \{1, \dots, T\} \quad (9)$$

$$s_{ij}(t) \leq s_{ij}(t+1) \quad \forall ij \in E, t = \{1, \dots, T\} \quad (10)$$

$$\sum_{(i,j) \in E'} [s_{ij}(t) - s_{ij}(t-1)] = 1 \quad \forall t = \{1, \dots, T\} \quad (11)$$

$$s_{ij}(t) \in \{0,1\}, s_{ij}(0) = 0 \quad \forall ij \in E', t = \{1, \dots, T\} \quad (12)$$

The objective (4) is to maximize the system resilience over the time horizon of recovery. Constraints (5)-(9) are typical network flow constraints over the links and supply/demand nodes in the network in period t . They ensure that: the flow generated at a supply node does not exceed its supply capacity (5); the amount of net injected flow at a transshipment node is zero (6); the amount of net injected flow at a demand node is equal to the received flow at the node (7) while not exceeding its requested demand (8); the flow on an operational link does not exceed its capacity and there is no flow passing through an arc if the arc is failed (9). Constraint (10) ensures that once an arc has been restored at time t , it will keep operational thereafter. Finally, constraint (11) ensures that only a single arc can be repaired at any given timeslot.

This resilience optimization above defined is a typical mixed integer programming (MIP) problem. A commercial optimization solver Cplex [39] is used in this study for its solution. It is noted that this resilience optimization model is only applied for the purpose of illustration of resilience-based component importance metrics. More complex optimization models (e.g., taking into account the cost and duration of repairing a particular failed link) can be adopted in other application cases. Besides, a specific application involving the 400kV French Power Transmission Network of the proposed resilience (restoration) optimization problem has been presented in [49]. While the present study mainly focuses on the definition of resilience-based CIMs and the analysis of their properties, as mentioned d before.

III. RESILIENCE-BASED COMPONENT IMPORTANCE MEASURES FOR INFRASTRUCTURE NETWORK SYSTEMS

A. Component Importance Measures Definition

As described in Section II.B, the analysis concerns a network $G(V, E)$ comprising a set of nodes V and a set of links E . The binary state variable of arc ij at time t is defined by $s_{ij}(t)$, $\forall ij \in E$. The initial impact experienced by the network after a disruptive event e at time $t = 0$ is represented by the removal of a subset of arcs, $E' \subset E$, from the network, setting $s_{ij}(0) = 0$, $\forall ij \in E'$. We introduce the failure probability of arc ij under event e , $p_e(ij)$

$$P[s_{ij}(0) = 0|e] = p_e(ij), \forall ij \in E \quad (13)$$

Eq. (13) describes how individual components (links) are initially affected by a disruptive event e ; Section II.B explains how these failed components optimally recover from the disruption state following the event; finally, Eq. (1) incorporates these dimensions to quantify system resilience.

When considering component criticality in a resilience setting, we are interested in understanding: (i) the optimal time to repair the failed components in order to maximize system resilience, and (ii) the effect that the timely recovery of the components have on the overall resilience of the system. These concepts are at the basis of the definition of the two resilience-based importance measures here proposed.

Given a particular initial failure state, the optimal repair time T_{ij}^{opt} of a failed arc ij is defined as the time when the arc ij is restored to operating status so to maximize the system resilience over the recovery time T , given by:

$$T_{ij}^{opt} = \arg \max_{T_{ij} \in [0, T]} R(T) \quad (14)$$

It can be obtained by firstly solving the MIP problem (4) - (12) and, then, computing it based on the state variables of the arc ij :

$$T_{ij}^{opt} = \sum_{t=0}^T (1 - s_{ij}(t)) \quad (15)$$

The timespan for restoration, T , is chosen as the time period necessary to restore the system functionality to the same level as the original system. It is noted that the optimal repair time T_{ij}^{opt} offers an explicit quantification of the priority that should be given to the reparation and installation of arc ij into the network. Low values of T_{ij}^{opt} indicate higher priority of being repaired and re-installed into the network, i.e. higher ranking of the component in the repair checklist.

To account for the delay in the restoration of a particular link ij , a resilience reduction worth (RRW) metric is introduced as

$$RRW_{ij}(\Delta t_0) = \frac{R^{opt}(T) - R^{opt}(T | T_{ij} \geq T_{ij}^{opt} + \Delta t_0)}{R^{opt}(T)} \quad (16),$$

where $R^{opt}(T)$ represents the optimal system resilience at restoration time T ; $R^{opt}(T | T_{ij} \geq T_{ij}^{opt} + \Delta t_0)$ corresponds to the optimal system resilience at time T if link ij cannot be repaired until time $(T_{ij}^{opt} + \Delta t_0)$, where Δt_0 is the delay with respect to its optimal repair time T_{ij}^{opt} given by Eq. (14). Eq. (16) quantifies the potential (normalized) loss in optimal system resilience due to a delay Δt_0 in the repair of link ij . This metric is comparable to the so-called reliability reduction worth [40], which measures the potential damage caused to the system reliability by the failure of a particular component. It can provide valuable information to guide the recovery process of a particular component. Components with high values of

$RRW_{ij}(\Delta t_0)$ should be given high priority in the restoration process, e.g. be assigned adequate restoration resources to avoid delays that would have a more significant impact on system restoration.

It is noted that the resilience-based CIMs in [35] are introduced to quantify the effect that the *disruption*, rather than the *recovery (behavior)*, of an individual component has on the system resilience (represented by the system recovery time). Specifically, the first metric in [35] measures the contribution that the *disruption* of link i has on the system recovery time, weighted by the ratio of system service loss due to the *disruption effect* on link i to the maximum loss among all the links; The second metric in [35] quantifies how the system recovery time is improved if link i is *invulnerable*. Both definitions imply that the resilience improvement is achieved by actions related to system protection and fortification efforts in system design. However, the two resilience-metrics introduced in the present paper measure the effect that the *recovery* of an individual component has on the global system resilience, by quantifying (i) the priority with which a failed component should be repaired and re-installed into the network, and (ii) the potential loss in the optimal system resilience due to a time delay in the recovery of a failed component, respectively. Hence, our resilience-based metrics are able to suggest the most effective way for system operation, i.e., to help the implementation of (i) recovery schedule planning, and (ii) restoration resources allocation.

B. Methodology for Component Importance Ordering

Ordering network links recovery on the basis of the values of the criticality measures described above, i.e., the optimal repair time T_{ij}^{opt} and resilience reduction worth RRW_{ij} (fixed Δt_0), requires quantifying the effect of timely repairing these links on the overall resilience of the system. Given the stochastic nature of disruptive events in terms of components failures after the event, the resilience-based criticality measures introduced are not represented by deterministic values, but rather by probability distributions. Therefore, given a network $G(V, E)$ under a disruptive event e , we first apply a Monte Carlo-based method to generate distributions of optimal repair time T_{ij}^{opt} and resilience reduction worth $RRW_{ij}(\Delta t_0)$ for all the links in the network; then, we rank links importance using a stochastic approach based on the Copeland's pairwise aggregation method [37]. The detailed steps of the algorithm are as follows:

- Step 1. A network $G(V, E)$ is initially operating with a given parameters setting: flow demand P_j^D of all the demand nodes in V_D , supply capacity P_i^S of all the supply nodes in V_S and link capacity $P(ij)$ for all the network arcs in E .
- Step 2. A failure configuration of the network is randomly sampled on the basis of the failure probabilities of each arc in the system given by Eq. (13), under a disruptive event e at initial time $t = 0$. The operation state variables of failed links are set to 0, i.e., $s_{ij}(0) = 0, \forall ij \in E'$.

- Step 3.* The resilience optimization model of Eq. (4) - (12) is applied and solved by Cplex to obtain the optimal strategy of network recovery, i.e., the optimal repair time T_{ij}^{opt} for each failed arc $ij \in E'$.
- Step 4.* In order to evaluate the second importance measure $RRW_{ij}(\Delta t_0)$, for each failed arc $ij \in E'$, the additional constraint that the restoration of arc ij should not be accomplished earlier than $T_{ij}^{opt} + \Delta t_0$ (i.e., $T_{ij} \geq T_{ij}^{opt} + \Delta t_0$) is added to the optimization model of Eq. (4) - (12). Then, $R^{opt}(T|T_{ij} \geq T_{ij}^{opt} + \Delta t_0)$ is obtained by solving this “modified” optimization model by Cplex. Finally, the resilience reduction worth $RRW_{ij}(\Delta t_0)$ for each arc ij is recorded.
- Step 5.* To account for the stochasticity of the disruptive event in terms of arcs failures, repeat Step 2 to Step 4 for a chosen number \aleph of iterations, generating probability distributions for T_{ij}^{opt} and $RRW_{ij}(\Delta t_0)$, for all the links in the network.
- Step 6.* Given the distributions of T_{ij}^{opt} (resp., $RRW_{ij}(\Delta t_0)$) for each arc ij , perform a stochastic ranking of links according to ascending (resp., descending) T_{ij}^{opt} values (see Section III.B.1).

C. Stochastic Ranking

In order to rank network links according to the distribution of their optimal repair time T_{ij}^{opt} (or resilience reduction worth $RRW_{ij}(\Delta t_0)$) obtained at step 6 of the algorithm above, an approach based on the Copeland's pairwise aggregation method [37] is proposed. The Copeland's method (CM) is a simple non-parametric Condorcet method used in the political field (voting) that does not require any information about decision maker preference and operates on a multi-indicator matrix formed by m objects characterized by Ω attributes [41]. CM relies on pair-wise comparisons between objects in the candidate pool, and the so-called Copeland score is defined for each object as the difference between the number of times that this object beats the other objects and the number of times that it is beat by other objects.

The CM-based ranking approach applied here corresponds to a modification proposed by Al-Sharrah [42]. It first examines the CDF of a given variable for all the candidates, e.g., the CDF of T_{ij}^{opt} , $\forall (i, j) \in E$; then, it compares the CDF of two candidates under analysis, i.e., links ij and $\bar{i}\bar{j}$, with respect to specific attributes q_k of the CDF: for example, attribute q_k may represent the k th percentile. Subsequently, a quantity $S_k(ij, \bar{i}\bar{j})$ is calculated based on a pairwise comparison between links ij and $\bar{i}\bar{j}$ with respect to (percentile) q_k of the corresponding distributions, $k = 1, \dots, \Omega$:

$$S_k(ij, \bar{i}\bar{j}) = \begin{cases} S_{k-1}(ij, \bar{i}\bar{j}) + 1, & \text{if } q_k(ij) \text{ beats } q_k(\bar{i}\bar{j}) \\ S_{k-1}(ij, \bar{i}\bar{j}) + 0.5, & \text{if } q_k(ij) \text{ and } q_k(\bar{i}\bar{j}) \text{ are tied} \\ S_{k-1}(ij, \bar{i}\bar{j}), & \text{if } q_k(ij) \text{ beats } q_k(\bar{i}\bar{j}) \end{cases} \quad (17),$$

where the sentence “ $q_k(ij)$ beats $q_k(\bar{i}\bar{j})$ ” means that $q_k(ij)$ dominates $q_k(\bar{i}\bar{j})$ with respect to the ranking rule of the variable considered, i.e., $q_k(ij) < q_k(\bar{i}\bar{j})$ for T_{ij}^{opt} , while $q_k(ij) > q_k(\bar{i}\bar{j})$ if $RRW_{ij}(\Delta t_0)$ is

considered. $S_0(ij, \bar{ij})$ is initialized at zero for the first (percentile) q_1 and Eq. (16) is iterated through all Ω attributes (percentiles). Then, the Copeland score for each link ij is defined as

$$C(ij) = \sum_{\bar{ij} \neq ij} S_{\Omega}(ij, \bar{ij}) \quad (18)$$

This Copeland score is finally used to rank all the links: the higher $C(ij)$, the higher the contribution of link ij to the overall resilience of the network.

IV. CASE STUDY

A. Resilience-Based Criticality Measures on The IEEE 30 Bus Test System

The IEEE 30 Bus test system [43] is taken as reference case study for the proposed resilience-based component importance measure approach. This system (Fig. 3) represents a portion of the American Electric Power System and is composed of 30 buses connected by 41 transmission lines. To carry out the analysis, each system component is transposed into a node or edge of the representative topological network, as it is shown in Fig. 4. Three different physical types of nodes are considered: generator nodes (where the electricity flow is fed into the network), demand nodes (where customers are connected) and transfer or transmission nodes (without customers or sources).

The simulation procedure introduced in Section III.B is, then, used to rank each component of the IEEE 30 Bus network according to the criticality metrics introduced. In normal conditions, the network is assumed to operate under the following parameters setting: the generation capacity is identical for all generation nodes and equal to 60, in arbitrary units (a.u.); the flow demands are 20 a.u. for all load nodes; the values of the transmission capacities are 20 a.u. for all the network links. The homogeneous assignments of generation capacity, demand and link capacity are here applied for the purpose of identifying the resilience criticalities of all the network arcs stemming from their different topological connections. For the same reason, a constant failure probability $p_e(ij)$ is assumed for all the network links under disruptive event e . The roulette wheel selection method [44] is used in step 2 for sampling a failure configuration by selecting a failed link at each spin until a certain number $\|E\| \cdot p_e(ij) = 12$ of arcs are selected.

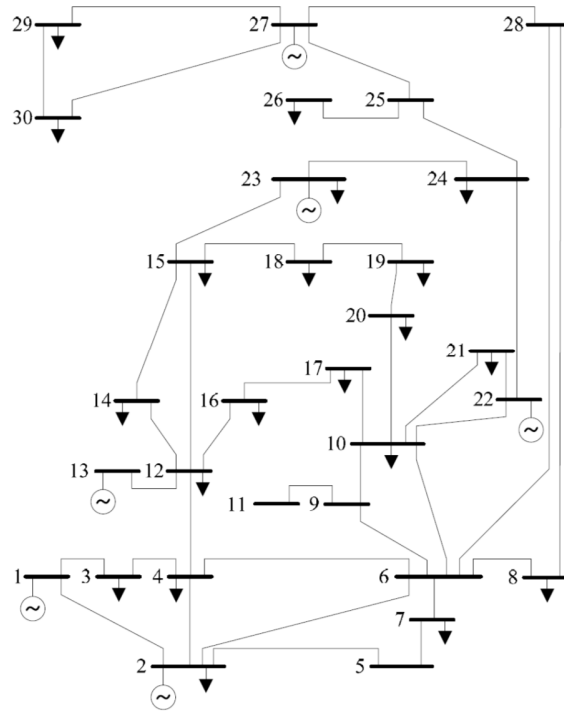


Fig. 3. Single line diagram of the IEEE 30 Bus test system.

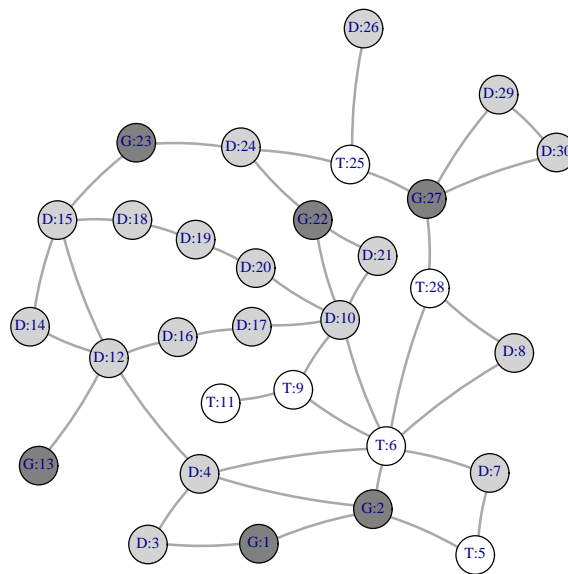


Fig. 4. Graph representation of the IEEE 30 Bus test system. The dark grey circles labeled with G represent the generator nodes, the white circles labeled with T represent transmission nodes and the light grey circles labeled with D represent the demand nodes.

Fig. 5 illustrates the Cumulative Distribution Functions (CDFs) of T_{ij}^{opt} for five representative links (<1, 3>, <5, 7>, <27, 30>, <8, 28> and <10, 21>), obtained at step 5 of the procedure by applying the simulation

algorithm proposed in Section III.B (for $\aleph = 1000$ samples). The Figure illustrates the probability that T_{ij}^{opt} is less than or equal to a target value x . It can be seen that the optimal repair time associated with link $\langle 1, 3 \rangle$, i.e. T_{13}^{opt} , will never be larger than 5 (square-line curve in Fig. 5). Moreover, the curve for link $\langle 1, 3 \rangle$ always “dominates” the other curves. Therefore, this link should have the highest priority to be repaired in order to maximize system resilience.

However, considering e.g. links $\langle 5, 7 \rangle$ (circle line) and $\langle 27, 30 \rangle$ (triangle line) in Fig. 5, it is not evident which one “dominates” the other, due to the intersection of their CDF curves. Thus, the CM-based ranking approach introduced in Section III.B.1 is applied to rank the importance of the links. Fig. 6 reports the Copeland scores of all the 41 links in the IEEE 30 Bus network, ordered in descending order, with link $\langle 1, 3 \rangle$ having the highest score, followed by links $\langle 2, 6 \rangle$, $\langle 2, 4 \rangle$, $\langle 10, 22 \rangle$ and so forth. Furthermore, Fig. 7 graphically illustrates the Copeland score of the optimal repair time T_{ij}^{opt} for all IEEE 30 Bus network links, where links with higher values of Copeland score are represented as thicker and darker edges. It is shown that two types of links are more important in terms of T_{ij}^{opt} : i) the links which connect the generator nodes with the other two types of nodes (transmission nodes and demand nodes), e.g. links $\langle 2, 6 \rangle$, $\langle 1, 3 \rangle$, $\langle 12, 13 \rangle$ etc., and ii) the links which are the only ones connected to demand nodes, e.g. link $\langle 25, 26 \rangle$. The restoration of these types of links is most likely able to augment the total amount of flow received by the demand nodes of the network: thus, high priority should be given to these links when considering the repair order of the failed links.

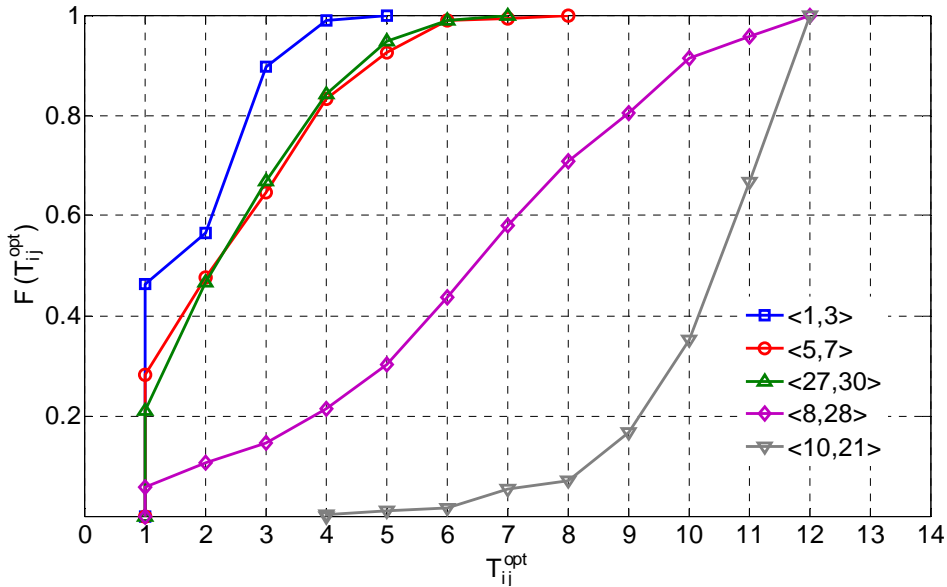


Fig. 5. Cumulative probability distributions of the optimal repair time T_{ij}^{opt} for five representative links.

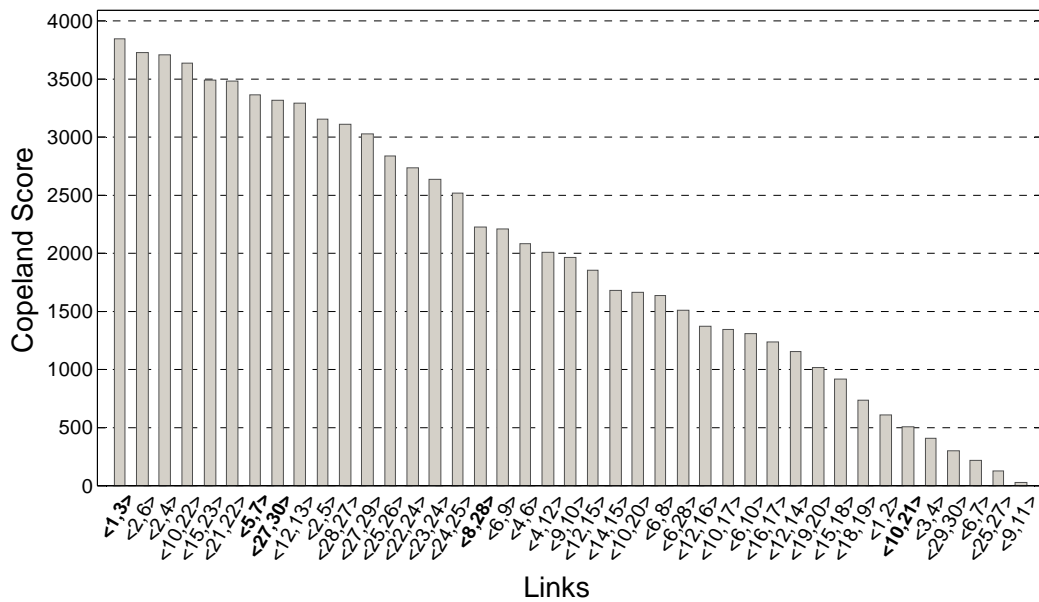


Fig. 6. Copeland score ranking of the optimal repair time T_{ij}^{opt} for all IEEE 30 Bus network links.

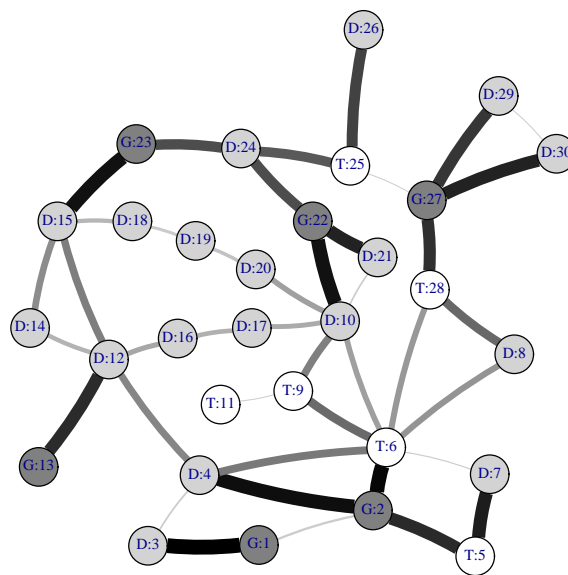


Fig. 7. Graphical illustration of the Copeland scores of the optimal repair time T_{ij}^{opt} for all IEEE 30 Bus network links. Links with higher value of Copeland score are represented as thicker and darker edges.

Fig. 8 and Fig. 9 illustrate the results based on the resilience reduction worth $RRW_{ij}(\Delta t_0)$ for all the links and for a delay time $\Delta t_0 = 3$ units, i.e., the Copeland score ranking and its graphical representation, respectively. It is noted that different sets of values of the resilience reduction worth $RRW_{ij}(\Delta t_0)$ for all network links can be obtained under different values of delay Δt_0 ; however, the ranking of $RRW_{ij}(\Delta t_0)$ for all the links by the Copeland method will keep consistent in our study. It is shown that <24, 25> is the most critical link in terms of RRW_{ij} , i.e. a delay in its restoration would cause the largest reduction in

system resilience among all the network links; thus, adequate resources should be given to make sure of its timely restoration. Besides, it is noted that the links with high Copeland scores in terms of the optimal repair time T_{ij}^{opt} also have high Copeland score ranking in terms of the resilience reduction worth RRW_{ij} : the correlation coefficient between the two Copeland scores is $r(C_{T_{ij}^{opt}}, C_{RRW_{ij}}) = 0.82$ for $\Delta t_0 = 3$.

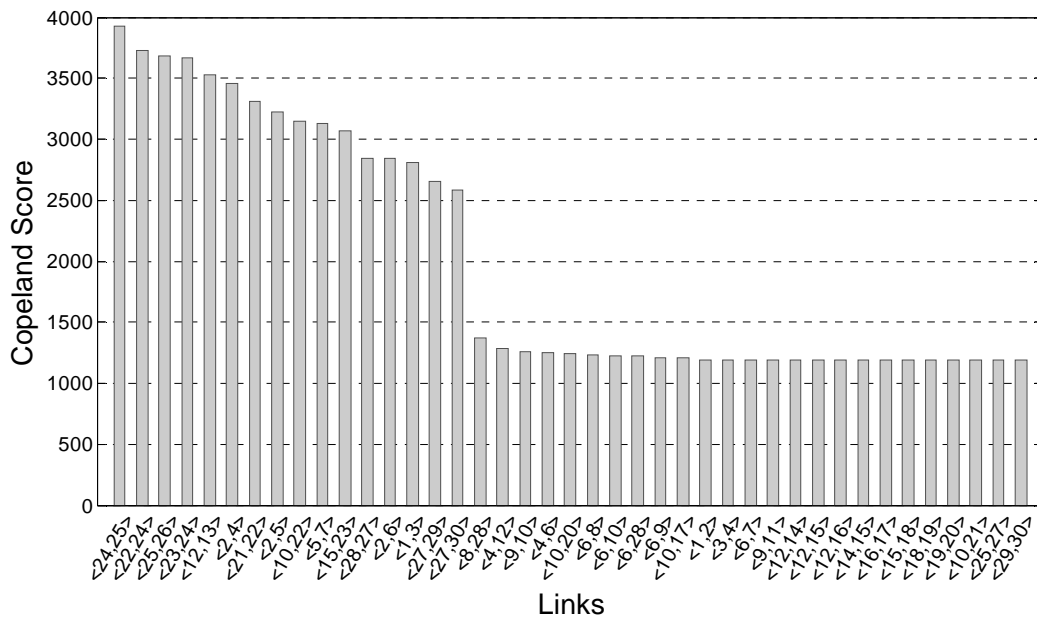


Fig. 8. Copeland score ranking of the resilience reduction worth $RRW_{ij}(\Delta t_0 = 3)$ for all IEEE 30 Bus network links.

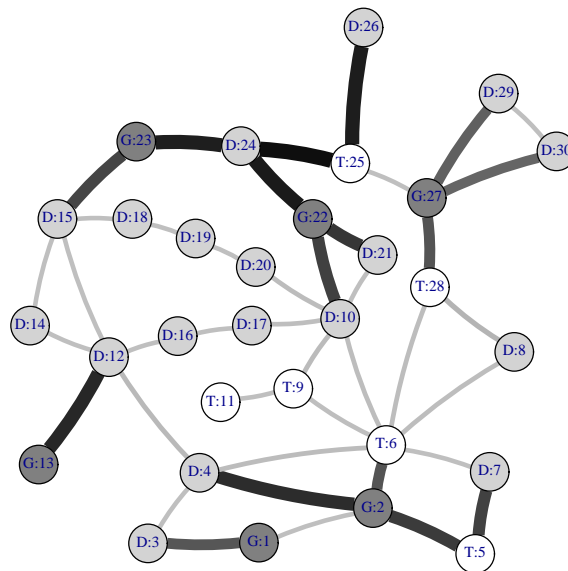


Fig. 9. Graphical illustration of the Copeland scores of the resilience reduction worth $RRW_{ij}(\Delta t_0 = 3)$ for all IEEE 30 Bus network links. Links with higher values of Copeland score are represented as thicker and darker edges.

B. Comparison with Betweenness Centrality Measures

Betweenness centrality indices have been introduced as measures of component importance in a network, taking into account the different ways in which a component interacts and communicates with the rest of the network [24], [32]. A classical centrality measure is the topological betweenness centrality introduced in the social network field, which is based on the idea that a component is central if it lies between many other components, in the sense that it is traversed by many of the shortest paths connecting pairs of nodes [24], usually called as *shortest path betweenness*. The topological betweenness centrality C_{ij}^B of a given link ij in a supply-demand-differentiated network $G(V_S \cup V_T \cup V_D, E)$ is given by [38]:

$$C_{ij}^B = \frac{1}{\|V_S\| \cdot \|V_D\|} \sum_{s \in V_S, d \in V_D} \frac{n_{sd}(ij)}{n_{sd}}, ij \in E \quad (19),$$

where n_{sd} is the number of topological shortest paths between supply nodes and demand nodes, and $n_{sd}(ij)$ is the number of supply-demand shortest paths passing through link ij .

To account for the issue that in some cases flow may not follow the ideal geodesic paths from supply to demand nodes, a betweenness centrality measure based on the idea of maximum network flow has been proposed [28], usually known as *flow betweenness*. The measure counts all independent paths that carry information when a maximum flow is pumped between each pair of vertices. The flow betweenness of a component is defined as the amount of flow through it when the maximum flow is transmitted from source s to demand d , averaged over all s and d . It is quantitatively defined as [28]

$$C_{ij}^F = \frac{\sum_{s \in V_S, d \in V_D} m_{sd}(ij)}{\sum_{s \in V_S, d \in V_D} m_{sd}}, ij \in E \quad (20),$$

where m_{sd} is the maximum flow from a source node s to a demand node d and $m_{sd}(ij)$ is the maximum flow from s to d that passes through link ij .

In practical terms, however, neither of the two betweenness measures introduced above is realistic. Both count only a small subset of possible paths between vertices, and both assume some kind of optimality in information transmission (shortest paths or maximum flow). Therefore, a new betweenness measure that counts essentially all paths between vertices and which makes no assumptions of optimality has been suggested, called *random walk betweenness* [29]. This measure is based on random walks between vertex pairs and asks, in essence, how often a given component will fall on a random walk between another pair of vertices. Roughly speaking, the random walk betweenness of a link ij is equal to the number of times that a random walk starting at s and ending at d passes through the link along the way, averaged over all

s and d . Let I_{ij}^{sd} be the current flow from s to d , through link ij . Then, the random walk betweenness of a link ij is defined as

$$C_{ij}^{RW} = \frac{1}{\|V_S\| \cdot \|V_D\|} \sum_{s \in V_S, d \in V_D} I_{ij}^{sd}, ij \in E \quad (21)$$

We are interested in comparing the ranking results of our resilience-based component importance measures to these betweenness centrality indices, i.e., *shortest path betweenness*, *flow betweenness* and *random walk betweenness* for the proposed IEEE 30 Bus network. Fig. 10 shows the values of the Copeland scores for the optimal repair time $C_{T_{ij}^{opt}}$ (left panel) and for the resilience reduction worth $C_{RRW_{ij}}$ (right panel) plotted with respect to the *shortest path betweenness* C_{ij}^B for all the links of IEEE 30 Bus network. No obvious correlation can be identified from the figures. Actually, the correlation coefficients between $C_{T_{ij}^{opt}}$, $C_{RRW_{ij}}$ and C_{ij}^B are $r(C_{T_{ij}^{opt}}, C_{ij}^B) = 0.08$ and $r(C_{RRW_{ij}}, C_{ij}^B) = 0.14$, respectively. Similarly, Fig. 11 plots the relationship between the Copeland scores for the optimal repair time $C_{T_{ij}^{opt}}$ (left panel) and the resilience reduction worth $C_{RRW_{ij}}$ (right panel) with the *flow betweenness* C_{ij}^F ; Fig. 12 shows the same scatterplots with respect to the *random walk betweenness* C_{ij}^{RW} . The correlation coefficients are $r(C_{T_{ij}^{opt}}, C_{ij}^F) = 0.002$, $r(C_{RRW_{ij}}, C_{ij}^F) = -0.24$, $r(C_{T_{ij}^{opt}}, C_{ij}^{RW}) = 0.24$ and $r(C_{RRW_{ij}}, C_{ij}^{RW}) = 0.32$, respectively.

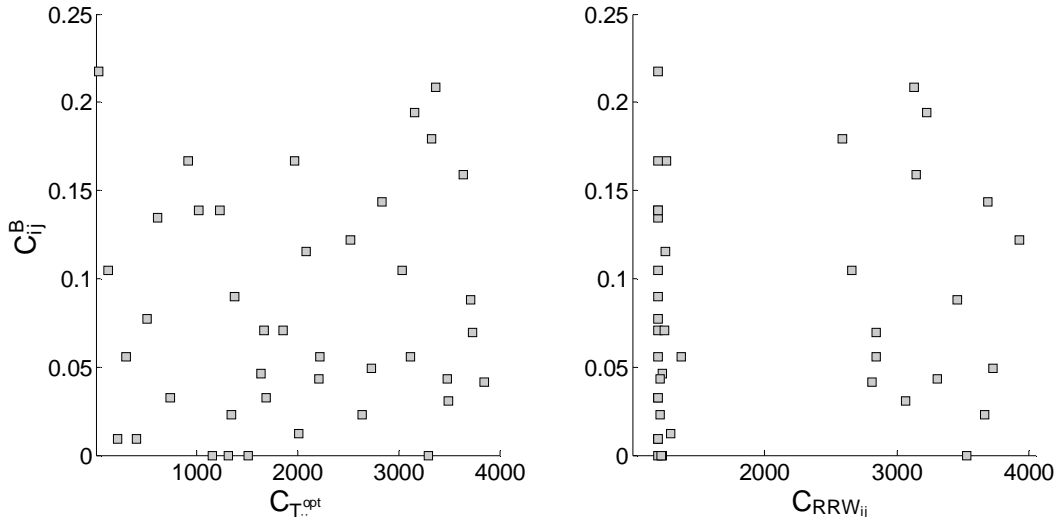


Fig. 10. Scatterplot of the Copeland scores of the optimal repair time $C_{T_{ij}^{opt}}$ (left panel) and resilience reduction worth $C_{RRW_{ij}}$ (right panel) with the shortest path betweenness C_{ij}^B for the links of the IEEE 30 Bus network.

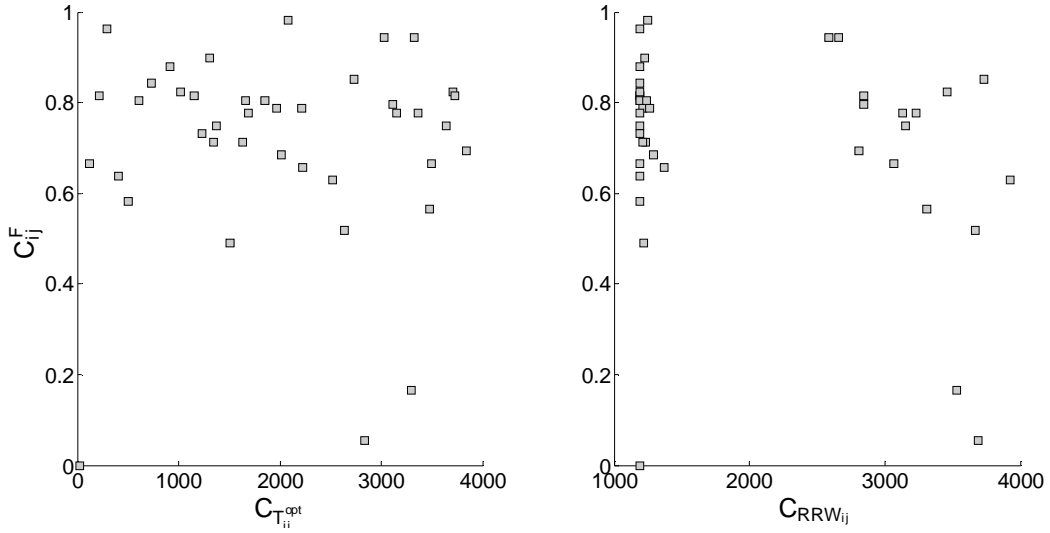


Fig. 11. Scatterplot of the Copeland scores of the optimal repair time $C_{T_{ij}^{opt}}$ (left panel) and resilience reduction worth $C_{RRW_{ij}}$ (right panel) with flow betweenness C_{ij}^F for the links of the IEEE 30 Bus network.

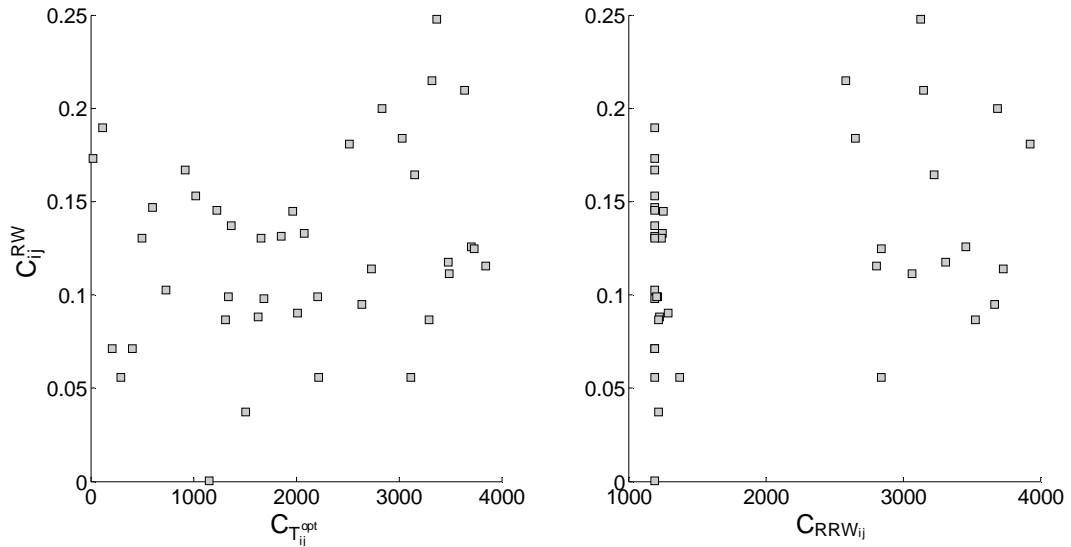


Fig. 12. Scatterplot of the Copeland scores of the optimal repair time $C_{T_{ij}^{opt}}$ (left panel) and resilience reduction worth $C_{RRW_{ij}}$ (right panel) with the random walk betweenness C_{ij}^{RW} for the links of the IEEE 30 Bus network.

These results show that the betweenness centrality indices (e.g., *shortest path betweenness*, *flow betweenness* and *random walk betweenness*) do not capture the component criticality with respect to resilience for the recovery of the IEEE 30 Bus network. This implies that these centrality measures (which are calculated under normal operation condition) are not applicable to guide the system restoration after a disruptive event, e.g., to prepare an efficient component repair priority checklist in the event of system failure.

V. CONCLUSIONS

This paper primarily contributes two metrics to measure the criticality of network components from the perspective of their contribution to system resilience, defined as the cumulative system functionality that has been restored at time t , normalized by the expected cumulative system functionality supposing that the system has not been affected by disruption during this time period.

The first resilience-based component importance measure, i.e. the optimal repair time T_{ij}^{opt} in Eq. (14), offers an explicit quantification of the priority that should be given to arc ij to be repaired and re-installed into the network. Lower values of T_{ij}^{opt} indicate higher priority, i.e. higher rank in the component repair checklist for system restoration in the event of system failure. The second resilience-based component importance measure, i.e. the resilience reduction worth $RRW_{ij}(\Delta t_0)$, quantifies the potential loss in optimal system resilience due to a delay Δt_0 in the repair time of link ij . This measure can provide valuable information to guide the recovery process of a particular component: components with high values of $RRW_{ij}(\Delta t_0)$ should be given high priority to their timely restoration, e.g. be assigned with adequate restoration resources.

Given the stochastic nature of disruptive events on an infrastructure network, a Monte Carlo-based method has been proposed to generate distributions of optimal repair time T_{ij}^{opt} and resilience reduction worth $RRW_{ij}(\Delta t_0)$ for all the components in the network; then, a stochastic ranking approach based on the Copeland's pairwise aggregation method has been applied to rank components importance.

The results of the two measures applied to the IEEE 30 Bus test network demonstrate some non-obvious and meaningful conclusions about the contributions of certain links to the resilience of the network. It is shown that two types of links are most important in terms of T_{ij}^{opt} : i) the links which connect generator nodes with the other two types of nodes (transmission nodes and demand nodes), e.g. links $\langle 2, 6 \rangle$, $\langle 1, 3 \rangle$, $\langle 12, 13 \rangle$ etc., and ii) the link which is the only arc connecting to demand nodes, i.e., link $\langle 25, 26 \rangle$. The restoration of these types of links is most likely able to augment the total amount of flow received by the demand nodes of the network so that high priority should be given to these links in the reparation list. Besides, those links with high Copeland scores in terms of T_{ij}^{opt} also have high Copeland scores ranking in terms of the resilience reduction worth RRW_{ij} : actually, the correlation coefficient between the two quantities is $r\left(C_{T_{ij}^{opt}}, C_{RRW_{ij}(3)}\right) = 0.82$.

It is noted that the differences of the resilience-based CIMs proposed in the present paper with respect to that in [35] mainly fall into the following two aspects: (i) the concept and definition of system resilience which the CIMs rely on is an non-memoryless one in this study by taking into account the cumulative restored system functionality; (ii) the focus of the proposed resilience-based CIMs is to quantify the effect

that the *recovery*, rather than the *disruption*, of individual components has on the global system resilience, thus, are valuable in suggesting the most effective way for system (recovery) operation.

Finally, it is shown that the classical betweenness centrality indices, such as the *shortest path betweenness*, *flow betweenness* and *random walk betweenness*, do not capture resilience criticality as do the resilience-based measures T_{ij}^{opt} and $RRW_{ij}(\Delta t_0)$. Actually, the existing centrality indices and the CIMs in the reliability engineering literature are not appropriate to help implement resilience planning because they do not take into account system recovery time. Instead, the two measures proposed in the present paper provide insights useful for practical restoration activities of infrastructure networks after suffering a disruptive event.

It is noted that in this study we consider a system suffering a specific type of events e (e.g., earthquakes of a certain magnitude) and rank the component importances by a stochastic ranking method. The results are valuable to help implement the system restoration if this type of event happens on the system. However, it might be possible that an infrastructure system is affected by different types of events concurrently (e.g., earthquake and tsunami). Future studies will concentrate on the application of the resilience-based component importance measures to different types of infrastructure networks subject to (possibly different types of correlated) disruptive events, in order to further demonstrate the practical effectiveness of the measures.

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