Filtering of Discrete-Time Switched Neural Networks Ensuring Exponential Dissipative and l_2–l_∞ Performances

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***Abstract*—This paper studies delay-dependent exponential dissipative and l_2–l_∞ filtering problems for discrete-time switched neural networks (DSNNs) including time-delayed states. By introducing a novel discrete-time inequality, which is a discrete-time version of the continuous-time Wirtinger-type inequality, we establish new sets of linear matrix inequality (LMI) criteria such that discrete-time filtering error systems are exponentially stable with guaranteed performances in the exponential dissipative and l_2–l_∞ senses. The design of the desired exponential dissipative and l_2–l_∞ filters for DSNNs can be achieved by solving the proposed sets of LMI conditions. Via numerical simulation results, we show the validity of the desired discrete-time filter design approach.**

***Index Terms*—l_2–l_∞ filtering, discrete Wirtinger-type inequality, discrete-time switched neural networks (DSNNs), dissipative filtering, exponential stability.**

I. Introduction

OVER the past years, neural networks (NNs) have attracted considerable attention since they are successfully applied in many fields, such as communication, computational optimization problems, pattern recognition, and signal processing [1]–[5]. The dynamic behavior of NNs has been successfully described in these applications. The stability criteria, state estimation, and control problems for NNs [6]–[11] have received significant attention in recent decades in the design of NNs for engineering applications. In practical applications, it is should be noted that time delay occurs frequently, and causes oscillation, divergence, and instability. Therefore, many researchers have been interested in studying time-delayed systems. In particular, with the successful introduction of the linear matrix inequality (LMI) approach, many issues on this topic have been addressed in the literature.

Hybrid systems which consist of continuous dynamics, discrete dynamics, and the interaction between them have been extensively used because of the rapid development of intelligent control. The purpose of researching hybrid systems originates from the fact that the hybrid control design gives a successful solution for highly nonlinear complex dynamical systems with uncertain parameters. Switched systems, which are composed of a family of dynamical subsystems, are classified as a significant branch of hybrid systems. Hence, many efforts have concentrated on the design and/or construction of switched systems [12]–[15]. Recently, switched systems have been integrated with NNs, because they can be used to model several practical applications. Actually, many complex nonlinear systems [16]–[18] are efficiently represented by switched NNs as a mathematical model. Therefore, some considerations in the stability analysis for switched NNs were studied in [16], [17], and [19]–[22].

On the other hand, the filtering problem is an important issue in control engineering due to the lack of sensors in real applications (see, for instance, [23]–[28]). In particular, practical considerations for utilizing relatively large-scale NNs in real-world applications include the problem of estimating neuron states. Often, neuron information is not completely accessible, and only a partial state variable of the network is obtained from the measurements of the network. Therefore, the estimation of neuron states has great importance for achieving certain objectives, and many studies have been conducted on the state estimation of NNs [29]–[31]. Recently, filter design techniques based on passivity [32]–[34], l_2–l_∞ [35], [36], and $\mathcal{H}_\infty$ [37]–[39] approaches for switched NNs were also proposed. Up to now, it should be noted that all the aforementioned studies have examined continuous-time NNs. In the case of discrete-time NNs, various stability and state estimation criteria have been introduced in [40]–[45], but relatively few studies have examined the exponential stability and state estimation problem [46]–[51]. A delay-dependent filter design focused on discrete-time switched NNs (DSNNs) was presented in [47]. Mathiyalagan *et al.* [48], Zhang *et al.* [49], and Zhang *et al.* [50] proposed exponential $\mathcal{H}_\infty$ filtering for DSNNs with delays. Although research on discrete-time filter design is in progress, due to its importance, approaches

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or results on discrete-time cases are lacking compared with continuous-time cases. As far as we are aware, the previous works do not completely cover the exponential dissipative and l_2–l_∞ filtering problems of DSNNs. Due to this consideration, the major objective of this paper is to study the problems of exponential dissipative and l_2–l_∞ filter design for DSNNs with time-delayed states. In this regard, we introduce new delay-dependent exponential dissipative and l_2–l_∞ filters for DSNNs with time delay. Based on a discrete-time Wirtinger-type inequality, which is a new summation inequality, and a discrete-time version of the continuous-time one, new sets of delay-dependent LMI conditions are established such that filtering error systems are exponentially stable with guaranteed performances in the exponential dissipative and l_2–l_∞ senses. By solving sets of the proposed LMIs, the desired filter gains for DSNNs with time delay can be obtained. It is demonstrated that the proposed filters are effective through numerical examples.

The contents of this paper are as follows. In Section II, we introduce the problem formulation. Sections III and IV, present the delay-dependent conditions of the proposed exponential dissipative and l_2–l_∞ filters, respectively, for DSNNs with time delay. Section V presents numerical simulations to validate the effectiveness of the developed filters, and conclusions are given in Section VI.

Notation: The superscript T is employed to represent matrix transposition. $P > 0$ means P is a positive definite matrix. $\varrho_{\max}(P)$ and $\varrho_{\min}(P)$ represent the maximum and minimum eigenvalues of P, respectively. An asterisk * is employed to represent matrix expressions to induce a symmetric structure. diag$\{\cdot\}$ indicates a block-diagonal matrix. It is assumed that matrices without precise definition of their dimensions are compatible for algebraic operation. $l_2[0, \mathcal{T}]$ is the space with the square-summable discrete-time vector sequences over $[0, \mathcal{T}]$ with the following norm: $\|x(k)\|_2 = \sqrt{\sum_{k=k_0}^{\mathcal{T}} x^{\mathrm{T}}(k)x(k)}$ for a vector $x(k)$.

II. Problem Formulation

As an effective approach of the hybrid system, DSNNs that consist of subsystems change their mode according to a switching rule and can be an efficient model for describing nonlinear dynamics. According to subsystems and the switching rule, the DSNN model is represented as

$$\begin{aligned} x(k+1) &= A_{\sigma(k)}x(k) + W_{\sigma(k)}\phi(x(k-d)) + J_{\sigma(k)}(k) \\ &\quad + G_{\sigma(k)}w(k) \qquad (1)\\ y(k) &= C_{\sigma(k)}x(k) + D_{\sigma(k)}x(k-d) + F_{\sigma(k)}w(k) \qquad (2)\\ z(k) &= Hx(k) \qquad (3)\end{aligned}$$

where $x(k) \in R^n$ denotes the discrete-time state; $y(k) \in R^m$ denotes the discrete-time output; $z(k) \in R^l$ denotes a discrete-time controlled output; $w(k) \in R^p$ denotes the discrete-time disturbance, which belongs to $l_2[0, \mathcal{T}]$; $d \geq 0$ denotes the discrete-time time delay; $\phi(x(k)) = [\phi_1(x(k)), \ldots, \phi_n(x(k))]^{\mathrm{T}} \in R^n$ denotes the discrete-time nonlinear vector function; $A = \mathrm{diag}\{a_1, \ldots, a_n\} \in R^{n\times n}$ ($|a_i| < 1, k = 1, \ldots, n$) denotes the feedback matrix; $W \in R^{n\times n}$ denotes the weight matrix; $G \in R^{n\times p}$, $C \in R^{m\times n}$, $D \in R^{m\times n}$, $F \in R^{m\times p}$, and $H \in R^{l\times n}$ are given matrices; $J(k) \in R^n$ denotes an exogenous input; and $\sigma(k)$ denotes the switching signal, which can be changed within the finite set $\mathbb{N} = \{1, 2, \ldots, N\}$. Let us assume that the switching signal, which may become autonomous, state- or time-dependent, or controlled, is not known previously, but its instant value can be obtained in real-time application. The holding time between $[k_t, k_{t+1}]$ is said to be the dwell time of the presently considered discrete-time subsystem for the switching time sequences $k_0 < k_1 < k_2 < \cdots$ of switching signal σ, where t is a non-negative integer. $\phi(\cdot)$ is assumed to satisfy the following assumption.

Assumption 1 [52]: For given constant scalars λ_i^-, λ_i^+, the neuron activation function holds the following condition such that:

$$\lambda_i^- \leqslant \frac{\phi_i(s_1) - \phi_i(s_2)}{s_1 - s_2} \leqslant \lambda_i^+ \qquad \forall s_1, s_2,\ s_1 \neq s_2 \qquad (4)$$

for $i = 1, 2, \ldots, n$. Hereafter, the notation of the constant bound matrices of the activation function is represented as

$$\begin{aligned} \Lambda_1 &\triangleq \mathrm{diag}\big(\lambda_1^-\lambda_1^+, \lambda_2^-\lambda_2^+, \ldots, \lambda_n^-\lambda_n^+\big) \\ \Lambda_2 &\triangleq \mathrm{diag}\left(\frac{\lambda_1^- + \lambda_1^+}{2} \frac{\lambda_2^- + \lambda_2^+}{2}, \ldots, \frac{\lambda_n^- + \lambda_n^+}{2}\right). \qquad (5)\end{aligned}$$

Remark 1: The constants λ_i^- and λ_i^+ in Assumption 1 can be negative, zero, or positive. This implies that the activation functions are nonmonotonic. Recently, they have been commonly considered Lipschitz conditions. For less conservative results, this assumption is introduced.

Remark 2: It is commonly accepted that the activation functions are continuous, differential, bounded, and monotonically increasing, such as a sigmoid-type function. This type denotes the class of global Lipschitz activation functions. On the other hand, Assumption 1 represents the nonmonotonic activation function, because the constants λ_i^- and λ_i^+ are assumed to be negative, zero, or positive. These nonmonotonic functions are more general ones [52] than the class of global Lipschitz functions and can be more suitable for describing the activation function of the neuron in designing and implementing an NN.

The ith switched subsystem model is expressed by

$$\begin{aligned} x(k+1) &= A_i x(k) + W_i\phi(x(k-d)) + J_i(k) \\ &\quad + G_i w(k) \qquad (6)\\ y(k) &= C_i x(k) + D_i x(k-d) + F_i w(k) \qquad (7)\\ z(k) &= Hx(k). \qquad (8)\end{aligned}$$

Our main interest is estimating the state vector. Thus, the filter for DSNNs (6)–(8) is designed by

$$\begin{aligned} \hat{x}(k+1) &= A_i\hat{x}(k) + W_i\phi\big(\hat{x}(k-d)\big) + J_i(k) \\ &\quad + L_i(y(k) - \hat{y}(k)) \qquad (9)\\ \hat{y}(k) &= C_i\hat{x}(k) + D_i\hat{x}(k-d) \qquad (10)\\ \hat{z}(k) &= H\hat{x}(k) \qquad (11)\end{aligned}$$

where $\hat{x}(k) \in R^n$ is the discrete-time state signal of the estimator and $\hat{y}(k) \in R^m$ is the discrete-time output signal of the estimator. $L_i \in R^{n\times m}$ is the filter gain, and our main objective is to obtain a suitable filter gain matrix L_i. It is assumed that the filter and system (1)–(3) share the same switching signal $\sigma(k)$ which is said to be mode-dependent. Let the filtering errors be denoted as $e(k) = x(k) - \hat{x}(k)$ and $\tilde{z}(k) = z(k) - \hat{z}(k)$. By combining the state estimator (9)–(11) and the switched NN (6)–(8), the discrete-time filtering error system is obtained as follows:

$$\begin{aligned} e(k+1) &= (A_i - L_iC_i)e(k) - L_iD_ie(k-d) \\ &\quad + W_i\tilde{\phi}(x(k-d)) + (G_i - L_iF_i)w(k) \qquad (12)\\ \tilde{z}(k) &= He(k) \qquad (13)\end{aligned}$$

where $\tilde{\phi}(x(k-d)) = \phi(x(k-d)) - \phi(\hat{x}(k-d))$.

Definition 1: Under a known switching signal $\sigma(k)$, the exponential stability result of the discrete-time filtering error system (12) is ascertained if there are some scalars $\delta > 0$ and $0 < \chi < 1$ satisfying

$$\|e(k)\|^2 < \delta\chi^{k-k_0}\|e(k_0)\|_L^2 \qquad k \geq k_0 \qquad (14)$$

where χ is the decay rate, $\|e(k_0)\|_L = \sup_{k_0-d\leqslant\theta\leqslant k_0}\|e(\theta)\|$, and k_0 is an initial time.

Definition 2 [47], [49]: Under a known switching signal $\sigma(k)$, $k \geq k_0$, and $k_0 \leqslant \tau \leqslant k$, let define $N_\sigma(k_0, k)$ as the number of times to switch within $[k_0, k]$. If there are $T_a > 0$ and $N_0 \geq 0$ such that $N_\sigma(k_0, k) \leqslant N_0 + (k - k_0)/T_a$ is satisfied, then N_0 and T_a are called the chatter bound and the average dwell time, respectively. As it is generally used, we select $N_0 = 0$.

Definition 3: Let the energy supply function be defined as $E(e(k), w(k)) = e^{\mathrm{T}}(k)\mathcal{Q}e(k) + 2e^{\mathrm{T}}(k)\mathcal{S}w(k) + w^{\mathrm{T}}(k)\mathcal{R}w(k)$. Given scalars $\alpha > 0$ and $0 < \rho < 1$, the filtering error system (12) is said to be strictly $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$ exponentially dissipative, under the zero initial condition, if the following condition is satisfied for every nonzero $w(k) \in l_2[0, \mathcal{T}]$:

$$\sum_{s=k_0}^{\mathcal{T}} E(e(s), w(s)) \geq \alpha\sum_{s=k_0}^{\mathcal{T}}(1-\rho)^s w^{\mathrm{T}}(s)w(s) \qquad (15)$$

where k_0 is an initial time and $\mathcal{Q}$, $\mathcal{S}$, and $\mathcal{R}$ are real matrices with symmetric $\mathcal{Q}$ and $\mathcal{R}$. For convenience, assume $\mathcal{Q} \leqslant 0$ and $-\mathcal{Q} = \mathcal{Q}_-^{\mathrm{T}}\mathcal{Q}_-$ for some $\mathcal{Q}_-$.

Definition 4: Under a known scalars $\gamma > 0$ and $0 < \rho < 1$, the filtering error system (12) is said to have a guaranteed exponential l_2–l_∞ index γ if the following condition is satisfied for every nonzero $w(k) \in l_2[0, \mathcal{T}]$ under a zero initial condition:

$$\sup_{k\in[k_0,\mathcal{T}]}(1-\rho)^k\tilde{z}^{\mathrm{T}}(k)\tilde{z}(k) \leqslant \gamma^2\sum_{s=k_0}^{\mathcal{T}} w^{\mathrm{T}}(s)w(s) \qquad (16)$$

where k_0 is an initial time.

The major focus of this paper is finding appropriate filter gain such that the exponential stability result of the discrete-time filtering error system (12) is ensured for $w(k) = 0$ with the performance indices given in Definitions 3 and 4 for $w(k) \neq 0$.

Before moving on, to propose a new summation inequality called the discrete form of the vector Wirtinger-type inequality, the following lemma is required.

Lemma 1 [53]: For some symmetric constant matrix $S \geq 0$, scalar $d > 0$, and a vector $\eta(k) = e(k+1) - e(k)$, we obtain

$$-\sum_{s=k-d}^{k-1}\eta^{\mathrm{T}}(s)S\eta(s) \leqslant -\frac{1}{d}\sum_{s=k-d}^{k-1}\eta^{\mathrm{T}}(s)S\sum_{s=k-d}^{k-1}\eta(s). \qquad (17)$$

Using Lemma 1, we present a discrete form of the vector Wirtinger-type inequality as follows.

Lemma 2 (Discrete Form of Vector Wirtinger-Type Inequality): Let $\eta(k) = e(k+1) - e(k) \in R^n$ be the discrete derived function on the interval $[a, b]$. Then, for any $n \times n$ matrix $R > 0$, scalar $d > 0$, the following relation is satisfied:

$$\begin{aligned}-\sum_{s=k-d}^{k-1}\eta^{\mathrm{T}}(s)R\eta(s) &\leqslant -\frac{2}{d^3}\left(\sum_{s=k-d}^{k-1}e(s)\right)^{\mathrm{T}}R\left(\sum_{s=k-d}^{k-1}e(s)\right)\\ &\quad -\frac{2}{d}e^{\mathrm{T}}(k-d)Re(k-d)\\ &\quad +\frac{4}{d^2}\left(\sum_{s=k-d}^{k-1}e(s)\right)^{\mathrm{T}}Re(k-d). \qquad (18)\end{aligned}$$

Proof: It is assumed that $2 < d < k$ holds, and the summation is only allowed to increase the index of summation. The result of the summations with regard to the decreasing direction is zero [e.g., $\sum_{s=a}^{b}\eta(s) \triangleq 0$ if $a > b$]. From Lemma 1, one can obtain

$$\begin{aligned}&\frac{1}{d}\left(\sum_{s=k-d}^{k-1}[e(s)-e(k-d)]\right)^{\mathrm{T}}R\left(\sum_{s=k-d}^{k-1}[e(s)-e(k-d)]\right)\\ &\leqslant \sum_{s=k-d}^{k-1}\left([e(s)-e(k-d)]^{\mathrm{T}}R[e(s)-e(k-d)]\right)\\ &= \sum_{s=k-d}^{k-1}\left(\left[\sum_{j=k-d}^{s-1}\eta(j)\right]^{\mathrm{T}}R\left[\sum_{j=k-d}^{s-1}\eta(j)\right]\right)\\ &\leqslant \sum_{s=k-d}^{k-1}\sum_{j=k-d}^{s-1}(s-k+d)\eta^{\mathrm{T}}(j)R\eta(j)\\ &= \sum_{j=k-d}^{k-2}\sum_{s=j+1}^{k-1}(s-k+d)\eta^{\mathrm{T}}(j)R\eta(j)\\ &= \sum_{j=k-d}^{k-2}\left(\frac{d^2}{2}-\frac{(j-k+d)^2}{2}-\frac{j-k+2d}{2}\right)\eta^{\mathrm{T}}(j)R\eta(j)\\ &\leqslant \frac{d^2}{2}\sum_{j=k-d}^{k-2}\eta^{\mathrm{T}}(j)R\eta(j)\\ &\leqslant \frac{d^2}{2}\sum_{s=k-d}^{k-1}\eta^{\mathrm{T}}(s)R\eta(s).\end{aligned}$$

Note that

$$\left(\sum_{s=k-d}^{k-1}[e(s)-e(k-d)]\right)^{\mathrm{T}} R\left(\sum_{s=k-d}^{k-1}[e(s)-e(k-d)]\right)$$
$$=\left(\sum_{s=k-d}^{k-1}e(s)\right)^{\mathrm{T}} R\left(\sum_{s=k-d}^{k-1}e(s)\right)+d^2e^{\mathrm{T}}(k-d)Re(k-d)$$
$$-\ 2d\left(\sum_{s=k-d}^{k-1}e(s)\right)^{\mathrm{T}} Re(k-d).$$

Thus, we have the discrete form of the vector Wirtinger-type inequality (18). This completes the proof. ■

III. Strictly $(\mathcal{Q},\mathcal{S},\mathcal{R})$ Exponentially Dissipative Filter Design for DSNNs With Time Delay

This section establishes a new set of sufficient conditions for a discrete-time strictly $(\mathcal{Q},\mathcal{S},\mathcal{R})$ exponentially dissipative DSNN filter design scheme (6)–(8). The designed filter gain matrix guarantees that the filtering error system (12) satisfies the performance index presented in Definition 3.

Theorem 1: Based on Assumption 1, for prescribed scalars $d>0$, $\sigma>0$, $\alpha>0$, $\mu\geq 1$, and $0<\rho<1$, if there are matrices $P_i=P_i^{\mathrm{T}}>0$, $Q_i=Q_i^{\mathrm{T}}>0$, $R_i=R_i^{\mathrm{T}}>0$, $S_i=S_i^{\mathrm{T}}>0$, and $N_i=[N_{1i}^{\mathrm{T}}\ N_{2i}^{\mathrm{T}}\ N_{3i}^{\mathrm{T}}\ N_{4i}^{\mathrm{T}}\ N_{5i}^{\mathrm{T}}\ N_{6i}^{\mathrm{T}}]^{\mathrm{T}}$; diagonal matrix $\Omega>0$; and M_i for certain switching signal $\sigma(k)$ satisfying the average dwell time with $T_a\geq T_a^*=-\ln\mu/\ln(1-\rho)$, such that

$$\begin{bmatrix}\Xi_{D_i} & \sqrt{d(1-\rho)^d}N_i & \Upsilon_i^{\mathrm{T}} & \sqrt{d}\Psi_i^{\mathrm{T}}\\ * & -S_i & 0 & 0\\ * & * & -P_i & 0\\ * & * & * & -2\sigma P_i+\sigma^2S_i\end{bmatrix}<0 \tag{19}$$

$$P_i\leqslant\mu P_j,\quad Q_i\leqslant\mu Q_j,\quad R_i\leqslant\mu R_j,\quad S_i\leqslant\mu S_j \tag{20}$$

for all $i,j\in\mathbb{N}$, $i\neq j$, where

$$\Xi_{D_i}=\begin{bmatrix}\Xi_{11_i} & \Xi_{12_i} & \Xi_{13_i} & \Xi_{14_i} & \Xi_{15_i} & \Xi_{16_i}\\ * & \Xi_{22_i} & \Xi_{23_i} & \Xi_{24_i} & \Xi_{25_i} & \Xi_{26_i}\\ * & * & \Xi_{33_i} & \Xi_{34_i} & 0 & 0\\ * & * & * & \Xi_{44_i} & \Xi_{45_i} & \Xi_{46_i}\\ * & * & * & * & \Xi_{55_i} & 0\\ * & * & * & * & * & -[\mathcal{R}-\alpha I]\end{bmatrix}$$

$$\begin{aligned}
\Xi_{11_i}&=-(1-\rho)P_i+Q_i+(1-\rho)^d\big(N_{1i}+N_{1i}^{\mathrm{T}}\big)\\
&\qquad+dR_i-\mathcal{Q}\\
\Xi_{12_i}&=(1-\rho)^d(N_{2i}^{\mathrm{T}}-N_{1i})\\
\Xi_{13_i}&=(1-\rho)^dN_{3i}^{\mathrm{T}},\ \ \Xi_{14_i}=(1-\rho)^d\big(N_{4i}^{\mathrm{T}}-N_{1i}\big)\\
\Xi_{15_i}&=(1-\rho)^dN_{5i}^{\mathrm{T}},\ \ \Xi_{16_i}=(1-\rho)^dN_{6i}^{\mathrm{T}}-\mathcal{S}\\
\Xi_{22_i}&=-(1-\rho)^dQ_i-\Lambda_1\Omega-\frac{2(1-\rho)^d}{d}S_i\\
&\qquad-(1-\rho)^d\big(N_{2i}+N_{2i}^{\mathrm{T}}\big)\\
\Xi_{23_i}&=\frac{2(1-\rho)^d}{d^2}S_i-(1-\rho)^dN_{3i}^{\mathrm{T}}\\
\Xi_{24_i}&=-(1-\rho)^d\big(N_{4i}^{\mathrm{T}}+N_{2i}\big)\\
\Xi_{25_i}&=\Lambda_2\Omega-(1-\rho)^dN_{5i}^{\mathrm{T}},\ \ \Xi_{26_i}=-(1-\rho)^dN_{6i}^{\mathrm{T}}\\
\Xi_{33_i}&=-\frac{2(1-\rho)^d}{d^3}S_i-\frac{(1-\rho)^d}{d}R_i\\
\Xi_{34_i}&=-(1-\rho)^dN_{3i},\ \ \Xi_{44_i}=-(1-\rho)^d\big(N_{4i}+N_{4i}^{\mathrm{T}}\big)\\
\Xi_{45_i}&=-(1-\rho)^dN_{5i}^{\mathrm{T}},\ \ \Xi_{46_i}=-(1-\rho)^dN_{6i}^{\mathrm{T}}\\
\Xi_{55_i}&=-\Omega\\
\Upsilon_i&=[(P_iA_i-M_iC_i)\ \ -M_iD_i\ \ 0\ \ P_iW_i\\
&\qquad(P_iG_i-M_iF_i)]\\
\Psi_i&=[(P_iA_i-M_iC_i-P_i)\ \ -M_iD_i\ \ 0\ \ P_iW_i\\
&\qquad(P_iG_i-M_iF_i)]
\end{aligned}\tag{21}$$

then the exponential stability result of the discrete-time filtering error system is ensured for $w(k)=0$ and the strictly $(\mathcal{Q},\mathcal{S},\mathcal{R})$ exponential dissipativity of the filtering error system is ensured for all nonzero $w(k)$. Then, the desired filter gain matrix in (9) is calculated by $L_i=P_i^{-1}M_i$ for $i\in\mathbb{N}$.

Proof: Choose the piecewise Lyapunov–Krasovskii functional

$$V_i(k)=\sum_{s=1}^{4}V_{si}(k) \tag{22}$$

where

$$\begin{aligned}
V_{1i}(k)&=e^{\mathrm{T}}(k)P_ie(k)\\
V_{2i}(k)&=\sum_{s=k-d}^{k-1}e^{\mathrm{T}}(s)(1-\rho)^{k-s-1}Q_ie(s)\\
V_{3i}(k)&=\sum_{j=-d}^{-1}\sum_{s=k+j}^{k-1}e^{\mathrm{T}}(s)(1-\rho)^{k-s-1}R_ie(s)\\
V_{4i}(k)&=\sum_{j=-d}^{-1}\sum_{s=k+j}^{k-1}\eta^{\mathrm{T}}(s)(1-\rho)^{k-s-1}S_i\eta(s)
\end{aligned}$$

where $\eta(s)=e(s+1)-e(s)$. Let the exponential forward differences of $V_i(k)$ and $V_{si}(k)$ be defined as $\Delta V_i(k)=V_i(k+1)-V_i(k)$ and $\Delta V_{si}(k)=V_{si}(k+1)-V_{si}(k)$ for $s=1,\ldots,3$ and $i\in\mathbb{N}$, respectively; then we obtain

$$\begin{aligned}
\Delta V_{1i}(k)+\rho V_{1i}(k)&=e^{\mathrm{T}}(k+1)P_ie(k+1)\\
&\quad-(1-\rho)e^{\mathrm{T}}(k)P_ie(k)
\end{aligned}\tag{23}$$

$$\begin{aligned}
\Delta V_{2i}(k)+\rho V_{2i}(k)&=e^{\mathrm{T}}(k)Q_ie(k)\\
&\quad-(1-\rho)^de^{\mathrm{T}}(k-d)Q_ie(k-d)
\end{aligned}\tag{24}$$

$$\begin{aligned}
\Delta V_{3i}(k)+\rho V_{3i}(k)&\leqslant de^{\mathrm{T}}(k)R_ie(k)\\
&\quad-(1-\rho)^d\sum_{s=k-d}^{k-1}e^{\mathrm{T}}(s)R_ie(s)\\
&\leqslant de^{\mathrm{T}}(k)R_ie(k)\\
&\quad-\frac{(1-\rho)^d}{d}\sum_{s=k-d}^{k-1}e^{\mathrm{T}}(s)R_i\sum_{s=k-d}^{k-1}e(s)
\end{aligned}\tag{25}$$

$$\Delta V_{4i}(k)+\rho V_{4i}(k)\leqslant d\eta^{\mathrm{T}}(k)S_i\eta(k) - (1-\rho)^d\sum_{s=k-d}^{k-1}\eta^{\mathrm{T}}(s)S_i\eta(s). \tag{26}$$

Based on Assumption 1, it is apparent that

$$\begin{bmatrix} e(k-d)\\ \tilde{\phi}(x(k-d))\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\lambda_j^-\lambda_j^+e_je_j^{\mathrm{T}} & -\frac{\lambda_j^-+\lambda_j^+}{2}e_je_j^{\mathrm{T}}\\ -\frac{\lambda_j^-+\lambda_j^+}{2}e_je_j^{\mathrm{T}} & e_je_j^{\mathrm{T}}\end{bmatrix} \times\begin{bmatrix} e(k-d)\\ \tilde{\phi}(x(k-d))\end{bmatrix}\leqslant 0,\qquad j=1,\ldots,n$$

where e_j is the unit column vector, which has a "1" element on its jth row and zeros elsewhere. Let $\Omega=\mathrm{diag}\{\omega_1,\omega_2,\ldots,\omega_n\}$; then

$$\sum_{j=1}^{n}\omega_j\begin{bmatrix} e(k-d)\\ \tilde{\phi}(x(k-d))\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\lambda_j^-\lambda_j^+e_je_j^{\mathrm{T}} & -\dfrac{\lambda_j^-+\lambda_j^+}{2}e_je_j^{\mathrm{T}}\\ -\dfrac{\lambda_j^-+\lambda_j^+}{2}e_je_j^{\mathrm{T}} & e_je_j^{\mathrm{T}}\end{bmatrix} \times\begin{bmatrix} e(k-d)\\ \tilde{\phi}(x(k-d))\end{bmatrix}\leqslant 0$$

or

$$\begin{bmatrix} e(k-d)\\ \tilde{\phi}(x(k-d))\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\Lambda_1\Omega & -\Lambda_2\Omega\\ -\Lambda_2\Omega & \Omega\end{bmatrix}\begin{bmatrix} e(k-d)\\ \tilde{\phi}(x(k-d))\end{bmatrix}\leqslant 0$$

which is rewritten as

$$e^{\mathrm{T}}(k-d)\Lambda_1\Omega e(k-d)-2e^{\mathrm{T}}(k-d)\Lambda_2\Omega\tilde{\phi}(x(k-d)) + \tilde{\phi}^{\mathrm{T}}(x(k-d))\Omega\tilde{\phi}(x(k-d))\leqslant 0. \tag{27}$$

By combining (23)–(27), we obtain

$$\begin{aligned}
&\Delta V_i(k)+\rho V_i(k)\\
&\quad\leqslant e^{\mathrm{T}}(k+1)P_ie(k+1)-(1-\rho)e^{\mathrm{T}}(k)P_ie(k)\\
&\qquad+e^{\mathrm{T}}(k)Q_ie(k)-(1-\rho)^de^{\mathrm{T}}(k-d)Q_ie(k-d)\\
&\qquad+de^{\mathrm{T}}(k)R_ie(k)-\frac{(1-\rho)^d}{d}\sum_{s=k-d}^{k-1}e(s)^{\mathrm{T}}R_i\sum_{s=k-d}^{k-1}e(s)\\
&\qquad+d\eta^{\mathrm{T}}(k)S_i\eta(k)-(1-\rho)^d\sum_{s=k-d}^{k-1}\eta^{\mathrm{T}}(s)S_i\eta(s)\\
&\qquad-e^{\mathrm{T}}(k-d)\Lambda_1\Omega e(k-d)\\
&\qquad+2e^{\mathrm{T}}(k-d)\Lambda_2\Omega\tilde{\phi}(x(k-d))\\
&\qquad-\tilde{\phi}^{\mathrm{T}}(x(k-d))\Omega\tilde{\phi}(x(k-d)).
\end{aligned}\tag{28}$$

According to the free-weighting matrix technique [54], the following expression is possible for any appropriately dimensional matrix N_i:

$$2(1-\rho)^d\xi^{\mathrm{T}}(k)N_i\left[e(k)-e(k-d)-\sum_{s=k-d}^{k-1}\eta(s)\right]=0 \tag{29}$$

where

$$\xi(k)=\left[e^{\mathrm{T}}(k),\ e^{\mathrm{T}}(k-d),\ \sum_{s=k-d}^{k-1}e^{\mathrm{T}}(s) \quad \sum_{s=k-d}^{k-1}\eta^{\mathrm{T}}(s),\ \tilde{\phi}^{\mathrm{T}}(x(k-d)),\ w^{\mathrm{T}}(k)\right]^{\mathrm{T}}. \tag{30}$$

If we add the terms on the left-hand side of (29) and $\Gamma(k)=\alpha w^{\mathrm{T}}(k)w(k)-E(e(k),w(k))$ to the upper bound of $\Delta V_i(k)+\rho V_i(k)$ in (28) and apply the change of variable such that $M_i=P_iL_i$, we can obtain

$$\begin{aligned}
&\Delta V_i(k)+\rho V_i(k)+\Gamma(k)\\
&\quad\leqslant\xi^{\mathrm{T}}(k)\Big\{\Phi_i+d(1-\rho)^dN_iS_i^{-1}N_i^{\mathrm{T}}\\
&\qquad\quad+d\Psi_i^{\mathrm{T}}P_i^{-1}S_iP_i^{-1}\Psi_i+\Upsilon_i^{\mathrm{T}}P_i^{-1}\Upsilon_i\Big\}\xi(k)\\
&\qquad\quad-(1-\rho)^d\sum_{s=k-d}^{k-1}\left[\xi^{\mathrm{T}}(k)N_i+\eta^{\mathrm{T}}(s)S_i\right]S_i^{-1}\\
&\qquad\quad\times\left[N_i^{\mathrm{T}}\xi(k)+S_i\eta(s)\right]\\
&\quad=\xi^{\mathrm{T}}(k)\Big\{\Phi_i+d(1-\rho)^dN_iS_i^{-1}N_i^{\mathrm{T}}\\
&\qquad\quad+d\Psi_i^{\mathrm{T}}P_i^{-1}S_iP_i^{-1}\Psi_i+\Upsilon_i^{\mathrm{T}}P_i^{-1}\Upsilon_i\Big\}\xi(k)\\
&\qquad\quad-d(1-\rho)^d\xi^{\mathrm{T}}(k)N_iS_i^{-1}N_i^{\mathrm{T}}\xi(k)\\
&\qquad\quad-2(1-\rho)^d\xi^{\mathrm{T}}(k)N_i\sum_{s=k-d}^{k-1}\eta(s)\\
&\qquad\quad-(1-\rho)^d\sum_{s=k-d}^{k-1}\eta^{\mathrm{T}}(s)S_i\eta(s)
\end{aligned}\tag{31}$$

where

$$\Phi_i=\begin{bmatrix}
\Phi_{11_i} & \Phi_{12_i} & \Phi_{13_i} & \Phi_{14_i} & \Phi_{15_i} & \Phi_{16_i}\\
* & \Phi_{22_i} & \Phi_{23_i} & \Phi_{24_i} & \Phi_{25_i} & \Phi_{26_i}\\
* & * & \Phi_{33_i} & 0 & 0 & 0\\
* & * & * & 0 & 0 & 0\\
* & * & * & * & \Phi_{55_i} & 0\\
* & * & * & * & * & -[\mathcal{R}-\alpha I]
\end{bmatrix}$$

$$\begin{aligned}
\Phi_{11_i}&=-(1-\rho)P_i+Q_i+(1-\rho)^d\big(N_{1i}+N_{1i}^{\mathrm{T}}\big)+dR_i-\mathcal{Q}\\
\Phi_{12_i}&=(1-\rho)^d\big(N_{2i}^{\mathrm{T}}-N_{1i}\big)\\
\Phi_{13_i}&=(1-\rho)^dN_{3i}^{\mathrm{T}},\quad \Phi_{14_i}=(1-\rho)^dN_{4i}^{\mathrm{T}}\\
\Phi_{15_i}&=(1-\rho)^dN_{5i}^{\mathrm{T}},\quad \Phi_{16_i}=(1-\rho)^dN_{6i}^{\mathrm{T}}-\mathcal{S}\\
\Phi_{22_i}&=-(1-\rho)^dQ_i-\Lambda_1\Omega-(1-\rho)^d\big(N_{2i}+N_{2i}^{\mathrm{T}}\big)\\
\Phi_{23_i}&=-(1-\rho)^dN_{3i}^{\mathrm{T}},\quad \Phi_{24_i}=-(1-\rho)^dN_{4i}^{\mathrm{T}}\\
\Phi_{25_i}&=\Lambda_2\Omega-(1-\rho)^dN_{5i}^{\mathrm{T}},\quad \Phi_{26_i}=-(1-\rho)^dN_{6i}^{\mathrm{T}}\\
\Phi_{33_i}&=-\frac{(1-\rho)^d}{d}R_i,\quad \Phi_{55_i}=-\Omega.
\end{aligned}\tag{32}$$

From Lemma 2, we can obtain

$$-(1-\rho)^d\sum_{s=k-d}^{k-1}\eta^{\mathrm{T}}(s)S_i\eta(s) \leqslant-\frac{2(1-\rho)^d}{d^3}\left(\sum_{s=k-d}^{k-1}e(s)\right)^{\mathrm{T}}S_i\left(\sum_{s=k-d}^{k-1}e(s)\right)$$

$$
\begin{aligned}
&- \frac{2(1-\rho)^d}{d} e^{\mathrm{T}}(k-d) S_i e(k-d) \\
&+ \frac{4(1-\rho)^d}{d^2} \left(\sum_{s=k-d}^{k-1} e(s) \right)^{\mathrm{T}} S_i e(k-d)
\end{aligned}
$$

which applies to (31). Then, we have

$$
\begin{aligned}
&\Delta V_i(k) + \rho V_i(k) + \Gamma(k) \\
&\quad \leqslant \xi^{\mathrm{T}}(k) \Big\{ \Xi_{D_i} + d(1-\rho)^d N_i S_i^{-1} N_i^{\mathrm{T}} \\
&\qquad + d\Psi_i^{\mathrm{T}} P_i^{-1} S_i P_i^{-1} \Psi_i + \Upsilon_i^{\mathrm{T}} P_i^{-1} \Upsilon_i \Big\} \xi(k) \\
&\qquad - d(1-\rho)^d \xi^{\mathrm{T}}(k) N_i S_i^{-1} N_i^{\mathrm{T}} \xi(k)
\end{aligned}
$$

where Ξ_{D_i} is defined in (21). Since $S_i > 0$, $-d(1-\rho)^d \xi^{\mathrm{T}}(k) N_i S_i^{-1} N_i^{\mathrm{T}} \xi(k) \leqslant 0$, if the inequality

$$
\begin{aligned}
\Xi_{D_i} + d(1-\rho)^d N_i S_i^{-1} N_i^{\mathrm{T}} + d\Psi_i^{\mathrm{T}} P_i^{-1} S_i P_i^{-1} \Psi_i \\
+ \Upsilon_i^{\mathrm{T}} P_i^{-1} \Upsilon_i < 0
\end{aligned} \tag{33}
$$

is satisfied for $i = 1, \ldots, N$, we obtain

$$
\Delta V_i(k) + \rho V_i(k) + \Gamma(k) \leqslant 0. \tag{34}
$$

According to the Schur complement, the inequality (33) changes to $\Pi_i < 0$ for $i = 1, \ldots N$, where

$$
\Pi_i = \begin{bmatrix} \Xi_{D_i} & \sqrt{d(1-\rho)^d} N_i & \Upsilon_i^{\mathrm{T}} & \sqrt{d}\Psi_i^{\mathrm{T}} \\ * & -S_i & 0 & 0 \\ * & * & -P_i & 0 \\ * & * & * & -P_i S_i^{-1} P_i \end{bmatrix} \tag{35}
$$

for $i = 1, \ldots, N$. Note that (35) is not an LMI condition due to the term $-P_i S_i^{-1} P_i$. Given the inequality $P_i S_i^{-1} P_i \geq 2\eta P_i - \eta^2 S_i$ resulting from

$$
(P_i - \eta S_i)^{\mathrm{T}} S_i^{-1} (P_i - \eta S_i) = P_i S_i^{-1} P_i - 2\eta P_i + \eta^2 S_i \geq 0
$$

the LMI condition (19) implies (35). The filter gain matrix is then calculated by $L_i = P_i^{-1} M_i$. Based on (34), it is easy to obtain

$$
V_i(k) \leqslant (1-\rho)^{k-k_0} V_i(k_0) - \sum_{s=k_0}^{k-1} (1-\rho)^{k-s-1} \Gamma(s). \tag{36}
$$

From (20) and (36), one can obtain

$$
\begin{aligned}
V_{\sigma(k)}(k) &\leqslant (1-\rho)^{k-k_t} V_{\sigma(k_t)}(k_t) - \sum_{s=k_t}^{k-1} (1-\rho)^{k-s-1} \Gamma(s) \\
&\leqslant (1-\rho)^{k-k_t} \mu V_{\sigma(k_{t-1})}(k_t) - \sum_{s=k_t}^{k-1} (1-\rho)^{k-s-1} \Gamma(s) \\
&\leqslant (1-\rho)^{k-k_t} \mu \Bigg[(1-\rho)^{k_t - k_{t-1}} V_{\sigma(k_{t-1})}(k_{t-1}) \\
&\qquad - \sum_{s=k_{t-1}}^{k_t - 1} (1-\rho)^{k_t - s - 1} \Gamma(s) \Bigg] \\
&\quad - \sum_{s=k_t}^{k-1} (1-\rho)^{k-s-1} \Gamma(s) \\
&\leqslant \cdots \leqslant (1-\rho)^{k-k_0} \mu^{N_\sigma(k_0,k)} V_{\sigma(k_0)}(k_0) \\
&\quad - (1-\rho)^{k-k_1} \mu^{N_\sigma(k_0,k)} \sum_{s=k_0}^{k_1-1} (1-\rho)^{k_1-s-1} \Gamma(s) \\
&\quad - (1-\rho)^{k-k_2} \mu^{N_\sigma(k_1,k)} \sum_{s=k_1}^{k_2-1} (1-\rho)^{k_2-s-1} \Gamma(s) \\
&\quad \cdots - \sum_{s=k_t}^{k-1} (1-\rho)^{k-s-1} \Gamma(s) \\
&= (1-\rho)^{k-k_0} \mu^{N_\sigma(k_0,k)} V_{\sigma(k_0)}(k_0) \\
&\quad - \sum_{s=k_0}^{k-1} \mu^{N_\sigma(s,k)} (1-\rho)^{k-s-1} \Gamma(s).
\end{aligned} \tag{37}
$$

Under a zero initial condition, (37) becomes

$$
\sum_{s=k_0}^{k-1} \mu^{N_\sigma(s,k)} (1-\rho)^{k-s-1} \Gamma(s) \leqslant 0. \tag{38}
$$

Substituting $\Gamma(s)$ into (38) yields

$$
\begin{aligned}
&\alpha \sum_{s=k_0}^{k-1} \mu^{N_\sigma(s,k)} (1-\rho)^{k-s-1} w^{\mathrm{T}}(s) w(s) \\
&\qquad \leqslant \sum_{s=k_0}^{k-1} \mu^{N_\sigma(s,k)} (1-\rho)^{k-s-1} E(e(s), w(s)).
\end{aligned} \tag{39}
$$

Multiplying both sides of (39) by $\mu^{-N_\sigma(0,k)}$ yields

$$
\begin{aligned}
&\alpha \sum_{s=k_0}^{k-1} \mu^{-N_\sigma(0,s)} (1-\rho)^{k-s-1} w^{\mathrm{T}}(s) w(s) \\
&\qquad \leqslant \sum_{s=k_0}^{k-1} \mu^{-N_\sigma(0,s)} (1-\rho)^{k-s-1} E(e(s), w(s)).
\end{aligned} \tag{40}
$$

Then, due to the fact that

$$
N_\sigma(0, s) \leqslant \frac{s}{T_a} \leqslant \frac{-s \ln(1-\rho)}{\ln \mu} \tag{41}
$$

we have

$$
\begin{aligned}
&\alpha \sum_{s=k_0}^{k-1} \mu^{\frac{s \ln(1-\rho)}{\ln \mu}} (1-\rho)^{k-s-1} w^{\mathrm{T}}(s) w(s) \\
&\qquad \leqslant \sum_{s=k_0}^{k-1} (1-\rho)^{k-s-1} E(e(s), w(s)).
\end{aligned} \tag{42}
$$

Therefore

$$
\begin{aligned}
&\alpha \sum_{s=k_0}^{k-1} (1-\rho)^s (1-\rho)^{k-s-1} w^{\mathrm{T}}(s) w(s) \\
&\qquad \leqslant \sum_{s=k_0}^{k-1} (1-\rho)^{k-s-1} E(e(s), w(s))
\end{aligned} \tag{43}
$$

which implies that

$$
\alpha \sum_{s=k_0}^{\mathcal{T}} (1-\rho)^s w^{\mathrm{T}}(s) w(s) \leqslant \sum_{s=k_0}^{\mathcal{T}} E(e(s), w(s)). \tag{44}
$$

Next, it will be shown that the filtering error system (12) and (13) for $w(k) = 0$ is exponentially stable. If $w(k) = 0$, from (34), we have

$$V_i(k+1) \leqslant (1-\rho)V_i(k) + E(e(k), w(k)) \leqslant (1-\rho)V_i(k) \tag{45}$$

which leads to

$$V_{\sigma(k)}(k) \leqslant (1-\rho)^{k-k_t} V_{\sigma(k_t)}(k_t) \leqslant \mu^{N_\sigma(k_0,k)}(1-\rho)^{k-k_0} V_{\sigma(k_0)}(k_0). \tag{46}$$

We know that $N(k_0, k) \leqslant (k-k_0)/T_a$, and then (46) becomes

$$V_{\sigma(k)}(k) \leqslant \left((1-\rho)\mu^{\frac{1}{T_a}}\right)^{k-k_0} V_{\sigma(k_0)}(k_0). \tag{47}$$

It can be established from (22) that

$$\beta_1 \|e(k)\|^2 \leqslant V_{\sigma(k)}(k) \tag{48}$$

and

$$\beta_2 \|e(k_0)\|_L^2 \geq V_{\sigma(k_0)}(k_0) \tag{49}$$

where $\beta_1 = \min_{\forall i \in \mathbb{N}} \varrho_{\min}(P_i)$ and $\beta_2 = \max_{\forall i \in \mathbb{N}} \varrho_{\max}(P_i) + d \max_{\forall i \in \mathbb{N}} \varrho_{\max}(Q_i) + d^2 \max_{\forall i \in \mathbb{N}} \varrho_{\max}(R_i) + 4d^2 \max_{\forall i \in \mathbb{N}} \varrho_{\max}(S_i)$. From (46)–(49), we have

$$\begin{aligned}\beta_1 \|e(k)\|^2 &\leqslant V_{\sigma(k)}(k) \\ &\leqslant \left((1-\rho)\mu^{\frac{1}{T_a}}\right)^{k-k_0} V_{\sigma(k_0)}(k_0) \\ &\leqslant \left((1-\rho)\mu^{\frac{1}{T_a}}\right)^{k-k_0} \beta_2 \|e(k_0)\|_L^2. \end{aligned} \tag{50}$$

Now, we have

$$\|e(k)\|^2 \leqslant \frac{\beta_2}{\beta_1}\left((1-\rho)\mu^{\frac{1}{T_a}}\right)^{k-k_0} \|e(k_0)\|_L^2. \tag{51}$$

From Definition 1, the exponential stability result of the discrete-time error system (12) and (13) is ensured with $\delta = \beta_2/\beta_1$ and $\chi = ((1-\rho)\mu^{(1/T_a)})$. Note that $\delta > 0$ and $0 < \chi < 1$. This completes the proof. ■

Remark 3: The optimal dissipativity performance bound α^* can be derived through the maximization problem of α subject to $P_i > 0$, $Q_i > 0$, $R_i > 0$, $S_i > 0$, and the LMI conditions in (19) and (20) for $i \in \mathbb{N}$.

Next, consider DSNNs without time delay. When there is no delay in the activation function, the DSNN filtering error system can be described as follows:

$$\begin{aligned} e(k+1) &= (A_i - L_iC_i)e(k) + W_i\tilde{\phi}(x(k)) \\ &\quad + (G_i - LF_i)w(k) \end{aligned} \tag{52}$$

$$\tilde{z}(k) = He(k) \tag{53}$$

where $\tilde{\phi}(x(k)) = \phi(x(k)) - \phi(\hat{x}(k))$. Then, we can show the exponential stability with the strictly $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$ dissipative performance of the discrete-time filtering error system (52) and (53) without delay in the following corollary.

Corollary 1: Based on Assumption 1, for prescribed scalars $\alpha > 0$, $\mu \geq 1$, and $0 < \rho < 1$, if there are matrices $P_i = P_i^{\mathrm{T}} > 0$, diagonal matrix $\Omega > 0$, and M_i for certain switching signal $\sigma(k)$ satisfying the average dwell time with $T_a \geq T_a^* = -\ln \mu / \ln(1-\rho)$, such that

$$\begin{bmatrix} -(1-\rho)P_i - \mathcal{Q} - \Lambda_1\Omega & \Lambda_2\Omega & -\mathcal{S} & (P_iA_i - M_iC_i)^{\mathrm{T}} \\ * & \Omega & 0 & P_iW_i^{\mathrm{T}} \\ * & * & -[\mathcal{R}-\alpha]I & (P_iG_i - M_iF_i)^{\mathrm{T}} \\ * & * & * & -P_i \end{bmatrix} < 0$$

$$P_i \leqslant \mu P_j$$

for all $i, j \in \mathbb{N}$, $i \neq j$, then the exponential stability result of the discrete-time filtering error system is ensured for $w(k) = 0$ and the strictly $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$ exponential dissipativity of the filtering error system is ensured for all nonzero $w(k)$. The desired filter gain matrix is calculated by $L_i = P_i^{-1}M_i$.

Proof: The proof of Corollary 1 can be derived by selecting the Lyapunov function $V(k) = e^{\mathrm{T}}(k)P_ie(k)$ and following the proof of Theorem 1. ■

IV. Exponential l_2–l_∞ Filter Design for DSNNs With Time Delay

Theorem 1 represents an exponentially dissipative filtering approach, which contains exponential passivity and exponential $\mathcal{H}_\infty$ filters as special cases. The exponential $\mathcal{H}_\infty$ filter design considers the filtering error for the worst possible bounded disturbance; that is, an energy-to-energy guaranteed gain for disturbance is considered. However, it cannot cover exponential l_2–l_∞ performance, which can enable the peak filtering error to be investigated for all possible bounded disturbances [40]. Thus, in this section, the design technique of an exponential l_2–l_∞ filter for DSNN (6)–(8) is introduced. The desired filter gain guarantees that the filtering error system (12) satisfies the performance index presented in Definition 4. Theorem 2 presents a new set of sufficient criteria for the presence of the proper filter for DSNN (6)–(8).

Theorem 2: Based on Assumption 1, for prescribed scalars $d > 0$, $\sigma > 0$, $\gamma > 0$, $\mu \geq 1$, and $0 < \rho < 1$, if there are matrices $P_i = P_i^{\mathrm{T}} > 0$, $Q_i = Q_i^{\mathrm{T}} > 0$, $R_i = R_i^{\mathrm{T}} > 0$, $S_i = S_i^{\mathrm{T}} > 0$, and $N_i = [N_{1i}^{\mathrm{T}}\ N_{2i}^{\mathrm{T}}\ N_{3i}^{\mathrm{T}}\ N_{4i}^{\mathrm{T}}\ N_{5i}^{\mathrm{T}}\ N_{6i}^{\mathrm{T}}]^{\mathrm{T}}$; diagonal matrix $\Omega_i > 0$; and M_i for certain switching signal $\sigma(k)$ satisfying the average dwell time with $T_a \geq T_a^* = -\ln \mu / \ln(1-\rho)$, such that

$$\begin{bmatrix} \Xi_{L_i} & \sqrt{d(1-\rho)^d}N_i & \Upsilon_i^{\mathrm{T}} & \sqrt{d}\Psi_i^{\mathrm{T}} \\ * & -S_i & 0 & 0 \\ * & * & -P_i & 0 \\ * & * & * & -2\sigma P_i + \sigma^2 S_i \end{bmatrix} < 0 \tag{54}$$

$$\begin{bmatrix} P_i & H^{\mathrm{T}} \\ H & \gamma^2 I \end{bmatrix} > 0 \tag{55}$$

$$P_i \leqslant \mu P_j, \quad Q_i \leqslant \mu Q_j, \quad R_i \leqslant \mu R_j, \quad S_i \leqslant \mu S_j \tag{56}$$

for all $i, j \in \mathbb{N}, i \neq j$, where

$$\Xi_{L_i} = \begin{bmatrix} \hat{\Xi}_{11_i} & \Xi_{12_i} & \Xi_{13_i} & \Xi_{14_i} & \Xi_{15_i} & \hat{\Xi}_{16_i} \\ * & \Xi_{22_i} & \Xi_{23_i} & \Xi_{24_i} & \Xi_{25_i} & \Xi_{26_i} \\ * & * & \Xi_{33_i} & \Xi_{34_i} & 0 & 0 \\ * & * & * & \Xi_{44_i} & \Xi_{45_i} & \Xi_{46_i} \\ * & * & * & * & \Xi_{55_i} & 0 \\ * & * & * & * & * & -I \end{bmatrix} \quad (57)$$

with

$$\begin{aligned} \hat{\Xi}_{11_i} &= -(1-\rho)P_i + Q_i + (1-\rho)^d\big(N_{1i} + N_{1i}^{\mathrm{T}}\big) + dR_i \\ \hat{\Xi}_{16_i} &= (1-\rho)^d N_{6i}^{\mathrm{T}} \end{aligned} \quad (58)$$

and other terms ($\{\Xi_{12_i} \cdots \Xi_{55_i}\}$) are the same as those in Theorem 1, then the exponential stability result of the discrete-time filtering error system is ensured with exponential l_2–l_∞ index γ and the desired filter gain matrix in (9) is obtained by $L_i = P_i^{-1} M_i$.

Proof: By replacing $\Gamma(k)$ in Theorem 1 with $\Gamma(k) = -w^{\mathrm{T}}(k)w(k)$, one can easily obtain

$$\hat{\Phi}_i = \begin{bmatrix} \hat{\Phi}_{11_i} & \Phi_{12_i} & \Phi_{13_i} & \Phi_{14_i} & \Phi_{15_i} & \hat{\Phi}_{16_i} \\ * & \Phi_{22_i} & \Phi_{23_i} & \Phi_{24_i} & \Phi_{25_i} & \Phi_{26_i} \\ * & * & \Phi_{33_i} & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & \Phi_{55_i} & 0 \\ * & * & * & * & * & -I \end{bmatrix} \quad (59)$$

instead of Φ_i presented in Theorem 1, where $\hat{\Phi}_{11_i} = -(1-\rho)P_i + Q_i + (1-\rho)^d(N_{1i} + N_{1i}^{\mathrm{T}}) + dR_i$, $\hat{\Phi}_{16_i} = (1-\rho)^d N_{6i}^{\mathrm{T}}$, and the other terms are the same as those in (32). Then, by following the proof in Theorem 1 after replacing Φ_i with $\hat{\Phi}_i$, one can show that discrete-time filtering error system (12) is globally exponentially stable for $w(k) = 0$. Next, we will show that the performance index prescribed in Definition 4 is satisfied using the Lyapunov–Krasovskii functional in (22). We introduce the following new performance index:

$$\begin{aligned} \mathcal{J}_i(k) &= V_i(k) - (1-\rho)^{k-k_0} V_i(k_0) \\ &\quad - \sum_{s=k_0}^{k-1} (1-\rho)^{k-s-1} w^{\mathrm{T}}(s)w(s) \\ &= \sum_{s=k_0}^{k-1} (1-\rho)^{k-s-1} \big(\Delta V_i(s) + \rho V_i(s) - w^{\mathrm{T}}(s)w(s)\big). \end{aligned}$$

Using the proof in Theorem 1, $\Delta V_i(k) + \rho V_i(k) - w^{\mathrm{T}}(k)w(k) < 0$ is satisfied under the condition of Theorem 2. Thus, we have $\mathcal{J}_i(k) < 0$, which leads to

$$V_i(k) \leqslant (1-\rho)^{k-k_0} V_i(k_0) + \sum_{s=k_0}^{k-1} (1-\rho)^{k-s-1} w^{\mathrm{T}}(s)w(s).$$

Hence, for any switching signal $\sigma(k)$, it holds that

$$\begin{aligned} V_{\sigma(k)}(k) &\leqslant (1-\rho)^{k-k_0} V_{\sigma(k)}(k_0) \\ &\quad + \sum_{s=k_0}^{k-1} (1-\rho)^{k-s-1} w^{\mathrm{T}}(s)w(s). \end{aligned} \quad (60)$$

From (56) and (60), we obtain

$$\begin{aligned} V_{\sigma(k)}(k) &\leqslant (1-\rho)^{k-k_t} V_{\sigma(k)}(k_t) \\ &\quad + \sum_{s=k_t}^{k-1} (1-\rho)^{k-s-1} w^{\mathrm{T}}(s)w(s) \\ &\leqslant (1-\rho)^{k-k_t} \mu V_{\sigma(k_t-1)}(k_t) \\ &\quad + \sum_{s=k_t}^{k-1} (1-\rho)^{k-s-1} w^{\mathrm{T}}(s)w(s) \\ &\leqslant (1-\rho)^{k-k_t} \mu \Bigg[(1-\rho)^{k_t-k_{t-1}} V_{\sigma(k_t-1)}(k_{t-1}) \\ &\qquad + \sum_{s=k_{t-1}}^{k_t-1} (1-\rho)^{k_t-s-1} w^{\mathrm{T}}(s)w(s) \Bigg] \\ &\quad + \sum_{s=k_t}^{k-1} (1-\rho)^{k-s-1} w^{\mathrm{T}}(s)w(s) \\ &\leqslant \cdots \leqslant (1-\rho)^{k-k_0} \mu^{N_\sigma(k_0,k)} V_{\sigma(k_0)}(k_0) \\ &\quad + \sum_{s=k_0}^{k-1} \mu^{N_\sigma(s,k)} (1-\rho)^{k-s-1} w^{\mathrm{T}}(s)w(s). \end{aligned} \quad (61)$$

Assuming the zero initial condition, the following is given by the above equation as:

$$V_{\sigma(k)}(k) \leqslant \sum_{s=k_0}^{k-1} \mu^{N_\sigma(s,k)} (1-\rho)^{k-s-1} w^{\mathrm{T}}(s)w(s). \quad (62)$$

Multiplying both sides of (62) by $\mu^{-N_\sigma(0,k)}$ gives

$$\begin{aligned} &\mu^{-N_\sigma(0,k)} V_{\sigma(k)}(k) \\ &\qquad \leqslant \sum_{s=k_0}^{k-1} \mu^{-N_\sigma(0,s)} (1-\rho)^{k-s-1} w^{\mathrm{T}}(s)w(s). \end{aligned} \quad (63)$$

Then, due to the fact that

$$N_\sigma(0,k) \leqslant \frac{k}{T_a} \leqslant \frac{-k\ln(1-\rho)}{\ln \mu} \quad (64)$$

we have

$$(1-\rho)^k V_i(k) \leqslant \sum_{s=k_0}^{k-1} w^{\mathrm{T}}(s)w(s). \quad (65)$$

From (55) and (65), we obtain

$$\begin{aligned} &(1-\rho)^k \tilde{z}^{\mathrm{T}}(k)\tilde{z}(k) - \gamma^2 \sum_{s=k_0}^{k-1} w^{\mathrm{T}}(s)w(s) \\ &\leqslant (1-\rho)^k \Big\{ \tilde{z}^{\mathrm{T}}(k)\tilde{z}(k) - \gamma^2 V_i(k) \Big\} \\ &\leqslant (1-\rho)^k e^{\mathrm{T}}(k) \Big(H^{\mathrm{T}}H - \gamma^2 P_i \Big) e(k) < 0 \end{aligned} \quad (66)$$

which further implies that the exponential l_2–l_∞ performance in Definition 4 is guaranteed under the zero initial condition. Then, the proof is completed. ■

Remark 4: The minimum exponential l_2–l_∞ performance bound γ^* can be derived from the minimization problem of

γ subject to $P_i > 0$, $Q_i > 0$, $R_i > 0$, $S_i > 0$, and the LMI conditions in (54)–(56) for $i \in \mathbb{N}$.

Remark 5: Compared with the normal l_2–l_∞ performance index, the exponential l_2–l_∞ performance index includes a weighting parameter $(1-\rho)$. If a ρ close to 1 is selected, the exponential l_2–l_∞ performance index (16) puts more weight on the filtering error peak value during the initial period of time. If a ρ close to zero is selected, the performance index (16) becomes the normal l_2–l_∞ performance. Thus, the exponential l_2–l_∞ performance can be regarded as an extension of the normal l_2–l_∞ performance.

Next, we consider DSNNs without delay. Based on the filtering error system (52) and (53), exponential stability with guaranteed exponential l_2–l_∞ performance for DSNNs without delay is presented in Corollary 2.

Corollary 2: Based on Assumption 1, for prescribed scalars $\gamma > 0$, $\mu \geq 1$, and $0 < \rho < 1$, if there exist matrices $P_i = P_i^{\mathrm{T}} > 0$, diagonal matrix $\Omega > 0$, and M_i for certain switching signal $\sigma(k)$ satisfying the average dwell time with $T_a \geq T_a^* = -\ln\mu/\ln(1-\rho)$, such that

$$\begin{bmatrix} -(1-\rho)P_i - \Lambda_1\Omega_i & \Lambda_2\Omega_i & 0 & (P_iA_i - M_iC_i)^{\mathrm{T}} \\ * & -\Omega_i & 0 & P_iW_i^{\mathrm{T}} \\ * & * & -I & (P_iG_i - M_iF_i)^{\mathrm{T}} \\ * & * & * & -P_i \end{bmatrix} < 0$$

$$\begin{bmatrix} P_i & H^{\mathrm{T}} \\ H & \alpha^2 I \end{bmatrix} > 0$$

$$P_i \leqslant \mu P_j$$

for all $i, j \in \mathbb{N}, i \neq j$, then we can say that the exponential stability of the discrete-time filtering error system (52) and (53) is ensured with exponential l_2–l_∞ index γ, and the desired filter gain is obtained by $L_i = P_i^{-1}M_i$.

Proof: The proof of Corollary 2 is easily derived by selecting the Lyapunov function $V(k) = e^{\mathrm{T}}(k)P_ie(k)$ and following the proof of Theorem 2. ∎

Remark 6: Based on a μ-dependent approach, Zhang *et al.* [49], [50] designed the exponential H_∞ filter and H_∞ controller for a discrete switched linear system under dwell time switching. However, these results do not deal with the exponential dissipative and l_2–l_∞ filtering problems for discrete NNs. Compared with these results, we propose exponential dissipative and l_2–l_∞ filtering for DSNNs with time delay. A new discrete-time Wirtinger-type inequality, whose counterpart is the continuous-time Wirtinger-type inequality, was first established and applied to filter design problems for DSNNs. Thus, new sets of delay-dependent LMI conditions are established such that filtering error systems are exponentially stable with guaranteed performances in the exponential dissipative and l_2–l_∞ senses.

Remark 7: Recently, some new results on filter design have been reported [23], [37], [39], [42], [44] for various kinds of NNs. These studies have focused on filtering problems with multiple missing measurements, switching regularities, and sensor nonlinearities. Our findings can be integrated with these studies to produce new results. In [38], [40], [41], [43], and [46], filter design methods were introduced for NNs by constructing suitable Lyapunov–Krasovskii functionals. However, these results were restricted to filtering problems based on H_∞ performance. Thus, this paper can be combined with the results presented in [38], [40], [41], [43], and [46] to produce new results on exponential dissipative and l_2–l_∞ filter design for DSNNs.

TABLE I
OPTIMAL DISSIPATIVITY PERFORMANCE BOUND α^* CORRESPONDING TO DIFFERENT VALUES OF ρ

ρ	0.001	0.005	0.01	0.05	0.1
T_a	96	20	10	2	1
α^*	2.4962	2.4951	2.4938	2.4825	2.4678

Remark 8: Recently, a new free-matrix-based integral inequality was proposed by Zeng *et al.* [55], [56] to further reduced the potential conservatism of conditions. However, the result was restricted to the stability criteria of continuous time-varying delayed systems. By extending this method to a discrete-time counterpart, a new exponential dissipative and l_2–l_∞ filter for DSNNs can be derived. Some recent issues on the stability and passivity of NNs have been addressed in [57]–[59]. Although these results focus on passivity analysis, they may be integrated with our results to obtain new filter for DSNNs.

Remark 9: Many practical systems (e.g., robotic systems), are governed by nonlinear dynamics. In this case, switched NNs can be used as an appropriate replacement for describing these nonlinear systems. However, the full state information of NNs cannot be directly obtained. Thus, to overcome this problem, the filter designed in this paper can be effectively utilized for estimating the network state variables. In addition, the proposed filter can be used for many applications, including filtering noisy signals, generating nonobservable states, and predicting future states. These results could be integrated with a discrete-time output-feedback controller design method for discrete-time nonlinear complex systems modeled by switched NNs.

V. NUMERICAL EXAMPLES

This section verifies the usefulness and capability of the proposed discrete-time results through two numerical simulations.

A. Example 1

Consider the following DSNN:

$$\begin{aligned} x(k+1) = A_ix(k) + W_i\phi(x(k-d)) + J_i(k) \\ + G_iw(k) \end{aligned} \tag{67}$$

$$y(t) = C_ix(k) + D_ix(k-d) + E_iw(k) \tag{68}$$

for $i = 1, 2$, where

$$x(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}, \quad w(k) = \begin{bmatrix} w_1(k) \\ w_2(k) \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 0.51 & 0 \\ 0 & 0.37 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.44 & 0 \\ 0 & 0.28 \end{bmatrix}$$

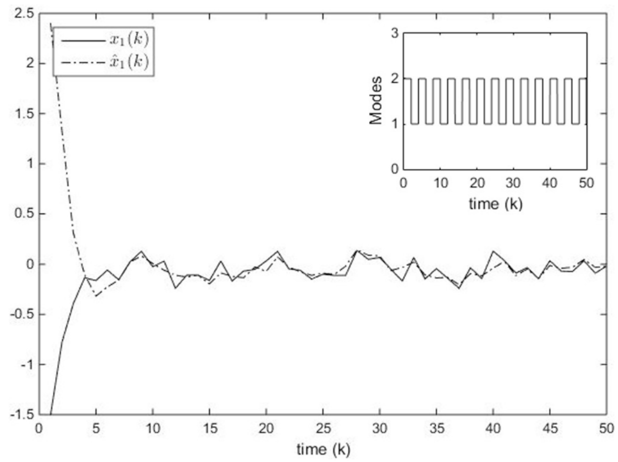

Fig. 1. State trajectory $x_1(k)$ and estimate $\hat{x}_1(k)$.

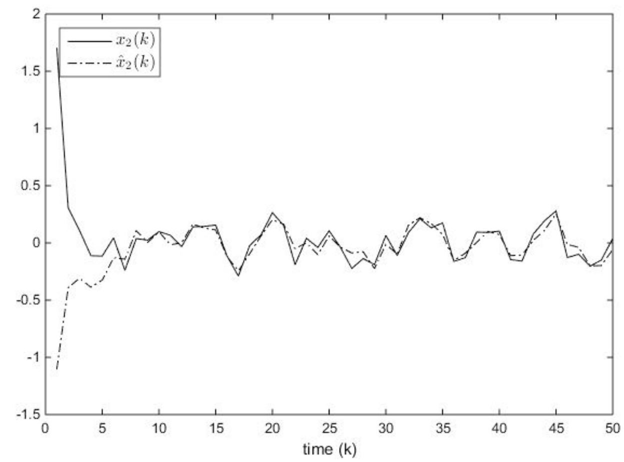

Fig. 2. State trajectory $x_2(k)$ and estimate $\hat{x}_2(k)$.

$$G_1 = \begin{bmatrix} 0.2 & 0.1 \\ 0 & 0.1 \end{bmatrix}, G_2 = \begin{bmatrix} 0.15 & 0 \\ 0.2 & 0.1 \end{bmatrix}$$
$$W_1 = \begin{bmatrix} -0.02 & 0.04 \\ 0 & 0.02 \end{bmatrix}, \; W_2 = \begin{bmatrix} 0.02 & -0.06 \\ 0.03 & 0.04 \end{bmatrix}$$
$$C_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}, \; C_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}$$
$$D_1 = \begin{bmatrix} -0.5 & 0.1 \end{bmatrix}, D_2 = \begin{bmatrix} -0.1 & 0.3 \end{bmatrix}$$
$$F_1 = \begin{bmatrix} 0.1 & -0.4 \end{bmatrix}, F_2 = \begin{bmatrix} -0.2 & -0.5 \end{bmatrix}.$$

Choose the activation function as $\phi(x(k)) = [\tanh(-1.6x_1(k)) \;\; \tanh(1.2x_2(k))]^{\mathrm{T}}$. Through Assumption 1, it is easily seen that $\Lambda_1 = \mathrm{diag}\{0, 0\}$ and $\Lambda_2 = \mathrm{diag}\{-0.8, 0.6\}$. For given values, $d = 1$, $\sigma = 2$, $\mu = 1.1$, $\mathcal{Q} = -0.1I$, $\mathcal{S} = I$, and $\mathcal{R} = 3I$, by solving the conditions in Theorem 1, the optimal dissipativity performance bounds α^* are given for several values of ρ in Table I. Let $\rho = 0.05$ for simulations; the filter gain matrices L_i $(i = 1, 2)$ are obtained as follows:

$$L_1 = \begin{bmatrix} -0.1872 \\ -0.0411 \end{bmatrix}, \quad L_2 = \begin{bmatrix} -0.1497 \\ -0.0533 \end{bmatrix}. \tag{69}$$

Moreover, we obtain the average dwell time $T_a = 2$ through the condition $T_a > T_a^* = 1.8581$. The decay rate of the given system is $\chi = 0.9964 < 1$, and we obtain

$$\|e(k)\| \leqslant 19.1130e^{-0.0036(k-k_0)}\|e(k_0)\|_L, \quad \forall k \geq k_0. \tag{70}$$

Hence, we can see that an exponential stability result of the discrete-time filtering error system is ensured. The external inputs are given by $J_1(k) = [0.1\sin(k) \quad 0.2\cos(k)]^{\mathrm{T}}$ and $J_2(k) = [-0.1\cos^2(2k) \quad 0.1\sin(5k)]^{\mathrm{T}}$. It is assumed that the external disturbance $w_i(k)$ $(i = 1, 2)$ is white noise with normal distribution The simulations for the discrete-time switched exponential dissipative filter design with initial conditions $x(0) = [-1.5 \;\; 1.7]^{\mathrm{T}}$ and $\hat{x}(0) = [2.4 \;\; -1.1]^{\mathrm{T}}$ are illustrated in Figs. 1–3. Figs. 1 and 2 illustrate the responses of the real states $x_1(k)$, $x_2(k)$ and the estimation states $\hat{x}_1(k)$, $\hat{x}_2(k)$, respectively. Fig. 3 depicts the response of the discrete-time filtering error $e(k)$. From the numerical simulations, the effectiveness of the developed LMIs for a strictly $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$ exponentially dissipative filter design for DSNNs is verified.

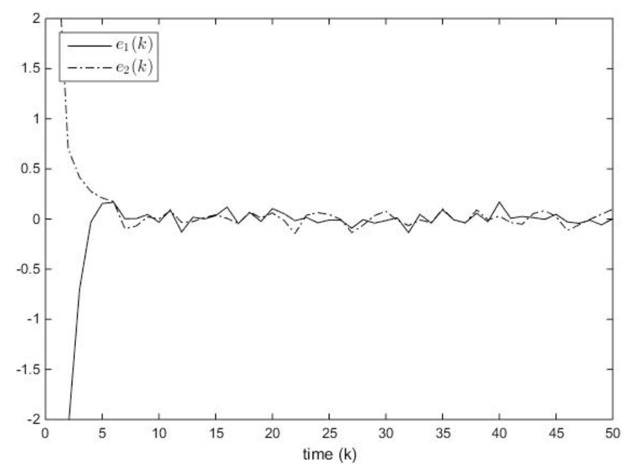

Fig. 3. Filtering error trajectory $e(k)$.

TABLE II
MINIMUM l_2–l_∞ PERFORMANCE BOUNDS γ^* CORRESPONDING TO SEVERAL VALUES OF ρ

ρ	0.001	0.005	0.01	0.05	0.1
T_a	140	28	14	3	2
γ^*	0.3017	0.3015	0.3015	0.3020	0.3027

B. Example 2

Consider the DSNN in Example 1. For given values $d = 1$, $\sigma = 2$, and $\mu = 1.15$, by solving the LMIs in Theorem 2, the minimum l_2–l_∞ performance bounds γ^* are listed for several values of ρ in Table II. Let $\rho = 0.05$ for simulations. The filter gain matrices L_i $(i = 1, 2)$ are obtained as follows:

$$L_1 = \begin{bmatrix} 0.0110 \\ -0.0879 \end{bmatrix}, \quad L_2 = \begin{bmatrix} -0.0477 \\ -0.0429 \end{bmatrix}. \tag{71}$$

Moreover, the average dwell time is obtained as $T_a = 3$ through the condition $T_a > T_a^* = 2.7248$. The decay rate of the above system is given as $\chi = 0.9953 < 1$, and we have

$$\|e(k)\| \leqslant 19.1130e^{-0.0047(k-k_0)}\|e(k_0)\|_L, \quad \forall k \geq k_0. \tag{72}$$

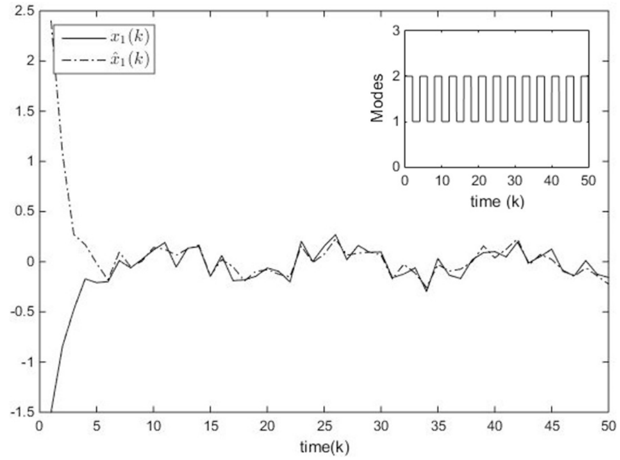

Fig. 4. State trajectory $x_1(k)$ and its estimation $\hat{x}_1(k)$.

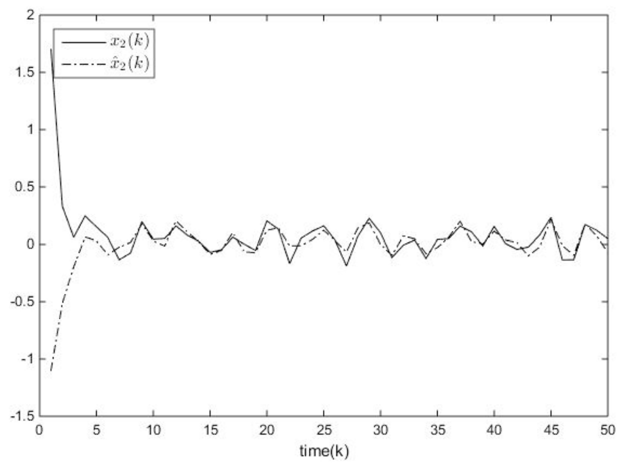

Fig. 5. State trajectory $x_2(k)$ and its estimation $\hat{x}_2(k)$.

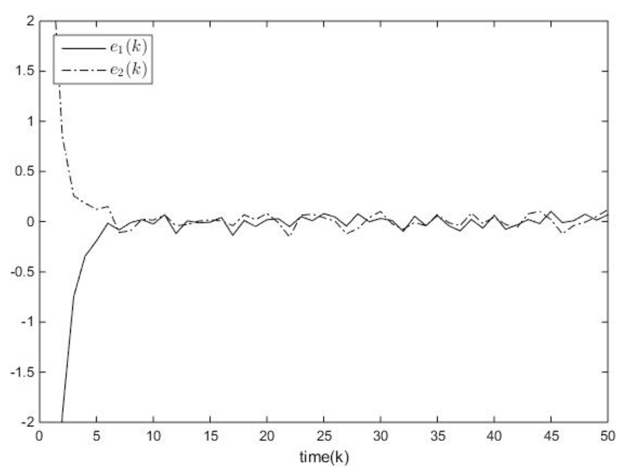

Fig. 6. Filtering error trajectory $e(k)$.

Therefore, we can conclude that an exponential stability result of the discrete-time filtering error system is ensured. The external inputs are given by $J_1(k) = [-0.1\sin(2k) \;\; 0.2\cos^2(2k)]^{\mathrm{T}}$ and $J_2(k) = [0.2\cos(2k) \quad -0.1\sin^2(k)]^{\mathrm{T}}$. The external disturbance and initial condition are the same as those in Example 1. The simulations for the discrete-time switched exponential l_2–l_∞ filter design approach are illustrated in Figs. 4–6. Figs. 4 and 5 show the real states $x_1(k)$, $x_2(k)$ and the estimation states $\hat{x}_1(k)$, $\hat{x}_2(k)$, respectively. Fig. 6 illustrates the filtering error trajectory $e(k)$. Through these numerical examples, it is shown that the exponential stability result of the discrete-time filtering error system is ensured with an exponential l_2–l_∞ performance γ.

VI. Conclusion

This paper has examined the exponential $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$ dissipative and l_2–l_∞ filtering problems for DSNNs including time-delayed states. New sets of LMI conditions were developed by employing the discrete-time Wirtinger-type inequality, which is a new summation vector inequality. By solving the proposed sets of LMIs, desired exponential $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$ dissipative and l_2–l_∞ filters for DSNNs were designed such that the filtering error systems had exponential stability with guaranteed performances in the strictly $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$ exponential dissipativity and exponential l_2–l_∞ senses. Numerical examples verified the effectiveness of the developed filter design.

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