

# 1        **Nonlinear discrete homogenized model for masonry walls out-of-plane loaded**

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## 13    **Abstract**

14    A simple and reliable homogenization approach coupled with rigid elements and  
15    homogenized interfaces for the analysis of out-of-plane loaded masonry panels is  
16    presented.

17    The homogenization approach proposed is a coarse FE discretization where bricks are  
18    meshed with a few elastic constant stress triangular elements and joints reduced to  
19    interfaces with elasto-plastic softening behavior with friction, tension cutoff and a cap in  
20    compression. Flexural behavior is deduced from membrane homogenized stress-strain  
21    relationships through thickness integration (Kirchhoff-Love plate hypothesis). The  
22    procedure is robust and allows obtaining homogenized bending moment/torque curvature  
23    relationships (also in presence of membrane pre-compression) to be used at a structural  
24    level within a Rigid Body and Spring Mass model (RBSM) implemented in the  
25    commercial code ABAQUS. The model relies in rigid quadrilateral elements

26 interconnected by homogenized bending/torque nonlinear springs. The possibility of  
27 extending the procedure to the FE-package ABAQUS, with standard built-in solution  
28 procedures, allows for a robust reproduction of masonry out-of-plane behavior beyond  
29 the peak load, in presence of global softening.

30 The procedure is tested on a set of windowed and full masonry panels in two-way  
31 bending. Excellent agreement is found both with experimental data and previously  
32 presented numerical approaches.

### 33 **Introduction**

34 Out-of-plane failure of masonry occurs at very low levels of the horizontal actions and  
35 there are three main features to deal with in a numerical model devoted to the analysis of  
36 masonry in bending: (1) the role of vertical membrane pre-compression, (2) masonry  
37 orthotropic behavior due to the arrangement of the units, and (3) possible failure due to  
38 out-of-plane shear in case of thick walls. A vertical membrane pre-compression, typically  
39 due to masonry self-weight and gravity loads in general, plays a fundamental role in the  
40 increase in the ductility and the out-of-plane strength, as extensively shown by Milani  
41 and Tralli (2011).

42 Masonry orthotropy is evident for walls exhibiting a regular texture. Masonry units  
43 staggering is responsible for a horizontal bending (i.e. with rotation along a vertical axis)  
44 stiffer and more resistant than the vertical one (i.e. with rotation along a horizontal axis),  
45 as the bed joint contributes in torque to increase stiffness and strength. Orthotropy tends  
46 to become more evident with the progressive degradation of the material. The different  
47 topology of the continuous horizontal joints with respect to the vertical ones, interrupted  
48 by the blocks, implies that tangential stresses acting on bed joints tend to play a significant  
49 role in the horizontal bending increase, while they are not relevant in vertical bending.  
50 Micro-modelling, relying into the distinct discretization of units and mortar (usually

51 reduced to interface to speed up computations) is certainly capable of well reproducing  
52 out-of-plane orthotropy, see for instance Macorini & Izzuddin (2011) and Macorini &  
53 Izzuddin (2013), but such procedure is characterized by long processing times and a large  
54 number of degrees of freedom, sometimes requiring parallelization.

55 Considering the difficulties, it can be affirmed that at present a macro-scale computational  
56 approach is still needed. Macro-modelling (Dhanasekar et al. 1985; Lourenço 1997, 2000;  
57 Pelà et al. 2013) allows studying large scale structures without the drawbacks exhibited  
58 by micro-modelling, because the heterogeneous assemblage of mortar and bricks is  
59 substituted at a structural scale with a fictitious homogeneous anisotropic material. The  
60 calibration of the model is however cumbersome, as a consequence of the high level of  
61 sophistication, usually needing several inelastic parameters to set, requiring expensive  
62 experimental campaigns and data (Lourenço et al. 1998).

63 It is noted that it is not straightforward to account for tangential stresses acting along the  
64 out-of-plane direction. This would require to deal with 3D models at the meso-scale, as  
65 well as to adopt 3D strength domains and 3D inelastic strain evolution laws for mortar  
66 joints reduced to interfaces. For running bond and generally for single or two-wythes  
67 walls (e.g. English or Flemish bond) with slenderness greater than 8-10, it has been shown  
68 by different authors (Casolo and Milani 2010; Cecchi et al. 2007; Cecchi and Milani 2008;  
69 Milani et al. 2006) that the assumption of the thin plate Kirchhoff–Love hypothesis is  
70 adequate and that out-of-plane sliding can occur on limited portions of the walls, mainly  
71 near corners or under concentrated loads. Therefore, at the macro-scale, damage  
72 mechanisms can be reasonably described assuming a thin plate hypothesis, i.e. where  
73 inelastic dissipation is mainly due to the combination of vertical, horizontal bending and  
74 torsion. Considering the aforementioned key issues characterizing masonry subjected to

75 out-of-plane loading, a simple two-step model is used here to analyze efficiently masonry  
76 panels in bending.

77 In such a framework, homogenization (see e.g Luciano and Sacco 1997; de Buhan and de  
78 Felice 1997; Mistler et al. 2007; Milani 2011) is probably the most efficient compromise  
79 between micro- and macro-modelling, because it allows in principle to perform nonlinear  
80 analyses of engineering interest without a distinct representation of bricks and mortar, but  
81 still taking into account their mechanical properties and masonry texture at a cell level.

82 Homogenization (or related simplified approaches) is essentially an averaging procedure  
83 performed at a meso-scale on a representative element of volume (RVE), which generates  
84 the masonry pattern by repetition. On the RVE, a Boundary Value Problem BVP is  
85 formulated, allowing an estimation of the expected average masonry behavior to be used  
86 at structural level. The resultant material obtained is orthotropic, with softening in both  
87 tension and compression. A straightforward approach to solve BVPs at the meso-scale is  
88 based on Finite Elements (FEs) (Massart et al. 2007; Mercatoris and Massart 2011), where  
89 bricks and mortar are either elasto-plastic with softening or damaging materials. It is also  
90 known as a multilevel finite element method (FE<sup>2</sup>), which essentially is a twofold  
91 discretization, the first for the unit cell and the second at structural level. However, FE<sup>2</sup>  
92 appears still rather demanding, because a new BVP has to be solved numerically for each  
93 load step, in each Gauss integration point.

94 In order to circumvent such a limitation, a two-step homogenization procedure is hereafter  
95 proposed. In the first step, masonry is substituted with a macroscopic equivalent material  
96 through a simplified homogenization model in which the unit cell is subdivided into  
97 several layers along the thickness. The choice of concentrating non-linearity on the  
98 interfaces appears particularly suitable because: (1) it allows limiting the computational  
99 effort required to perform full scale analyses to a great extent, and; (2) it seems in

100 agreement with experimental evidence, clearly showing a damage propagation  
101 zigzagging along joints. Considering a single masonry layer, the RVE is discretized  
102 through triangular elastic plane stress elements (blocks) and nonlinear interfaces (mortar  
103 joints). The procedure is robust and allows obtaining homogenized bending  
104 moment/torque curvature relationships (also in presence of membrane pre-compression)  
105 to be used at a structural level.

106 In the second step, entire masonry walls are analyzed in the nonlinear range by means of  
107 a Rigid Body and Spring Mass model (RBSM) implemented in the commercial code  
108 Abaqus (2006). The RBSM model relies into a discretization with rigid quadrilateral  
109 elements interconnected by homogenized bending/torque nonlinear springs. It is stressed  
110 that the RBSM model is not available in ABAQUS, but it can be easily implemented  
111 utilizing the FEs gallery available in any commercial code. Standard arc-length routines  
112 already built in Abaqus (2006) allow for a robust reproduction of out-of-plane masonry  
113 behavior beyond the peak load, in presence of global softening. The latter addresses the  
114 main drawback of previous work (Milani and Tralli 2011) whereby an energy-based  
115 formulation at a structural scale was used, through a quadratic-programming approach,  
116 which assumed linear piecewise discontinuous functions for the homogenized bending  
117 curves to be able to account for material softening. The main novelty of the present study  
118 is that it allows using homogenized curves, derived from the foregoing scale, without the  
119 need of further simplifications to reproduce softening.

120 Two sets of structural comparisons are discussed here to show the capabilities of the  
121 procedure proposed, the first on solid walls and the second on windowed panels in two-  
122 way bending, for which global pressure-displacement and crack patterns are available  
123 from both experimental data and previously presented numerical models.

## 124 **Out-of-plane homogenized model**

125 A multi-scale approach is presented for the out-of-plane study of running-bond masonry  
126 panels, as schematically described in Fig. 1a. The figure briefly shows the proposed flow-  
127 work and the two-step strategy that firstly relies in a homogenization procedure at a meso-  
128 scale. This theory focuses on the periodicity feature of a given media and it is therefore a  
129 proper strategy for masonry (Pegon and Anthoine 1997). Again, the concept is based on  
130 the mechanical characterization of a representative volume element (hereafter, RVE) by  
131 solving a boundary value problem. Then, the study of the structure is accomplished  
132 through the assemblage of these RVE units. The strategy allows defining the mechanical  
133 properties of each material at the unit cell only, and obtaining the damage stress and strain  
134 response by introducing considerations at the component level.

135 Several studies showed the clear advantages of this process. It allows a good trade-off  
136 between consumed time and results accuracy and enables the study of real scale buildings,  
137 see Milani and Tralli (2011), Milani and Venturini (2011), Casolo and Milani (2013),  
138 Akhaveissy and Milani (2013) and Milani et al. (2007). The present out-of-plane  
139 homogenization model is based on the initial in-plane identification of an elementary cell.  
140 The main features of the in-plane homogenized model will be explained in what follows,  
141 for further information the reader is recommended to Milani and Tralli (2011).

142 The RVE  $Y$  (or elementary cell) contains all the information necessary for describing the  
143 macroscopic behavior of an entire wall. In brief, homogenization consists in introducing  
144 averaged quantities for macroscopic strain and stress tensors ( $\mathbf{E}$  and  $\mathbf{\Sigma}$ , respectively). This  
145 is the main concept of the homogenization process and implies that the macroscopic  
146 stress  $\mathbf{\Sigma}$  and strain  $\mathbf{E}$  tensors are calculated as given by Eq. (1):

$$147 \quad \mathbf{E} = \langle \boldsymbol{\varepsilon} \rangle = \frac{1}{V} \int_Y \boldsymbol{\varepsilon}(\mathbf{u}) dY ; \quad \mathbf{\Sigma} = \langle \boldsymbol{\sigma} \rangle = \frac{1}{V} \int_Y \boldsymbol{\sigma} dY \quad (1)$$

148 where  $\langle * \rangle$  is the average operator,  $\boldsymbol{\varepsilon}$  is the local strain value, which is directly dependent  
 149 on the displacements field  $\mathbf{u}$ ,  $\boldsymbol{\sigma}$  is the local stress value and  $V$  is the volume of the  
 150 elementary cell.

151 The homogenization procedure allows to describe the macroscopic level through the  
 152 meso-scale by means of an upward scheme. All the mechanical quantities are considered  
 153 as additive functions and periodicity conditions are imposed on the stress field  $\boldsymbol{\sigma}$  (see  
 154 Eq.(2) and the displacement field  $u$  (see Eq.(3)) (Anthoine 1995), so that:

$$155 \quad \boldsymbol{\sigma} \text{ periodic on } \partial Y \text{ and } \boldsymbol{\sigma} \mathbf{n} \text{ antiperiodic on } \partial Y_1 \quad (2)$$

$$156 \quad \mathbf{u} = \mathbf{E} \mathbf{y} + \mathbf{u}^{per} \text{ periodic on } \partial Y_1 \quad (3)$$

157 where  $\mathbf{u}^{per}$  stands for a periodic displacement field. It may be noted that the periodic  
 158 displacement fluctuation  $\mathbf{u}^{per}$  in Eq.(3) enforces the boundary segments of the RVE to  
 159 have the same deformed configuration, see Fig. 1b.

160 In the present model, the RVE is constituted by joints reduced to interfaces with zero  
 161 thickness and elastic bricks. Bricks are discretized by means of a coarse mesh constituted  
 162 by plane-stress triangles, Fig. 1b. Likewise, brick-brick interfaces are elastic and therefore  
 163 they do not contribute on the inelastic deformation of the unit cell. The utilization of  
 164 brick-brick interfaces may be useful when dealing with low strength units. Here, it is  
 165 assumed that all the nonlinearity in the RVE is concentrated exclusively on joint  
 166 interfaces. The elastic domain of joints is bounded by a composite yield surface that  
 167 includes tension, shear and compression failure with softening. A multi-surface plasticity  
 168 model is adopted, with softening, both in tension and compression (see Fig. 1b). The  
 169 joints failure is ruled by a classical Mohr-Coulomb type strength criterion, with a tension  
 170 cut-off and a linear compression cap. The parameters  $f_t$  and  $f_c$  are, respectively, the tensile  
 171 and compressive strength of the mortar,  $c$  is the cohesion,  $\Phi$  is the friction angle, and  $\Psi$   
 172 is the angle which defines the linear compression cap. For the tension mode, exponential

173 softening on the tensile strength is assumed with an associated flow-rule. The yield  
 174 function reads:

$$175 \quad f_1(\boldsymbol{\sigma}, \kappa_1) = \boldsymbol{\sigma} - f_0 e^{\frac{f_{t0}}{G_f} \kappa_1} \quad (4)$$

176 where  $f_{t0}$  is the initial joint tensile strength,  $G_f^I$  is the mode-I fracture energy and  $\kappa_1$  is a  
 177 scalar that controls the amount of softening. For the shear mode, a Mohr-Coulomb yield  
 178 function with a non-associated flow rule is considered:

$$179 \quad f_2(\boldsymbol{\sigma}, \kappa_2) = |\tau| + \boldsymbol{\sigma} \times \left( \tan(\phi_0) + \frac{(\tan(\phi_t) - \tan(\phi_0))(c_0 - c)}{c_0} \right) - c_0 e^{\frac{c_0}{G_f^{II}} \kappa_2} \quad (5)$$

180 where  $c_0$  is the initial cohesion,  $\tan(\phi_0)$  the initial friction angle,  $\tan(\phi_t)$  the residual  
 181 friction angle and  $G_f^{II}$  is the mode-II fracture energy. For the compression mode, an  
 182 associated elastic-perfectly plastic behavior is assumed, with a yield function described  
 183 as follows:

$$184 \quad f_3(\boldsymbol{\sigma}) = |\tau| + (\boldsymbol{\sigma} + f_c) \tan(\Psi) \quad (6)$$

185 where  $f_c$  is the uniaxial compressive strength and  $\Psi$  is the angle that defined the linear  
 186 compression cap. The properties adopted for the present study are gathered on Table 1.  
 187 The latter information is related with the experimental data used for the validation step at  
 188 a structural level of the proposed discrete model.

189 The response of the RVE under out-of-plane actions is obtained subdividing the thickness  
 190 into several  $n$  layers (40 layers are assumed). A displacement driven approach is adopted,  
 191 meaning that macroscopic curvature increments  $\Delta\chi_{11}$ ,  $\Delta\chi_{22}$ ,  $\Delta\chi_{12}$  are applied through  
 192 suitable periodic boundary displacement increments. Thus, each layer undergoes only in-  
 193 plane displacements and may be modelled through plane stress FEs. Each increment  
 194 defines the number of discrete data points of  $\sigma$ - $\varepsilon$  and  $M$ - $\theta$  curves.

195 Thus, a bending moment-curvature relationship is obtained for each interface angle;  
 196 through the obtained RVE macroscopic mode-I stresses. The latter failure mode



197 assumption is valid once masonry presents in general low compressive stresses at failure.  
 198 Being a low-tensile strength material, the cross-section failure is ruled by tensile cracking  
 199 and a linearized behavior in compression is considered, with stiffness degradation present  
 200 only in tension. Towards the derivation of the  $M$ - $\theta$  curve for each interface, the cross-  
 201 section equilibrium is iteratively calculated accounting for potential pre-compression  
 202 states. The bending moment capacity  $M$  of the cross section is calculated by the  
 203 summation of each  $n_i$  layer contribution by means of the following equation:

$$204 \quad M = \sum_{i=1}^n \sigma_i \bar{d}_L dA_i \quad (7)$$

205 where  $\sigma_i$  is the mean stress at each layer,  $\bar{d}_L$  is the distance between the centroid of each  
 206 layer and the neutral axis and  $dA_i$  is the area of each layer. The resultant moment  $M$  can  
 207 also be simply written as the integral of stress multiplied by its distance from the middle  
 208 section through the wall thickness:

$$209 \quad M = \langle \sigma y_3 \rangle = \frac{1}{A} \int_Y \sigma y_3 dY \quad (8)$$

210 In this way, homogenized curves are approximated to define the nonlinear flexural  
 211 behavior of the interfaces. The on-thickness integration hypothesis allows evaluating  
 212 moment-curvature diagrams for solid brick masonries, but can be easily adapted to hollow  
 213 bricks assuming different mechanical properties for, e.g. internal and external layers. The  
 214 latter procedure is represented in Fig. 2 for a horizontal interface, hereafter labelled with  
 215 orientation  $\theta = 90$  degrees, i.e. vertical bending. A similar strategy is performed to derive  
 216 the torsion moment curve. Interface orientations are guided by the mesh representation of  
 217 the discrete model at a structural scale. So, the implementation in a finite element package  
 218 at a macro-scale allows to represent and study three-dimensional structures under out-of-  
 219 plane actions.

## 220 **Structural discrete model**

221 On a macro-scale level, the out-of-plane analysis of the masonry walls is performed  
222 through a novel discrete element mechanical system. The latter has support and  
223 background in the works by Kawai (1977) and employs the information of the  
224 homogenized curves at a structural scale. Simply, the discrete model is described as the  
225 assemblage of quadrilateral rigid plates inter-connected on interface vertices by a set of  
226 rigid beams and deformable trusses. The system of deformable trusses carries the material  
227 information required for interfaces. A decoupled characterization of flexural and torsional  
228 actions is adopted. In the mid-span of each interface a spherical hinge is positioned. The  
229 aim is to allow the rotation for torsional movements as well as to guarantee the deformed  
230 shape compatibility between adjoining elements. For a clear understanding of the model,  
231 the discrete system is represented in Fig. 3.

232 Such discrete element approach is implemented into a commercial finite element  
233 software, namely Abaqus (2006). The inherent advantages are mainly two. Firstly, the  
234 robustness of the software to solve nonlinear static problems in presence of material  
235 softening is obtained by means of an established arc-length procedure (Memon and Su  
236 2004). Secondly, this allow a great potential to extend the model to structural applications  
237 in any finite element software and the possibility to be used by professionals and  
238 researchers.

## 239 **Material Properties: from meso- to macro-scale**

240 The masonry behavior when out-of-plane loaded is highly dependent on its anisotropy at  
241 failure (Gilbert et al. 2006; Milani and Lourenço 2010). Experimental information  
242 conducted on masonry walls in two-way bending shows that failure occurs for a relatively  
243 ductile behavior and forming a well-defined path, see Chong et al. (1994) and  
244 Southcombe et al. (1995).

245 Aiming at developing the required material information at a macro-scale, an identification  
 246 of the desired mesh dimensions and geometrical characteristics of the walls may be  
 247 performed. Bearing in mind that quadrilateral elements are assumed, two different angles  
 248 are considered for the interfaces: 0 and 90 degrees. The behavior of the interfaces is  
 249 obviously orthotropic with softening, because it derives from the aforementioned  
 250 homogenization strategy. In this way, the homogenized bending moment-curvature and  
 251 torsional moment-curvature curves of the interfaces is depicted in Fig. 4.

252 The procedure described in what follows is required to convert the latter information in  
 253 valid input data for the FE package used at a structural scale. To accomplish this goal,  
 254 obtaining stress and strain curves for each angle of the interface and for each bending  
 255 moment direction is mandatory. Thus, the approach offers the possibility to reproduce the  
 256 material orthotropy by defining different input stress-strain relationships according to the  
 257 trusses' plane. The conversion between bending and torsion moment and stress values is  
 258 achieved by Eq.(9) and (10):

$$259 \quad \sigma_{Axial\ truss} = \frac{M l_{influence}}{A_{Axial} t} \quad (9)$$

$$260 \quad \sigma_{Torque\ truss} = \frac{M l_{influence}}{A_{Torque} H} \quad (10)$$

261 Here,  $M$  is the bending moment,  $l_{influence}$  is the influence length of each truss,  $t$  is the  
 262 thickness of the wall,  $H$  the length of each quadrilateral panel,  $A_{Axial}$  is the axial truss area  
 263 given by  $0.25 \times t \times H$  and  $A_{Torque}$  is the torque truss area given by  $0.5 \times e \times H$ , where  $e$  (value  
 264 of 10 mm) is the gap between the rigid plates, which ideally should be zero but in practice  
 265 is assumed small enough to be able to place trusses between elements.

266 At last, the stress homogenized input curves may be properly calibrated. An elastic  
 267 calibration for the stress curves is conducted. Briefly, by assuring the energy equivalence  
 268 between the discrete mechanism and a homogeneous (for the masonry data, see Table 1)  
 269 continuous shell element. The latter is guaranteed separately for both flexural and

270 torsional movements and so, a decoupled behavior is derived. For the sake of conciseness,  
 271 the theoretical demonstration is not shown, but it can be easily derived that the Young's  
 272 moduli of axial ( $E_{flexural}$ ) and torque trusses ( $E_{torque}$ ) are:

$$273 \quad E_{flexural} = \frac{E_{masonry}e}{12l_{influence}+6e} \frac{E_{masonry}}{(1-\nu^2)} \text{ and } E_{torque} = \frac{t^4}{3(2l_{influence}+e)H^2e} \frac{E_{masonry}}{(1+\nu)} \quad (11)$$

274 It is important to state that the present study focuses on the nonlinear static analysis of  
 275 two sets of masonry panels. The walls under study were already experimentally out-of-  
 276 plane tested at the University of McMaster and Plymouth by Gazzola and Drysdale (1986)  
 277 and Chong et al. (1994), respectively. Also, it is highlighted that a refined mesh was  
 278 defined for both case studies. The size of the interfaces (H), i.e. the side length of each  
 279 quadrilateral panel, is only 100 mm.

280 In the first step, the holonomic homogenization model allows obtaining the macroscopic  
 281 masonry material properties accounting for the strain softening regime. In the second step,  
 282 this information should serve as input for the analysis at a structural level. Thus, the novel  
 283 discrete element model implemented in the finite element package ABAQUS must be  
 284 able to receive such data. The concrete damage plasticity model is selected for this  
 285 purpose, as it allows to fully represent the inelastic behavior of masonry, by defining  
 286 stress-strain curves for axial and torque trusses of the system. For further details  
 287 concerning the model and its implementation, see Wahalathantri et al. (2011).

288 Simplified softening curves are considered for each truss, see for instance Fig. 5. To avoid  
 289 convergence and run time problems, a small plateau near the peak of the curves is adopted  
 290 in order to avoid abrupt stiffness losses. For the simulations, the post-failure stress-strain  
 291 behavior must be introduced in the material information parameters. Specifically,  
 292 ABAQUS requires the introduction of the cracking strain  $\tilde{\epsilon}_t^{ck}$ , which can be obtained for  
 293 each point of the homogenized curve by Eq.(12):

$$294 \quad \tilde{\epsilon}_t^{ck} = \epsilon_t - \epsilon_0^{el} \quad (12)$$

295 where  $\varepsilon_o^{el}$  is the elastic strain corresponding to the undamaged material and  $\varepsilon_t$  is the total  
296 strain of the holonomic curve. Damage parameters  $d_t$  should also be introduced, which  
297 link the undamaged elastic modulus with that of the damaged material in the unloading  
298 phase, as  $E_d = E(1 - d_t)$ , see also Fig. 5.

### 299 **Macro-scale validation: out-of-plane loaded masonry panels**

300 The macro-scale validation of the homogenization model is achieved by analyzing  
301 masonry panels subjected to out-of-plane loads. The aim is to conclude about the ability  
302 of the model to reproduce the nonlinear out-of-plane response of masonry. Available  
303 experimental data of windowed and full panels in two-way bending are used. The panels  
304 result from the studies of Gazzola and Drysdale (1986) at the University of McMaster  
305 and Chong et al. (1994) at the University of Plymouth.

306 The first set of panels that are being studied refers to three running bond masonry panels  
307 tested at the University of McMaster (Gazzola and Drysdale 1986). The panels are  
308 designated as WII, WF and WPI. The geometry of the panels is similar, being the  
309 boundary conditions the main difference, see Fig. 6. Such analyses allow to conclude  
310 about the ability of the model to describe the response in terms of pressure vs. out-of-  
311 plane displacements, and if the homogenized model is able to reproduce a pre-  
312 compression state (due to the analysis in WPI panel).

313 Information concerning the assumed mechanical properties is reported in Table 1. The  
314 out-of-plane behavior of a masonry wall is essentially ruled by the flexural strengths along  
315 vertical and horizontal directions, which are available for both studied panels. The  
316 properties identification is achieved by fitting the flexural strengths values with the ones  
317 reported by Lourenço (1997). The same values for the horizontal flexural strength,  $f_{tx} =$   
318  $0.81 \text{ (N/mm}^2\text{)}$ , and for the vertical flexural strength,  $f_{ty}=0.40 \text{ (N/mm}^2\text{)}$ , are adopted. The  
319 bricks dimensions are  $390 \times 190 \times 150 \text{ mm}^3$  and the thickness of the joints is 10 mm. The

320 same strategy is conducted for the Plymouth panels. Assuming bricks elastic and that the  
321 non-linearity is restricted to the tensile regime, only mortar tensile strength and cohesion  
322 can be tuned, with a fixed softening with pre-assigned fracture energy. It is believed that  
323 the model is able to reproduce and predict well the response of masonry in the cases where  
324 sufficient experimental information on its constituents is available.

325 The refined mesh with 100 mm of size has 1196 discrete elements for each panel (each  
326 discrete element has 4 quadrilateral rigid plates). Whilst only collapse loads are reported  
327 in Gazzola and Drysdale (1986), the results discussion addresses also the obtained  
328 capacity curves. For each studied panel, Fig. 7 illustrates a comparison on global force-  
329 displacement curves between the present model and: (i) the experimental collapse load  
330 (McMaster university data), (ii) an anisotropic macro-model by Lourenço (2000) and (iii)  
331 an upper and lower bond limit analysis by Milani et al. (2006).

332 For all the panels and regarding the collapse load, the present model allows to reach an  
333 acceptable maximum error of 11% on peak experimental loads. Moreover, the pushover  
334 curves present a similar shape when compared with those provided by the macro-model  
335 proposed by Lourenço (2000). As aforementioned, the conducted analyses include a pre-  
336 compression state only for the panel WPI. The homogenized model was prepared also to  
337 compute the final stress-strain curves bearing a defined pre-compression state, assuming  
338 that it is maintained constant during the out-of-plane loading.

339 The second set of out-of-plane experimental data is constituted by the panels tested at the  
340 University of Plymouth by Chong et al. (1994). Five panels in running bond masonry  
341 texture using solid clay bricks were tested and designated by SB (Chong et al. 1994;  
342 Southcombe et al. 1995). The panels SB01 and SB05 have the same geometry, thus only  
343 four panels (SB01-SB04) are considered and represented in Fig. 8. The boundary  
344 conditions are the same for the four panels, i.e. laterally simply supported and fixed at the

345 base. The experimental investigation aimed at a better insight on the role played by the  
346 openings size and shape.

347 The panels were loaded by air-bags until failure, whereas both the pressure and  
348 displacement at the middle span of the free edge were monitored. Thus, the comparison  
349 is here done in terms of pressure load and displacement in each masonry panel.

350 At a meso-scale, the mechanical properties adopted for the RVE characterization were  
351 already presented in Table 1. Bearing that according to the experimental data (Chong et  
352 al. 1994; Southcombe et al. 1995), the flexural uniaxial strengths  $f_{tx}$  and  $f_{ty}$  are 2.28 and  
353  $0.97 \text{ N/mm}^2$ , respectively, the mechanical properties adopted were tuned in order to fit  
354 the latter values. The bricks dimensions are  $215 \times 65 \times 102.5 \text{ mm}^3$  and the thickness of the  
355 joints is 10 mm.

356 The refined mesh with 100 mm of size has 1122 discrete elements for panel SB01/05,  
357 892 elements for panel SB02, 987 elements for panel SB03 and 960 elements for panel  
358 SB04. It is important to stress that the mesh at the macro-scale is independent from the  
359 mesh adopted in the RVE at a meso-scale and from the masonry texture, i.e. units'  
360 geometry. Each nonlinear analysis, with the present refined mesh, took around 9 minutes  
361 in a computer with an Intel Core i7-4710MQ 2.50 GHz processor. This running time  
362 accounts for the pre-homogenization and calibration steps required before the analysis  
363 and could be minimized, if (1) a coarser mesh is adopted or (2) by analyzing a half part  
364 of the wall due to symmetry conditions. It is also important to understand that softening  
365 is being represented and the associated convergence problems cannot be avoided.

366 Fig. 9 shows the comparison between the numerical and experimental results (Chong et  
367 al. 1994), concerning pressure load and displacement at the middle node of the free edge.

368 In addition to the present model, other results are represented, namely an anisotropic  
369 macro-model (Lourenço 2000), an elastic perfectly-plastic homogenized model

370 designated as EPP-model (Milani and Tralli 2011), a simplified deteriorating model based  
371 on homogenized limit analysis designated as SD model (Milani and Tralli 2011) and  
372 finally a simplified quadratic programming elastic-plastic model by Milani and Tralli  
373 (2011), in which deterioration of interfaces (ultimate bending moment) is considered. For  
374 the sake of conciseness, the reader is referred to Lourenço (2000) and Milani and Tralli  
375 (2011), in order to analyze with further detail each of the aforementioned models.

376 In general, the comparison allows concluding that the obtained results are good, both in  
377 terms of collapse load and displacements prediction, see Fig. 9. For the panel SB01/05  
378 the failure pattern indicates that cracking occurs as expected due to flexural failure at the  
379 fixed base of the wall, see Fig. 10. The cracking formation near the lateral supports, i.e.  
380 diagonal cracks, is also clear. For further comparison with the experimental failure modes,  
381 see Lourenço (1997). The peak load results are similar to the ones obtained  
382 experimentally, even if the softening range starts slightly before than the other reference  
383 curves.

384 For the second panel, designated as SB-02, the initial stiffness is marginally  
385 overestimated. This panel is the one with the largest opening in height. Nevertheless,  
386 reasonable agreement is found regarding the obtained peak load with a relative error of  
387 around 20% with the experimental curve. The damage patterns show cracking due to  
388 horizontal bending in the fixed base, vertical bending above the opening and the  
389 formation of diagonal cracks surrounding the corners and lateral supports.

390 To what concerns panel SB03, both peak load and curve shape are quite similar to the  
391 results by Lourenço (1997). The post-peak behavior is again characterized by the  
392 formation of the vertical crack above the opening. Also, as expected, the formation of  
393 diagonal cracks is evident at the opening sides and with the direction of the lateral  
394 supports.



395 At last, the present model leads to a capacity curve with a reasonable agreement for the  
396 panel SB04, in which the peak load has a relative error of around 10% with the macro-  
397 model by Lourenço (1997). Similarly, a vertical crack above the opening is developed.  
398 Failure due to torsional movements is also visible around the lateral supports, as well as  
399 failure due to flexion at the base fixed support. The model is not able to directly follow  
400 diagonal yield lines (zig-zag instead). Even so, the used quadrilateral mesh is refined  
401 enough to minimize the mesh dependence and the differences concerning the  
402 experimental results are not significant.

403 The results show the capacity of the model to obtain good representations of the nonlinear  
404 behavior in panels with complex geometries, using refined meshes. The analyses of the  
405 Plymouth panels are repeated with less refined meshes, see Fig. 11. The goal is to evaluate  
406 the mesh dependence both in terms of results accuracy and running time duration. For the  
407 first panel (SB-01/05) three medium-high refinement meshes (in respect with the brick  
408 size) with edge size equal to 100, 150 and 200 mm, and two very coarse meshes, with  
409 edge size equal to 500 and 1000 mm, are compared. Fig. 11a demonstrates that the mesh  
410 dependency is low as the obtained difference on the pressure-displacement curve among  
411 the meshes is less than 15%, for such large variation of mesh sizes, which is acceptable  
412 from an engineering standpoint. In addition, it is worth noting that the required  
413 computational time is impressively reduced for the coarse meshes (less than one minute),  
414 but still reasonable for a strong mesh refinement, Fig. 11a (exponential reduction with the  
415 increase of mesh size). The deformed shapes of panel SB-01/05 for the four refinement  
416 levels studied are also presented in Fig. 11b.

417 On the other hand, only two refined meshes ( $150 \times 150 \text{ mm}^2$  and  $200 \times 200 \text{ mm}^2$ ) were  
418 considered for the SB-02-04 panels to avoid geometrical misrepresentations, due to the  
419 existence of openings. Regarding the running time duration, the coarser mesh ( $200 \times 200$

420 mm<sup>2</sup>) allows to obtain analyses times within 3 minutes only. For the peak load, the  
421 differences between the studied meshes are lower than 5%, being therefore not relevant  
422 for engineering applications. Some difference may be noted in the post-peak behavior,  
423 but it is well known that rigid elements, where nonlinearity is concentrated on interfaces,  
424 intrinsically suffer from limited mesh dependence on softening.

## 425 **Conclusions**

426 A two-step procedure was presented to study the nonlinear static behavior of masonry  
427 panels subjected to out-of-plane loading, and allowing the use of any standard advanced  
428 nonlinear finite element code. The first step concerns the homogenization model based  
429 on an elastoplastic approach. This is performed at a meso-scale through a FE  
430 discretization of the unit cell, the so-called representative volume element (RVE) and  
431 allows obtaining the curvature-bending moment diagrams for each direction, i.e. masonry  
432 orthotropy. For each layer, a plane-stress boundary problem was solved in which the  
433 nonlinearity is concentrated only on joint interfaces, accounting for both tensile and  
434 compressive strength and strain softening.

435 Being a new methodology, at a structural scale, the simulations were done within a novel  
436 discrete element model implemented in the Finite Element software package Abaqus  
437 (2006). The latter is composed by quadrilateral rigid plates connected by a system of rigid  
438 beams, axial and torque trusses. This system represents the behavior of the homogenized  
439 interfaces obtained previously. The obtained homogenized curves were calibrated and  
440 then scaled in order to be readable by the software.

441 The validation of the model was performed through nonlinear static analyses on masonry  
442 panels. The obtained peak loads have a good agreement with the experimental values with  
443 an error less than 20% for the peak load. Also, the shape of the capacity curves was  
444 compared with an anisotropic model. Good agreement was obtained between the capacity

445 curves and damage patterns between the complex anisotropic model and the new discrete  
446 model, whereas a maximum peak load error of about 10% may be observed for the panel  
447 SB-02. In addition, a mesh dependency test was conducted to deepen the knowledge on  
448 refinement issues. One may note the importance of addressing the two following  
449 recommendations to practitioners interested in a fast and reliable analysis of masonry  
450 panels out-of-plane loaded: (i) the proposed homogenization-discrete element model does  
451 not show critical mesh dependence issues. Very coarse meshes proved to predict well the  
452 initial stiffness, ultimate load carrying capacity and ultimate ductility. The advantage of  
453 the utilization of coarse meshes is certainly the considerable reduced computation effort  
454 needed, see Fig. 11a. The only constraint is obviously in the correct definition of the  
455 possible location of yield lines compatible with the real ultimate behavior of the walls.  
456 On the other hand, (ii) as far as the previous precautions on the mesh generation are kept,  
457 the only limitation in the utilization of few rigid elements is the impossibility to obtain a  
458 detailed description of the actual crack patterns, to be compared with either experimental  
459 ones or those obtained from expensive micro-modelling strategies. When such output is  
460 needed, the user is recommended to refine the discretization.

461 At last, it is important to note the advantage of the procedure and its efficiency in respect  
462 with a detailed heterogeneous micro-modelling strategy (i.e. a separate discretization of  
463 bricks and mortar). The use of rigid plates minimizes the complexity regarding inelastic  
464 phenomena problems. Using standard commercial FE packages, the effectiveness and  
465 robustness of the software to solve problems accounting for the post-elastic behavior with  
466 softening can be used. This also allows the possibility to extend the use of the proposed  
467 model at professional level to fields such as earthquake or blast engineering. Regarding  
468 the former, the use of truss beam elements that reproduces the homogenized behavior of  
469 interfaces within a Concrete Damage Plasticity model at a macro-scale allows, in

470 principle, to conduct numerical analyses in the non-linear dynamic range. In addition, the  
471 utilization of a robust commercial code like ABAQUS allows running analyses in the  
472 non-linear dynamic range without any special difficulty, because the ex-novo  
473 implementation of global solvers is not needed and proper hysteresis models are  
474 available. On the other hand, in what concerns the latter, the application of the model in  
475 the field of blast and impact engineering deserves a separate discussion because, in such  
476 case, mechanical properties of the constituent materials are rate-dependent. A practical  
477 way of proceeding would be to define the material properties using dynamic increase  
478 factors.

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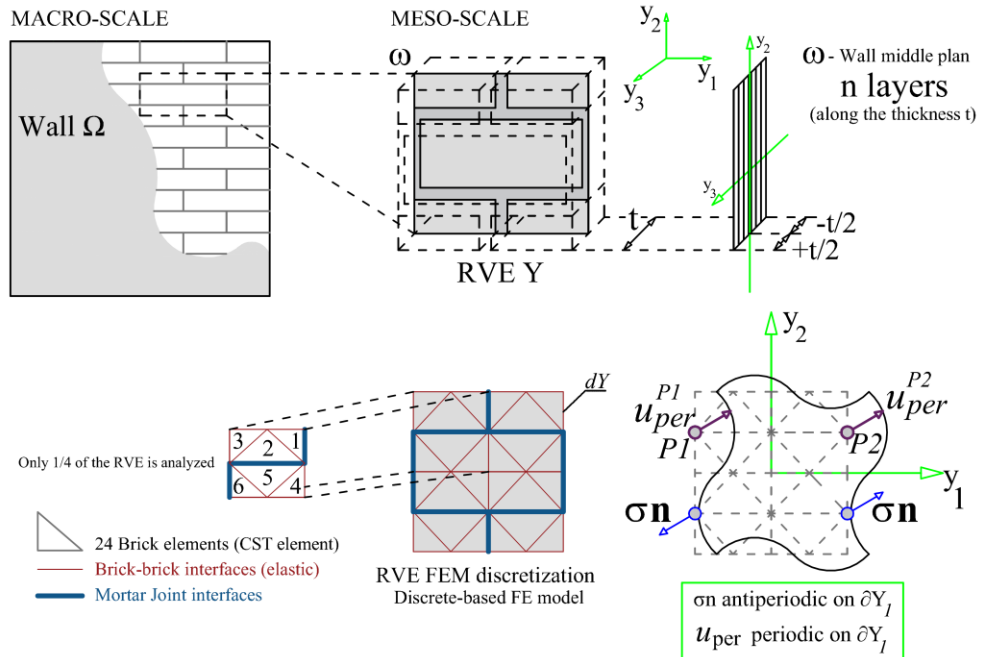


(a)

FLOW-CHART

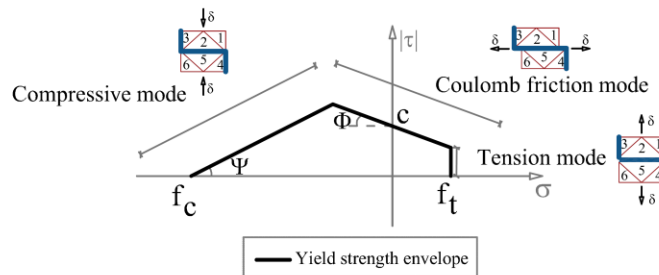


(b)



(c)

Modified Mohr-Coulomb for mortar joints

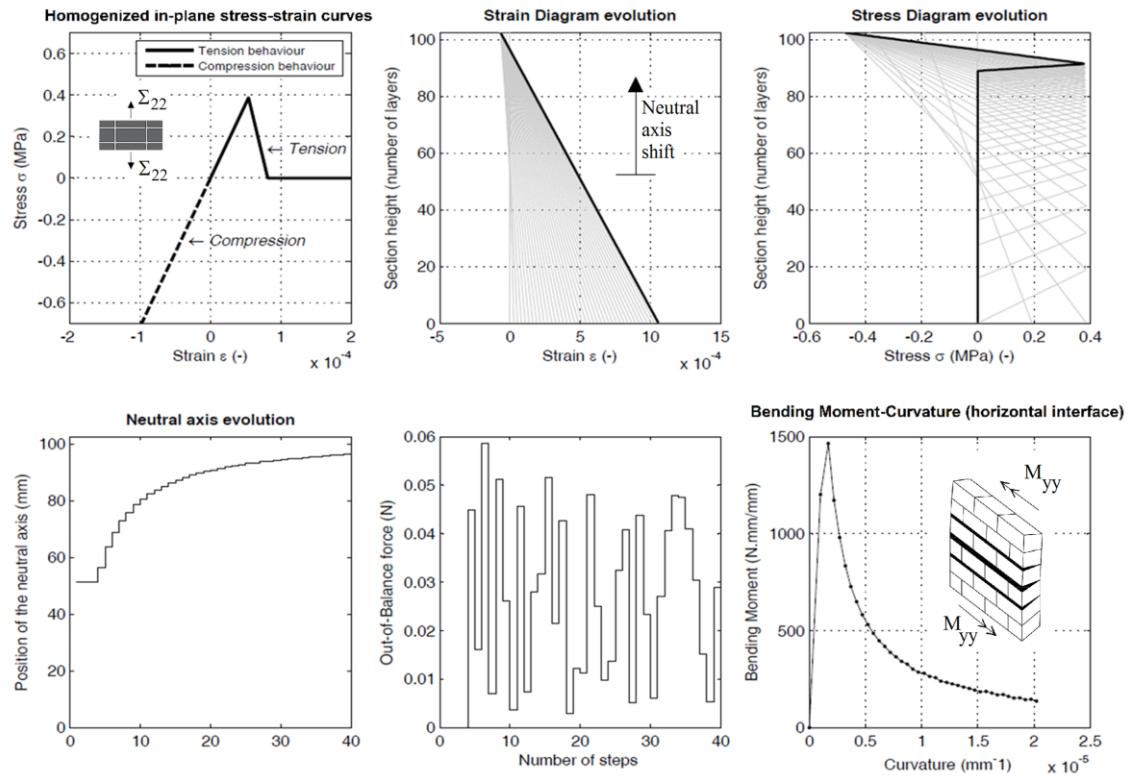


585

586 **Fig. 1.** (a) Flow-chart of the present two-step procedure; (b) Micro-mechanical model

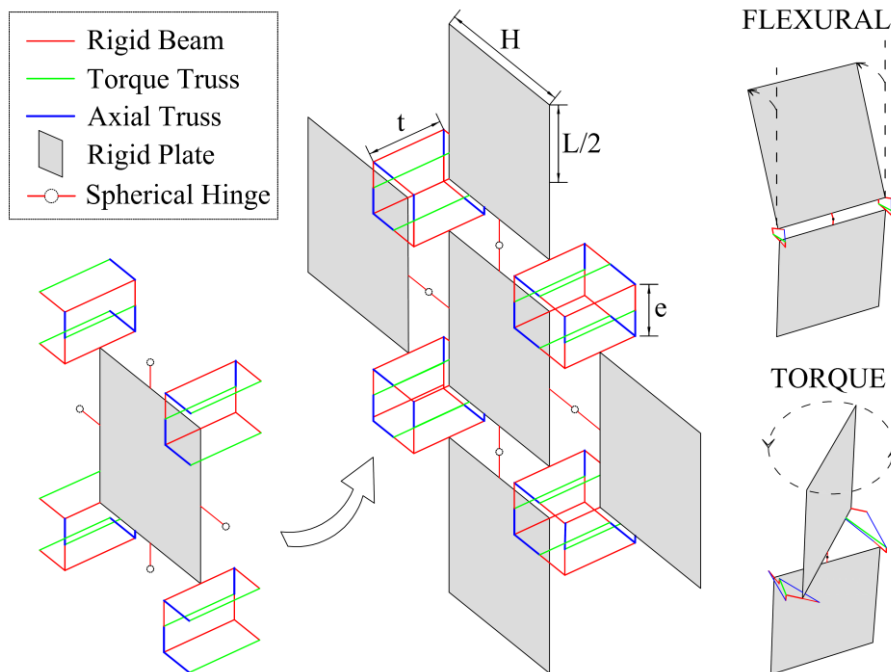
587 adopted for the present homogenized model; and (c) strength domain for joints reduced

588 to interfaces.



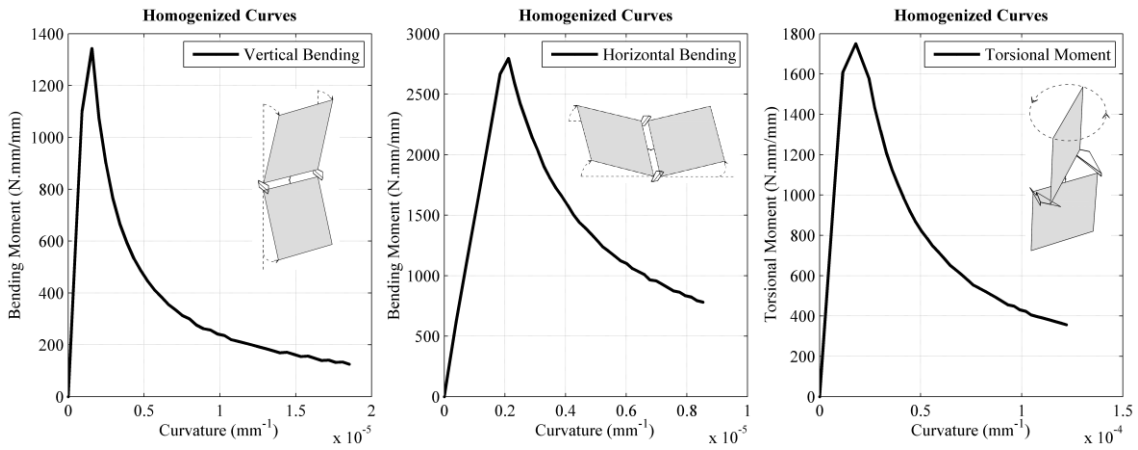
589

590 **Fig. 2.** Adopted procedure to derive out-of-plane homogenized bending moment-  
 591 curvature curves (e.g. vertical bending).



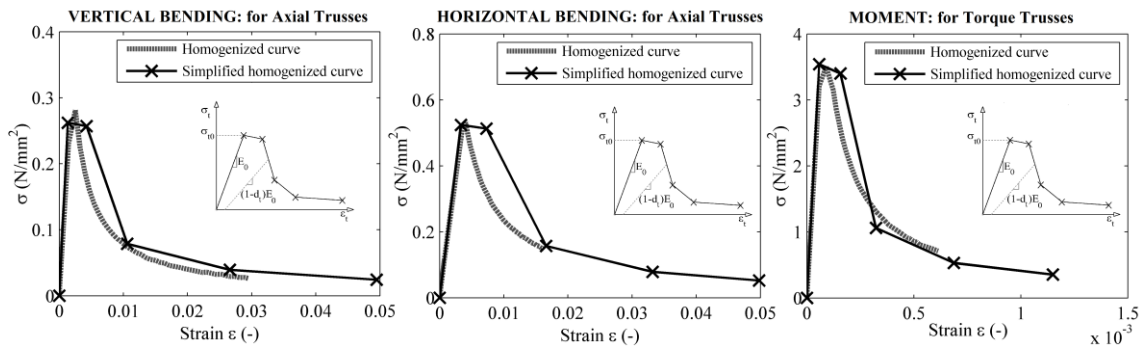
592

593 **Fig. 3.** Description of the novel discrete element system proposed.



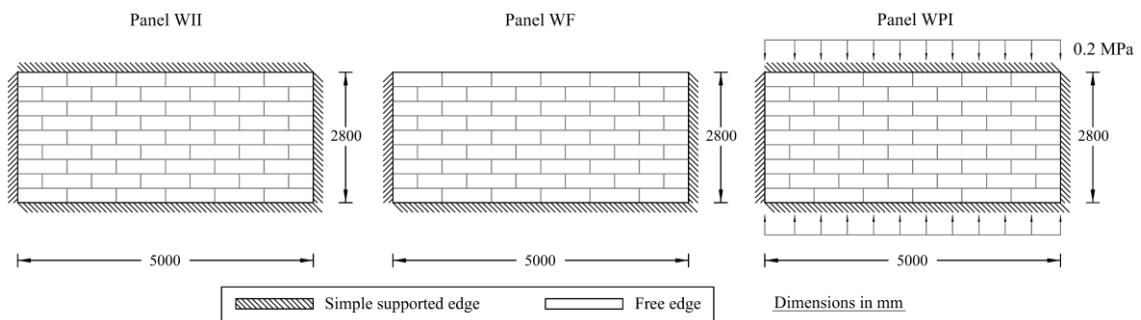
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595 **Fig. 4.** Calibrated bending moment and torsional moment homogenized curves for the  
 596 study of the panels tested by Chong et al. (1994).



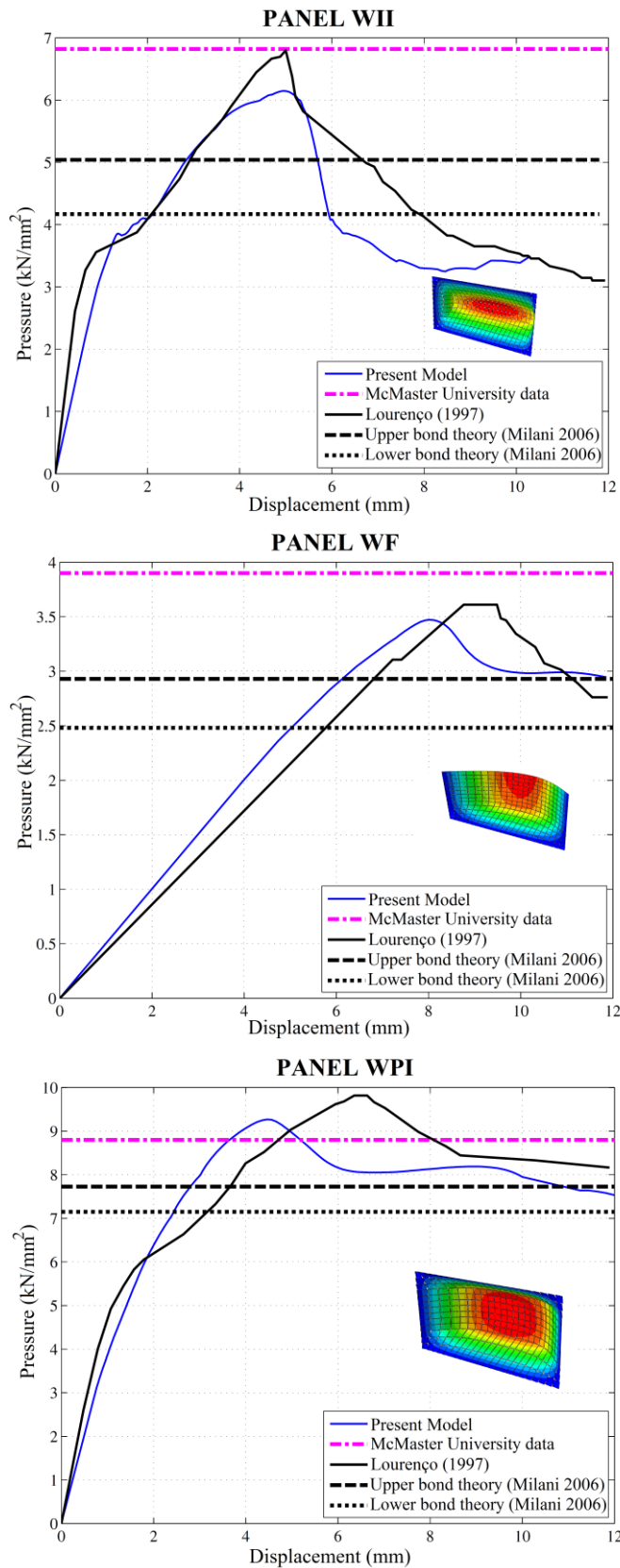
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598 **Fig. 5.** The calibrated stress-strain curves obtained for the panels tested experimentally  
 599 by Chong et al. (1994) at the University of Plymouth; input curves for each truss beam of  
 600 the discrete system.



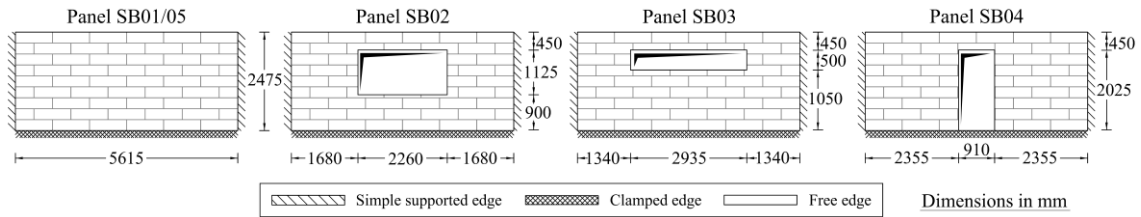
601

602 **Fig. 6.** Masonry panels out-of-plane loaded at University of McMaster (Gazzola and  
 603 Drysdale 1986); description of the geometry and boundary conditions.



604

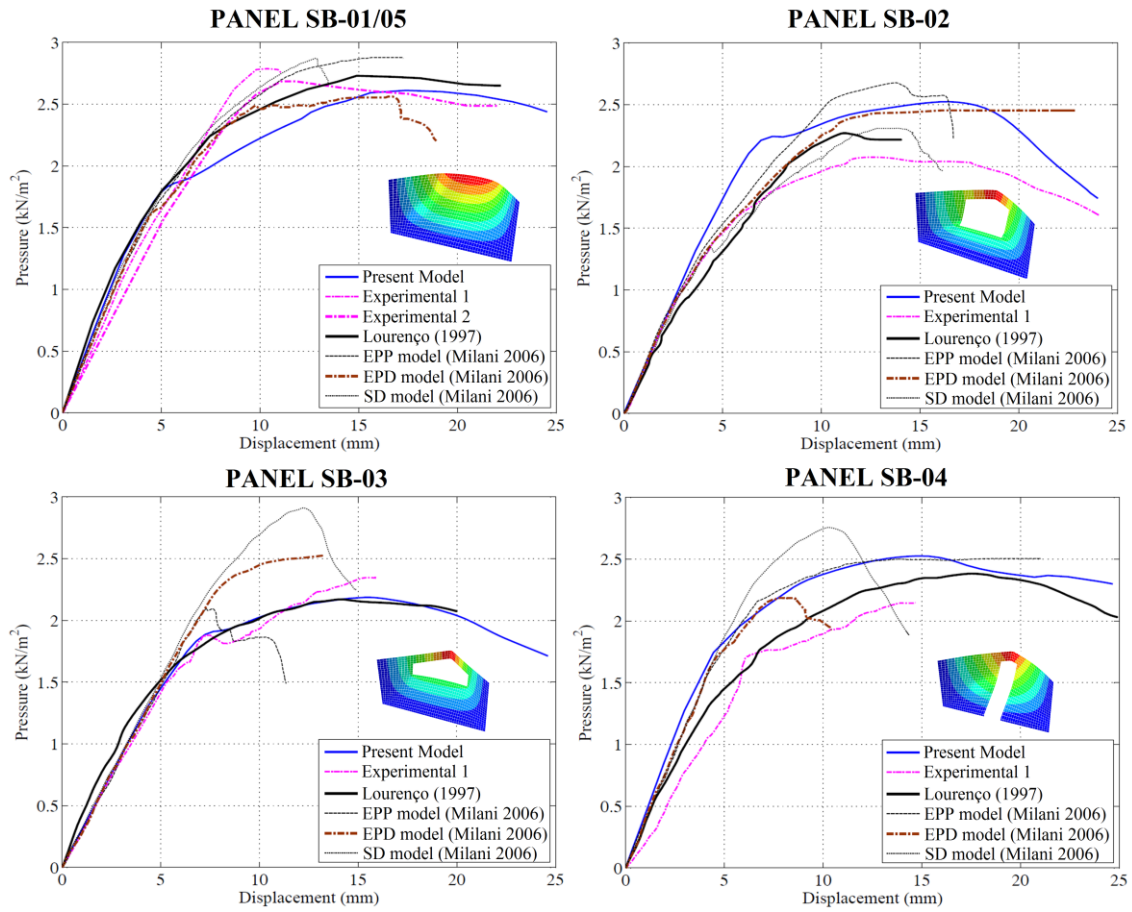
605 **Fig. 7.** Numerical and experimental curves of the panels experimentally tested by Gazzola  
 606 and Drysdale (1986): pressure load vs displacement.



607

608 **Fig. 8.** Masonry panels out-of-plane loaded at University of Plymouth (Chong et al.

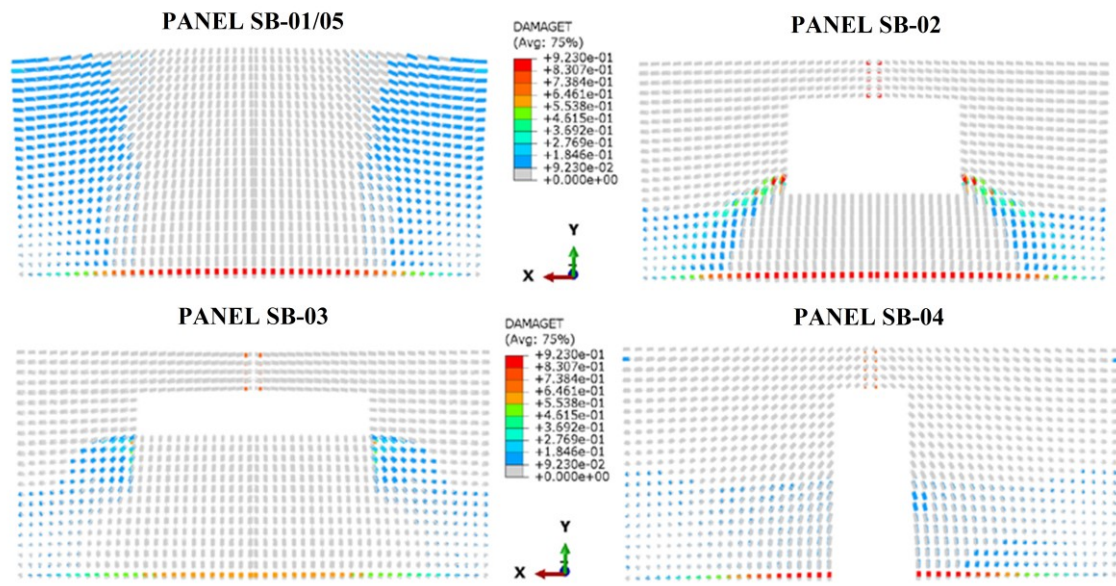
609 1994); description of the geometry and boundary conditions.



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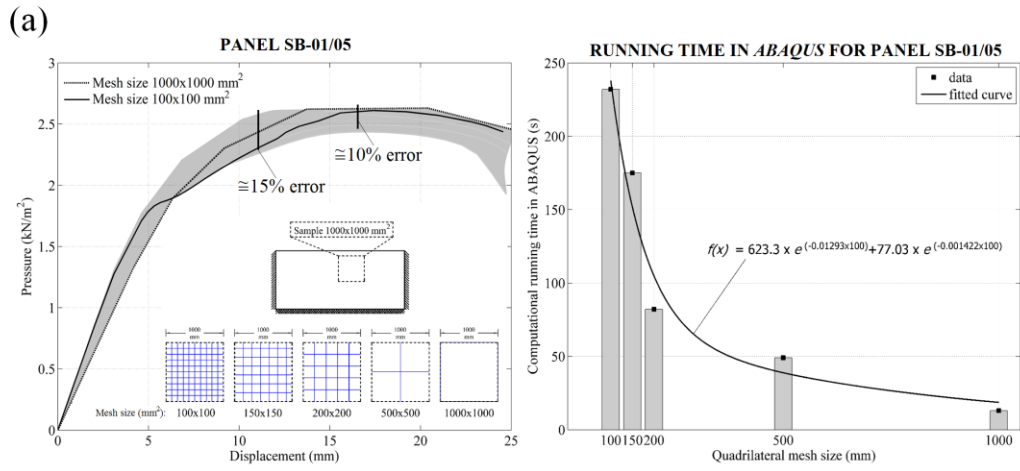
611 **Fig. 9.** Numerical and experimental curves of the panels experimentally tested by Chong

612 et al. (1994): pressure load vs displacement and deformed shapes at ultimate load level.

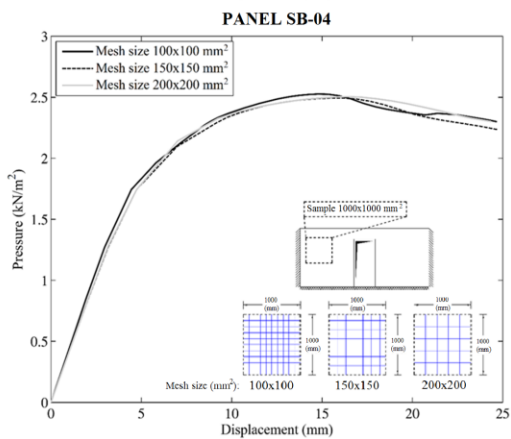
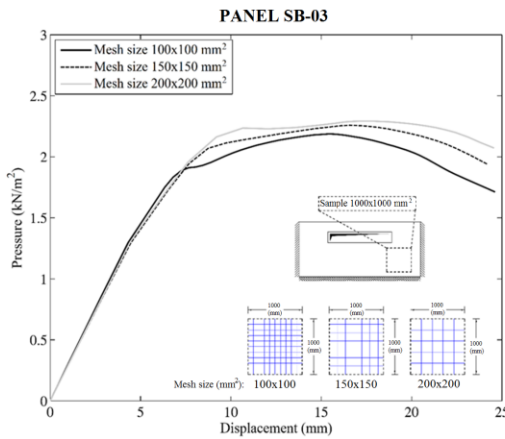
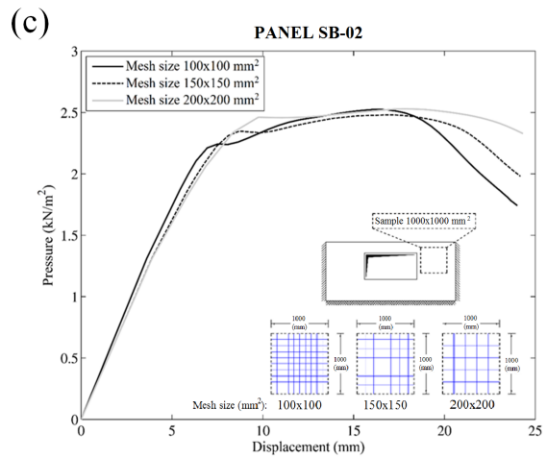
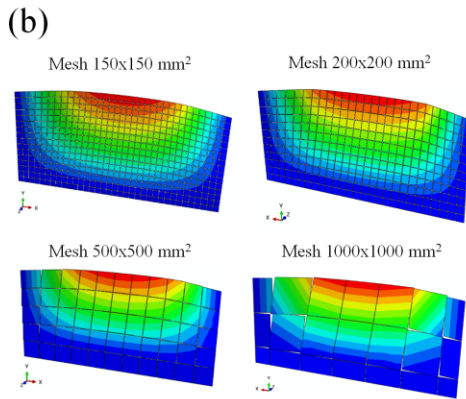
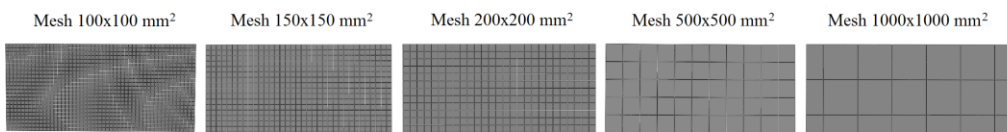


613

614 **Fig. 10.** Damage patterns obtained from the numerical analyses (ultimate load).



Panel SB01/05:



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618

**Fig. 11.** (a) Mesh dependence for the SB-01/05 panel; (b) deformed shapes for the less refined meshes for Panel SB-01/05; (c) mesh dependence study for the SB-02, SB-03 and SB-04 panels.



619 **Table 1.** Mechanical properties adopted for the homogenization step for both McMaster  
 620 and Plymouth University panels.

Parameter	Panels	
	McMaster	Plymouth
Young's Modulus of the mortar (MPa)	4000	3500
Young's Modulus of the brick (MPa)	15000	10000
Poisson coefficient (-)	0.20	0.20
Shear Modulus (MPa)	2000	1500
Cohesion, c (MPa)	1.6 x $f_t$	1.2 x $f_t$
Tensile strength $f_t$ (MPa)	0.35	0.52
Compressive strength $f_c$ (MPa)	20.0	2.0
Friction angle ( $\phi$ ) (degrees)	30.0	30.0
Linearized compressive cap angle ( $\psi$ ) (degrees)	45.0	50.0
Mode I fracture energy, $G_f^I$ (N/mm)	0.018	0.010
Mode II fracture energy, $G_f^{II}$ (N/mm)	0.022	0.012
Elastic Parameters (for a mesh size: H = 100 mm; e=10 mm)		
$K_n$ - axial truss (MPa)	236.74	157.83
$K_n$ - torque truss (MPa)	191761	27874
Axial truss area (mm <sup>2</sup> )	3750	2562.5
Torque truss area (mm <sup>2</sup> )	500	500

621