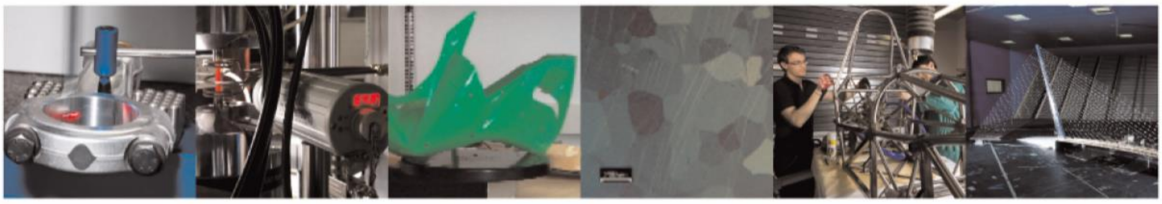

Force analysis as a support to computer-aided tolerancing of planar linkages

Armillotta, Antonio

Force analysis as a support to computer-aided tolerancing of planar linkages

Antonio Armillotta

Dipartimento di Meccanica, Politecnico di Milano
Via La Masa 1, 20156 Milano, Italy
Phone: +39 02 23998296
Fax: +39 02 23998585
E-mail: antonio.armillotta@polimi.it

Abstract

The paper proposes a method for tolerance analysis on planar structures and mechanisms. As for generic assemblies, deviations on a functional requirement involving different parts are calculated from tolerance specifications on individual part dimensions. For a linkage the requirement is usually the position or orientation of a given element (link, joint) with respect to a fixed frame, and the solution of the problem has some additional difficulties such as the correct treatment of joint clearances. The paper points out that, for underconstrained or exactly constrained linkages, tolerance analysis can be treated by analogy with a force analysis problem. The static analogy is first introduced and justified from well-known concepts of classical mechanics, and then translated into a calculation procedure that is able to evaluate the propagation of deviations from different types of dimensions (lengths of links, diameters of pins and holes at joints), as well as from linear and angular actuators. Problems of high complexity are shown to be easily solvable by the proposed method by means of available software tools for the structural analysis of planar frames and trusses.

Keywords: tolerance analysis, truss structure, mechanism, joint clearance, finite element analysis.

1. Introduction

In a mechanical assembly, the design of parts and connections is generally subject to a set of functional requirements defined as geometric entities. These can include the position or orientation of a body supported by a static structure, or the trajectory of the endpoint of a mechanism. It is often needed that some requirement is satisfied within a given tolerance, which however cannot be directly specified on individual parts because it involves features on different parts. To overcome this difficulty, tolerance analysis evaluates the resulting deviation on a requirement as a stackup of deviations on the dimensions of individual parts, for which either worst-case limits or statistical distributions are given [1]. Several methods are available in literature for this task under a wide range of assumptions on part geometries and connection types (see [2] for a recent survey).
The stackup of deviations is of special concern for mechanical linkages, which are the building blocks of many high-precision mechanisms and structures. Such assemblies are typically made of slender parts (links), connected at their ends by some types of kinematic pairs (joints). Tolerance analysis on linkages can rely upon general-purpose methods, possibly adapted to the specific domain of geometric configurations through dedicated rules and procedures. More often, however, the problem is treated with due consideration to related design tasks (kinematic, static and dynamic analysis of mechanisms) that can in turn be influenced by geometric deviations.

This paper illustrates a method for tolerance analysis on structures and mechanisms, which extends a basic calculation procedure proposed for generic mechanical assemblies [3]. The method is based on a static analogy, which evaluates the influence of manufacturing errors on links and joints by solving an equivalent problem of force analysis. As a potential advantage over existing methods, the approach allows to calculate tolerance stackups by means of well-known methods and tools for structural analysis, such as finite-element software. The method applies to planar linkages composed by rigid links connected by translational and rotational kinematic pairs; at current stage of development, it assumes that the linkage assembly is not overconstrained and considers only deviations on linear and angular dimensions, with the exclusion of otherwise defined geometric characteristics (e.g. orientation, location and profile).
The remainder of the paper is organized as follows. Section 2 reviews the literature on tolerance analysis for mechanisms, including a few existing methods based on similar analogies. Section 3 illustrates the basic procedure based on force analysis and introduces some extensions on its previous formulation. Section 4 verifies the correctness of the method by comparison to explicit analytic solutions on two simple examples. Section 5 demostrates the application of the whole tolerance analysis procedure on a more complex example. Section 6 discusses the advantages and limitations of the proposed approach and outlines its future developments.

2. Background

Tolerance analysis has been extensively studied in the context of linkages. These are generally defined as mechanisms with one or more degrees of freedom (DOF), considering static structures as special cases. The complexity of the problem depends on the range of geometric properties that a mechanism is allowed to have. Basic assumptions include dimensionality (planar or spatial mechanisms) and mobility (number of DOF, degree of overconstraining). Joint types can either be limited to lower pairs with fixed contacts (e.g. cylindrical, prismatic, helical and spherical pairs) or include higher pairs with continuously changing contact surfaces (e.g. gear and cam profiles) where geometric deviations may cause jamming or other degenerate configurations. Further difficulties arise when the analysis is intended to cope with elastic deformations of parts or uncertainty regions around nominal trajectories, often relevant in robotic and function-generating mechanisms.
Under the same assumptions of the present paper (planar linkages, rigid parts, fixed contacts, no overconstraining, linear and angular dimensions), stackups are usually calculated by methods developed for generic assemblies. As most standard approaches apply to static assemblies, mechanisms are regarded as stationary and analyzed over a representative sample of configurations. Functional requirements are usually linearized, thus reducing the problem to the evaluation of sensitivities (partial derivatives of a requirement with respect to individual dimensions). As pointed out in [4], analytic calculation of sensitivities is feasible for very simple mechanisms, while realistic problems can only be solved by algorithmic procedures suitable for software implementation. One of these is the direct linearization method, which builds a Jacobian matrix of the requirements from one or more vector loops identified in the linkage; the approach has been applied to planar and spatial mechanisms in [5, 6, 7] and recently revised in [8, 9] with some modifications allowing to deal with elastic deformations. Planar mechanisms with deformable links are also treated in [10, 11] by Monte Carlo simulation guided by statistical methods to limit computational efforts. In [12, 13] Monte Carlo simulation is avoided by using explicit sets of inequalities to identify regions of given probability of requirement deviations (mechanism reliability) in the space of the geometric

variables. Alternatives to simulation are sought especially for mechanisms with local overconstrainings; these have been treated by analytic evaluation of direct kinematics on lower-mobility manipulators [14, 15] and by other reliability-based methods [16, 17, 18, 19, 20].
Kinematic tolerance analysis is an extended formulation developed to release the above assumptions. Functional requirements are not limited to relative positions and orientations, but can include complex kinematic functions such as the drive ratio or the amount of backlash. The problem is thus regarded as an extension of the kinematic analysis of mechanisms, and can take advantage of the graphical and analytical methods developed for that task (e.g. [21]). The concept has been laid down in early studies such as [22, 23, 24], which have proposed solutions based on Monte Carlo simulation and sensitivity analysis; however, such approaches are computationally inefficient and can be difficult to apply in the presence of higher pairs. These have been recently treated by a simulation-based approach where contact conditions are evaluated by perturbed geometric representations of joint features according to the skin model of ISO standards [25, 26]. Another approach suitable for higher pairs is based on the concept of configuration space, originally used for kinematic analysis [27, 28] and later extended to consider deviations on geometric variables [29]. By algorithms of computational geometry, the domain of valid configurations of the mechanism is built in the space of the DOF of individual links; in ideal conditions the configuration space is made of continuous lines, which grow and warp in the presence of deviations creating areas or volumes (clearances) and discontinuities (degenerate configurations). In [30, 31, 32] the approach has been demonstrated on general planar mechanisms, including some real cases of high complexity, while further developments and modifications have been later introduced in [33, 34, 35, 36, 37].
Even for mechanisms including only lower pairs, a further need recognized by many researchers is the correct treatment of joint clearances. As a consequence of deviations on the dimensions involved in a joint, e.g. diameters of holes and pins in cylindrical pairs, translational and rotational displacement (shifts) arise between the connected links; unlike deviations on link lengths, their amount and direction change continuously and cannot be compensated. A few methods for tolerance analysis, mostly developed for robotic mechanisms, provide a specific treatment of joint clearances by introducing modifications in well-known models of kinematic analysis (see [38] for a comparison). The vector-loop model has been enhanced by virtual links corresponding to clearances (clearance vector model) possibly considering the effect of lubrication [39]. The Denavit-Hartenberg model has been extended with the addition of either differential terms [40], stochastic terms (kinematic reliability [41]) and composition rules among tolerance intervals (interval arithmetic [42]). The screw-theory model has been extended with virtual links [43, 44, 45, 46] and by an analogy with Kirchhoff's laws of electrical circuits [47]. Other approaches emphasizing the treatment of joint clearances include rotatability [48, 49, 50], dual algebra [51] and inverse kinematic [52, 53]. Beside influencing the stackup of deviations, joint clearances have also a complex influence on jerk, impact and vibration of mechanisms, as widely discussed in the contexts of kinematic and dynamic analysis [54, 55, 56, 57, 58, 59, 60, 61].
The method proposed here is based on a static analogy of the tolerance analysis problem, which draws its justification by the principle of virtual work of rigid bodies [3]. Approaches relying on statics have been previously proposed for tolerance analysis on selected types of mechanisms, such as spatial linkages and closed-loop manipulators. In [62, 63, 64, 65], the principle of virtual work allows to predict the actual shifts at joints from the external forces acting on a mechanism. In [66], the sensitivities are calculated from the internal forces induced in the links by unit loads along

reference directions: although not extensively discussed and developed into application rules, that method is actually very similar to the one proposed here.

Tolerance analysis is a prerequisite to other design problems on mechanisms, which have received limited attention so far. These include optimal tolerance allocation [67], kinematic synthesis in the presence of deviations [68], and robust design of link dimensions to mimimize variation in functional requirements [69].

3. Static analogy

After a brief definition of the tolerance analysis problem, the method based on static analogy is described in the following. A previous formulation [3] is recalled and then extended to cover a wide class of planar structures and mechanisms.

3.1 Prior results

A linkage is an assembly of parts (mainly links and hinge pins), which collectively fulfill a static or kinematic function depending on a set of dimensions

$$x_i = x_{0i} + \delta x_i \quad (i = 1, 2, \ldots n)$$

where x_{0i} is the nominal value and δx_i is the deviation on a dimension. A deviation is caused by errors in the manufacturing process of a part and is assumed to take random values within a symmetric tolerance interval:

$$|\delta x_i| \le T_i$$

Tolerance T_i is specified at design stage for each dimension. The linkage is subject to a functional requirement, defined as the position of a given point of the system with respect to a fixed coordinate axis y. The deviation δy on the requirement is to be controlled according to an additional design specification, which can in turn be defined as a tolerance although it is referred to the whole assembly rather than on an individual part:

$$|\delta y| \le T_y$$

In the tolerance analysis problem, the tolerance T_y on the requirement must be calculated from the tolerances T_i on individual dimensions. For this purpose, the deviation δy is expressed as a function of the deviations δx_i. This is not a trivial task as the functional equation

$$y = f(x_1, x_2, \ldots x_n)$$

which relates the requirement to the dimensions, is nonlinear and usually not explicitly known. If deviations are small, however, an approximate linear relation can be assumed among the deviations:

$$\delta y \approx \sum_{i=1}^{n} s_i \delta x_i \,, \quad s_i = \frac{\partial y}{\partial x_i}$$

The method calculates the sensitivities s_i of the dimensions on the requirement under the assumptions that the assembly is rigid and exactly constrained, and that all dimensions (y and x_i) are linear distances. The calculation is based on a static analogy, which is described below in two steps.

The first one considers only deviations on the lengths of links, while the second one broadens the analysis to the deviations on the dimensions of rotational joints (diameters of pins and holes).

The linkage in Fig. 1a is an exactly constrained truss, where each link has length l_i and the position of a selected point (here coinciding with the center of a joint) is to be controlled along the direction y. Tolerance analysis aims at calculating the sensitivity s_i of each dimension l_i on the requirement y, i.e. the ratio of deviation δy to deviation δl_i. For this purpose an equivalent static model is built, where an external force F is applied along direction y to the control point (Fig. 1b). As the truss is in equilibrium, known methods of force analysis allow to calculate the support reactions and the internal forces F_i of the links (tensile or compressive depending on the sign). The desired sensitivities are then equal to

$$s_i = \frac{F_i}{F}$$

A formal justification of the above result can be given with reference to related concepts of statics. The deviation δl_i can be regarded as a virtual displacement of one end of the link with respect to the other along the lengthwise direction; similarly, the deviation δy corresponds to a virtual displacement of the control point along the reference direction, which is congruent to the link displacements since it is determined by the assembly relations of the system. For the principle of virtual work for rigid bodies, the system is in equilibrium if and only if the total virtual work of the external forces equals zero. If the i-th link is removed from the system, such condition is satisfied if

$$F\delta y - F_i \delta l_i = 0$$

This can be rewritten as:

$$\delta y = \frac{F_i}{F}\delta l_i \quad \rightarrow \quad \frac{\partial y}{\partial l_i} = \frac{F_i}{F}$$

A rotational joint is now considered on a similar linkage. The dimensions involved in the joint (Fig. 2a) are the diameter d of the pin and the diameters D_j of the holes at the ends of the connected links (in this case j = 1, 2, 3). Due to the radial clearances of the pin with the holes, the deviations on these dimensions contribute to the deviation on the functional requirement of the whole system. Their sensitivities can again be calculated through an equivalent static model, where the force F applied to the control point induces a force on each contacting feature. Specifically, each hole is subject to an outwardly-directed force F_j along the lengthwise direction of its link, while the pin is subject to three inwardly-directed forces F_j (Fig. 2b). If the F_j are regarded as unsigned force intensities, the desired sensitivities are equal to:

$$s_{Dj} = \frac{1}{2}\frac{F_j}{F}$$

$$s_d = -\frac{1}{2}\sum_j \frac{F_j}{F}$$

To justify that, it can be observed that the force F brings about translational shifts h_j of the three links with respect to the pin along the directions of the forces F_j (Fig. 2c). Considering these shifts as virtual displacements and removing the pin from the system, the equilibrium condition is

$$F\delta y - \sum_j F_j h_j = 0$$

This gives the sensitivities of the shifts on the functional requirement:

$$\frac{\partial y}{\partial h_j} = \frac{F_j}{F}$$

But the shifts are related to the dimensions through the following expression (Fig. 2c):

$$h_j = \frac{D_j}{2} - \frac{d}{2}$$

which can be differentiated to find the sensitivities of the dimensions:

$$\frac{\partial y}{\partial D_j} = \sum_j \frac{\partial y}{\partial h_j}\frac{\partial h_j}{\partial D_j} = \frac{1}{2}\frac{F_j}{F}$$

$$\frac{\partial y}{\partial d} = \sum_j \frac{\partial y}{\partial h_j}\frac{\partial h_j}{\partial d} = -\frac{1}{2}\sum_j \frac{F_j}{F}$$

The rule applies for any number of connected links. If the pin is connected to an external support, the intensity of the reaction is an additional force F_j to be included in the summation. If the external load is applied to the pin, its intensity must obviously be excluded from the summation.

3.2 Extension

The original intent of the static analogy was to allow an easy analysis of tolerances on assemblies where the internal forces on individual parts can be calculated by graphical constructions or direct application of equilibrium equations. This imposed the restrictive assumptions of rigid parts and exactly constrained assemblies. Although they cover only a small subdomain of applications for structural analysis, the same assumptions are consistent with all the cases that can be treated by commercial software tools for tolerance analysis. Precision requirements on overconstrained and compliant assemblies depend on built-in stresses created during assembly; tolerance analysis methods that deal with this complication are currently limited to very simple and specific assembly configurations.

For planar linkages, force analysis is widely supported by dedicated methods and software tools. This is a further advantage for tolerance analysis, since it can allow to easily treat highly complex structures and mechanisms. To exploit such opportunity, the method needs to include additional cases that were not considered in the original formulation. Some useful extensions in this direction are introduced below.

In the truss of Fig. 3a the requirement y depends not only on the lengths of the links, but also on the horizontal coordinate z of an external support whose sensitivity is to be calculated. The equivalent static model (Fig. 3b) includes an additional link with nonzero length along direction z. The link is connected to any support that can provide a reaction along the same direction; in this case the new link might also be connected to the other existing support as it is aligned to the link along z. Under the external force F, the additional link is subject to an internal force F_z that can be calculated by force analysis. Considering the deviation δz as a virtual displacement and removing the additional link, the virtual work of the external forces now includes also a term equal to $-F_z\delta z$. As in the case of link lengths, the desired sensitivity is then equal to

$$\frac{\partial y}{\partial z} = \frac{F_z}{F}$$

All the above examples are related to structures with rotational joints. This assumption can be released by also considering translational joints, which have special interest for mechanisms. Fig. 4a shows a mechanism with one DOF given by a slider, which is driven by an actuator to control the total length l_{m} of the connected links with random deviation δl_{m}. The effect of the positioning accuracy on the requirement y in a given configuration of the mechanism can be evaluated by a static model (Fig. 4b), where the two connected links are consolidated into a single link with length l_{m}. The structure is exactly constrained, thus force analysis easily provides the internal force F_{m} to the solid link. The usual application of the principle of virtual work gives the sensitivity associated to the dimension l_{m}:

$$\frac{\partial y}{\partial l_{\mathrm{m}}} = \frac{F_{\mathrm{m}}}{F}$$

Another assumption to be released is that only linear distances are considered for dimensions and requirements. Extending the static analogy to angular dimensions is essential for analyzing mechanisms with rotary actuators. In Fig. 5a the configuration of a four-bar linkage depends not only on the lengths of the links, but also on the angular position ξ_1 of the crank, which is controlled with random deviation $\delta\xi_1$. In the equivalent static model (Fig. 5b) the rotational DOF is removed by introducing a fixed end on the crank at the joint. The system is now exactly constrained and force analysis provides the internal forces on the crank in the fixed end section. These include the normal force F_1 and the torque M_1 (the shear force T_1 will not considered further). Removing the fixed connection on the crank, the equilibrium condition gives

$$F\delta y - M_1\delta\xi_1 = 0$$

the sensitivity of ξ_1 (i.e. of the angular positioning accuracy of the actuator) is thus equal to

$$\frac{\partial y}{\partial \xi_1} = \frac{M_1}{F}$$

A similar reasoning applies to structures or mechanisms where a functional requirement is associated to the angular dimension φ of a link. In this cases the external load acting on the equivalent static model would be a torque M applied to the considered link with the same orientation as φ. Again, force analysis would provide the internal forces F_i at the links (or possible bending moments corresponding to angular dimensions). As the virtual work of the external torque is $M\delta\varphi$, the sensitivities of link lengths would then be given by the following expression:

$$\frac{\partial \varphi}{\partial l_i} = \frac{F_i}{M}$$

4. Validation

The above justification of the static analogy rests upon the association of deviations with virtual displacements, which is a reasonable abstraction but cannot be regarded as a proof of correctness. For this reason, the method will be now verified on two simple examples by comparison to explicit analytic solutions. This will also allow a first demonstration of the calculation of sensitivities on both structures and mechanisms.

4.1 Structure example

Fig. 6a shows an exactly constrained truss including three links with lengths A, B and C, arranged as a right-angle triangle in the nominal configuration. The external supports (a hinge at joint 2 and a slider at joint 3) define the reference direction y along which the position of joint 1 is to be controlled. The equivalent static model (Fig. 6b) can be analyzed by solving the equilibrium equations to find the support reactions and the internal forces at the links (Fig. 6c). These allow to calculate the sensitivities of either link lengths:

$$\frac{\partial y}{\partial A}=0\ ,\quad \frac{\partial y}{\partial B}=-\frac{B}{A}\ ,\quad \frac{\partial y}{\partial C}=\frac{C}{A}$$

pin diameters:

$$\frac{\partial y}{\partial d_1}=-\frac{1}{2}\frac{B+C}{A}\ ,\quad \frac{\partial y}{\partial d_2}=-\frac{C}{A}\ ,\quad \frac{\partial y}{\partial d_3}=-\frac{B}{A}$$

and link holes:

$$\frac{\partial y}{\partial D_{2A}}=\frac{\partial y}{\partial D_{3A}}=0\ ,\quad \frac{\partial y}{\partial D_{1B}}=\frac{\partial y}{\partial D_{3B}}=\frac{1}{2}\frac{B}{A}\ ,\quad \frac{\partial y}{\partial D_{1C}}=\frac{\partial y}{\partial D_{2C}}=\frac{1}{2}\frac{C}{A}$$

where all lengths have nominal values, d_i (i = 1, 2, 3) is the diameter of the pin at joint i, and D_{ij} (i = 1, 2, 3; j = A, B, C) is the diameter of the hole in link j at joint i.
The sensitivities of link lengths can be verified by finding the explicit relation of y with A, B and C. The diagram of the truss is redrawn considering a generic configuration with deviations (Fig. 7a), whose inspection gives the following expression:

$$y=\frac{A^2-B^2+C^2}{2A}$$

where A, B and C are the actual values of link lengths; the partial derivatives of y at nominal values of dimensions (i.e. assuming again $A^2 + B^2 = C^2$), coincide with the sensitivities calculated by the static analogy.
For a geometric verification of the sensitivities of pin and hole diameters, the only dimensions D_{2C} e d_2 will be considered. In the nominal configuration the holes in links A and C at joint 2 can be assumed as perfectly aligned, due to either zero clearance with the pin or the centering effect of lubrication. Due to deviations on diameters, a shift occurs between the centers of the two holes. Considering the hole on link A as fixed, the displacement of the hole on link C has the same effect as an increase of the length C whose sensitivity is known.
Specifically, a deviation δD_{2C} (increase of hole diameter) involves a shift h_{C2} of the hole on C with respect to the pin (Fig. 7b). Its effect is equivalent to the following increase of length C:

$$\delta C'_{\mathrm{eq}} = h_{C2} = \frac{\delta D_{2c}}{2}$$

A deviation -δd_2 (decrease of pin diameter) involves again a shift h_{C2} as well as an additional shift h_{2A} of the pin with respect to the hole on A (Fig. 7c). The effect of the two shifts is equivalent to the following increase of length C:

$$\delta C''_{\mathrm{eq}} = h_{C2} + h_{2A} = -\frac{\delta d_2}{2} - \frac{\delta d_2}{2} = -\delta d_2$$

The total increase of length C equivalent to the two deviations is then

$$\delta C_{\mathrm{eq}} = \delta C'_{\mathrm{eq}} + \delta C''_{\mathrm{eq}} = \frac{\delta D_{2C}}{2} - \delta d_2$$

and its contribution to the deviation on the requirement is given by

$$\delta y = \frac{\partial y}{\partial C}\delta C_{\mathrm{eq}} = \frac{C}{A}\delta C_{\mathrm{eq}} = \frac{C}{A}\left(\frac{\delta D_{2C}}{2} - \delta d_2\right) = \frac{1}{2}\frac{C}{A}\delta D_{2C} - \frac{C}{A}\delta d_2$$

This confirms the the sensitivities of D_{2C} and d_2 calculated by the static analogy.

4.2 Mechanism example

Fig. 8a shows a crank-slider mechanism with crank length A and rod length B, in the configuration where the crank angle is β. The position of the slider is to be controlled along the translational direction y. The tolerance analysis will be limited to the sensitivities of A, B and β, while the sensitivities of joint-related dimensions can be separately calculated as in the previous example.
The equivalent static model (Fig. 8b) can be easily analyzed to determine explicit analytic expressions for the internal forces (Fig. 8c). These include the rod angle α, an additional variable related to the input dimensions:

$$\sin\alpha = \frac{A}{B}\sin\beta$$

According to the static analogy, the desired expressions of sensitivities are equal to the normal forces F_A and F_B on the two links and to the bending moment M_β in the fixed end section, divided by the external force F:

$$\frac{\partial y}{\partial A} = \frac{F_A}{F} = \frac{\cos(\alpha+\beta)}{\cos\alpha}$$
$$\frac{\partial y}{\partial B} = \frac{F_B}{F} = \frac{1}{\cos\alpha}$$
$$\frac{\partial y}{\partial \beta} = \frac{M_\beta}{F} = -\frac{A\sin(\alpha+\beta)}{\cos\alpha}$$

The analytic expression of y is needed for a verification of the above results. The additional variable gives the following expression:

$$\cos\alpha = \sqrt{1 - \frac{A^2}{B^2}\sin^2\beta}$$

which is differentiated with respect to the dimensions:

$$\frac{\partial(\cos\alpha)}{\partial A} = -\frac{1}{B}\frac{\sin\alpha\sin\beta}{\cos\alpha}$$
$$\frac{\partial(\cos\alpha)}{\partial B} = \frac{1}{B}\frac{\sin^2\alpha}{\cos\alpha}$$
$$\frac{\partial(\cos\alpha)}{\partial \beta} = -\frac{A}{B}\frac{\sin\alpha\cos\beta}{\cos\alpha}$$

Considering also that

$$y = A\cos\beta + B\cos\alpha$$

the partial derivatives of y are equal to the sensitivities calculated by the static analogy:

$$\frac{\partial y}{\partial A} = \cos\beta + B\frac{\partial(\cos\alpha)}{\partial A} = \cos\beta - \frac{\sin\alpha\sin\beta}{\cos\alpha} = \frac{\cos(\alpha+\beta)}{\cos\alpha}$$
$$\frac{\partial y}{\partial B} = \cos\alpha + B\frac{\partial(\cos\alpha)}{\partial B} = \cos\alpha + \frac{\sin^2\alpha}{\cos\alpha} = \frac{1}{\cos\alpha}$$
$$\frac{\partial y}{\partial \beta} = -A\sin\beta + B\frac{\partial(\cos\alpha)}{\partial \beta} = -A\sin\beta - A\frac{\sin\alpha\cos\beta}{\cos\alpha} = -\frac{A\sin(\alpha+\beta)}{\cos\alpha}$$

5. Application

The examples discussed so far were simple enough for an analytic solution. Practical design cases often involve a larger set of dimensions, multiple functional requirements and relations among tolerances of different parts of the assembly. A further example is provided below to demonstrate the application of the proposed procedure in such situations.

Fig. 9 shows a linkage for the generation of exact straight-line motion, of the type referred to as Peaucellier cell [70]. The mechanism has 7 links and 6 rotational joints, two of which (1 and 2) are connected to an external frame. An input rotation of link 1-3 (crank) about joint 1 determines a displacement of joint 6 along the normal to line 1-2. The coordinate system xy is fixed to the external frame with origin at joint 1, x-axis along line 1-2, and thus y-axis parallel to the direction of generated motion. The tolerance analysis problem consists in evaluating the deviations on the position of point 6 along x and y from the deviations on the dimensions of individual parts of the mechanism. These include the distance between the two external supports, the lengths and the hole diameters for each link, the pin diameter for each joint, and the angular position of the crank. The only nominal values of link lengths will be assumed for the moment ($L_{12} = L_{13} = 1$ m, $L_{24} = L_{25} = 3$ m, $L_{34} = L_{34} = L_{46} = L_{56} = 1.5$ m), postponing further design specifications to a later phase.

The equivalent static model is the exactly constrained frame in Fig. 10. To include the distance between the supports in the analysis, the frame has an additional link 1-2 and replaces the hinge at joint 2 with a slider. All the joints allow free rotation as in the original mechanism, except for the fixed connection between the two beams converging at joint 1 which corresponds to the angular dimension α imposed to the crank. The model should be replicated for a representative sample of configurations of the mechanism, which in this case will be limited to just two values of α (0, 30°). Unit loads along the two reference directions ($F_x = F_y = 1$ kN) are alternatively applied to joint 6 for each configuration.

Although the frame might be easily analyzed by direct inspection of the free-body diagrams, software calculations are more advisable in practice to speed up the analysis and to avoid mistakes. The finite element method with plane-truss and plane-frame elements is suitable for the force analysis in the whole range of applications of the proposed approach. Figs. 11 and 12 show the results of the analyses for both configurations under the two loads F_x e F_y, calculated by the freely available software Ftool [71]. The diagrams include the data needed for the evaluation of sensitivities from the static analogy, namely the support reactions (R_1, R_2) and the normal forces in the links (F_{12}, F_{13}, F_{24}, F_{25}, F_{34}, F_{35}, F_{46}, F_{56}) in kN, and the bending moment at joint 1 (M_1) in KNm.

Tab. 1 shows the calculation of the sensitivities of individual dimensions on the position of the control point along directions x and y in the two configurations. Each sensitivity is expressed as a function of the above calculated forces; all equations are independent on the configuration and apply for any value of α within the physical limits of motion. Due to the symmetry of the mechanism, it is immediate to deduce the sensitivities for α = -30° and thus the variation in the absolute value of each parameter. As a further note, if the mechanism is scaled without changing the proportions of link lengths, all the sensitivities remain unchanged except for that of angle α which derives from a bending moment and is thus proportional to the scaling factor.

The analysis can be enhanced by considering additional design constraints. It is reasonable to assume that all the pins and all the links with the same length are manufactured as a common part type. This choice leads to aggregating the tolerance specifications into as few as 7 distinct values, which involve the following dimensions: distance of the two supports in the external frame (T_{LA}), lengths of the three types of links (T_{LB}, T_{LC}, T_{LD}), common diameter of the holes in all links (T_{D}), common diameter of all pins (T_{d}); crank angle imposed by the actuator (T_{α}).

The sensitivities of the aggregate dimensions depend on the stackup criterion adopted in the tolerancing study. A suitable choice in this case is the root sum square (RSS) stackup, which calculates the resulting tolerance T_y on a requirement from the tolerances T_i specified on n related dimensions by the following equation:

$$T_y = \sqrt{\sum_{i=1}^{n} s_i^2 T_i^2}$$

This criterion is theoretically valid if every dimension has a normal distribution with mean equal to the nominal value and standard deviation in a constant ratio to the tolerance. These conditions may appear too limiting for linkages, as the shifts due to joint clearances (which are implicitly included in the sensitivities) are likely to violate the normality assumption if the centering effect of the lubricant films is disrupted by heavy loads and inertia forces. However, in the presence of a large number of dimensions such violations have a limited effect on the normality of the requirement. The RSS stackup can then be reasonably assumed, possibly with an inflation factor not considered here. The sensitivity of an aggregate dimension s_{a} can thus be calculated from the sensitivities of the related dimensions s_j (j = 1,… m) as

$$s_{\mathrm{a}} = \sqrt{\sum_{j=1}^{m} s_j^2}$$

Tab. 2 shows the aggregate tolerances and their sensitivities.

Tab. 3 shows the results of the analysis from a given set of tolerance specifications, selected in the medium tolerance class according to the ISO 2768-1 standard [72]. The individual contributions of each tolerance to the RSS stackups of the two requirements is also listed both in absolute value (not additive):

$$\sqrt{s_i^2 T_i^2} = s_i T_i$$

and in percentage value (additive):

$$100 \frac{s_i^2 T_i^2}{\sum_{i=1}^{n} s_i^2 T_i^2}$$

Some final observations can be made within the limits of the specified tolerances and of the range of analyzed configurations. The tolerances on the two requirements are in the order of 2 to 4 times the tolerances on link lengths, with slightly higher values along the normal to the generated motion. For both of them, the main contribution (about half of the total amount) comes from the tolerance on the length of the links that form the four-sided polygon next to the control point. The tolerance on hole diameters has a negligible contribution on the requirements. The tolerance on pin diameters (which is not completely independent on the former for cost reasons) has a stronger yet limited contribution; however, it would have a predominant effect along the normal to the generated motion if the actual deviations on link lengths were statically compensated on the assembled mechanism. The angular deviation due to the actuator has an effect only along the motion direction, where it contributes to the deviation for about a third of the total amount (or for almost the whole amount if link length deviations are compensated). Except for verifications on a wider range of configurations, the symmetrical configuration of the mechanism ($\alpha = 0$) has the minimum deviation along the direction of generated motion and the maximum deviation along the normal direction.

6. Conclusions

As demonstrated in the last example, the proposed method allows straightforward application to practical cases of tolerance analysis. In perspective the static analogy should be particularly suitable for highly complex structures and mechanisms, which are customarily designed with the aid of computer-aided tools for structural analysis; the opportunity to use the same tools for tolerance analysis as well should encourage the correct and systematic treatment of tolerance specifications throughout the design process. Another advantage of the static analogy is the wide availability of algorithms for finite element analysis, which can help the development of dedicated software tools in order to streamline and extend the tolerance analysis procedure. Also attractive from a theoretical side is the idea that some optimization problems in the context of tolerancing, such as tolerance allocation and robust design of assembly configurations, might be treated by borrowing concepts and methods from the emerging research field of structural optimization.

Further developments of the work will try to improve the validation of the method, possibly by comparison with commercial software tools, and to remove some limitations of its current formulation. The main needed extensions (spatial mechanisms, higher pairs, overconstraining) seem to be naturally consistent with an approach based on static analogy.

Acknowledgements

This research received no specific grant from any funding agency.

References

[1] P.K. Singh, P.K. Jain, S.C. Jain, Important issues in tolerance design of mechanical assemblies. Part 1: tolerance analysis, Proc. Inst. Mech. Eng. Part C 223 (2009) 1225-1247.

[2] W. Polini, Geometric tolerance analysis, in: B.M. Colosimo, N. Senin (Eds.), Geometric tolerances, Springer, London, 2011, pp. 39-68.

[3] A. Armillotta, A static analogy for 2D tolerance analysis, Assem. Autom. 34 (2014) 182-91.

[4] A.A. Fogarasy, M.R. Smith, The influence of manufacturing tolerances on the kinematic performance of mechanisms, Proc. Inst. Mech. Eng. Part C 212 (1998) 35-47.

[5] C.D. Carr, A comprehensive method for specifying tolerance requirements for assemblies, ADCATS Report 93-1, Brigham Young University, 1993.

[6] K.W. Chase, A.R. Parkinson, A survey of research in the application of tolerance analysis to the design of mechanical assemblies, Res. Eng. Des. 3 (1991) 23-37.

[7] J.W. Wittwer, K.W. Chase, L.L. Howell, The direct linearization method applied to position error in kinematic linkages, Mech. Mach. Theory 39 (2004) 681-693.

[8] B.M. Imani, M. Pour, Tolerance analysis of flexible kinematic mechanism using DLM method. Mech. Mach. Theory 2009; 44:445-456.

[9] D.J.A. Villaroman, 2D tolerance analysis of truss-like assembly structure, Study paper 09564798, Hong Kong University of Science and Technology, 2013.

[10] J. Stuppy, H. Meerkamm, Tolerance analysis of a crank mechanism by taking into account different kinds of deviation, Proc. CIRP Conference on Computer Aided Tolerancing, Annecy, France, 2009.

[11] M. Walter, T. Sprügel, S. Wartzack, Tolerance analysis of systems in motion taking into account interactions between deviations, Proc. Inst. Mech. Eng. Part B 227 (2013) 709-719.

[12] Z. Shi, Synthesis of mechanical error in spatial linkages based on reliability concept, Mech. Mach. Theory 32 (1997) 255-259.

[13] Z. Shi, X. Yang, W. Yang, Q. Cheng, Robust synthesis of path generating linkages, Mech. Mach. Theory 40 (2005) 45-54.

[14] T. Huang, D.G. Chetwynd, J.P. Mei, X.M. Zhao, Tolerance design of a 2-DOF overconstrained translational parallel robot, IEEE Trans. Robot. 22 (2006) 167-172.

[15] H. Liu, T. Huang, D.G. Chetwynd, A general approach for geometric error modeling of lower mobility parallel manipulators, ASME J. Mech. Robot. 3 (2011) 021013.

[16] A. Ballu, J.Y. Plantec, L. Mathieu, Geometrical reliability of overconstrained mechanisms, CIRP Ann. Manuf. Technol. 57 (2008) 159-162.

[17] P. Beaucaire, N. Gayton, E. Duc, M. Lemaire, J.Y. Dantan, Statistical tolerance analysis of a hyperstatic mechanism using system reliability methods, Comput. Ind. Eng. 63 (2012) 1118-1127.

[18] P. Beaucaire, N. Gayton, E. Duc, J.Y. Dantan, Statistical tolerance analysis of a mechanisms with gaps based on system reliability methods, Proc. CIRP Conference on Computer Aided Tolerancing, Huddersfield, UK, 2012.

[19] P. Beaucaire, N. Gayton, E. Duc, J.Y. Dantan, Statistical tolerance analysis of over-constrained mechanisms with gaps using system reliability methods, Comput. Aided Des. 45 (2013) 1547-1555.
[20] K. Luo, X. Du, Probabilistic mechanism analysis with bounded random dimension variables, Mech. Mach. Theory 60 (2013) 112-121.
[21] R.L. Norton, Design of machinery, 2nd ed., McGraw-Hill, Boston, 1999.
[22] R.S. Hartenberg, J. Denavit, Kinematic synthesis of linkages, McGraw-Hill, New York, 1964.
[23] S. Dobowsky, F. Freudenstein, Dynamic analysis of mechanical systems with clearances, Part 1: Formulation of dynamic model, ASME J. Eng. Ind. 90 (1971) 305-316.
[24] R.E. Garret, A.S. Hall, Effects of tolerance and clearance in linkage design, ASME J. Eng. Ind. 91 (1969) 198-202.
[25] B. Schleich, S. Wartzack, Tolerance analysis of rotating mechanism based on skin model shapes in discrete geometry, Proc. CIRP Conference on Computer Aided Tolerancing, Hangzhou, China, 2014.
[26] B. Schleich, N. Anwer, W. Zhang, L. Mathieu, S. Wartzack, Skin model based simulation of contact and mobility for mechanical assemblies, Proc. CIRP Conference on Computer Aided Tolerancing, Hangzhou, China, 2014.
[27] L. Joskowicz, Mechanism comparison and classification for design, Res. Eng. Des. 1 (1990) 149-166.
[28] L. Joskowicz, E. Sacks, Computational kinematics, Artif. Intell. 51 (1991) 381-416.
[29] L. Joskowicz, E. Sacks, V. Srinivasan, Kinematic tolerance analysis, Comput. Aided Des. 29 (1997) 147-157.
[30] E. Sacks, L. Joskowicz, Parametric kinematic tolerance analysis of planar mechanisms, Comput. Aided Des. 29 (1997) 333-342.
[31] E. Sacks, L. Joskowicz, Parametric kinematic tolerance analysis of general planar systems, Comput. Aided Des. 30 (1998) 707-714.
[32] E. Sacks, L. Joskowicz, R. Schultheiss, U. Hinze, Computer-assisted kinematic tolerance analysis of a gear selector mechanism with the configuration space method, Computer Science Technical Reports, Paper 1434, Purdue University, 1999.
[33] M.H. Kyung, E. Sacks, Nonlinear kinematic tolerance analysis of planar mechanical systems, Comput. Aided Des. 35 (2003) 901-911.
[34] Y. Ostrovsky-Berman, L. Joskowicz, Tolerance envelopes of planar mechanical parts with parametric tolerances, Comput. Aided Des. 37 (2005) 531-544.
[35] Y. Ostrovsky-Berman, L. Joskowicz, Relative positioning of planar parts in toleranced assemblies, Comput. Aided Des. Appl. 2 (2005) 675-684.
[36] Y. Ostrovsky-Berman, L. Joskowicz, Relative position computation for assembly planning with planar toleranced parts, Int. J. Robot. Res. 25 (2006) 147-170.
[37] E. Sacks, L. Joskowicz, R. Schultheiss, M. Kyung, Towards robust kinematic synthesis of mechanical systems, in: P. Bourdet, L. Mathieu (Eds.), Geometric product specification and verification: integration of functionality, Springer, Berlin, 2003, pp. 135-144.
[38] C.R. Rocha, C.P. Tonetto, A. Dias, A comparison between the Denavit-Hartenberg and the screw-based methods used in kinematic modeling of robot manipulators, Robot. Comput. Integr. Manuf. 27 (2011) 723-728.
[39] J.H. Choi, S.J. Lee, D.H. Choi, Tolerance optimization for mechanisms with lubricated joints, Multibody Syst. Dyn. 2 (1998) 145-168.

[40] J. Kim, W.J. Song, B.S. Kang, Stochastic approach to kinematic reliability of open-loop mechanism with dimensional tolerance, Appl. Math. Model. 34 (2010) 1225-1237.

[41] S.S. Rao, P.K. Bhatti, Probabilistic approach to manipulator kinematics and dynamics, Reliab. Eng. Syst. Saf. 72 (2001) 47-58.

[42] W. Wu, S.S. Rao, Uncertainty analysis and allocation of joint tolerances in robot manipulators based on interval analysis, Reliab. Eng. Syst. Saf. 92 (2007) 54-64.

[43] W.T. Chang, L.I. Wu, Tolerance analysis and synthesis of cam-modulated linkages, Math. Comput. Model. 57 (2013) 641-660.

[44] U. Kumaraswamy, M.S. Shunmugam, S. Sujatha, A unified framework for tolerance analysis of planar and spatial mechanisms using screw theory, Mech. Mach. Theory 69 (2013) 168-184.

[45] M.J. Tsai, T.H. Lai, Kinematic sensitivity analysis of linkage with joint clearance based on transmission quality, Mech. Mach. Theory 39 (2004) 1189-1206.

[46] M.J. Tsai, T.H. Lai, Accuracy analysis of a multi-loop linkage with joint clearances, Mech. Mach. Theory 43 (2008) 1141-1157.

[47] C.R. Tischler, A.E. Samuel, Prediction of the slop in general spatial linkages, Int. J. Robot. Res. 18 (1999) 845-858.

[48] X. Huang, Y. Zhang, Robust tolerance design for function generation mechanisms with joint clearances, Mech. Mach. Theory 45 (2010) 1286-1297.

[49] K.L. Ting, J. Zhu, D. Watkins, The effects of joint clearance on position and orientation deviation of linkages and manipulators, Mech. Mach. Theory 35 (2000) 391-401.

[50] J. Zhu, K.L. Ting, Uncertainty analysis of planar and spatial robots with joint clearances, Mech. Mach. Theory 35 (2000) 1239-1256.

[51] E. Pezzuti, R. Stefanelli, P.P. Valentini, L. Vita, Computer-aided simulation and testing of spatial linkages with joint mechanical errors, Int. J. Numer. Method. Eng. 65 (2006) 1735-1748.

[52] H.P. Jawale, H.T. Thorat, Investigation of positional error in two degree of freedom mechanism with joint clearance, ASME J. Mech. Robot. 4 (2012) 011002.

[53] H.P. Jawale, H.T. Thorat, Positional error estimation in serial link manipulator under joint clearances and backlash, ASME J. Mech. Robot. 5 (2013) 021003.

[54] O. Altuzarra, J. Aginaga, C. Pinto, X. Iriarte, Discontinuity analysis of position error due to clearances in parallel manipulators, Proc. World Congress in Mechanism and Machine Science, Guanajiuato, Mexico, 2011.

[55] D.W. Choi, J. Wickert, H.H. Yoo, Effect of manufacturing tolerances on dynamic equilibria of multibody systems undergoing prescribed rotational motion, J. Mech. Sci. Technol. 22 (2008) 1747-1756.

[56] H. Chun, S.J. Kwon, T. Tak, Multibody approach for tolerance analysis and optimization of mechanical systems, J. Mech. Sci. Technol. 22 (2008) 276-286.

[57] S.M. Eom, B.S. Kim, H.H. Yoo, Tolerance effects on natural frequencies of multibody systems undergoing constant rotational motion, J. Mech. Sci. Technol. 22 (2008) 2163-2170.

[58] P. Flores, J. Ambrósio, Revoulte joints with clearance in multibody systems, Comput. Struct. 82 (2004) 1359-1369.

[59] P. Flores, J. Ambrósio, J.C. Pimenta Claro, H.M. Lankarani, Kinematic and Dynamic of multibody systems with imperfect joints, Springer, Berlin, 2008.

[60] O. Muvengei, J. Kihiu, B. Ikua, Dynamic analysis of multi-body mechanical systems with imperfect kinematic joints: a literature survey and review, Sustain. Res. Innov. Proc. 3 (2011).
[61] A.L. Schwab, J.P. Meijaard, P. Meijers, A comparison of revolute joint clearance models in the dynamic analysis of rigid and elastic mechanical systems, Mech. Mach. Theory 37 (2002) 895-913.
[62] A.H. Chebbi, Z. Affi, L. Romdhane, Prediction of the pose errors produced by joint clearance for a 3-UPU parallel robot, Mech. Mach. Theory 44 (2009) 1768-1783.
[63] C. Innocenti, Kinematic clearance sensitivity analysis of spatial structures with revolute joints, ASME J. Mech. Des. 124 (2002) 52-57.
[64] V. Parenti-Castelli, S. Venanzi, Clearance influence analysis on mechanisms, Mech. Mach. Theory 40 (2005) 1316-1329.
[65] S. Venanzi, V. Parenti-Castelli, A new technique for clearance influence analysis in spatial mechanisms, ASME J. Mech. Des. 127 (2005) 446-455.
[66] A. Pott, M. Hiller, A force based approach to error analysis of parallel kinematic mechanisms, in: J. Lenarčič, C. Galletti (Eds.), On advances in robot kinematics, Springer, Berlin, 2004, pp. 293-302.
[67] P. Krishnaswami, A.G. Kelkar, Optimal design of controlled multibody dynamic systems for performance, robustness and tolerancing, Eng. Comput. 19 (2003) 26-34.
[68] S. Mutawe, Y.M. Al-Smadi, R.S. Sodhi, Planar four-bar path generation considering worst case joint tolerances, Proc. World Congress on Engineering and Computer Science, San Francisco, USA, 2011.
[69] B.K. Rout, R.K. Mittal, Tolerance design of robot parameters using Taguchi method, Mech. Syst. Signal Process. 20 (2006) 1832-1852.
[70] N. Chironis, N.P. Slater, Mechanisms and mechanical devices sourcebook, 2nd ed, McGraw-Hill, New York, 1996.
[71] L.F. Martha, Ftool: interactive graphics program for structural analysis, version 3.00, Pontifícia Universidade Católica do Rio de Janeiro, 2012.
[72] ISO 2768-1, General tolerances. Part 1: Tolerances for linear and angular dimensions without individual tolerance indications. International Organization for Standardization, Geneva, Switzerland, 1989.

List of figures

Fig. 10: Equivalent static model
Fig. 11: Force analysis with horizontal load
Fig. 12: Force analysis with vertical load

Tables

Tab. 1: Sensitivities of the dimensions on the requirements

Dimension	*Equation*	*Value, x*		*Value, y*	
		$\alpha = 0$	$\alpha = 30°$	$\alpha = 0$	$\alpha = 30°$
L_{12}	$s_{L12} = F_{12}/F$	-2.688	-2.567	0	-0.904
L_{13}	$s_{L13} = F_{13}/F$	-1.688	-1.809	0	0
L_{24}	$s_{L24} = F_{24}/F$	1.500	1.745	-0.774	-0.513
L_{25}	$s_{L25} = F_{25}/F$	1.500	1.255	0.774	1.317
L_{34}	$s_{L34} = F_{34}/F$	-1.841	-1.952	0.949	0.574
L_{35}	$s_{L35} = F_{35}/F$	-1.841	-1.403	-0.949	-1.473
L_{46}	$s_{L46} = F_{46}/F$	1.091	1.079	-0.563	-0.317
L_{56}	$s_{L56} = F_{56}/F$	1.091	0.776	0.563	0.814
D_{13}	$s_{D13} = 1/2\ \lvert F_{13}\rvert/F$	0.844	0.905	0	0
D_{24}	$s_{D24} = 1/2\ \lvert F_{24}\rvert/F$	0.750	0.873	0.387	0.257
D_{25}	$s_{D25} = 1/2\ \lvert F_{25}\rvert/F$	0.750	0.628	0.387	0.659
D_{34}	$s_{D34} = 1/2\ \lvert F_{34}\rvert/F$	0.921	0.976	0.475	0.287
D_{35}	$s_{D35} = 1/2\ \lvert F_{35}\rvert/F$	0.921	0.702	0.475	0.737
D_{46}	$s_{D46} = 1/2\ \lvert F_{46}\rvert/F$	0.546	0.540	0.282	0.159
D_{56}	$s_{D56} = 1/2\ \lvert F_{56}\rvert/F$	0.546	0.338	0.282	0.407
d_1	$s_{d1} = -1/2\ (\lvert F_{12}\rvert+\lvert F_{13}\rvert+\lvert R_1\rvert)\ /F$	-1.688	-2.682	-1.688	-2.140
d_2	$s_{d2} = -1/2\ (\lvert F_{12}\rvert+\lvert F_{24}\rvert+\lvert F_{25}\rvert+\lvert R_2\rvert)\ /F$	-2.844	-3.236	-1.962	-2.555
d_3	$s_{d3} = -1/2\ (\lvert F_{13}\rvert+\lvert F_{34}\rvert+\lvert F_{35}\rvert)\ /F$	-2.865	-2.582	-0.949	-1.024
d_4	$s_{d4} = -1/2\ (\lvert F_{24}\rvert+\lvert F_{34}\rvert+\lvert F_{46}\rvert)\ /F$	-2.216	-2.388	-1.143	-0.702
d_5	$s_{d5} = -1/2\ (\lvert F_{25}\rvert+\lvert F_{35}\rvert+\lvert F_{56}\rvert)\ /F$	-2.216	-1.717	-1.143	-1.802
d_6	$s_{d6} = -1/2\ (\lvert F_{46}\rvert+\lvert F_{56}\rvert)\ /F$	-1.091	-0.928	-0.563	-0.566
α	$s_{\alpha} = \lvert M_1\rvert/F$ (m/rad)	0	0	1.688	1.809

Tab. 2: Aggregation of sensitivities

Tolerance	*Related dimensions*	*Sensitivity, x*		*Sensitivity, y*	
		$\alpha = 0$	$\alpha = 30°$	$\alpha = 0$	$\alpha = 30°$
T_{LA}	L_{12}	2.688	2.567	0	0.904
T_{LB}	L_{13}	1.688	1.809	0	0
T_{LC}	L_{24}, L_{25}	2.121	2.149	1.095	1.413
T_{LD}	L_{34}, L_{35}, L_{46}, L_{56}	3.026	2.747	1.560	1.806
T_D	D_{13}, D_{24}, D_{25}, D_{34}, D_{35}, D_{46}, D_{56}	2.032	1.956	0.954	1.148
T_d	d_1, d_2, d_3, d_4, d_5, d_6	5.942	5.818	3.245	4.027
T_α	α	0	0	1.688	1.809

Tab. 3: Results of the tolerance analysis

Tolerance	*Value*	*RSS stackup, x* (mm)		*RSS stackup, y* (mm)	
		$\alpha = 0$	$\alpha = 30°$	$\alpha = 0$	$\alpha = 30°$
T_{LA}	± 0.8 mm	2.15 (19.4%)	2.05 (19.5%)	0	0.72 (5.0%)
T_{LB}	± 0.8 mm	1.35 (7.6%)	1.45 (9.7%)	0	0
T_{LC}	± 2 mm	1.70 (12.1%)	1.72 (13.7%)	0.88 (10.1%)	1.13 (12.2%)
T_{LD}	± 1.2 mm	3.63 (55.2%)	3.30 (50.2%)	1.87 (46.3%)	2.17 (44.9%)
T_D	± 0.2 mm	0.41 (0.7%)	0.39 (0.7%)	0.19 (0.5%)	0.23 (0.5%)
T_d	± 0.2 mm	1.10 (5.0%)	1.16 (6.3%)	0.65 (5.6%)	0.81 (6.2%)
T_α	± 1 mrad	0	0	1.69 (37.6%)	1.81 (31.2%)
Total		4.888	4.652	2.753	3.236

Figure 1

Click here to download high resolution image

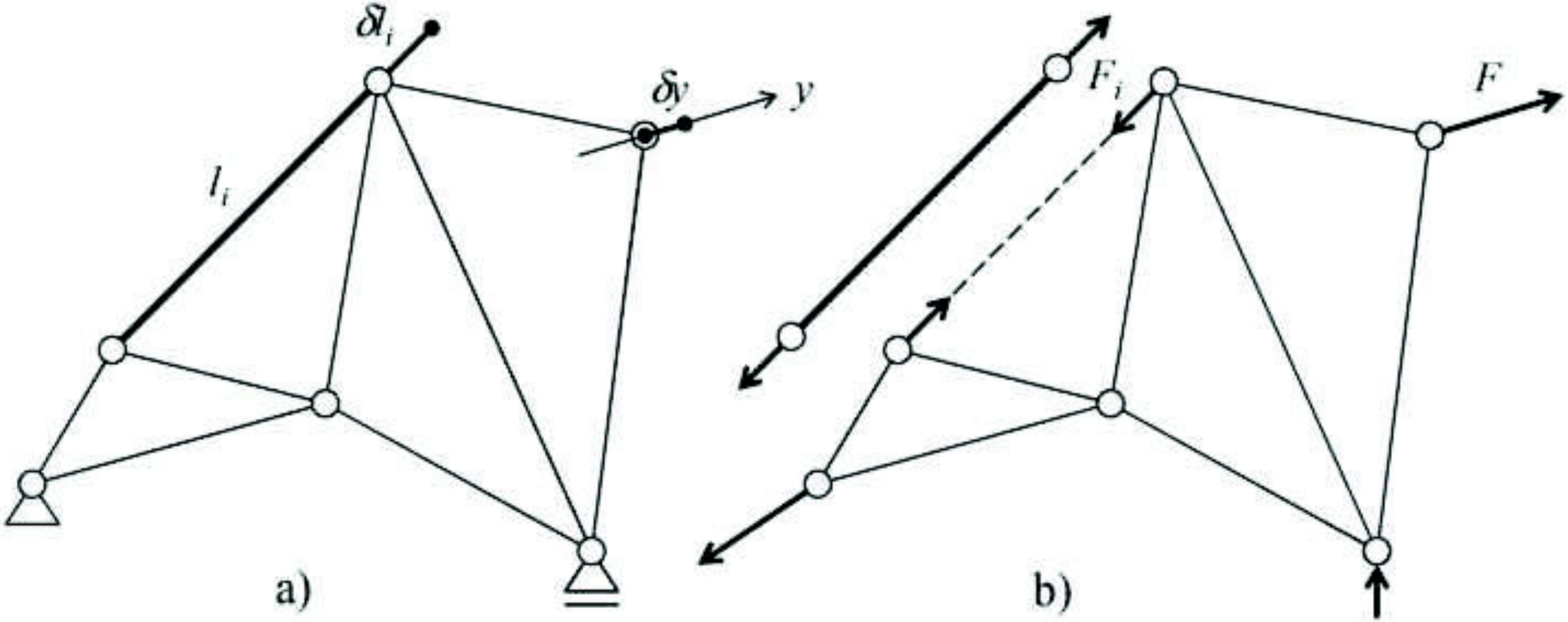

Figure 2
Click here to download high resolution image

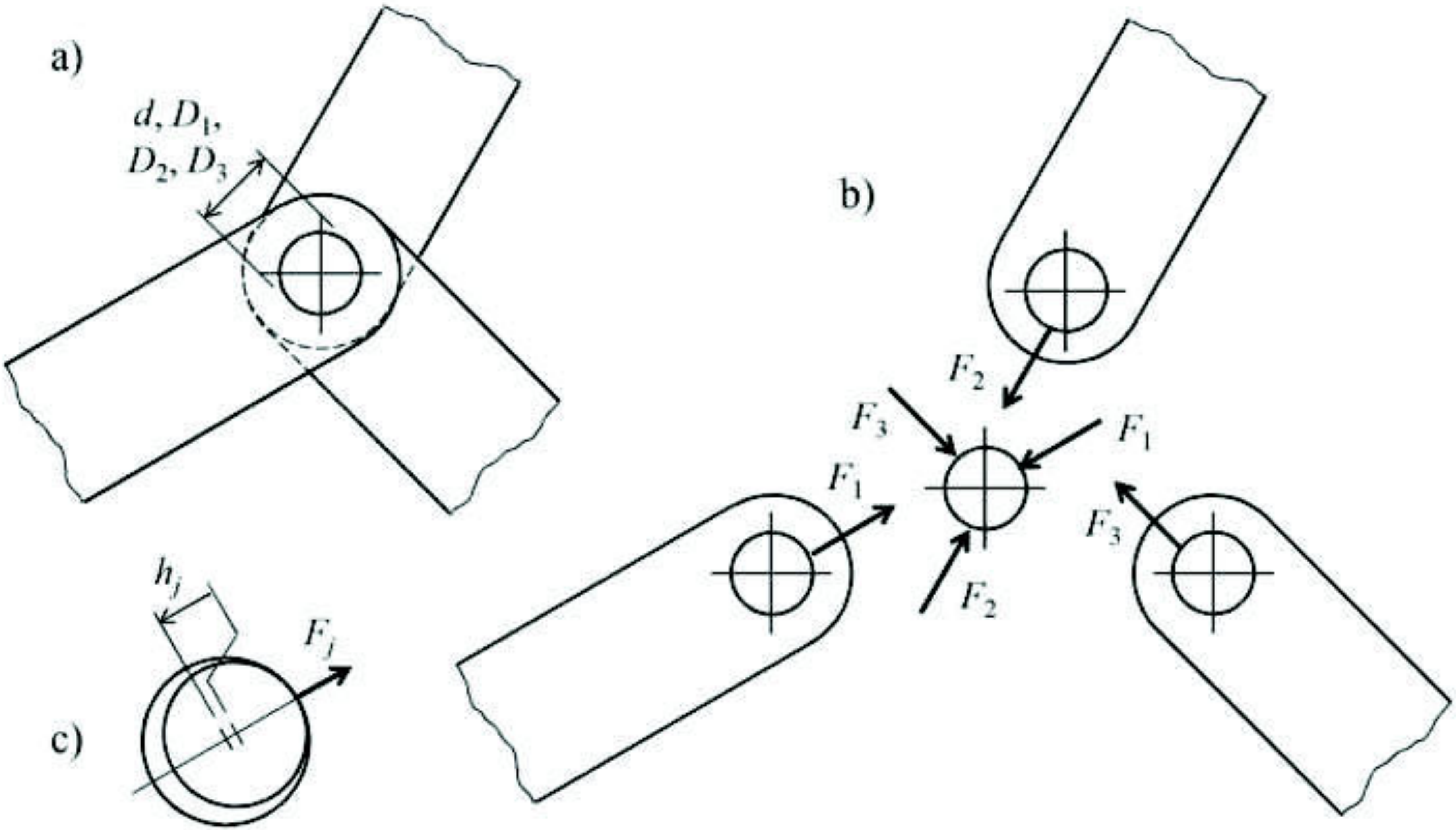

Figure 3
Click here to download high resolution image

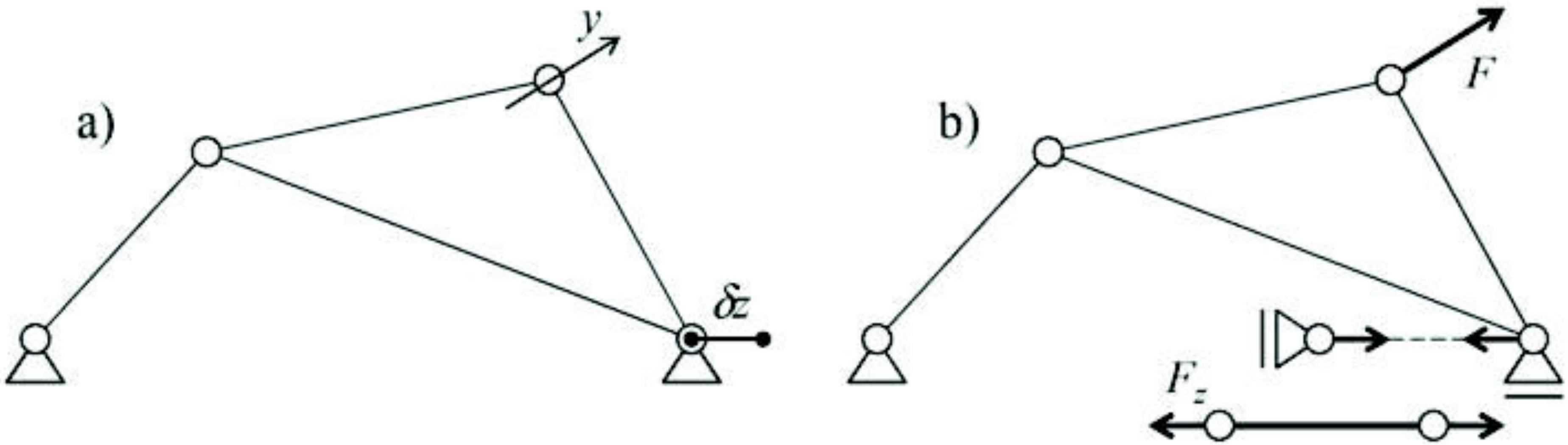

Figure 4
Click here to download high resolution image

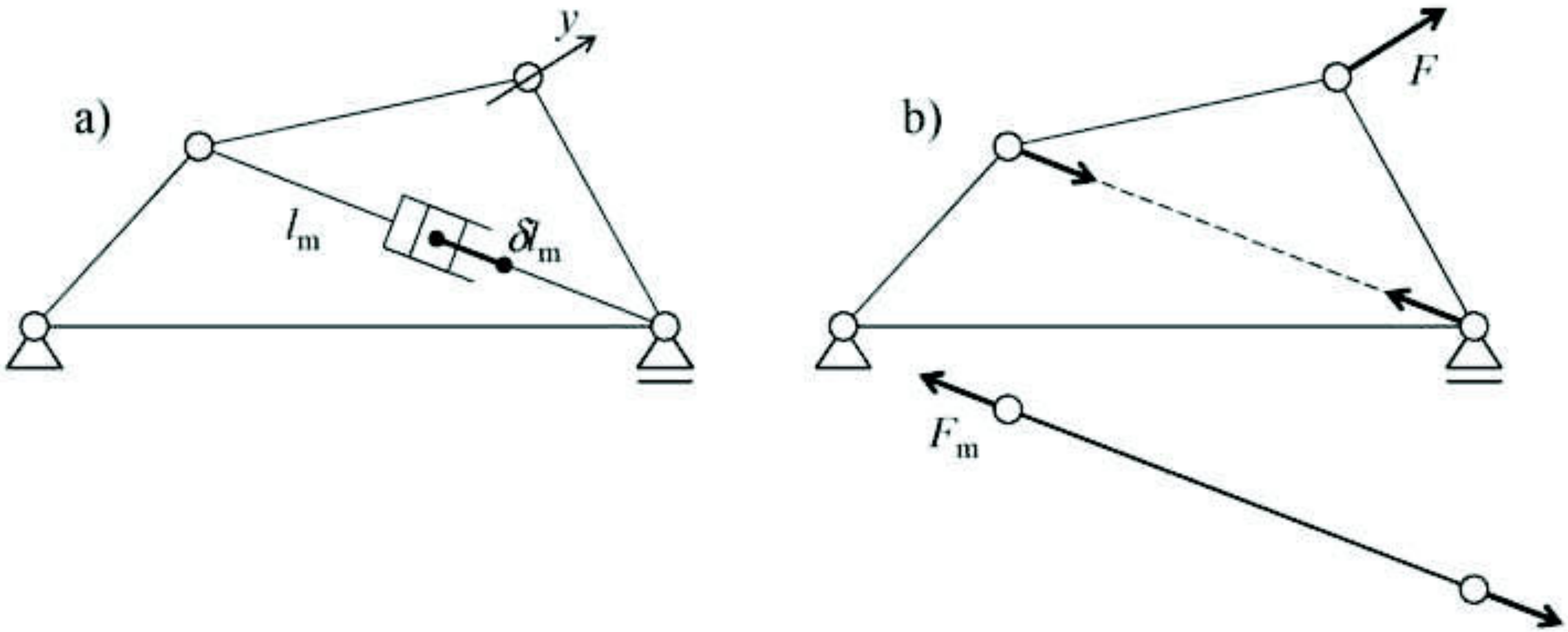

Figure 5
Click here to download high resolution image

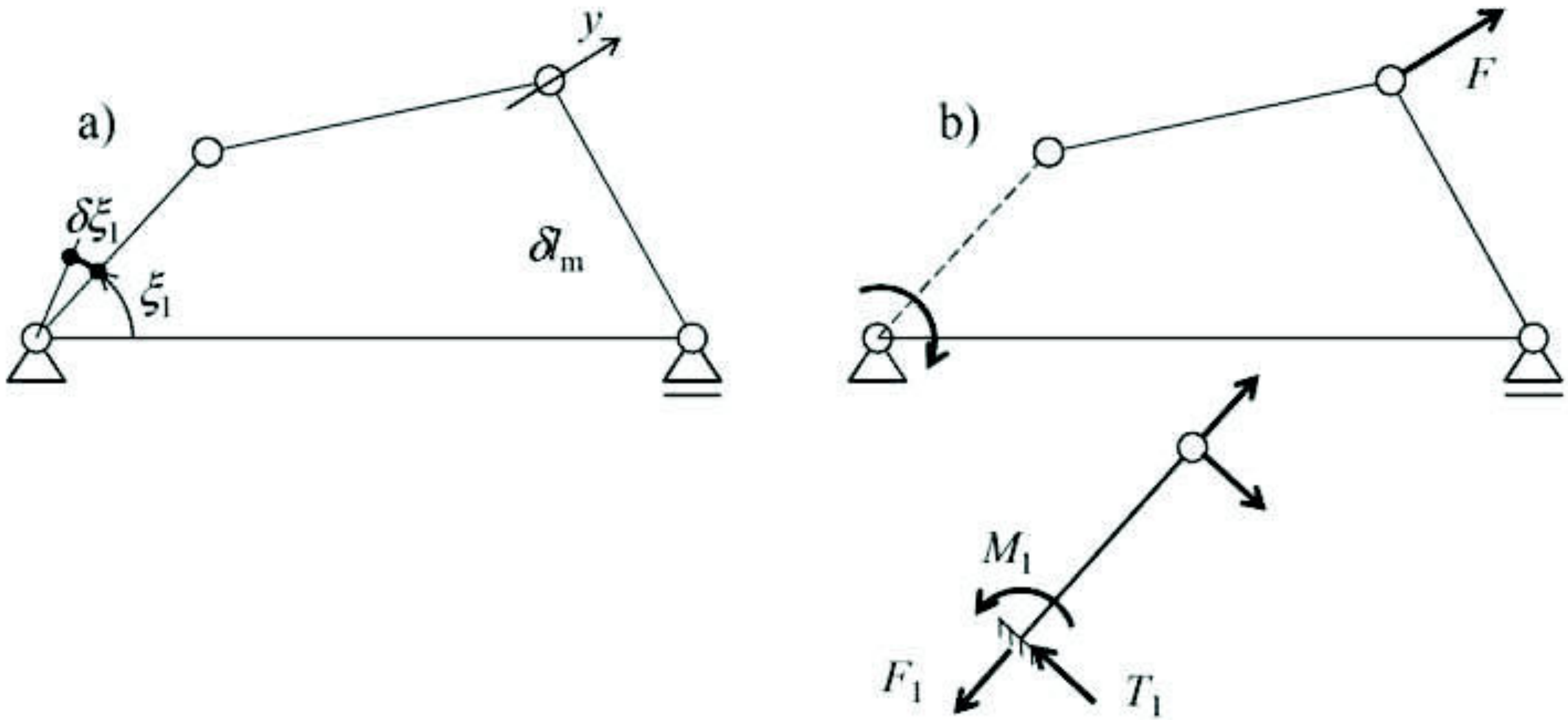

Figure 6

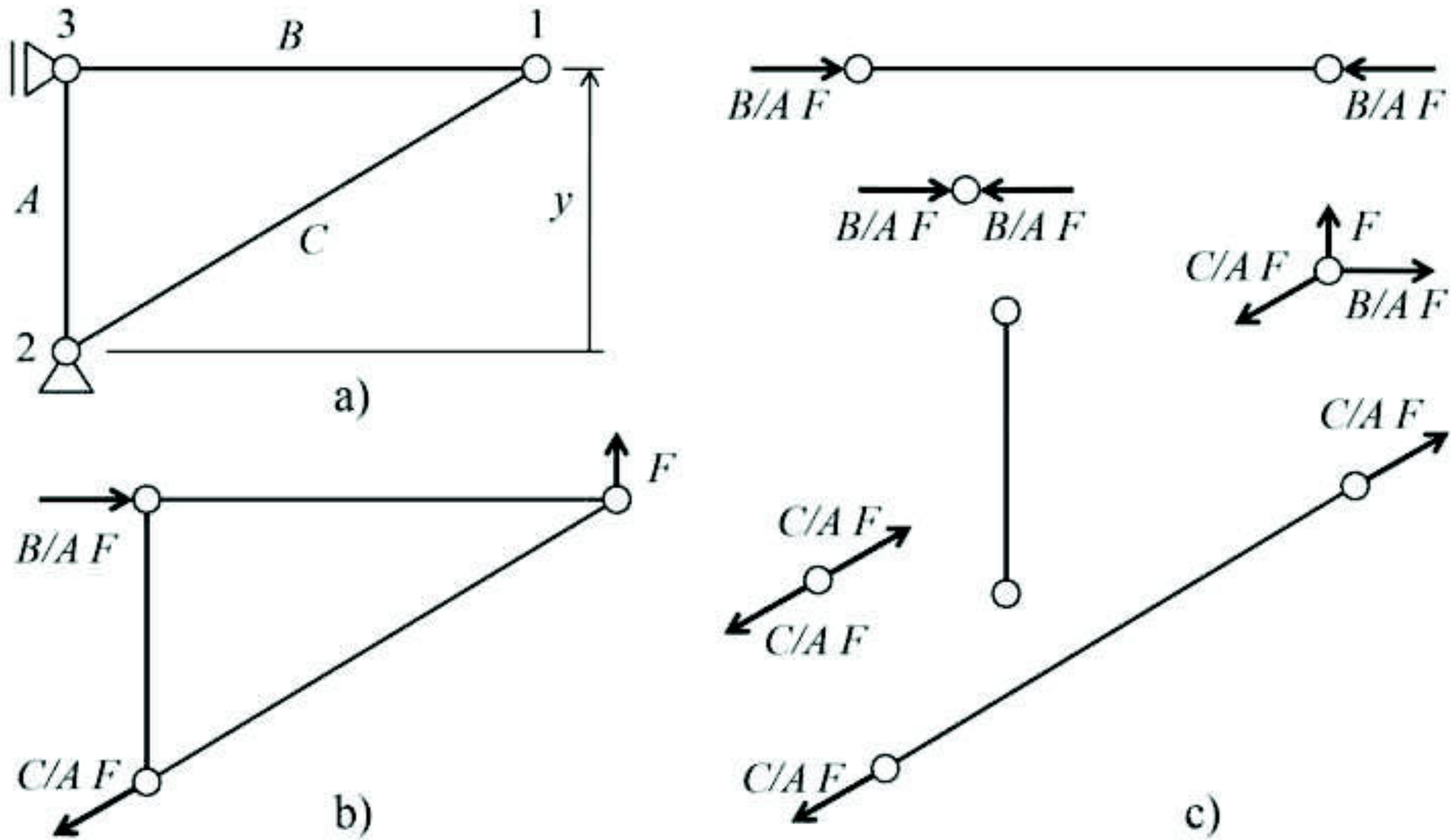

Figure 7

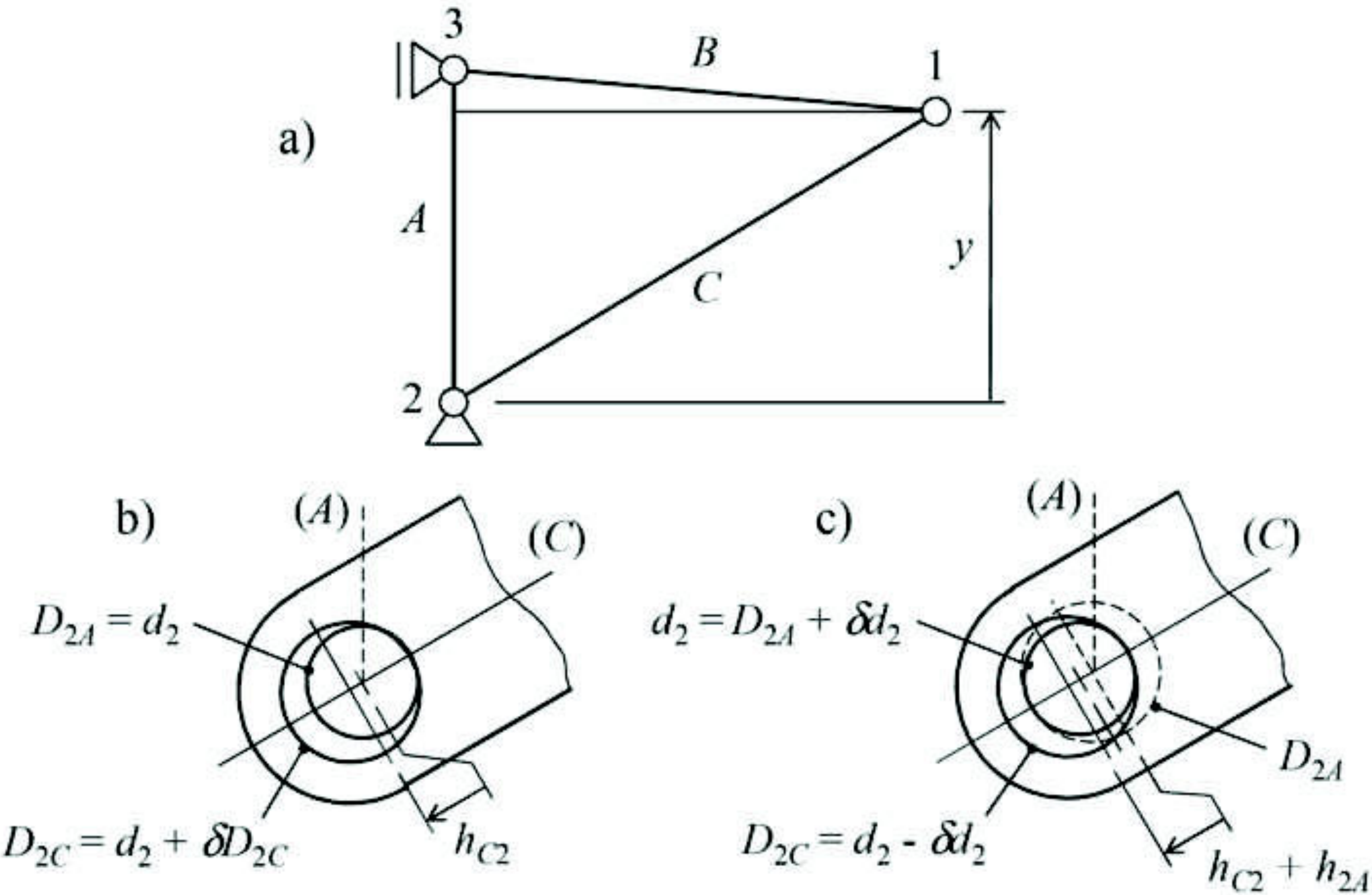
a)
3
B
1
A
y
C
2
b)
(A)
(C)
$D_{2A} = d_2$
$D_{2C} = d_2 + \delta D_{2C}$
h_{C2}
c)
(A)
(C)
$d_2 = D_{2A} + \delta d_2$
D_{2A}
$D_{2C} = d_2 - \delta d_2$
$h_{C2} + h_{2A}$

Figure 8
Click here to download high resolution image

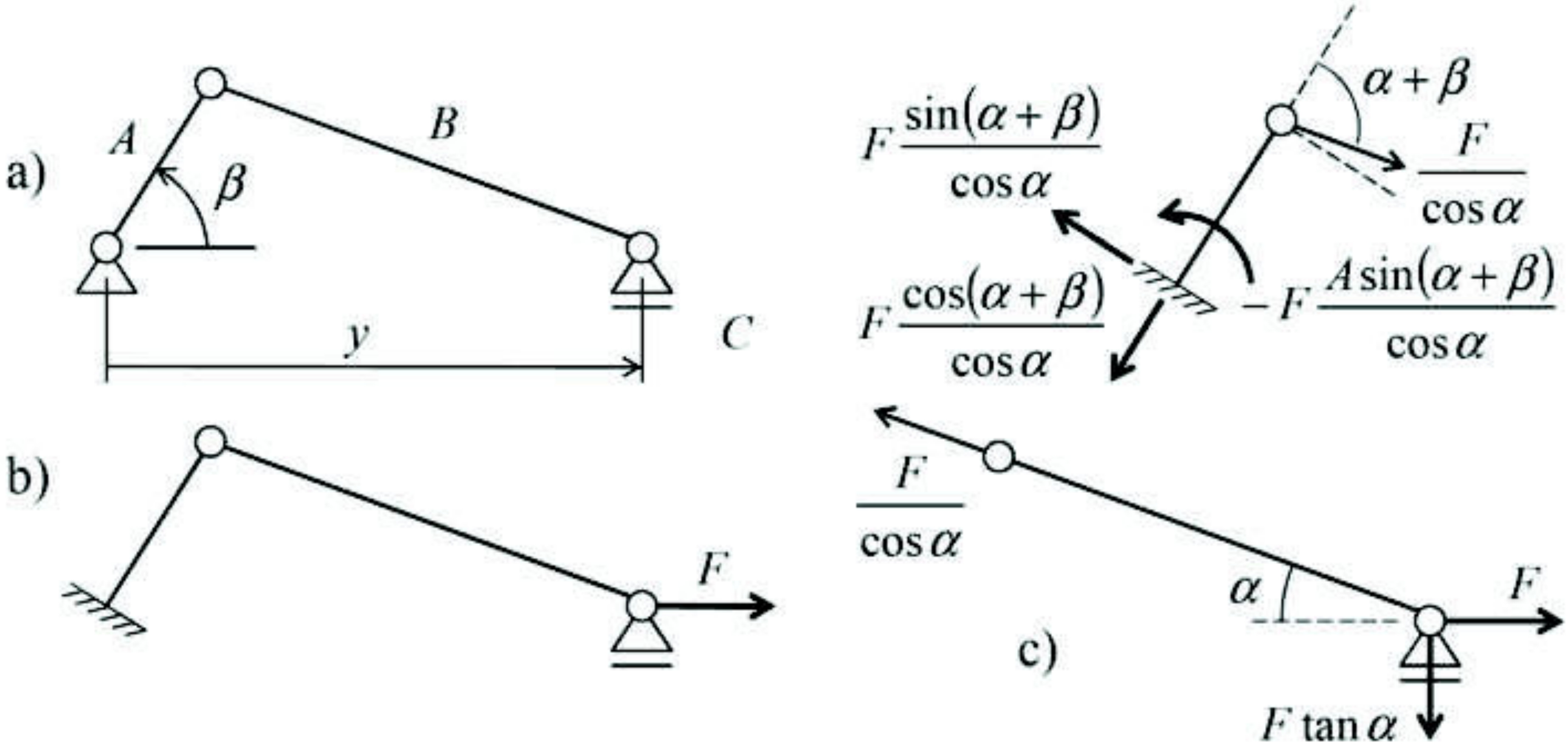

Figure 9
Click here to download high resolution image

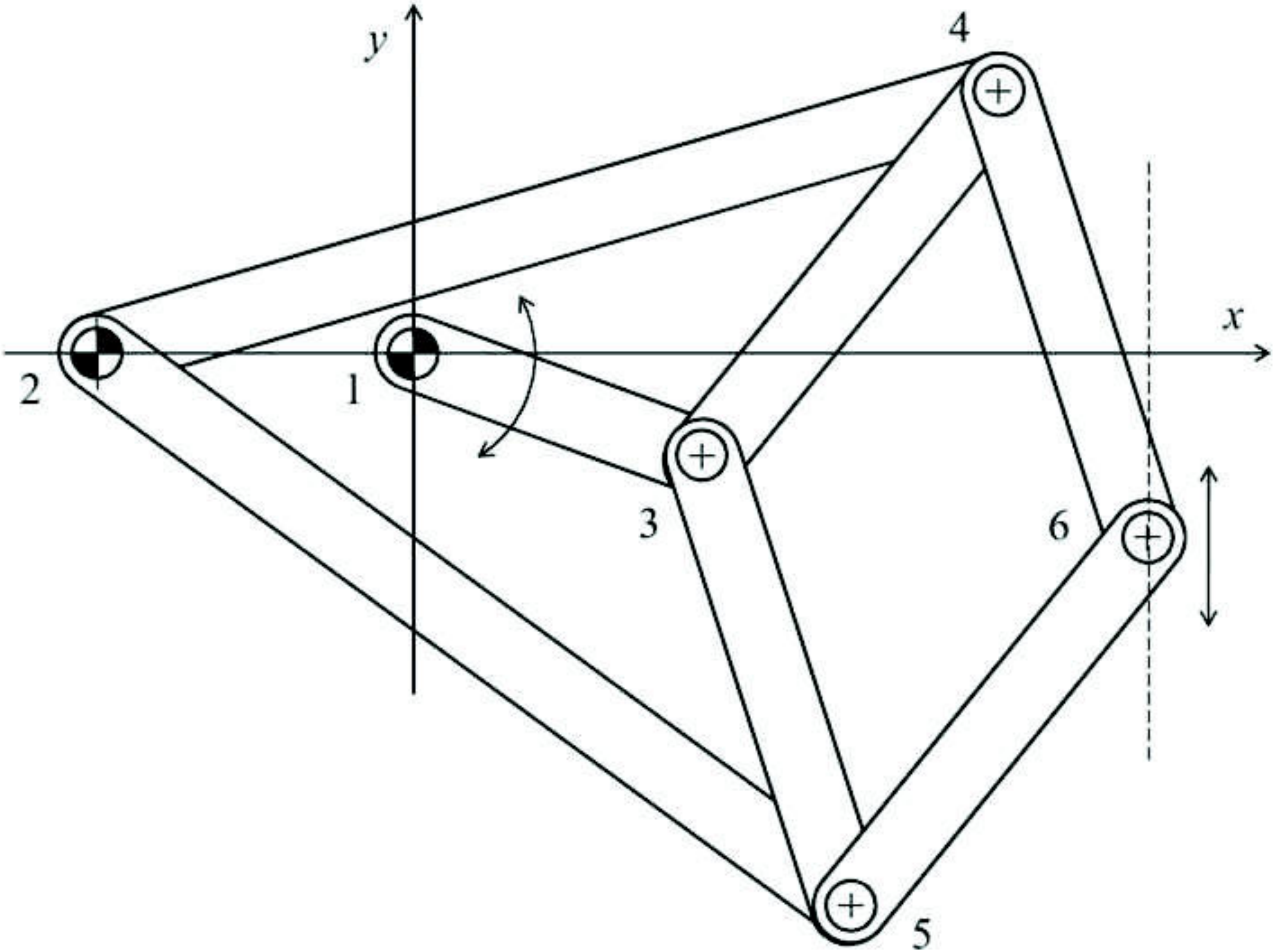

Figure 10
Click here to download high resolution image

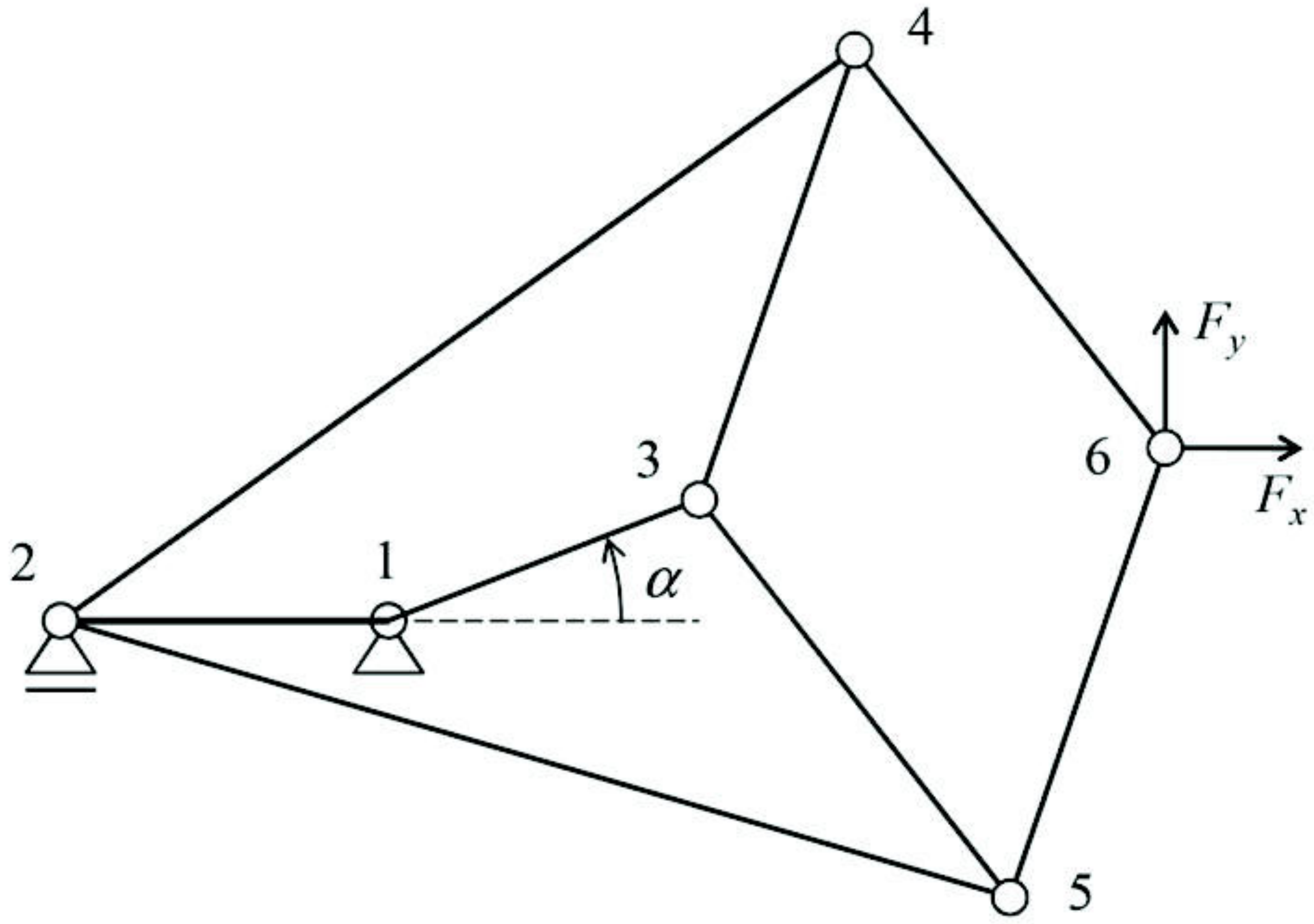

Figure 11

Click here to download high resolution image

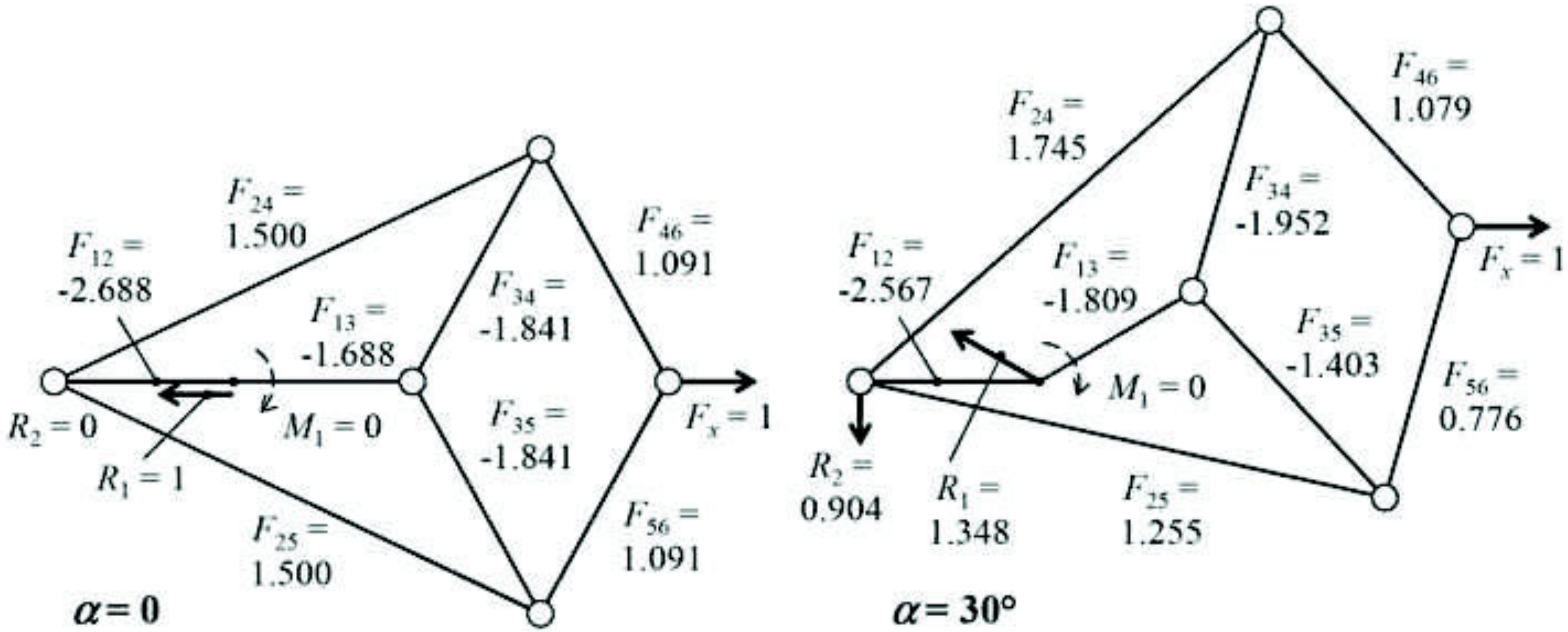

Figure 12

Click here to download high resolution image