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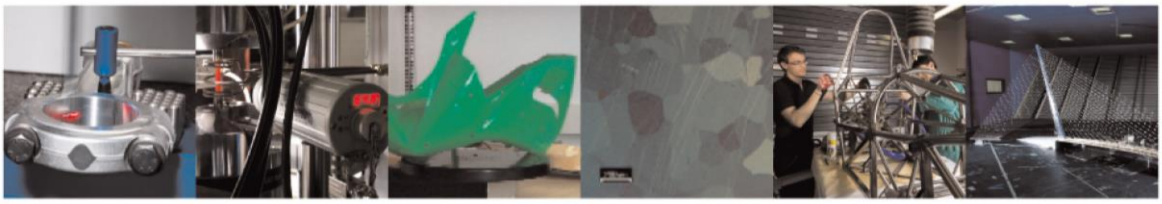

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Application of the Kalai-Smorodinsky approach in multi-objective optimization of metal forming processes

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Abstract. *Multi-objective optimization problem (MOP) is approached from the theoretical background of the Game Theory, which consists in finding a compromise between two rational players of a bargaining problem. In particular, the Kalai and Smorodinsky (K-S) model offers a balanced and attractive solution resulting from cooperative players. This approach allows avoiding the computationally expensive and uncertain reconstruction of the full Pareto Frontier usually required by MOPs. The search for the K-S solution can be implemented into methodologies with useful applications in engineering MOPs where two or more functions must be minimized. This paper presents an optimization algorithm aimed at rapidly finding the K-S solution where the MOP is transformed into a succession of single objective problems (SOP). Each SOP is solved by meta-model assisted evolution strategies used in interaction with FEM simulation software for metal forming applications. The proposed method is first tested and demonstrated with known mathematical multi-objective problems, showing its ability to find a solution lying on the Pareto Frontier, even with a largely incomplete knowledge of it. The algorithm is then applied to the FEM optimization problem of wire drawing process with one and two passes, in order to simultaneously minimize the pulling force and the material damage. The K-S solutions are compared to results previously suggested in literature using more conventional methodologies and engineering expertise. The paper shows that K-S solutions are very promising for finding quite satisfactory engineering compromises, in a very efficient manner, in metal forming applications.*

Keywords: Multi-Objection Optimization, MOP, Game theory, FEM simulation, Pareto Frontier, Wire Drawing.

Introduction

A large number of optimization problems in engineering do not aim at a single objective function, but at multiple and conflicting objective functions. In these cases, the solution is not unique and a set of optimal solutions form the so-called Pareto Frontier (PF). Computing the whole Frontier is often expensive, as it requires solving a multi-dimensional optimization problem. Virtually, all the available literature on the search on multi-objective optimal solution aims at the reduction of the computational cost for the approximation of the Pareto Frontier. Once the PF is available, the problem of ultimately selecting, with objectivity, one single solution out of the Frontier is still left open and this difficult and subjective choice is given to the engineer. If in 2D and 3D problems the PF is respectively a curve and a surface, which can be plotted and visualised, in problems with more than 3 functions its visualization is not possible and the selection of the best compromise becomes a challenging task. For this reason, the scientific literature has developed additional techniques or criteria to aid the selection, where necessary, e.g. as in reference [2].

In order to reduce computational cost and simplify the selection of solutions, other approaches aim at directing the search only towards interesting regions of the PF, such as its knees [1]. In this stream of thought, the Kalai and Smorodinsky approach provides “objective” criteria to select only one solution within the PF in a central region of the solution space; this eliminates the selection problem and dramatically reduces the computational cost to find the whole PF. In this paper, we will investigate how the K-S compromise fits with the engineering choices on some previously studied metal forming optimization problems.
As defined by Nash, the Game Theory provides the framework for finding an interesting compromise between two players of a multi-objective problem. It is developed according to either a cooperative or not non-cooperative approach [3]. The Kalai and Smorodinsky (K-S) solution belongs to the cooperative approach. It can be considered as an alternative to the non-cooperative equilibrium solution of Nash’s bargaining problem where players make decisions to maximize their own utility, while taking into account that other players are doing the same and that decisions made by players impact each other’s utilities, with a cooperative approach [4]. Understandably, the game theory has frequently been used in social and economic studies, with many articles in the literature that upgrade or modify the Kalai and Smorodinsky method, e.g. [5-6]. The K-S approach allows finding a unique solution on the Pareto Frontier by following some axioms: *Pareto optimality, symmetry, invariance with respect to affine transformation utility, monotonicity*. For further information the reader can refer to [4]. The symmetric (or egalitarian) property forces the solution to lie on a central region of the solution space, as stated above. It is consequently located on the straight line connecting the Utopia (F_u) and Nadir (F_n) points in correspondence with the Pareto Frontier of the surface of domain solutions (trade-off surface). The Utopia point is a utopic solution (out of the feasible region) consisting of the set of the minimal values obtained for all objective functions using different values of parameters. Instead, the coordinates of Nadir point correspond to the worst values obtained for each objective function when the solution set is restricted to the trade-off surface [7] (Fig. 1).
The Kalai and Smorodinsky method has been applied, in very few cases, to engineering problems, in which the players of bargaining problem are conceived as process variables or physical parameters. One of the earliest applications, in a supply chain context, can be found in Kohli and Park [8]. The authors study a model in which buyer and seller negotiate the terms of a quantity discount contract in an Economic Order Quantity setting; they study the allocations as a function of risk aversion and bargaining power. The solution of Kalai Smorodinsky has been used also for groundwater studies in Mexico, in which the exploitation of scarce water resources, particularly in areas of high demand, inevitably produces conflicts among disparate stakeholders, each of whom may have their own set of priorities [9]. A very recent application can be found in multi-objective design optimization of an aerosol can [10].
An approach to the problem used in [9] is to consider the linear segment between the disagreement point (Nadir) and the ideal point (Utopia); then the solution is the unique intercept of this segment with the Pareto Frontier. Similar methods can be found also in civil constructions, especially in the risk allocation field [11]. Another application of Kalai and Smorodinsky solution has been on Signal Processing Optimization [12]. The authors develop two optimization algorithms aimed at finding the Kalai and Smorodinsky solution on the straight line that connect the two points specified above. Like other authors, they tend to find the point on the Kalai and Smorodinsky line iteratively by substituting, from time to time, the new point found with the upper or lower limit, until the difference between two points falls below a tolerance value. They show that with this iterative technique a unique Kalai and Smorodinsky solution can be found.
From the very short list of papers mentioned so far, it appears that while the K-S paradigm (derived from the game theory) has received little attention from engineers and no attention (to the author’s knowledge) in the field of manufacturing processes. Therefore, one important purpose of this paper is to evaluate whether this kind of trade-off also applies to the problems of process design, i.e. to demonstrate the applicability of the Kalai and Smorodinsky not only as a general criterion, but also as a design methodology, in the field of metal forming processes. To this aim, multi-objective

optimization of metal forming problems is considered through the performances of the forming processes as the players of the bargaining game, expressed through objective functions. We will show the implementation of an original routine which iteratively interacts with an FEM solver, in order to solve multi-objective metal forming optimization problems. Metal forming processes are often designed by means of (computationally expensive) numerical simulations, hence it is important that the implementation of the method be simple and efficient. The proposed implementation is innovative and efficient, since the K-S solution will be found without evaluating the Pareto Frontier, by transforming a multi-objective optimization (MOP) problem into a sequence of single-objective optimizations (SOP).

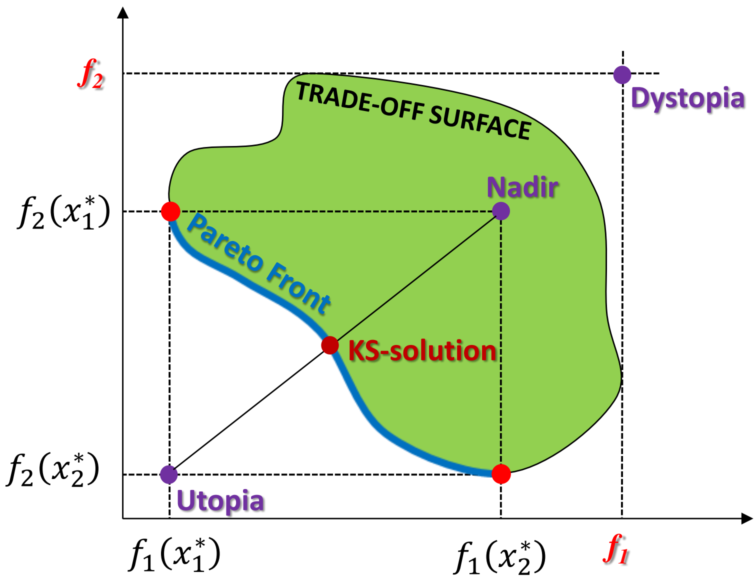

Fig. 1: Example of domain of solution for f_1 vs f_2, Utopia and Nadir point and KS solution on the Pareto Frontier [7].

The present work is a wider and more detailed extension of the work briefly presented by the authors at ESAFORM 2015 [13]. In the following section, the algorithm will be presented and explained. Then, its use will be demonstrated on mathematical problems with known analytical functions. Finally, it will be applied on the optimization of a single-step and a two-step wire drawing operations.

Mathematical formulation

The mathematical formulation is here presented in the case of two objective functions, f_1 and f_2. The starting point of the algorithm consists in defining the Nadir and the Utopia points. Let x_1^* be the vector value of design variables which minimises f_1 alone and x_2^* be the vector value of design variables which minimises f_2 alone. With reference to the space defined by Figure 1, the Utopia point is that (unfeasible, utopian) point where both functions take their absolute minimum value, $[f_1(x_1^*), f_2(x_2^*)]$. As defined in Figure 1, the Nadir point is obtained by calculating each function with the optimal arguments of the other function $[f_1(x_2^*), f_2(x_1^*)]$. Another relevant point in this theory is the Dystopia point, which combines the individual worst values of each function. The Dystopia point is also shown in Figure 1, but it has not been used in the present formulation. Table 1 summarises the mathematical definitions of the involved variables for 2D problems (i.e. when only two conflicting objective functions must be minimised) and in 3D problems, (similar definitions apply also for higher dimensional problems). When more than two objective functions are involved, a so-called pseudonadir definition must be instead of the Nadir [14].

Table 1: Definition of Utopia and Nadir Points with two and three objective functions.

$x_1^* = \min_x f_1(x)$	➔ Unconstrained minimization of $f_1(x)$
$x_2^* = \min_x f_2(x)$	➔ Unconstrained minimization of $f_2(x)$
$x_3^* = \min_x f_3(x)$	➔ Unconstrained minimization of $f_3(x)$
$F_u = [f_1(x_1^*), f_2(x_2^*)]$	➔ Utopia point in 2D problems
$F_n = [f_1(x_2^*), f_2(x_1^*)]$	➔ Nadir point in 2D problems
$F_u = [f_1(x_1^*), f_2(x_2^*), f_3(x_3^*)]$	➔ Utopia point in 3D problems
$F_n = [\max(f_1(x_2^*), f_1(x_3^*)), \max(f_2(x_1^*), f_2(x_3^*)), \max(f_3(x_1^*), f_3(x_2^*))]$	➔ Pseudonadir point in 3D problems

After the coordinates of the two points and their connecting line have been defined, a method must be found for localizing the intersection with the PF. An approach has been proposed in [9], where the authors propose a functional of the two conflicting objective functions. A similar approach was also proposed in [15]. Here a similar albeit different functional formulation is proposed, to be minimized in order to quickly find a solution very close to the Kalai and Smorodinsky point:

$$\min_{x,t} \ t \qquad s/c \begin{cases} F_u + t \cdot \tau = F(x) \\ a_{i_{inf}} \le x_i \le a_{i_{sup}}; \ i = 1; \dots; m \end{cases} \tag{1}$$

where,

$$\tau = \frac{F_n - F_u}{\|F_n - F_u\|} \tag{2}$$

and where F_u and F_n are respectively the Utopia and Nadir points, as defined in Table 1. $a_{i_{inf}} \le x_i \le a_{i_{sup}}$ are the boundary conditions of the design variables. Constraints could also be added to the formulation in order to define a more generalized minimization problem. The objective function of the minimization problem is then as follows:

$$F(x) = F_u + t \cdot \tau \tag{3}$$

The minimization of Equation (3) brings the F(x) value as close as possible to the Utopia point, while laying on the K-S connecting line, allows finding the K-S point. For using this optimization algorithm on engineering problems, some modifications are preferable. First, every variable should be normalized with respect to the coordinates of the Nadir and Utopia points, as follows:

$$f'_1(x) = \frac{f_1(x)}{f_1(x_2^*) - f_1(x_1^*)} \tag{4}$$

$$f'_2(x) = \frac{f_2(x)}{f_2(x_1^*) - f_2(x_2^*)} \tag{5}$$

$$\tau'_1 = \frac{f'_1(x_2^*) - f'_1(x_1^*)}{\sqrt{\left(f'_1(x_2^*) - f'_1(x_1^*)\right)^2 + \left(f'_2(x_1^*) - f'_2(x_2^*)\right)^2}} \tag{6}$$

$$\tau'_2 = \frac{f'_2(x_1^*) - f'_2(x_2^*)}{\sqrt{\left(f'_1(x_2^*) - f'_1(x_1^*)\right)^2 + \left(f'_2(x_1^*) - f'_2(x_2^*)\right)^2}} \tag{7}$$

After this normalization, it is possible to rewrite the equation (4) in matrix form:

$$\begin{bmatrix} f'_1(x) \\ f'_2(x) \end{bmatrix} = \begin{bmatrix} f'_1(x_1^*) \\ f'_2(x_2^*) \end{bmatrix} + t \begin{bmatrix} \tau'_1 \\ \tau'_2 \end{bmatrix} \tag{8}$$

Eliminating the auxiliary "t" variable gives the following equation:

$$t = \frac{f'_1(x) - f'_1(x_1^*)}{\tau'_1} = \frac{f'_2(x) - f'_2(x_2^*)}{\tau'_2} \tag{9}$$

which must be enforced in order to find the K-S solution. The two members of equation (9) can be transformed into two functions, defined as follows:

$$g_1(x) = \frac{f'_1(x) - f'_1(x_1^*)}{\tau'_1} \tag{10}$$

$$g_2(x) = \frac{f'_2(x) - f'_2(x_2^*)}{\tau'_2} \tag{11}$$

Equation (10) rewrites as the implicit constraint: $g_1(x) = g_2(x)$ on the minimization problem of $g_1(x)$ or $g_2(x)$ with respect to x. In order to formulate this problem in a symmetric way, the two functions (11) and (12) are combined and constrained using a penalty coefficient "ρ", as follows:

$$g_\rho(x) = g_1(x) + g_2(x) + \rho|g_1(x) - g_2(x)| \tag{12}$$

If "ρ" is large enough, the minimization of $g_\rho(x)$ (13) provides the localization of the Kalai and Smorodinsky solution. To summarise, a general bi-objective optimization problem can be divided into the solution of three SOPs steps as follows:

$$\begin{cases} x_1^* = \min_x f_1(x) \\ x_2^* = \min_x f_2(x) \\ x^* = \min_x g_\rho(x) \\ \quad z_j(x) \geq 0 \qquad j = 1; \dots; J \\ \quad h_k(x) = 0 \qquad k = 1; \dots; K \\ a_{i_{inf}} \leq x_i \leq a_{i_{sup}} \quad i = 1; \dots; m \end{cases} \tag{13}$$

The algorithm used for the mathematical validation of the proposed method is schematically described in Fig. 2. It is based on the minimizer *Optim-Engine* integrated inside the FEM software Forge®. This penalty approach has been preferred over an enforcement via Lagrange multipliers, because it is already implemented in the Forge® *Optim-Engine* which has been used in the application to metal forming problems. The solution search method is based on an Evolutionary Algorithm (EA) assisted by local Gaussian Random Field Metamodels (GRFM) [16]. The Metamodel Assisted Evolution Strategy (MAEA) selects the most promising members in each generation and carries out exact (and costly) evaluations only for them. The extensive use of metamodeling for screening the candidate solutions makes it possible to significantly reduce the computational cost of the EA. Moreover, taking into account the uncertainty information during the predictions allows selecting candidates with best potential ([16-17]).

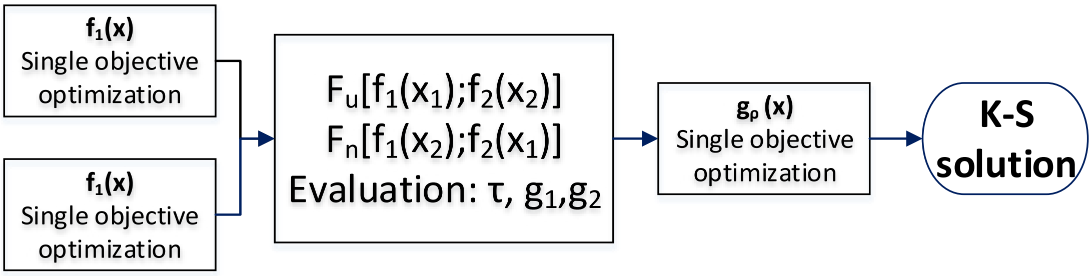

Fig. 2: Optimization algorithm for finding Kalai and Smorodinsky solution.

The validation of the optimization algorithm has been carried out by solving some MOPs based on known mathematical functions [18-20], listed in Table 2. The method has been applied to different problems with the main goal of better understanding the role of the penalty coefficient ρ. The Schaffer's function (SCH1), used by Deb [18], is an easy multi-objective problem initially used in this research work in order to check the results of the algorithm by hand. The Viennet's problem (VNT) [20] has been used in order to verify the extension of the algorithm to MOPs with more than two functions and more than one variable.

Table 2: Properties of the analyzed optimization problems.

Analysed problems		**Number of functions**	**Number of variables**
label	reference		
SCH1 (Schaffer)	[18]	2	1
POL (Poloni)	[19]	2	2
VNT (Viennet)	[20]	3	2

In the following Section, only the VNT problem studied by Viennet [20] will be presented, in order to clarify the application procedure of the optimization algorithm. The results of the other two tested problems are reported only synthetically. The results obtained with the POL problem had been presented in [13].

Mathematical validation and penalty coefficient range definition

The Viennet problems has been selected because it is rather complex, dealing with 3 functions and 2 design variables. It allows to demonstrate how the proposed methodology can be extended to MOP with larger dimensionality. Like the other two tested problems, the mathematical in known analytically, hence the quality of the solution found by the algorithm can be immediately assessed.

A more general form of equation (12) must be formulated in order to deal with three functions:

$$g_\rho(x) = \sum_{i=1}^{n} g_i(x) + \rho \sum_{i=1}^{n-1} |g_i(x) - g_{i+1}(x)| \quad (14)$$

Equation (14) is very important because it allows obtaining a reliable solution located on the Pareto Frontier whereas its graphical representation is complex or impossible. This $g_\rho(x)$ formulation leads to a solution close to the line connecting the Utopia to the pseudonadir point. Viennet's tri-objective minimization problem is formulated by equations (15), and formalized in form of equation (14) by equations (16):

$$
VNT:\begin{cases} f_1(x) = 0.5({x_1}^2 + {x_2}^2) + \sin({x_1}^2 + {x_2}^2) \\ f_2(x) = \frac{(3x_1 - 2x_2 + 4)^2}{8} + \frac{(x_1 - x_2 + 1)^2}{27} + 15 \\ f_3(x) = \frac{1}{{x_1}^2 + {x_2}^2 + 1} - 1.1\exp\left[-({x_1}^2 + {x_2}^2)\right] \\ -3 \leq x_1, x_2 \leq 3 \end{cases} \tag{15}
$$

$$
KS(VNT) = \begin{cases} \mathbf{g}_{\boldsymbol{\rho}}(\mathbf{x}) = g_1(x) + g_2(x) + g_3(x) + \rho(|g_1(x) - g_2(x)| + |g_2(x) - g_3(x)|) \\ g_1(x) = \frac{f'_1(x) - f'_1(x_1^*)}{\tau'_1} = \frac{f'_1(x) - 1{,}503\times10^{-5}}{0.577} \\ g_2(x) = \frac{f'_2(x) - f'_2(x_2^*)}{\tau'_2} = \frac{f'_2(x) - 7{,}384}{0.577} \\ g_3(x) = \frac{f'_3(x) - f'_3(x_3^*)}{\tau'_3} = \frac{f'_2(x) - (-0{,}428)}{0.577} \\ \rho = 1 \end{cases} \tag{16}
$$

The three SOPs listed in equation (15) and the SOP given by equation (16) have each been performed with 100 exact computations (iterations) resulting from 25 generations of the EA with a population of 4 individuals. The iteration number which yielded the minimum for each of the four functions (f_1, f_2, f_3 and g_ρ) is shown in Table 3. The minimum of function f_1 was found at the last iteration. Despite $g_\rho(x)$ being a function that is more complex than the previous three, the number of iterations required for finding the minimum of $g_\rho(x)$ does not seem to be larger than the number of iterations required for the other three functions. The results of the four single-function optimizations are shown in Figure 3.

Table 3: Iteration number of the minimum solution found for each function of the VNT problem.

function	iteration #
$\boldsymbol{f_1(x)}$	100
$\boldsymbol{f_2(x)}$	86
$\boldsymbol{f_3(x)}$	16
$\boldsymbol{g_\rho(x)}$	46

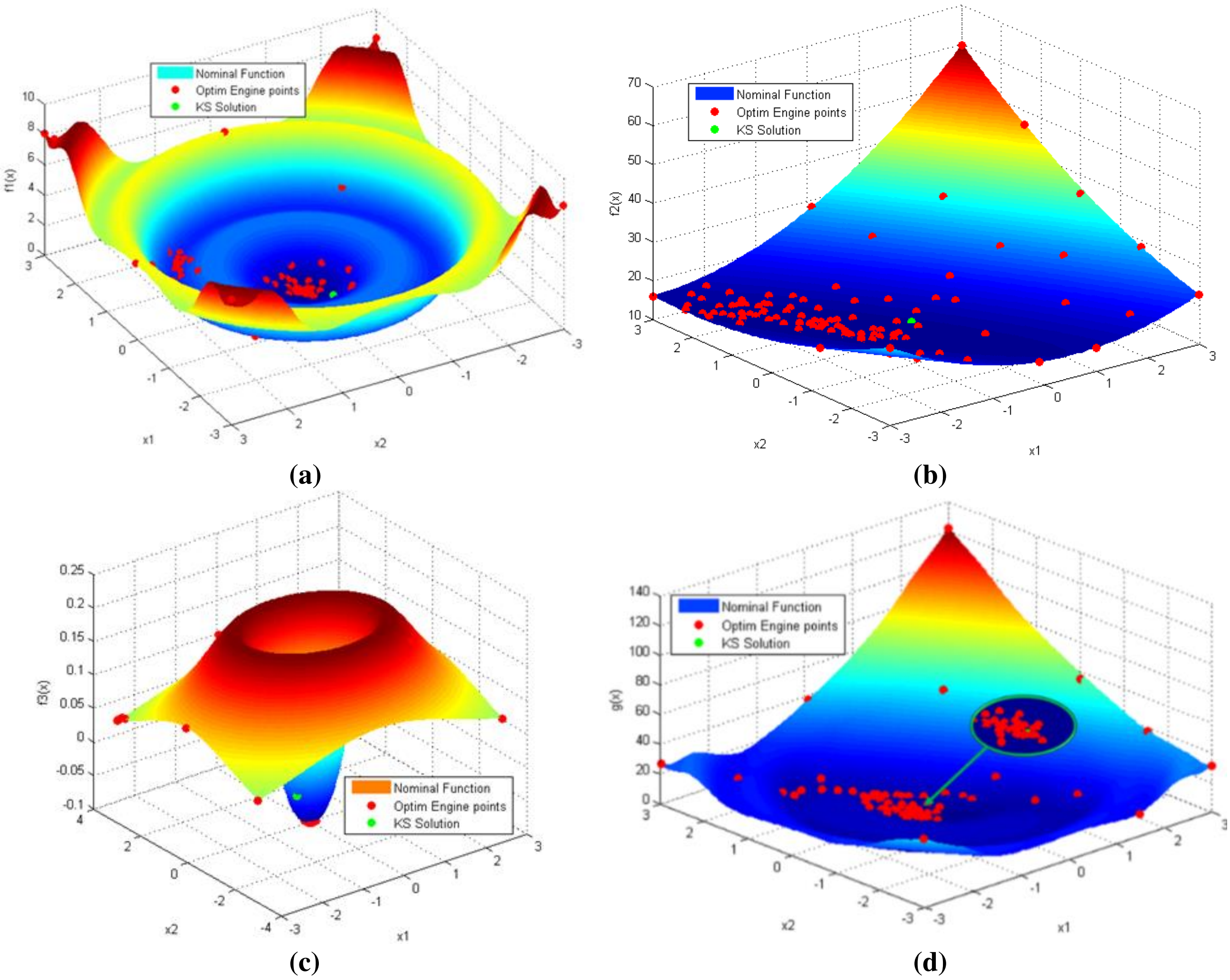

Fig. 3: Optimization results for VNT problem: single objective optimization for $f_1(x)$ (a), $f_2(x)$ (b), $f_3(x)$ (c) and $g_\rho(x)$ (d).

In Figure 4a, the results are plotted in the space of the solutions, which is 3-dimensional. For this reason only the PF and the KS point are plotted. The different projections of all evaluated solution points are shown in Figures 4b and 4c, respectively in the $f_2(x)$ vs. $f_1(x)$ space and in the $f_3(x)$ vs. $f_2(x)$ space. The optimization algorithm allows finding an interesting solution (the green K-S points in Fig. 4), in the centre zone of the Pareto Frontier.

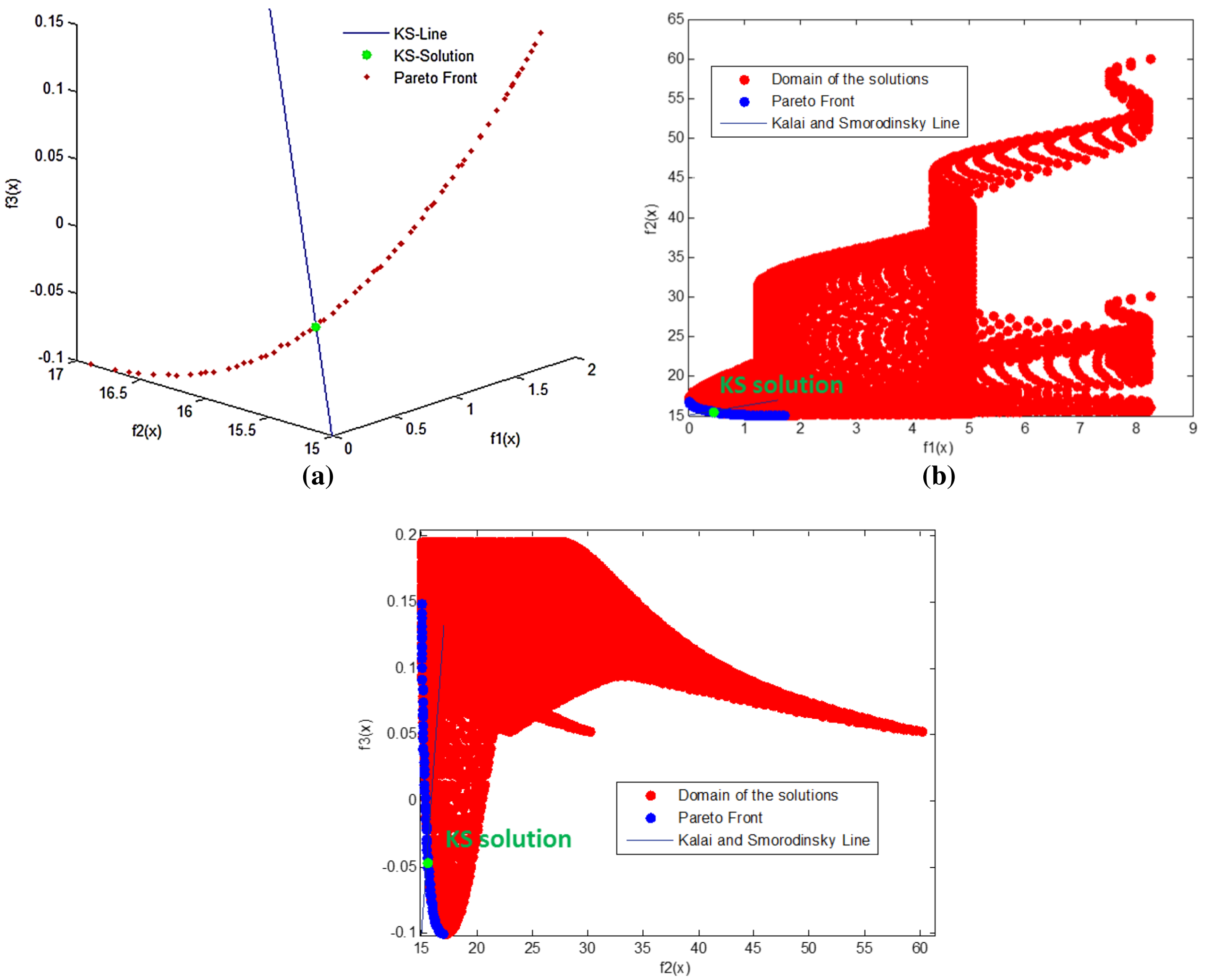

Fig. 4: VNT Pareto Frontiers projected at: f2(x) vs f1(x) (a), f3(x) vs f2(x) (b), f3(x) vs f1(x) (c) and resultant 3D Pareto Frontier (d).

This example shows how the method can be applied to multi-objective optimization problems with more than two functions and more than one variable. It allows obtaining a reliable solution located on the Pareto Frontier, which is especially interesting when its representation is very complex due to the problem dimensionality.

The quality of the solution, in terms of the normalized distance of the K-S solution to the K-S Line, and the computational efficiency of the algorithm, in terms of the number of iterations, are evaluated with respect to the ρ-value. These results are presented in Table 4 along with those obtained by the other two test problems listed in Table 2. The ρ-value plays an important role (13) because it balances the two mechanisms of minimization:

1. the localization of the solution on the Pareto Frontier, given by the minimization of $g_1(x) + g_2(x) + g_3(x)$, which is the first part of equation (16);
2. the positioning of the solution on the Kalai and Smorodinsky line, by the minimization of $|g_1(x) - g_2(x)| + |g_2(x) - g_3(x)|$, which is the second part of equation (16).

It is expected that the first mechanism is dominant when the ρ-value is smaller than 1, and the opposite when it is larger than or equal to 1. Table 4 confirms this expectations, since the only two values where the solution is found outside of the PF are for the largest ρ-value. On the contrary, the normalized distance of the found solution to the K-S Line decreases as ρ increases.

Table 4: Optimization results for different ρ values (numbers in bold font indicate the best performances).

Performance measure	*Test case*	*ρ=0.1*	*ρ=0.5*	*ρ=1*	*ρ=2*
normalized distance of the found solution to KS Line	SCH1	7.35E-04	**3.60E-04**	**3.60E-04**	**2.73E-04**
	POL	2.04E-01	**2.12E-03**	**4.83E-04**	OPF
	VNT	3.204 E-01	**1.08E-02**	1.39E-02	1.36E-02
number of iterations for solving the optimization problem	SCH1	41	**18**	**18**	19
	POL	**14**	93	97	OPF
	VNT	61	82	**46**	87

(OPF: **O**utside the **P**areto **Frontier**)

For a budgeted number of iterations of 100, the interval of acceptable ρ values is quite narrow. For values lower than 0.5, the constraint $g_1(x) = g_2(x)$ is not satisfied with sufficient accuracy in the VNT and POL problems. As stated above, for larger ρ-values, the solution obtained after 100 iterations is not always satisfactory: it is outside the Pareto Frontier for the Polini's problem. In conclusion, it appears that a good compromise range for ρ lays between 0.5 and 1.

The K-S method applied to a wire-drawing optimization problem

In metal wire-drawing operations, the optimization of the die angle with respect to the pulling force (**Errore. L'origine riferimento non è stata trovata.**) is a classical and common problem, whose solution has been solved for years by either empirical rules or analytical methods [21], before current use of the finite element simulation. In case of low friction, the optimum semi-die angle is 6°. However, the engineering practice may give account of wire damage resulting into failure, so damage is to be considered within the optimization, as done by the authors of [22]. The objective of reducing damage conflicts with the need for a low pulling force. In fact, angles smaller than 6° would allow a reduction of the damage. Therefore, a compromise between these two objectives is to be found. The problem becomes more complex in a two-steps wire drawing problem, not only because there are two design variables instead of only one, but also because the pulling force is not linear to the angle values of the two drawing stations. In symbols, the optimization objectives are to minimize the pulling Force (F_{max}) and the maximum computed Damage (D_{max}) on the wire at the end of a double step wire-drawing operation. The design variables are the two semi-die angles α_1, α_2 of the first and second dies (Table 6), so the MOP minimization can be formulated as:

$$\begin{cases} \min f_1(\alpha_1, \alpha_2) = D_{max}(\alpha_1, \alpha_2) \\ \min f_2(\alpha_1, \alpha_2) = F_{max}(\alpha_1, \alpha_2) \\ \quad 1.20° \leq \alpha_1, \alpha_2 \leq 22.50° \end{cases} \tag{21}$$

The general geometry of the problem is described in Figure 5, with an initial billet with radius R_i = 9.5 mm. the initial geometry is reported in Table 5.

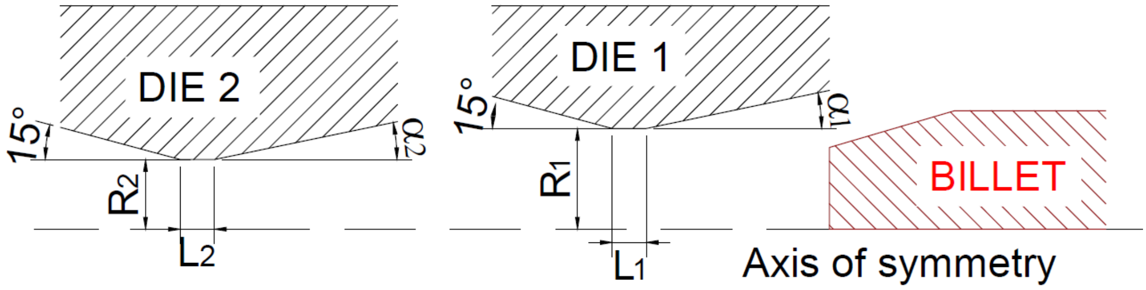

Fig. 5: Wire-drawing minimization problem with two stations.

Table 5: Initial geometry of the two wire drawing stations.

Geometrical parameter	Symbol	Value 1	Value 2
Length [mm]	L	2.9	3.4
Angle [deg]	α	1.2	1.2
Radius [mm]	R	8.5	7.5

Indeed, the problem could be formulated not as a MOP, but as a SOP on pulling force, with a constraint on damage. Unfortunately, ductile damage criteria in FEM cannot be used as deterministic indicators of threshold between safe and failed parts. More wisely, they are taken as indicators of risk. In wire drawing, an exact quantification of damage is also a very difficult problem [24]. In this case, the well-known Latham and Cockroft criterion is used, but not associated to any threshold nor critical value. Consequently, it is useful to reduce the damage formation as much as possible.
The D_{max} value is evaluated according to the Cockcroft and Latham ductile damage model:

$$D_{max} = \underset{\Omega}{\mathrm{Max}}\left(\int_0^{\varepsilon_f} \frac{\max(\sigma_I,0)}{\sigma_{eq}} d\varepsilon_p\right) \tag{18}$$

where Ω: computational domain

ε_f: final strain

σ_{eq}: equivalent stress

σ_I: principal stress

ε_p: plastic strain

The wire is made of steel C72 which is modelled by the following constitutive law:

$$\sigma_{eq} = \sqrt{3}K\left(1 + \alpha\left(\varepsilon_{eq}^{p}\right)^{n}\right) \tag{19}$$

with the following values of the coefficients: $\begin{cases} \alpha = 7.32 \\ K = 100.32\ \text{MPa} \\ n = 0.13 \end{cases}$

The Tresca law is used for modelling friction:

$$\tau_c = \bar{m}\frac{\sigma_0}{\sqrt{3}}\frac{\Delta v_t}{\|\Delta v_t\|} \tag{20}$$

where the friction factor is m=0.02, Δv_t is the relative tangent velocity and σ_0 is the Von Mises stress. Friction has a dramatic effect on the wire-drawing process, hence any change in the friction factor or in it formulation would lead to a different process and a different optimal solution. However, the optimization methodology would be unchanged, whichever the friction formulation. In this paper, we have selected a particular friction value for which there were available

optimization results in the literature [22]. The selected low m-value is in agreement also with reference [24].
The Kalai and Smorodinsky method has been implemented in the Forge® FEM Package. The proposed algorithm is schematically described in **Errore. L'origine riferimento non è stata trovata.**. First, two separate SOPS are solved, using the *Optim-Engine* algorithm, in order to find the coordinates [$D_{max\_U}(\alpha_1)$; $F_{max\_U}(\alpha_2)$] of the Utopia point. Then, two single simulations are run to obtain the coordinates [$D_{max\_N}(\alpha_2)$; $F_{max\_N}(\alpha_1)$] of the Nadir point. Finally, the KS point is found by minimising the $g_\rho(\alpha_1, \alpha_2)$ function, using the penalty coefficient. The results will be presented with ρ=0.8, but the same analysis has been run also with ρ=1 and a comparison will be presented.

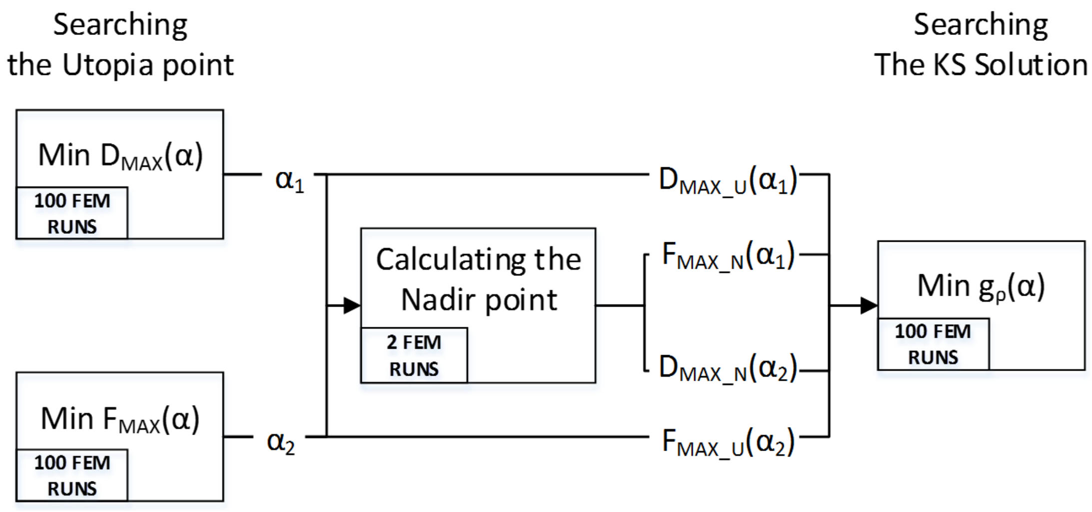

Fig. 6: Procedure for implementing Kalai and Smorodinsky method in Forge®.

A budget of N_{runs}=100 function evaluations (i.e. FEM simulation runs) has been used for each of the three optimizations, resulting into a total of 302 individual FEM runs ($3*N_{runs}+2$). The design space generated by the SOP problems is shown in Figure 7, along with the values of the two objective functions.
It must be observed that the plotted Pareto Frontier is only an approximation of the exact Frontier, which has not been explicitly and fully computed. The dotted line is only a by-product of the optimization procedure. The solution found, i.e. the estimated K-S point, is calculated independently of the full and accurate knowledge of the whole Pareto Frontier. The best approximation of the K-S point has been found after 76 runs of the $g_\rho(\alpha_1, \alpha_2)$ optimizer. It suggests the 3.14° and 2.14° values for α_1 and α_2 respectively. This is also close to the results found in [23], where the values obtained in a similar problem were respectively 2.65° and 1.80°.

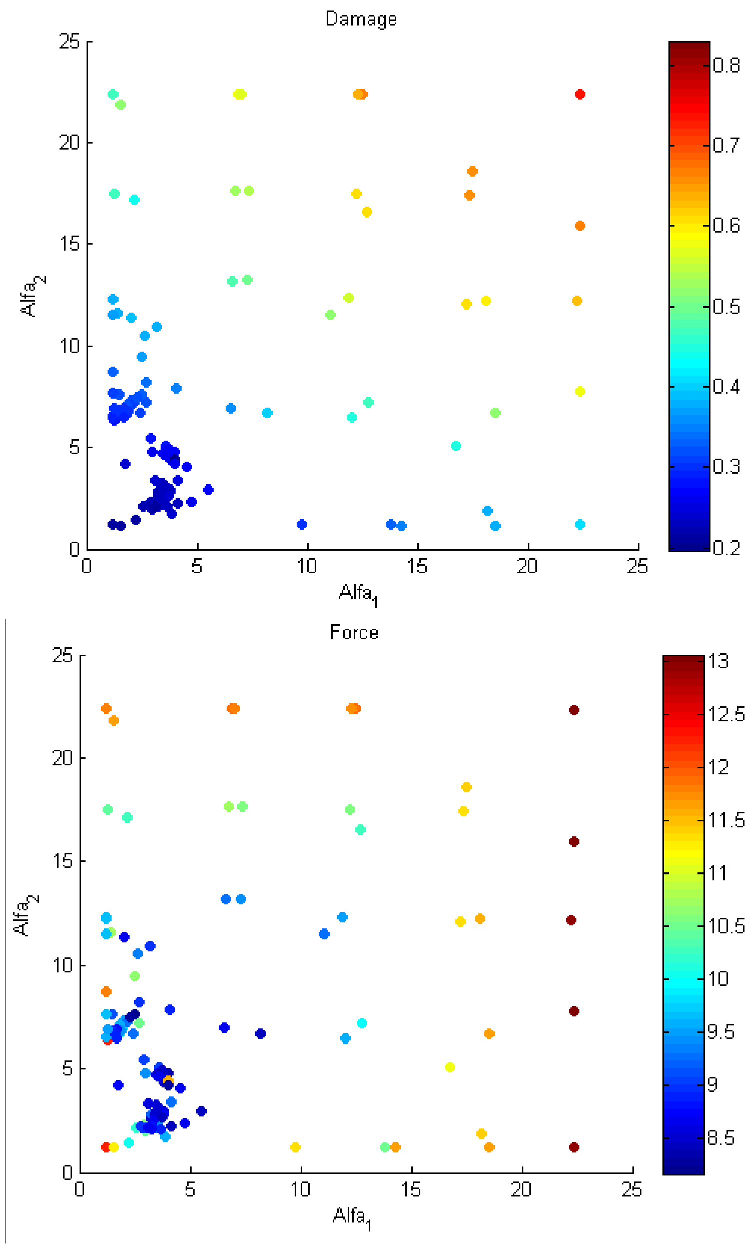

Fig. 7: Force [??] and damage [??] values plotted vs. the design points generate by the Optim-Engine.

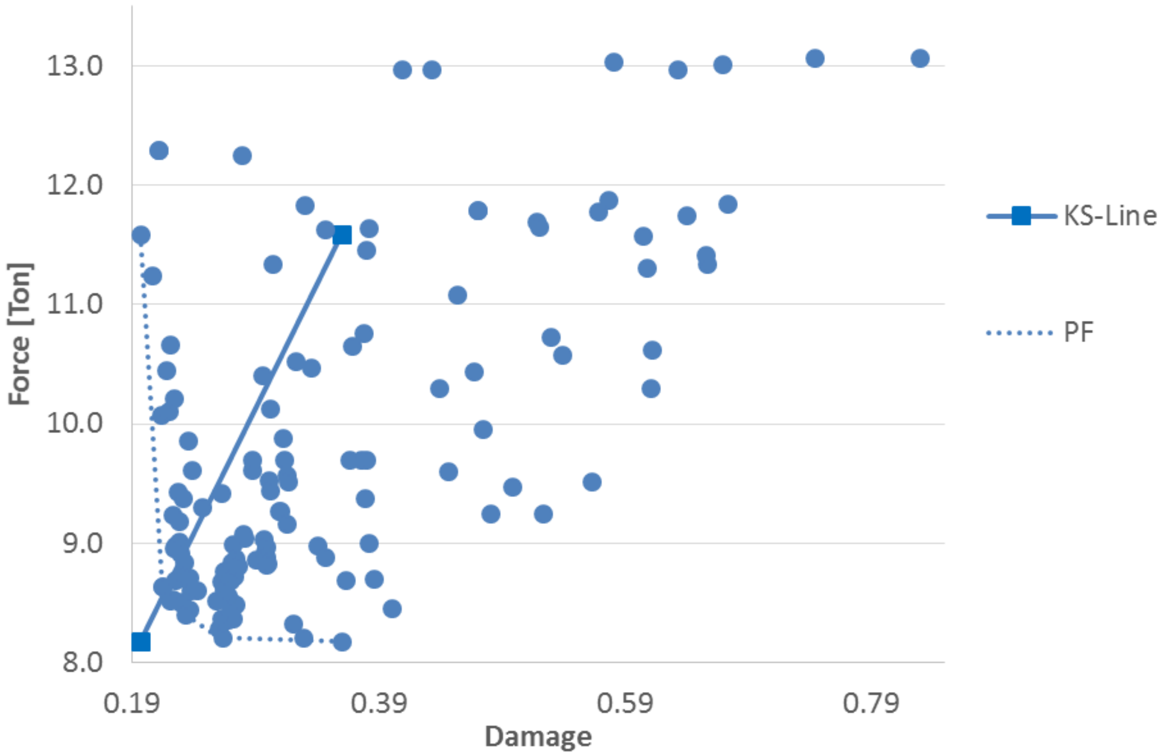

Fig. 8: Evaluation of Kalai and Smorodinsky solution with Forge® for the two-stage wire drawing problem.

The solutions found in this wire-drawing minimization problems, using the K-S paradigm, are very close to the solutions already suggested in the literature, which were obtained from the computation of the entire Pareto Frontier and with engineering expertise. This suggests that the K-S solution properties (symmetry, monotonicity and invariance with respect to affine transformation) properly model the expert expectations. These results encourage to apply the K-S approach to FEM-based metal forming multi-objective optimization problems, with reduced computational cost and no need of a posteriori selection of a solution.
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Conclusions

In this paper, a general methodology has been presented for computing the Kalai and Smorodinsky multi-objective solution by solving a series of mono-objective optimization problems. This constrained formulation, based on a penalty method, shows quite robust and efficient, and was validated on several analytical multi-objective problems. This approach can be easily implemented into any existing mono-objective algorithm, as was done in this paper with the Optimization Engine of Forge® software. A by-product of this approach is the knowledge of the minimum solution with respect each objective function, independently from the other, which can be useful in the engineering practice, for instance if there is a strong hierarchy between the functions and if one function plays a greater role than the others. The Kalai and Smorodinsky approach does not require computing the entire Pareto Frontier of the multi-objective optimization problem, consequently reducing the computational cost of the optimization procedure. By providing a single solution, it avoids an a posteriori tricky choice of a solution on the PF, which turns out to be quite cumbersome with more than two functions.
This paradigm has been successfully applied to well-known metal forming problems, with an efficient convergence of the algorithm. The computed solutions are very close to those proposed in the literature, where they have been deduced from the knowledge of the entire Pareto Frontier and the engineering expertise. This shows that the different properties of the Kalai and Smorodinsky

compromise provide a kind of natural choice which is in good agreement with the engineer logic. In any case, the K-S approach always provides an interesting solution among all possible ones, and the K-S paradigm suits well to metal forming problems.
In studied examples, the little differences between the K-S solutions and the engineer choices lying on the PF mainly results from the implicit criterion used by the engineer. In the studied cases, the pulling Force was regarded as a more traditional and consequently more important criterion to satisfy. Therefore, the two objectives were not quite considered in a symmetric manner. Such specificities, if known in advance, could be introduced in the construction of K-S solution by turning its symmetry property into an asymmetric criterion modelling the hierarchy between the functions. It is expected that such asymmetric K-S solution would be even closer to the engineering choice. Therefore, the results obtained in this paper open the doors of the proposed methodologies to future implementation inside other software packages, as a useful support to the decisions of process engineers.

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