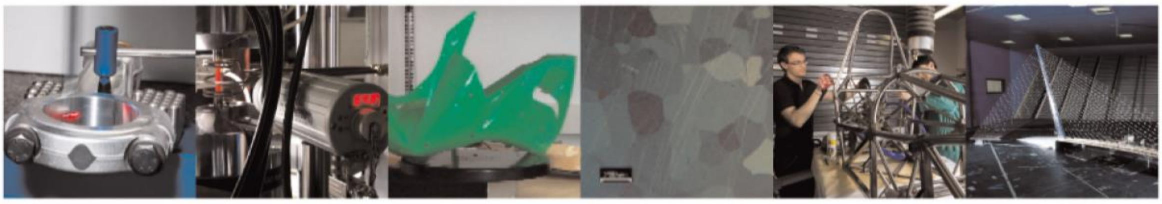

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Energy-Efficient Control Strategies for Machine Tools With Stochastic Arrivals

Nicla Frigerio and Andrea Matta

I. Introduction

IN the last few years, there has been a growing interest in technical solutions to reduce the energy consumption in manufacturing. This trend is mainly driven by governments which are conceiving new regulations to reduce the environmental impact of manufacturing. In this scenario, the machine tool energy consumption reduction is becoming a challenging goal. The most relevant measures for reducing energy consumption at machine level are the eco-design of components aiming at minimizing power demand, the kinetic energy recovery systems (KERS) to reuse and recover energy, and the implementation of control strategies for the efficient usage of components by minimizing processing time and nonvalue tasks [1].

This paper proposes a framework that integrates several control policies in a general one for deciding the most suitable energy state for machine tools during their nonproductive phases. Particularly, the general policy includes two controls: the first control command switches the machine *off* when production is not needed because the part flow is interrupted. The second command switches the machine *on* when the part flow is resuming or the machine has to be ready before the arrival of a part. The control parameters define the time instants wherein the machine tool is switched off and on, respectively.

II. Brief Literature Survey

The power requirement of a machine tool can be divided into two main components: a *Fixed Power*, demanded for the operational readiness of the machine and independent from the process and a *Load Dependent Power*, demanded to distinctively operate components enabling and executing the main process [2]–[6]. Among the energy-saving measures at the machine level, it is possible to differentiate between *process control* and *state control*. The power demanded for a manufacturing process depends on the selection of the process parameters. Thus, controlling the process itself towards the optimum tradeoff between cycle time and power demand, the machine load-dependent energy consumption can be reduced [7], [8]. The state control aims at reducing the fixed power consumption, which is required even if the production is not requested. Based on the results of experimental measurements, Schmitt *et al.* [9] deduced that auxiliary equipment—i.e., suppliers—can easily require more energy than necessary. Indeed, the suppliers keep consuming energy during not productive states, because they must be available when the production has to be resumed. This generates a supply exceed that could be reduced by controlling the suppliers state.

Other research efforts focused on the problem of scheduling startup and shutdown of machines to minimize total energy consumption. A first group of studies does not consider any warm-up transitory when the machine tool is switched off. In order to give some examples, Prabhu *et al.* developed an analytical model by combining an M/M/1 model with an energy control policy [10]. Firstly for a station and then for a production line, they calculate the time interval for switching the

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machine off during idle period with respect to a target energy waste limit. In this study, the machine switch-off accounts for a certain idling power, but the switch-on is instantaneous once the part arrives. Chang *et al.* analyzed several real-time machine switching strategies using energy saving opportunities windows in a machining line under random failures [11], but no warm-up time is considered.

A second group of studies considers a deterministic and constant warm-up duration whenever the machine is switched off. Mouzon *et al.* presented several switch-off dispatching rules for a non-bottleneck machine in a job shop [12]. Chen *et al.* formulated a constrained optimization problem for scheduling machines *on* and *off* modes in a production line based on Markov chain modelling and considering machines having Bernoulli reliability model [13], [14]. Sun and Li proposed an algorithm to estimate opportunity windows for real-time energy control in a machining line [15]. Random failures are considered, whereas the cycle time and warm-up time are deterministic and constant. Mashahei and Lennartson proposed a control policy to switch-off machine tools in a pallet-constrained flow shop. The policy aims to minimize energy consumption under design constraints and considering two idle modes with deterministic warm-up durations [16].

Currently, most of machine tools do not have "green" functionalities, and in the industrial market there are only few energy-saving control systems available. Most of them have been developed by machine tool builders in order to support the final users. As examples among the existing options in the industrial market, some companies provide devices for shutdown the machine tool, or some functional modules, once the machine idle period exceeds a user-defined limit [17]–[19]. In other cases, the switch-off command, that switches off the machine tool when a certain time interval has elapsed from the last departure, is embedded in the PLC of the machine, e.g., the machine studied for the numerical case of this work. However, the selection of the policy with its parameter is not supported by any tool or method.

The literature analysis points out a lack of theoretical modelling concerning the machine energy-efficient control problem for systems under uncertainty. This paper studies a framework for energy oriented control of machine tools in manufacturing. A general policy is assessed in terms of expected energy consumed by the machine under the assumption of stochastic arrivals and no input buffer. Under quite general assumptions, the paper shows that an optimal control always exists, and the equations for its numerical calculation are also provided. The policy and its optimal conditions are studied on the basis of a set of numerical cases built to provide useful guidelines for practical implementation of energy saving control policies. Numerical cases refer to a real CNC machining center that was experimentally characterized to estimate its power demand.

III. Assumptions

A single machine working a single part type is considered. This assumption is valid for machines specialized for one single part type or for a family of similar items, and machines working large batches while considering the single batch.

The machine can be in one of the following states: out-of-service, on-service, warm-up and working. In the *out-of-service* state—i.e., the stand-by state— some of the machine modules are not ready, indeed only the emergency services of the machine are active while all the others are deactivated. In this state, the machine cannot process a part being in a kind of "sleeping" mode. The power consumption of the machine when out-of-service is denoted with $x_{\rm out}$, generally lower compared to the other machine states. In the *on-service* state—i.e., the idle state—the machine is ready to process a part upon its arrival. The machine power consumption when on-service, denoted with $x_{\rm on}$, is due to the activation of all its modules that have to be ready for processing a part. From the out-of-service to the on-service state the machine must pass through the *warm-up* state—i.e., the transitory start-up state between the out-of-service and the on-service states—where a procedure is executed to make the modules suitable for processing. The warm-up procedure has duration and power consumption equal to $t_{\rm wu}$ and $x_{\rm wu}$, respectively. The value $x_{\rm wu}$ is generally greater than in the other machine states. The way in which the warm-up time is decided depends on the part tolerances and is not a decision variable of the control policy. Therefore, we assume the user of the control policy has already validated the warm-up time and power, and, at this level, they can be assumed as constant because they are considered as an input given to the control policy. In the *working* state the machine is processing a part and the requested power changes according to the process. Any non-cutting state with the part loaded on the machine is integrated in the working state.

The interarrival time is a random variable $(t_p + T)$, where the machine processing time t_p is deterministic and constant, and T is randomly distributed. The probability density function (PDF) $f(t)$ models the time T between a part departure from the machine and the next arrival—where t is the realization of T. Moreover, we assume the random variable T is not affected by the control policy applied, as a consequence the PDF $f(t)$ does not vary. This assumption can represents a production case where the machines are synchronized, whereas in other production cases, it yields to approximated results.

We assume the machine has an input buffer. However, this buffer is outside of the machine that cannot observe when parts arrive. There is an input mechanism that controls the release of parts to the machine: one part is sent to the machine only during its idle or not productive periods. The machine can see only the single part released by the input mechanism. Note that, in the real system, parts can arrive any time and stay (unobserved by machine) in the buffer in front of the machine. Indeed, if the part arrives while the machine is busy, the input mechanism keeps the part until the machine becomes idle, i.e., $t = 0$. When the machine is not working, the part immediately starts being processed if the machine is ready, otherwise it has to wait until the machine is warmed up. If the warm-up is long enough to ensure machine proper conditions to work, the quality of processed parts is guaranteed. After the completion of the process the part leaves the system. An infinite buffer is assumed downstream the machine. For simplicity, the machine is assumed to be perfectly reliable, thus failures cannot occur; this assumption can be relaxed without requiring large extensions to the developed analysis.

The transition between two states can be triggered by the occurrence of an uncontrollable event, e.g., the part arrival, or a controllable event. During the idle periods of the machine it is not necessary to keep all the machine modules active, and the machine can be moved, with a proper control, into the out-of-service state characterized by a low power consumption. Nevertheless, if a part arrives when the machine is not on-service—this can happen when the machine is in out-of-service or executing the warm-up procedure—there is a penalty. We express this penalty by the power consumption x_q necessary for keeping the part waiting until the on-service state is reached. However, once in out-of-service, the machine can be warmed up in advance in order to avoid x_q.

The following section describes a policy that can be used to control the state of the machine by activating a transition from the on-service to the out-of-service state, i.e., *Switch-off command*, and from the out-of-service to the warm-up state, i.e., *Switch-on command*.

IV. Control Policies

The control policy is now presented describing the machine behavior in terms of states visited and transitions triggered. The total energy consumed in a cycle is the sum of the product $power \times time$ for each state visited by the machine. We consider a cycle starting from the departure of a part and finishing when the machine starts processing the next part. The part processing, i.e., the machine working state, is not considered in the cycle because it does not affect the selection of the policy parameters, being not dependent on control actions. Indeed, the machine is modeled in the way that when arrival occurs the machine passes through the working state after maximum t_{wu}. Therefore, it is not allowed by the model to shut down the machine ignoring the working request. Since the time spent in a certain state by the machine during a cycle is the output of a stochastic process—indeed, the arrivals are random—the expected value of the energy consumed in a cycle by the machine is the objective function to be minimized. Other objective functions related to the risk of incurring in high consumptions are not considered in this work. The most common practices in manufacturing are also reported.

A. Switching Policy

The general policy studied in the paper is now defined.

Policy 1 (Switching): Switch off the machine after a time interval τ_{off} *has elapsed from the last departure* ($t = 0$). *Then, switch on the machine after a time interval* τ_{on} *has elapsed from the last departure, i.e., when* $\tau_{\mathrm{on}} - \tau_{\mathrm{off}}$ *has elapsed from the switch-off command, or when a part arrives.*

After a part departure, the machine remains in the on-service state in order to immediately process parts coming in the short time, and the time instant τ_{off} is properly set for avoiding too frequent warm-up procedures. Moreover, in order to be consistent, the switch-off command has to be issued before the switch-on command as follows:

$$\tau_{\mathrm{on}} \geq \tau_{\mathrm{off}}. \tag{1}$$

When the policy is applied, the machine evolution may follow one among four different paths depending on the random arrival T of the part (Fig. 1). The energy consumed through each path h_i can be calculated as the conditional expectation of $power \times time$ for all of the states visited through the path i, given a certain event A_i that represents the occurrence of the path i over the feasible ones (see [20] and [21]). The cycle starts with the

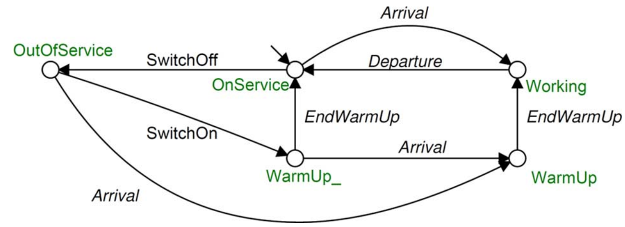


Fig. 1. Machine state model with *Switching* policy active.

machine in the on-service state waiting for the part arrival, then four different events may occur, outlined here.

1) *The part arrives before the machine is switched off*: $A_1 = \{0 \leq t < \tau_{\mathrm{off}}\}$. In this case, the machine spends the whole cycle in the on-service state, and the expected value of the energy consumed is

$$h_1 = \mathbb{E}\left[x_{\mathrm{on}} T \mid A_1\right]. \tag{2}$$

2) *The part arrives when the machine is out-of-service*: $A_2 = \{\tau_{\mathrm{off}} \leq t < \tau_{\mathrm{on}}\}$. This situation represents the fact that the machine has been switched off at τ_{off}. The machine is waiting either for a part arrival or for the switch-on command to start the warm-up. During the time in which the part has not arrived yet, the machine consumes $x_{\mathrm{on}}\tau_{\mathrm{off}}$ before being switched off, and it requires x_{out} once in the out-of-service state. When the part arrives, the machine starts executing the warm-up during which the part has to wait, and the related power requested is $(x_{\mathrm{wu}} + x_q)$, given as follows:

$$h_2 = x_{\mathrm{on}}\tau_{\mathrm{off}} + (x_{\mathrm{wu}} + x_q)t_{\mathrm{wu}} + \mathbb{E}\left[x_{\mathrm{out}}(T - \tau_{\mathrm{off}}) \mid A_2\right]. \tag{3}$$

3) *The part arrives when the machine is executing the warm-up procedure*: $A_3 = \{\tau_{\mathrm{on}} \leq t < \tau_{\mathrm{on}} + t_{\mathrm{wu}}\}$. This situation represents the fact that the machine has been switched off at τ_{off}, then it has been switched on at τ_{on} from the beginning of the cycle. Thus, the machine has already started the warm-up procedure. The machine consumes $x_{\mathrm{on}}\tau_{\mathrm{off}}$ before being switched off, $x_{\mathrm{out}}(\tau_{\mathrm{on}} - \tau_{\mathrm{off}})$ until the warm-up starts, and $x_{\mathrm{wu}}t_{\mathrm{wu}}$ during the warm-up. Furthermore, the part has to wait requiring x_q for the rest of the warm-up that the machine has not executed yet. The energy consumption is

$$\begin{aligned} h_3 =& x_{\mathrm{on}}\tau_{\mathrm{off}} + x_{\mathrm{out}}(\tau_{\mathrm{on}} - \tau_{\mathrm{off}}) + x_{\mathrm{wu}}t_{\mathrm{wu}} \\ &+ \mathbb{E}\left[x_q(\tau_{\mathrm{on}} + t_{\mathrm{wu}} - T) \mid A_3\right]. \end{aligned} \tag{4}$$

4) *The part arrives when the machine has completed the warm-up procedure and is in the on-service state*: $A_4 = \{t \geq \tau_{\mathrm{on}} + t_{\mathrm{wu}}\}$. This situation represents the fact that the machine has been switched off after τ_{off}, it has been switched on after τ_{on}, and the warm-up has been completed. The energy consumed by the machine is similar to h_3, but, in this case, the part will never wait for machine readiness, whereas the machine requires x_{on} after the warm-up waiting for the part arrival as follows:

$$\begin{aligned} h_4 =& x_{\mathrm{on}}\tau_{\mathrm{off}} + x_{\mathrm{out}}(\tau_{\mathrm{on}} - \tau_{\mathrm{off}}) \\ &+ x_{\mathrm{wu}}t_{\mathrm{wu}} + \mathbb{E}\left[x_{\mathrm{on}}(T - \tau_{\mathrm{on}} - t_{\mathrm{wu}}) \mid A_4\right]. \end{aligned} \tag{5}$$

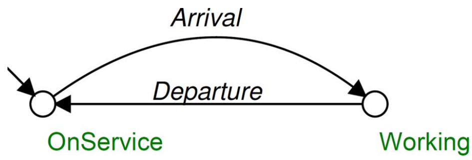


Fig. 2. Machine state model with *always on* policy active.

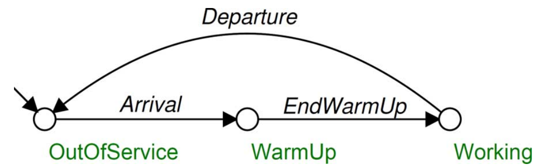


Fig. 3. Machine state model with *off* policy active.

Once defined, the sample space $\mathcal{A} = \{A_1, A_2, A_3, A_4\}$ composed by events A_i mutually exclusive and collectively exhaustive, by letting $\mathbb{P}(A_i)$ be the probability of occurrence of the event A_i, the Total Expectation Theorem [22] states that

$$\mathbb{E}[T] = \sum_{A_i \in \mathcal{A}} \mathbb{E}[T \mid A_i] \, \mathbb{P}(A_i). \tag{6}$$

Given (2)–(6), the expected value of the energy consumption in an average cycle under the *switching* policy control is

$$g_1 = h_1 \mathbb{P}(A_1) + h_2 \mathbb{P}(A_2) + h_3 \mathbb{P}(A_3) + h_4 \mathbb{P}(A_4). \tag{7}$$

Using T and $f(t)$ as defined in Section III, we have

$$\begin{aligned} g_1 = & \int_0^{\tau_{\text{off}}} x_{\text{on}} t f(t) dt \\ & + \int_{\tau_{\text{off}}}^{\tau_{\text{on}}} \left[x_{\text{out}}(t - \tau_{\text{off}}) + x_q t_{\text{wu}} \right] f(t) dt \\ & + \int_{\tau_{\text{on}}}^{\tau_{\text{on}} + t_{\text{wu}}} x_q (\tau_{\text{on}} + t_{\text{wu}} - t) f(t) dt \\ & + \int_{\tau_{\text{on}} + t_{\text{wu}}}^{\infty} x_{\text{on}} (t - \tau_{\text{on}} - t_{\text{wu}}) f(t) dt \\ & + (x_{\text{on}} \tau_{\text{off}} + x_{\text{wu}} t_{\text{wu}}) \int_{\tau_{\text{off}}}^{\infty} f(t) dt \\ & + x_{\text{out}} (\tau_{\text{on}} - \tau_{\text{off}}) \int_{\tau_{\text{on}}}^{\infty} f(t) dt. \end{aligned} \tag{8}$$

B. Always On and Off Policies

In order to compare the policy described, two simple ways of managing a machine are discussed. These policies are extreme situations in which the machine is always kept on-service or it is switched off as soon as the part process ends.

Policy 2 (Always on): Stay in the on-service state after the departure of a part.

The machine visits only the states on-service and working as illustrated in Fig. 2. The *always on* policy is apt to maintain the machine in the proper condition to work avoiding x_q, thus it is used for high utilized machines. This simple policy is commonly adopted in practice because, actually, most of the machine tools do not have "green" functionalities. In Policy 2, neither the switch-off control command nor the switch-on command have never been issued ($\tau_{\text{off}} = \tau_{\text{on}} = \infty$). As a consequence, the machine stays in the on-service state for the whole cycle and only the h_1 component affects the machine consumption g_2 because $\mathbb{P}(A_1) \to 1$:

$$g_2 = x_{\text{on}} \mathbb{E}[T]. \tag{9}$$

Policy 3 (Off): Switch off the machine after the departure of a part.

The machine visits the states out-of-service, warm-up and working as illustrated in Fig. 3. After the departure of a part the machine moves from the working state to the out-of-service state, and the warm-up state is visited after every arrival. With the *off* policy, the part must always wait for a period equal to t_{wu}. This policy is adopted for low utilized machines or with not relevant warm-up duration. In Policy 3 the switch-off command immediately follows the departure ($\tau_{\text{off}} = 0$), and the switch-on command never occurs ($\tau_{\text{on}} \to \infty$) because the machine is warmed up upon the arrival. Given that $\mathbb{P}(A_2) \to 1$ and $\tau_{\text{off}} = 0$, the energy consumption is

$$g_3 = x_{\text{out}} \mathbb{E}[T] + (x_{\text{wu}} + x_q) t_{\text{wu}}. \tag{10}$$

C. Switch-Off and Switch-On Policies

The *switch-off* policy represents a special case of Policy 1 where the switch-on command is never issued ($\tau_{\text{on}} \to \infty$).

Policy 4 (Switch Off): Switch off the machine after a time interval τ_{off} has elapsed from the last departure.

Indeed, after a part departure, the machine remains in the on-service state in order to immediately process parts coming in the short time, but at proper time instant τ_{off} the machine is switched off. Once in out-of-service, the machine is warmed up only after an arrival. Given that $\mathbb{P}(A_1 \cup A_2) \to 1$, the expected value of energy consumption is

$$\begin{aligned} g_4 = & x_{\text{on}} \mathbb{E}[T|A_1] \, \mathbb{P}(A_1) \\ & + \left[x_{\text{out}} (\mathbb{E}[T|A_2] - \tau_{\text{off}}) + (x_{\text{wu}} + x_q) t_{\text{wu}} \right] \mathbb{P}(A_2). \end{aligned} \tag{11}$$

Similarly to Policy 4, the *switch-on* policy is a special case of Policy 1 where the machine is immediately switched off after the part departure ($\tau_{\text{off}} = 0$). If the arrival occurs when the machine is still out-of-service, the machine enters in the warm-up state and the part must wait for a period equal to t_{wu}. Otherwise, the machine is warmed up in advance. If the part arrives while the machine is in the warm-up state, the part must wait until the on-service state is reached, whereas the part is immediately processed if the machine is ready.

Policy 5 (Switch On): Switch on the machine after a time interval τ_{on} has elapsed from the last departure or when a part arrives at time $t < \tau_{\text{on}}$.

With Policy 5, $\mathbb{P}(A_2 \cup A_3 \cup A_4) \to 1$ and the expected value of energy consumption is

$$\begin{aligned} g_5 = & x_{\text{wu}} t_{\text{wu}} + \mathbb{P}(A_2) \left[x_{\text{on}} \mathbb{E}[T|A_2] + x_q t_{\text{wu}} \right] \\ & + \mathbb{P}(A_3) \left[x_q \mathbb{E}[T|A_3] + (x_{\text{out}} - x_q) \tau_{\text{on}} \right] \\ & + \mathbb{P}(A_4) \left[(x_{\text{on}} \mathbb{E}[T|A_4] - \tau_{\text{on}} - t_{\text{wu}}) + x_{\text{out}} \tau_{\text{on}} \right]. \end{aligned} \tag{12}$$

V. Optimization and Structural Properties

Policies 2 and 3 represent simple ways of managing a machine, and they can profitably be used only in extreme situations. In all of the situations in which there is not a clear advantage of triggering the machine immediately off or always keeping the machine on, the *switching* policy must be applied with a properly selected control, in order to be effective.

A. Unconstrained Problem

Given the control parameters in the non-negative orthant $\mathbb{R}^+$, in this section, we derive $\boldsymbol{\tau}^*$ that is the vector composed by the two optimal control parameters that minimize the machine expected energy consumption with Policy 1:

$$\boldsymbol{\tau}^* = \{\tau_{\text{off}}^*, \tau_{\text{on}}^*\} = \underset{\boldsymbol{\tau}\in\mathcal{U}}{\arg\min}\ g_1 \tag{13}$$

$$\mathcal{U} \equiv \{\boldsymbol{\tau} \in \mathbb{R}^2 | \tau_{\text{off}}, \tau_{\text{on}} \in \mathbb{R}^+ \wedge \tau_{\text{on}} \geq \tau_{\text{off}}\} \tag{14}$$

where $\mathcal{U}$ is the feasible set of values that τ_{off} and τ_{on} can jointly assume because of constraints (1). Function (8) is continuous on the closed and bounded set $\mathcal{U}$ and presents finite limits, thus the function has both a maximum and a minimum on this interval, and the extremum (∞) occurs at a critical point (Generalized Extreme Value Theorem). As a consequence, the optimal solution $\boldsymbol{\tau}^*$ can be found, and the optimal control α for the machine is

$$\alpha = \begin{cases} \text{on}, & 0 \leq t < \tau_{\text{off}}^* \\ \text{off}, & \tau_{\text{off}}^* \leq t < \tau_{\text{on}}^* \\ \text{on}, & t \geq \tau_{\text{on}}^* \end{cases}. \tag{15}$$

If the minimum of (8) occurs for $\tau_{\text{off}}^* \to \infty$, it means that the machine is never switched off, or formally this can happen only after an infinite time, and the policy behaves as the *always on*. Further, if the minimum occurs for $\tau_{\text{on}}^* \to \infty$, it means that the machine is switched on only when the arrival occurs, and the policy behaves as the *switch off*. Moreover, if the minimum occurs for $\tau_{\text{off}}^* = 0$, it means that the machine is switched off immediately after the part departure, and the policy behaves as the *off* if $\tau_{\text{on}}^* \to \infty$, or as the *switch on* otherwise.

Theorem 1: The optimal parameters can be calculated independently if constraint (1) is relaxed, and the values $\tau_{\text{off,free}}^*$ and $\tau_{\text{on,free}}^*$ exist and are unique:

$$\tau_{\text{off,free}}^* = \underset{\tau_{\text{off}}\in\mathbb{R}^+}{\arg\min}\ g_1 \tag{16}$$

$$\tau_{\text{on,free}}^* = \underset{\tau_{\text{on}}\in\mathbb{R}^+}{\arg\min}\ g_1. \tag{17}$$

Proof: The control parameters are independent because the mixed partial derivative of g_1 in (8), with respect to τ_{off} and τ_{on}, is *zero*. Moreover, the partial derivative $\partial g_1/\partial\tau_{\text{off}} = 0$ either for $\tau_{\text{off}} \to \infty$ or for τ_{off} satisfying the following equation:

$$q(\tau_{\text{off}}) = \frac{x_{\text{on}} - x_{\text{out}}}{(x_{\text{wu}} + x_q)t_{\text{wu}}} = \gamma \tag{18}$$

where q is the hazard ratio of the random variable T. This happens only once both for decreasing hazard rate (DHR) and increasing hazard rate (IHR) PDF, because $q(t)$ is monotone in t. Similarly, the partial derivative $\partial g_1/\partial\tau_{\text{on}} = 0$ either for $\tau_{\text{on}} \to \infty$ or for τ_{on} satisfying the following relationship:

$$r(\tau_{\text{on}}) \equiv \frac{F(\tau_{\text{on}} + t_{\text{wu}}) - F(\tau_{\text{on}})}{1 - F(\tau_{\text{on}})} = \frac{x_{\text{on}} - x_{\text{out}}}{x_{\text{on}} + x_q} = \beta. \tag{19}$$

This happens only once both for DHR and IHR PDF, because $r(t)$ can be proved to be monotone in t. See the Appendix for a formal proof.

Theorem 2: For unimodal arrival distributions, the general Switching policy degenerates into simpler strategies (Policy 2 to Policy 5).

Proof: Due to Theorem 1, for DHR the value $\tau_{\text{on,free}}^*$ is either $\tau_{\text{on,free}}^* = 0$ or $\tau_{\text{on,free}}^* \to \infty$. As a consequence, the optimal strategy can be Policy 2 to Policy 4. Similarly, for IHR the value $\tau_{\text{off,free}}^*$is either $\tau_{\text{off,free}}^* = 0$ or $\tau_{\text{off,free}}^* \to \infty$ As a consequence, the optimal strategy can be Policy 2, Policy 3 or Policy 5. See the Appendix for a formal proof.

Corollary 1: The optimal value $\tau_{\text{off}}^* \in \mathbb{R}^+$ exists uniquely, and it is always equal to $\tau_{\text{off,free}}^*$. If $\gamma \to \infty$, then $\tau_{\text{off,free}}^* = 0$; if $\gamma = 0$, then $\tau_{\text{off,free}}^* \to \infty$. Moreover, for DHR and $\gamma > 0$, it is $\tau_{\text{off,free}}^* = q^{-1}(\gamma)$. For IHR and $\gamma > 0$, either $\tau_{\text{off,free}}^* = 0 (t_a < 1/\gamma)$ or $\tau_{\text{off,free}}^* \to \infty (t_a \geq 1/\gamma)$.

Corollary 2: The optimal value $\tau_{\text{on}}^* \in \mathbb{R}^+$ exists uniquely and is equal to $\tau_{\text{on,free}}^* = r^{-1}(\beta)$ if the value $\tau_{\text{on,free}}^*$ satisfies the constraint (1). Otherwise, the value $\tau_{\text{on}}^* \to \infty$ because it never happens that $g_1(\tau_{\text{off}}^*, \tau_{\text{off}}^*) < g_2$.

The following algorithm is proposed in order to assure the respect of the constraint (1) and to find the optimal solution over the set $\mathcal{U}$. Algorithm 1 can be decomposed in three steps exploiting the structural properties of function g_1. The value $t_{\max}$ should be set according to the mean arrival time, e.g., considering a certain threshold ε for the risk of having an arrival at $t > t_{\max}$ as follows:

$$\mathbb{P}(t > t_{\max}) \leq \varepsilon. \tag{20}$$

As a consequence, when the optimal control parameter τ_{off}^* (or τ_{on}^*) is equal to the time limit $t_{\max}$, it is not a strong approximation to say that the control parameter tends to *infinity*. The algorithm can be used profitably instead of directly calculating $\boldsymbol{\tau}^*$ with (13), indeed it requires less computational effort yielding to a more efficient resolution.

Algorithm 1:

Step 0 Set a value $t_{\max}$ such that $t_{\max} \gg t_a$ where t_a is the mean of the arrival time distribution;

Step 1 If the arrival distribution has monotone hazard rate:

1a) Compute numerically $q(\tau_{\text{off}})$ for $\tau_{\text{off}} \in [0, t_{\max}]$;

1b) Find the optimal value of the switch-off instant τ_{off}^* using Corollary 1.

Else find numerically the optimal value of τ_{off}^* using **(16)**.

Step 2 If the arrival distribution has monotone hazard rate:

2a) Compute numerically $r(\tau_{\text{on}})$ for $\tau_{\text{on}} \in [0, t_{\max}]$;

2b) Find the optimal value of the switch-on instant τ_{on}^* using Corollary 2.

Else find numerically the optimal value of $\tau_{\text{on}}^* \in [\tau_{\text{off}}^*, t_{\max}]$ using (17).

B. Constrained Problem

The most common and significant performance indicator for machine tool in production is the expected throughput θ. Energy-saving policies do not have to compromise the effectiveness of the production system.

When the machine is kept always in the on-service state, the part is processed immediately, and the machine productivity is the maximum reachable, according to the production case. In this case, the part will never wait for machine readiness, under the assumption of the machine being perfectly reliable.

Whenever a control strategy is applied, the control leads to a reduction of system performances because, in some cases, the part has to wait until the end of the warm-up procedure. Particularly, the worst case occurs whenever a part waits for the entire warm-up procedure, i.e., for t_{wu}. This case happens when the machine is triggered in the warm-up state by the occurrence of an arrival. As a consequence, the throughput is maximum when the waiting time t_q is *zero*, and it is minimum when it reaches t_{wu} as follows:

$$\theta = \frac{1}{t_p + t_a + t_q}. \tag{21}$$

The expected machine utilization u can be calculated as

$$u = \frac{t_p}{t_p + t_a + t_q}. \tag{22}$$

Particularly, the waiting time t_q depends on the control parameters as well as on the PDF $f(t)$ of the arrival distribution

$$\begin{aligned} t_q &= \mathbb{P}(A_2)t_{\text{wu}} + \mathbb{P}(A_3)\mathbb{E}\left[(\tau_{\text{on}} + t_{\text{wu}} - t)|A_3\right] \\ &= t_{\text{wu}} \int_{\tau_{\text{off}}}^{\tau_{\text{on}}} f(t)dt + \int_{\tau_{\text{on}}}^{\tau_{\text{on}}+t_{\text{wu}}} (\tau_{\text{on}} + t_{\text{wu}} - t) f(t)dt. \end{aligned} \tag{23}$$

Property 1: The time t_q monotonically decreases over τ_{off}.

Proof: The derivative of t_q on τ_{off} is

$$\frac{\partial t_p}{\partial \tau_{\text{off}}} = -t_{\text{wu}} f(\tau_{\text{off}}) \le 0 \; \forall \tau_{\text{off}}. \tag{24}$$

Property 2: The time t_q monotonically increases over τ_{on}.

Proof: The derivative of t_q on τ_{on} is

$$\frac{\partial t_p}{\partial \tau_{\text{on}}} = F(\tau_{\text{on}} + t_{\text{wu}}) - F(\tau_{\text{on}}) \ge 0 \; \forall \tau_{\text{on}}. \tag{25}$$

Further, common machine tools are not designed to be frequently switched off or on. For this reason, the number of switch-off commands issued per cycle $N_{\text{s.o.}}$ may affect machine reliability and wearing out. The number $N_{\text{s.o.}}$ is represented by the probability of having an arrival after τ_{off} as follows:

$$N_{\text{s.o.}} = \mathbb{P}(A_2 \cup A_3 \cup A_4) = \int_{\tau_{\text{off}}}^{\infty} f(t)dt. \tag{26}$$

This indicator is minimum for Policy 2 and reaches the maximum, i.e., *one* command per cycle, for Policy 5, when the machine is switched off immediately after the departure. The effect of $N_{\text{s.o.}}$ on machine reliability and wearing out is out of scope of this research.

Property 3: The number of commands issued is not depending on $\tau_{\text{on}}$and monotonically decreases over τ_{off}

Proof: The derivative of $N_{\text{s.o.}}$ on τ_{on} is null, and the derivative $\partial N_{\text{s.o.}}/\partial \tau_{\text{off}} = -f(\tau_{\text{off}}) \le 0 \; \forall \tau_{\text{off}}$

As a consequence a minimum level of utilization, i.e., a minimum u should be guaranteed, and, in order to avoid frequently switching off or on, $N_{\text{s.o.}}$must be kept under a certain limit as follows:

$$u \ge u_{\min} \tag{27}$$

$$N_{\text{s.o.}} \le N_{\text{s.o.},\max}. \tag{28}$$

Both utilization u and number of switch-off commands $N_{\text{s.o.}}$ are functions of the control parameters as well as of the PDF $f(t)$ of the arrival distribution. We give more details here.

- Constraint (27) yields to a lower bound L_u for τ_{off}, because of Property 1.
- Constraint (27) yields to an upper bound U for τ_{on}, because of Property 2.
- Constraint (28) yields to a lower bound L_n for τ_{off}, because of Property 3.

The optimal solution under constraints (27) and (28) can be calculated with the following algorithm.

Algorithm 2:

Step 0 Set a value $t_{\max}$ such that $t_{\max} \gg t_a$;

Step 1 Compute numerically **(22)** for $\tau_{\text{off}}, \tau_{\text{on}} \in [0, t_{\max}]$ and find the lower bound L_u associated to $u_{\min}$;

Step 2 Compute numerically **(26)** for $\tau_{\text{off}} \in [0, t_{\max}]$ and find the lower bound L_n associated to $N_{\text{s.o.},\max}$;

Step 3 Given $L = \max\{L_u, L_n\}$, compute numerically $g_1(\tau_{\text{off}}, \infty)$ for $\tau_{\text{off}} \in \mathcal{U}_{\text{off}} = [L, t_{\max}]$, if $\mathcal{U}_{\text{off}} \neq \{\emptyset\}$;

Step 4 Find the optimal value of τ^*_{off} using **(16)**;

Step 5 Given the value τ^*_{off}, find the upper bound U associated to $u_{\min}$ using the values of **(22)** calculated at Step 1;

Step 6 Find the optimal value of the switch-on instant $\tau^*_{\text{on}} \in \mathcal{U}_{\text{on}} = [\tau^*_{\text{off}}, U]$ using **(17)**, if $\mathcal{U}_{\text{on}} \neq \{\emptyset\}$.

If $\mathcal{U}_{\text{off}} = \{\emptyset\}$ or $\mathcal{U}_{\text{on}} = \{\emptyset\}$ the constraints cannot be satisfied or $t_{\max}$ has to be increased.

VI. Special Cases

The optimal policy α (15) is case-dependent. However, two special cases can be identified: the case of a negligible warm up, when the machine tool can directly handle an operation without demanding a warm-up, and the case of exponentially distributed arrivals T. These cases are analysed relaxing constraints (27) and (28), i.e., in the feasible set $\mathcal{U}$.

A. Negligible Warm-Up

A machine without a warm-up procedure can be seen as a border situation where the warm-up time t_{wu} and the warm-up power x_{wu} are equal to *zero*. In this case, the function (8) is monotonically increasing on τ_{off} for all arrival distribution functions $f(t)$. As a consequence, the optimal control parameter for the *switch off* policy will always correspond to *zero* ($\tau^*_{\rm off}=0$). Vice versa, it is monotonically decreasing on $\tau_{\rm on}$ for all $f(t)$, and the optimal control parameter for the *switch on* policy will always approach *infinity* ($\tau^*_{\rm on}\to\infty$). As a consequence, the optimal control $\alpha^{(\rm no\ wu)}$ is always the *off* policy application

$$\alpha^{(\rm no\ wu)}=\{\text{off}\ \forall t\}. \tag{29}$$

B. Exponential Distribution

In the special case of exponentially distributed arrivals, i.e., $T\sim Exp(\lambda)$ where λ is the arrival rate, with mean $t_a=1/\lambda$, the expected values $g_j^{\rm exp}$ with $j=\{1..5\}$ can be calculated in a closed form.

Property 4: For exponentially distributed arrivals, the optimal control parameter minimizing Policy 4 is

$$\tau^{*,\rm exp}_{\rm off,free}=\begin{cases}0, & \text{if } (x_{\rm on}-x_{\rm out})t_a>(x_{\rm wu}+x_q)t_{\rm wu}\\ \infty, & \text{else.}\end{cases} \tag{30}$$

Proof: The derivative of $g_4^{\rm exp}$ is monotone on $\tau_{\rm off}$ and reaches *zero* as $\tau_{\rm off}$ approaches *infinity*. Thus, the resulting optimal control is not dependent on $\tau_{\rm off}$ because of the memoryless property of the exponential distribution. See the Appendix for a formal proof.

Property 5: For exponentially distributed arrivals, the optimal control parameter minimizing Policy 5 is

$$\tau^{*,\rm exp}_{\rm on,free}=\begin{cases}\infty, & \text{if } (x_{\rm on}+x_q)e^{-t_{\rm wu}/t_a}\geq x_{\rm out}+x_q\\ 0, & \text{else.}\end{cases} \tag{31}$$

Proof: The derivative of $g_5^{\rm exp}$ is monotone on $\tau_{\rm on}$ and reaches *zero* as $\tau_{\rm on}$ approaches *infinity*. Thus, the resulting optimal control is not dependent on $\tau_{\rm on}$. See the Appendix for a formal proof.

Given the structural properties detailed in Section V in the special case of the exponentially distributed arrivals, the optimal control $\alpha^{\rm exp}$ for Policy 1 can be found as the combination between the two possible values of $\tau^{*,\rm exp}_{\rm off,free}$ for Policy 4 and the two possible values of $\tau^{*,\rm exp}_{\rm on,free}$ for Policy 5 provided in Properties 1 and 2, respectively. As a consequence, four different cases can be identified according to the mean of the arrival distribution t_a, given here.

1) The optimal control is immediately switching the machine out-of-service ($\tau^{*,\rm exp}_{\rm off,free}=0$), and leaving the machine in out-of-service until the part arrives ($\tau^{*,\rm exp}_{\rm on,free}\to\infty$). The values are consistent with constraint (1) and the optimal control $\alpha^{\rm exp}$ degenerates in the *off* (Policy 3):

$$\alpha^{\rm exp}=\{\text{off}\ \forall t\}. \tag{32}$$

2) The optimal control is leaving the machine on-service ($\tau^{*,\rm exp}_{\rm off,free}\to\infty$) and never switch on because it was never switched off ($\tau^{*,\rm exp}_{\rm on,free}\to\infty$). The values are consistent with (1) and the optimal control $\alpha^{\rm exp}$ degenerates in the *always on* (Policy 2):

$$\alpha^{\rm exp}=\{\text{on}\ \forall t\}. \tag{33}$$

3) The two control parameters are $\tau^{*,\rm exp}_{\rm off,free}\to\infty$, and $\tau^{*,\rm exp}_{\rm on,free}=0$ which is nonconsistent with (1). Adding such a constraint to the optimization, the solution yields $\tau^*_{\rm on}=\tau^*_{\rm off}\to\infty$, and the machine will be kept on-service and never switched off.

4) The fourth case ($\tau^{*,\rm exp}_{\rm off,free}=0$, and $\tau^{*,\rm exp}_{\rm on,free}=0$) yields to a nonsense case, i.e., to switch-off the machine and to immediately switch-on it again. However, this case never happens in the practice. The proof is provided in the Appendix.

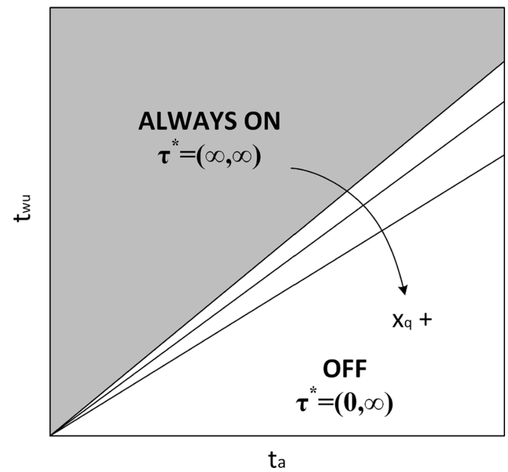


Fig. 4. Optimal strategy with exponential arrivals over the mean arrival time t_a and the warm-up duration $t_{\rm wu}$.

Therefore, the *switching* policy yields to two possible optimal strategies according to the machine power parameters ($x_{\rm out}$, $x_{\rm on}$, $x_{\rm wu}$, and x_q), the warm-up duration $t_{\rm wu}$, and the mean arrival time t_a. As a consequence, the optimal strategy to be implemented in the production case can be represented along the values of t_a and $t_{\rm wu}$ as in Fig. 4. The optimal control $\alpha^{\rm exp}$ is the *always on* policy if the following condition holds:

$$t_{\rm wu}\geq\frac{x_{\rm on}-x_{\rm out}}{x_{\rm wu}+x_q}t_a. \tag{34}$$

Otherwise the optimal policy is the *off*. Furthermore, by increasing the value of the power x_q, the area associated to the *always on* grows larger, indeed it becomes less profitable to switch off the machine due to the high part waiting energy.

VII. Test Cases

In order to model different machines operating in different situations and environments, we consider different power consumptions ($x_{\rm on}, x_{\rm out}, x_q, x_{\rm wu}$), warm-up duration $t_{\rm wu}$, and processing time t_p. Moreover, we assume the arrival of parts T follows a Weibull distribution with mean t_a and shape parameter k. The probability of having an arrival decreases over time if the shape parameter of the Weibull distribution is $k\leq 1$, i.e., DHR, whereas for $k>1$ the probability of having an arrival increases while approaching the mode of the distribution, i.e., IHR. The special case of $k=1$ represents the exponential distribution.

We considered eight factors varying within a certain range according to assumption in Section III. The values in Table I represent the range of the inference space relative to this study. Using the Algorithm 1 described in Section V-A and the limit $t_{\max}$ fixed at 1000 s, we optimize the *switching* policy for $m=3000$ different treatments randomly generated with a Latin hypercube design. The optimal strategy α has been classified in Policies 1–5 as in Table II.

TABLE I

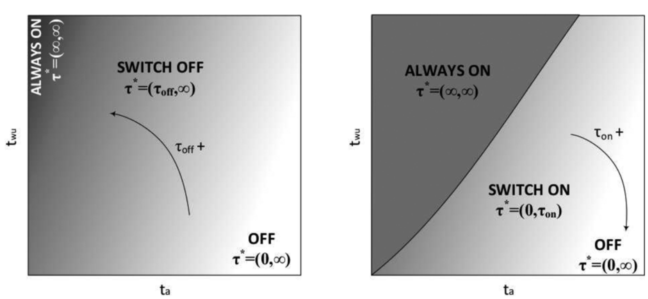
RANGE OF THE INFERENCE SPACE FOR THE LATIN HYPERCUBE DESIGN

Factor	Min Value	Max Value
x_{out} [kW]	0.2	5
x_{on} [kW]	1.1 x_{out}	10 x_{out}
x_q [kW]	1	10
x_{wu} [kW]	1.1 x_{on}	5 x_{on}
t_{wu} [s]	0	500
t_a [s]	0	300
k	0.5	10
t_p [s]	0.1 t_a	100 t_a

TABLE II

OPTIMAL STRATEGY CLASSIFICATION

	P1	P2	P3	P4	P5
Treatments	0	2606	94	254	46
$k < 1$	0	730	16	254	0
$k = 1$	0	942	58	0	0
$k > 1$	0	934	20	0	46



(a) (b)

Fig. 5. Qualitative representation of the optimal strategy with Weibull arrivals over the mean arrival time t_a and the warm-up duration t_{wu}. (a) $k < 1$. (b) $k > 1$.

For DHR ($k \leq 1$), the optimal control α for the *switching* policy degenerates into three policies: either the *off*, the *switch off*, or the *always on*. As represented qualitatively in Fig. 5(a), when the warm-up duration t_{wu} is much greater then the mean arrival time t_a, the optimal strategy is the *switch off* policy. The shorter the warm-up duration t_{wu}, the more the optimal control tends to the *off* policy (τ^*_{off} decreases toward *zero* with $\tau^*_{\text{on}} \to \infty$). In both cases, once the switch-off command is issued, there is no advantage in switching on the machine because the arrival probability is decreasing. For $t_a = 0$, i.e., the t_{wu} axis, the optimal policy is the *always on*, because it means that the next part is immediately available.

Similarly, for IHR ($k > 1$), the *switching* policy degenerates into one out of the three strategies represented in Fig. 5(b). Qualitatively, when the mean arrival time t_a is much greater then the warm-up duration t_{wu}, the optimal strategy tends to the *off* policy (τ^*_{on} increases toward *infinity* with $\tau^*_{\text{off}} = 0$). Otherwise, the optimal control is the *switch on* because the arrival probability increases while approaching the mode of the arrival distribution. Moreover, as far as t_{wu} increases it becomes not advantageous to switch off the machine, thus the optimal control is the *always on*.

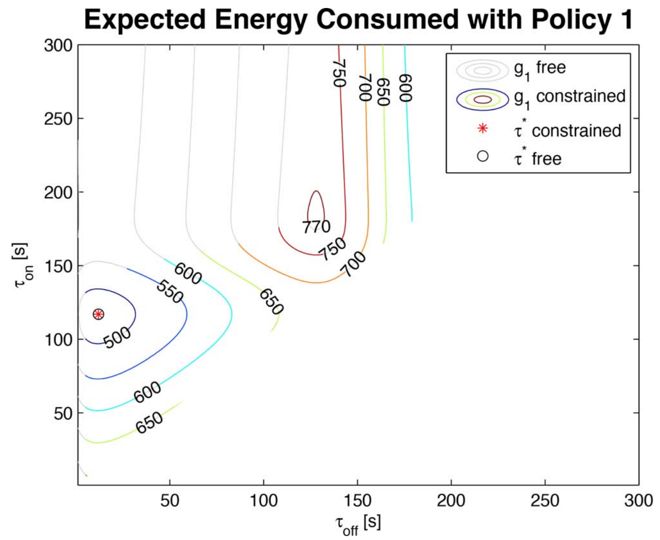


Fig. 6. Expected energy with a multimodal arrival distribution ($x_{\text{out}} = 0.5$ kW, $x_{\text{on}} = 5$ kW, $x_q = 10$ kW, $x_{\text{wu}} = 8$ kW, $t_{\text{wu}} = 30$ s, and $t_p = 100$ s).

TABLE III

REAL CASE MACHINE: POWER CONSUMPTION ESTIMATED FROM EXPERIMENTAL TESTS

	x_{out}	x_{on}	x_q	x_{wu}	t_{wu}	t_p
Value	0.52 kW	3.35 kW	1.00 kW	6.08 kW	24 s	168 s

For multimodal arrival distributions, the Theorem 2 does not hold. Indeed, it is possible to obtain an optimal control α with both τ^*_{off} and τ^*_{on} different from *zero* and superiorly bounded. To give an example, let $f(t)$ be the sum of two Weibull distributions: one distribution has the mean arrival time $\lambda_1 = 53.17$ and the shape parameter $k = 0.6$, the other has $\lambda_2 = 168.18$ and $k = 10$. The expected value of the energy consumed with Policy 1 is represented in Fig. 6 under the constraints of $u_{\min} = 52\%$ and $N_{\text{s.o.,max}} = 0.9$. The contour plot of energy nonconstrained is also provided. In this case, the constraints (27) and (28) do not affect the optimal solution that is feasible in the constrained set $\mathcal{U}'$. In more details, the machine is initially kept on-service because the probability of having an arrival is high, then it is switched off at $\tau^*_{\text{off}} = 12$ s because the probability decreases mainly due to the shape of the first Weibull. Moreover, the arrival distribution $f(t)$ has a probability peak around the mode of the second Weibull, thus the control switches on the machine as soon as the probability increases ($\tau^*_{\text{on}} = 117\ s$) in order to warm up the machine in advance. As in Fig. 6, it is possible to meet with a risk of high energy consumption around $\boldsymbol{\tau} = \{128\text{ s}; 183\text{s}\}$ because the control would switch off the machine after a time $\tau_{\text{off}} = 128$ s close to the peak of the arrival time distribution, that is not profitable.

VIII. REAL CASE

The CNC machining center ComauSmartDrive700L has been analyzed, and the data reported in Table III have been acquired with dedicated experimental measurements in Comau S.p.A. —according to the ISO normative [23], [24]. Particularly, power consumption information has been acquired in no-load operation mode according to the states identified in Section III. As

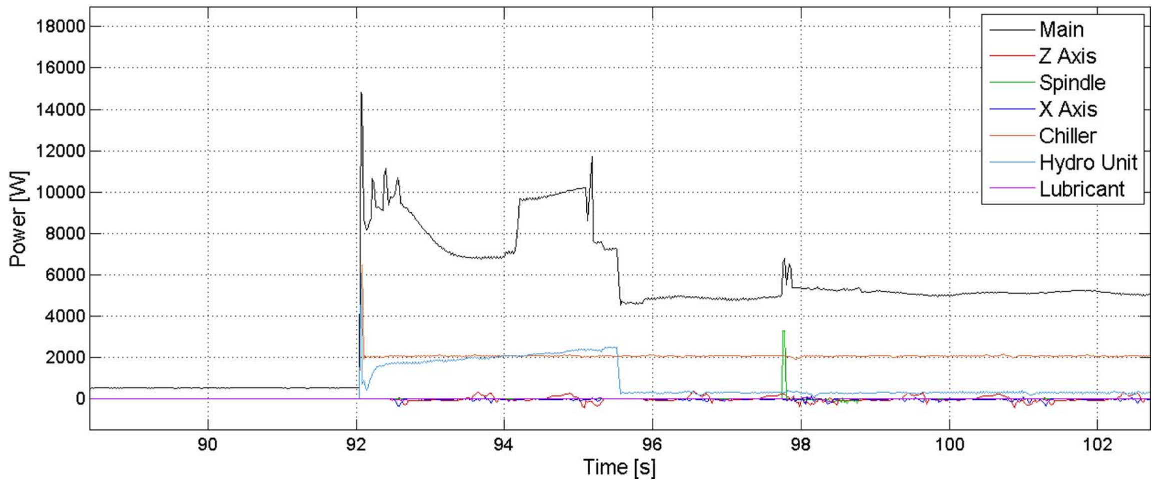


Fig. 7. Machine power requested during a progressive switch-on: out-of-service, warm-up, and on-service state are visited sequentially.

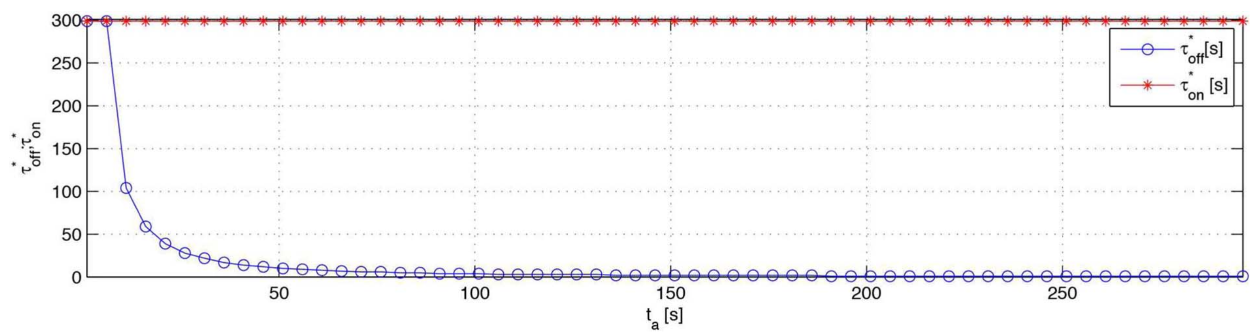


Fig. 8. Optimal solution $\boldsymbol{\tau}^*$ with a Weibull arrival distribution and different values of the mean arrival time t_a $(k = 0.6)$.

an example, the power signals acquired in a measurement test are represented in Fig. 7. The part waiting power is assumed to be $x_q = 1$ kW as the power consumed by auxiliary equipment in the line station, e.g., part-handling system, to keep the part waiting.

A. Machine Controls

In order to model the machine operating in different production systems, we assume the arrival of parts T follows a Weibull distribution with mean t_a and shape parameter k, and we optimize the *switching* policy for increasing values of the mean arrival time t_a. Given a certain value of k, the optimal solution $\boldsymbol{\tau}^*$, calculated using the Algorithm 1 described in Section V-A, is represented in Figs. 8 and 9 as the frequency of part arrival decreases (from $t_a = 1$ s to $t_a = 300$ s).

1) Case $\mathbf{k} < 1$*:* As already discussed in Section VII, for DHR distributions the machine is kept on-service in the short period, i.e., while the probability of an arrival is high, then the machine is switched off because the part flow is probably interrupted. As a consequence, the optimal control α for the machine is either the *switch off*, the *off*, or the *always on*. Referring to $k = 0.6$ in Fig. 8, for values of t_a much lower then the value of $t_{\text{wu}} = 24\ s$ the optimal control tends to keep the machine always on-service. Indeed, for high probability of an arrival in the first seconds, there is no advantage in switching the machine off—due to the warm-up energy request. However, the higher is the value of the t_a compared with the value of t_{wu}, the sooner the machine is switched off, because the importance of the warm-up on the energy consumed is decreasing. Once switched off the machine, it is better to wait for the part arrival before warming up the machine $(\tau_{\text{on}}^* \rightarrow \infty)$ because the arrival probability decreases over time.

2) Case $\mathbf{k} > 1$*:* For IHR distributions, as discussed in Section VII, the arrival probability is low in the short time, thus it is advantageous to switch off the machine immediately after a departure, and warm it up as the probability of having an arrival increases. As a consequence, the optimal control α for the machine is either the *switch on*, the *off*, or the *always on*. Referring to $k = 10$ in Fig. 9, for values of t_a lower or closed to the value of t_{wu}, it is better to keep the machine always on-service due to the warm-up energy request whereas, if $t_a \gg t_{\text{wu}}$, it is better to switch off the machine immediately and to switch it on in advance. Furthermore, the optimal value τ_{on}^* increases in t_a, thus it becomes more profitable to postpone the switch-on command as soon as the probability of an arrival approaches its peak.

B. Production Case

In order to give an example, a real case from Comau system in Bielskobiala (Poland) is presented into details. The real empirical arrival distribution has been approximated with a Weibull distribution having parameters $\lambda = 15.73$ and $k = 0.45$. The minimum value of the energy consumed with Policy 1 is reported together with the energy consumed with Policy 2 and Policy 3. Throughput and command constraints [(27) and (28), respectively] are not considered in the optimization. However,

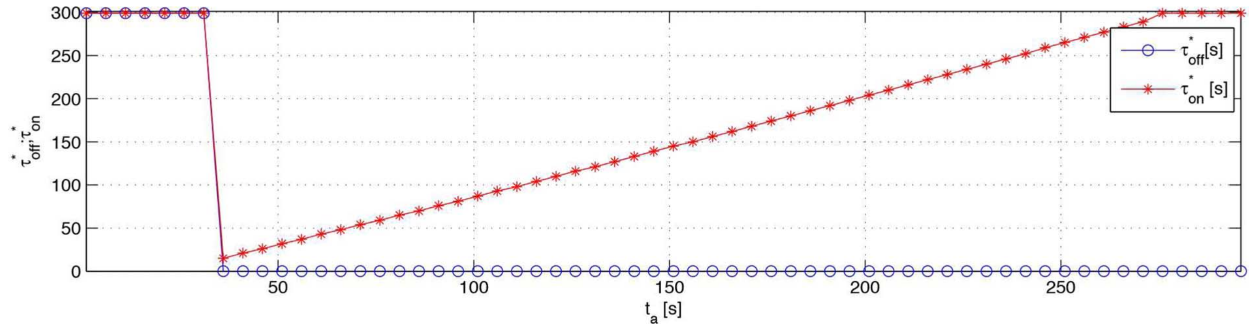


Fig. 9. Optimal solution $\boldsymbol{\tau}^*$ with a Weibull arrival distribution and different values of the mean arrival time t_a $(k = 10)$.

TABLE IV
ACHIEVABLE RESULTS: CASE 1 ($t_a = 39$ s; $k = 0.45$)

	P1	P2	P3	P1 vs P2	P1 vs P3
Energy [kJ]	145.2	208.6	190.2	-30.4 %	-23.6 %
u	0.7805	0.8115	0.7272	- 3.8 %	7.3 %

the utilization and the number of switch-off commands are also provided as performance indicators.

This case correspond to $t_a = 39\ s$ and the results are reported in Table IV. Particularly, Policy 2 is not advantageous compared with Policy 3: the former would consume 208.6 kJ throughout a cycle, while the latter 190.2 kJ. After the optimization, the Policy 1 yields to a *switch off* with $\tau_{\text{off}} = 16$ s, and it would achieve 30.4% of energy savings compared to Policy 2, and 23.6% compared with Policy 3. The utilization is reduced up to 3.8% with Policy 1 compared with the machine utilization with Policy 2 (0.8115), that is inside the minimum range accepted by Comau, i.e., $u_{\min} = 0.75$. Moreover, the average number of switch-off is 0.36 commands issued per part.

IX. CONCLUSION

A general control policy has been studied analytically in this paper, considering different warm-up duration, together with stochastic arrival times. The independency of the two control parameters has been demonstrated and, based on the structural properties of the control policy, an efficient algorithm has been proposed in order to find the optimal control strategy for a machine tool. The influence of two constraints related to production performances has been investigated and a second algorithm has been proposed to solve the constrained problem.

The policy has been deeply analyzed for DHR and IHR arrival distributions, and it has been demonstrated that the optimal strategy can never be Policy 1 but it always degenerates into simpler strategies. Moreover, several cases has been studied numerically in order to support the analysis.

The influence of arrival time distribution as well as the warm-up time over the optimal solution has been discussed although machine parameters remain fixed. Assuming a Weibull distribution for arrivals, we can make the following conclusions.

- Increasing the mean arrival time originates decreasing optimal control parameter for the switch-off command and increasing optimal control parameter for the switch-on command independently from the shape parameter;
- The shorter the warm-up time t_{wu} is, the faster the machine should be switched off. The effect of the warm-up time on the optimal control is reduced as the mean arrival time increases.

The remarks above hold also for other distributions different from the Weibull; the only condition that must be satisfied is that the arrival probability distribution is unimodal and does not depend on the control policy. However, since the production system is assumed to have one single part type, the arrival process is not renewal in general [25]. Hence, our analysis only gives guidance to decision making in an approximate sense. More reliable models and further improvement which considers the nonrenewal arrival process in practical production lines is left for future researches.

A real case application has showed that the benefits achievable by implementing such policies are meaningful. Particularly, the effects on the expected value of the machine energy consumption change significantly according to the strategy applied. This means that the control should be optimized according to the actual production performances and workload of the system.

The limitations of this research will inspire future activities. The risk of incurring in high consumptions is not considered in this work. A warm-up duration that is time dependent is an important issue that has to be investigated in future researches, indeed the warm-up might depend from the time interval that the machine spends in the out-of-service state. Future developments will also be devoted to extend the boundaries of the system considered by including the information of the number of parts waiting in the machine input buffer, and to investigate the dependencies between the switching off-on and the machine reliability. The extension of the policy to several *sleep modes* according to the machine subsystem state should also be considered, because they can theoretically have their own control policies for energy saving. Finally, a cost analysis could also be developed for a general assessment of the proposed control policy.

APPENDIX
EXTENDED PROOFS

Proof of Theorem 1:

The partial derivative of (8), with respect to τ_{off}, is

$$\frac{\partial g_1}{\partial \tau_{\text{off}}} = [1-F(\tau_{\text{off}})][(x_{\text{wu}}+x_q)t_{\text{wu}}q(\tau_{\text{off}})+x_{\text{on}}-x_{\text{out}}] \quad (35)$$

where $q(\tau_{\text{off}})$ is the hazard rate of the PDF evaluated in τ_{off}. The derivative (35) is greater than *zero* if

$$q(\tau_{\text{off}}) = \frac{f(\tau_{\text{off}})}{1-F(\tau_{\text{off}})} \geq \frac{x_{\text{on}} - x_{\text{out}}}{(x_{\text{wu}} + x_q)t_{\text{wu}}} = \gamma. \quad (36)$$

However, the function $q(\tau_{\text{off}})$ is monotonically decreasing on τ_{off} for DHR and monotonically increasing on τ_{off} for IHR.

Similarly, the partial derivative of (8), with respect to τ_{on}, is

$$\frac{\partial g_1}{\partial \tau_{\text{on}}} = [1 - F(\tau_{\text{on}})][r(\tau_{\text{on}})(x_{\text{on}} + x_q) - x_{\text{on}} + x_{\text{out}}]. \quad (37)$$

The derivative (37) is greater than *zero* if

$$r(\tau_{\text{on}}) \geq \frac{x_{\text{on}} - x_{\text{out}}}{x_{\text{on}} + x_q} = \beta. \quad (38)$$

The function $r(\tau_{\text{on}})$ is defined in $[0, 1]$ as well as β and it monotonically increases on τ_{on} if

$$\frac{\partial r(\tau_{\text{on}})}{\partial \tau_{\text{on}}} = q(\tau_{\text{on}} + t_{\text{wu}}) - q(\tau_{\text{on}}) \geq 0. \quad (39)$$

As a consequence, on τ_{on}, for DHR the function $r(\tau_{\text{on}})$ monotonically decreases, whereas for IHR it monotonically increases. Computing the mixed partial of (35), we obtain

$$\frac{\partial}{\partial \tau_{\text{on}}}\left(\frac{\partial g_1}{\partial \tau_{\text{off}}}\right) = \frac{\partial^2 g_1}{\partial \tau_{\text{off}} \partial \tau_{\text{on}}} = 0. \quad (40)$$

Proof of Property 4:

Given exponentially distributed arrival times, the function g_4^{exp} is

$$g_4^{\text{exp}} = x_{\text{on}} t_a - e^{-\lambda \tau_{\text{off}}}((x_{\text{on}} - x_{\text{out}})t_a - (x_{\text{wu}} + x_q)t_{\text{wu}}). \quad (41)$$

It is possible to demonstrate that the derivative of g_4^{exp}(41) is monotone on τ_{off} and reaches *zero* as τ_{off} approaches *infinity*, thus the resulting optimal control is not dependent on τ_{off} because of the memoryless property of the exponential distribution:

$$\tau_{\text{off,free}}^{*,\text{exp}} = \begin{cases} 0, & \text{if } \frac{\partial g_4^{\text{exp}}}{\partial \tau_{\text{off}}} > 0 \\ \infty, & \text{else.} \end{cases} \quad (42)$$

Indeed, when the expected energy saved by switching the machine off is greater than the energy required by the warm-up with the part waiting, the best application is always the immediate shutdown ($\tau_{\text{off}}^* = 0$). Otherwise, the best policy will never trigger the machine out-of-service (i.e., $\tau_{\text{off}}^* \to \infty$).

Proof of Property 5:

Given exponentially distributed arrival times, the function g_5^{exp} is

$$\begin{aligned} g_5^{\text{exp}} &= x_{\text{out}} t_a + (x_{\text{wu}} + x_q)t_{\text{wu}} \\ &+ e^{-\lambda(\tau_{\text{on}} + t_{\text{wu}})}(x_q + x_{\text{on}})t_a - e^{-\lambda \tau_{\text{on}}}(x_{\text{out}} + x_q)t_a. \end{aligned} \quad (43)$$

It is possible to demonstrate that the derivative of g_5^{exp} (43) is monotone on τ_{on} and reaches *zero* as τ_{on} approaches *infinity*, thus the resulting optimal control is not dependent on τ_{on} because of the memoryless property of the exponential distribution

$$\tau_{\text{on,free}}^{*,\text{exp}} = \begin{cases} \infty, & \text{if } \frac{\partial g_5^{\text{exp}}}{\partial \tau_{\text{on}}} \leq 0 \\ 0, & \text{else.} \end{cases} \quad (44)$$

The condition reflects the comparison between the energy consumed in two cases:

- the control leaves the machine out-of-service, thus the part waits during the machine warm-up, i.e., the *off* policy;
- the control switches on the machine immediately, thus the part arrives or during the warm-up or when the machine is already in the proper condition to work.

If the second case is expected to be more consuming than the first one, the best strategy is to turn on the machine upon the arrival. Otherwise, the machine should be switched on immediately. Notice that the warm-up energy x_{wu} is not affecting the optimal control because it will be requested in any case.

Proof of Theorem 2:

Due to Theorem 1, for DHR the value $\tau_{\text{off,free}}^*$ is $\tau_{\text{off,free}}^* = 0$ if $\gamma \to \infty$, $\tau_{\text{off,free}}^* \to \infty$ if $\gamma = 0$, and $\tau_{\text{off,free}}^* > 0$ otherwise. In parallel, the value of $\tau_{\text{on,free}}^*$ changes according to β and is either $\tau_{\text{on,free}}^* = 0$ or $\tau_{\text{on,free}}^* \to \infty$. Due to constraint (1), the optimal strategy is given as follows.

- If $\tau_{\text{on,free}}^* \to \infty$, constraint (1) always holds and the optimal strategy becomes P2 if $\tau_{\text{off,free}}^* \to \infty$, P3 if $\tau_{\text{off,free}}^* = 0$, or P4 otherwise;
- The optimal policy is P2 if $\tau_{\text{on,free}}^* = 0$ and $\tau_{\text{off,free}}^* \to \infty$, because it never happens that $g_1(\tau_{\text{off,free}}^*, \tau_{\text{off,free}}^*) < g_2$ and the value $\tau_{\text{on}}^* \to \infty$;
- The optimal policy is either P3 or P4 if $\tau_{\text{on,free}}^* = 0$ and $\tau_{\text{off,free}}^* \geq 0$, because the value $\tau_{\text{on}}^* \to \infty$.

Similarly, for IHR the value $\tau_{\text{off,free}}^*$ is either $\tau_{\text{off,free}}^* = 0$ or $\tau_{\text{off,free}}^* \to \infty$. Parallely, the value of $\tau_{\text{on,free}}^*$ is: $\tau_{\text{on,free}}^* = 0$ if $\beta = 0$, $\tau_{\text{on,free}}^* \to \infty$ if $\beta = 1$, and $\tau_{\text{on,free}}^* > 0$ otherwise. Due to constraint (1), the optimal strategy is given as follows.

- P2 if $\tau_{\text{off,free}}^* \to \infty$ because the value $\tau_{\text{on}}^* \to \infty$.
- If $\tau_{\text{off,free}}^* = 0$ and the value $\tau_{\text{on,free}}^* > 0$ the constraint (1) always holds and the optimal strategy becomes P3 if $\tau_{\text{on}}^* \to \infty$, or P5 otherwise.
- If $\tau_{\text{off,free}}^* = 0$ and $\tau_{\text{on,free}}^* = 0$, the value $\tau_{\text{on}}^* \to \infty$ because it never happens that $g_1(0, 0) < g_2$ and the optimal policy is P2.

The special case where $g_1(\tau_{\text{off,free}}^*, \tau_{\text{off,free}}^*) = g_2$ is actually the special case where the warm-up is negligible.

Proof of Condition 4:

For exponentially distributed arrivals, the case where $\tau_{\text{off,free}}^{*,\text{exp}} = 0$, and $\tau_{\text{on,free}}^{*,\text{exp}} = 0$ never happens in the practice. Indeed, if $\tau_{\text{off,free}}^{*,\text{exp}} = 0$, the following condition holds:

$$g_2^{\text{exp}} > g_3^{\text{exp}}. \quad (45)$$

Similarly, if $\tau_{\text{on,free}}^{*,\text{exp}} = 0$, the following condition is satisfied:

$$g_3^{\text{exp}} > g_1^{\text{exp}}|_{\tau_{\text{off}}^{*,\text{exp}}}. \quad (46)$$

However, combining (45) and (46) results in

$$g_2^{\text{exp}} > g_1^{\text{exp}}|_{\tau_{\text{off}}^{*,\text{exp}}}. \quad (47)$$

Condition (47) is never satisfied in the practice because the following system has no solution:

$$\begin{cases} \frac{t_{\text{wu}}}{t_a} < \frac{x_{\text{on}} + x_q}{x_{\text{wu}} + x_q}(1 - e^{-t_{\text{wu}}/t_a}) \\ x_{\text{wu}} \geq x_{\text{on}}. \end{cases} \quad (48)$$

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