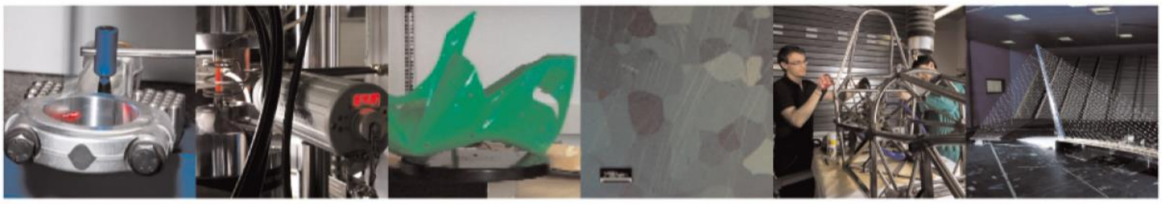

Models and algorithms for throughput improvement problem of serial production lines via downtime reduction

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Models and Algorithms for Throughput Improvement Problem of Serial Production Lines via Downtime Reduction

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Appendix I: Cost function formulation

The formulation of discrete cost function is as follows. For each of the proposal h on failure mode k at machine j, the resulting downtime reduction coefficient $U^{'}_{j,k,h}$ and the cost $C^{h}_{j,k}$ is estimated. Auxiliary 0-1 variables $y^{h}_{j,k}$ are introduced to represent whether proposal h is selected. The cost function will be as follows:

$$
\begin{aligned}
B = \sum_{j=1}^{M} \sum_{k \in \mathcal{F}_j} \sum_{h \in \mathcal{H}_{j,k}} & C^{h}_{j,k} y^{h}_{j,k} \\
x_{j,k} = \sum_{h \in \mathcal{H}_{j,k}} y^{h}_{j,k} U^{'}_{j,k,h} & \qquad \forall j = 1, ..., M,\ k \in \mathcal{F}_j \\
\sum_{h \in \mathcal{H}_{j,k}} y^{h}_{j,k} \leq 1 & \qquad \forall j = 1, ..., M,\ k \in \mathcal{F}_j \\
y^{h}_{j,k} \in \{0, 1\} & \qquad \forall j = 1, ..., M,\ k \in \mathcal{F}_j,\ h \in \mathcal{H}_{j,k},
\end{aligned}
$$

where $\mathcal{H}_{j,k}$ represent the set of proposals on failure mode k at machine j.

The piecewise linear cost function is formulated as follows. For a failure (j, k), its cost function has $G_{j,k}$ pieces, and the values of $x_{j,k}$ where the slope changes denoted by $x^{g}_{j,k}$ and its related cost $C(x^{g}_{j,k})$ are also provided as parameters. Specifically, $x^{0}_{j,k} = 0$, and $C(x^{0}_{j,k})$ is the fixed cost for improving the failure. Since there is discontinuity at zero, for $x_{j,k} = 0$, the cost is equal to zero, and for any positive $x_{j,k}$, the cost is at least $C(x^{0}_{j,k})$.

$$
\begin{aligned}
B = \sum_{j=1}^{M} \sum_{k \in \mathcal{F}_j} \sum_{g=0}^{G_{j,k}} \lambda_{j,k}^{g} C(x_{j,k}^{g}) \\
x_{j,k} = \sum_{g=1}^{G_{j,k}} \lambda_{j,k}^{g} x_{j,k}^{g} & \qquad \forall j = 1, ..., M,\ \ k \in \mathcal{F}_j \\
x_{j,k} \leq y_{j,k}^{0} U_{j,k} & \qquad \forall j = 1, ..., M,\ \ k \in \mathcal{F}_j \\
\sum_{g=0}^{G_{j,k}} \lambda_{j,k}^{g} = y_{j,k}^{0} & \qquad \forall j = 1, ..., M,\ \ k \in \mathcal{F}_j \\
\sum_{g=1}^{G_{j,k}} y_{j,k}^{g} = y_{j,k}^{0} & \qquad \forall j = 1, ..., M,\ \ k \in \mathcal{F}_j \\
\lambda_{j,k}^{0} \leq y_{j,k}^{1} & \qquad \forall j = 1, ..., M,\ \ k \in \mathcal{F}_j \\
\lambda_{j,k}^{g} \leq y_{j,k}^{g} + y_{j,k}^{g+1} & \qquad \forall j = 1, ..., M,\ \ k \in \mathcal{F}_j,\ \ g = 1, ..., G_{j,k} - 1 \\
\lambda_{j,k}^{G_{j,k}} \leq y_{j,k}^{G_{j,k}} & \qquad \forall j = 1, ..., M,\ \ k \in \mathcal{F}_j \\
0 \leq \lambda_{j,k}^{g} \leq 1 & \qquad \forall j = 1, ..., M,\ \ k \in \mathcal{F}_j,\ \ g = 0, ..., G_{j,k} \\
y_{j,k}^{g} \in \{0, 1\} & \qquad \forall j = 1, ..., M,\ \ k \in \mathcal{F}_j,\ \ g = 0, ..., G_{j,k}.
\end{aligned}
$$

The binary variables $y_{j,k}^{g}$ for $g = 1, ..., G_{j,k}$ represent if $x_{j,k} \in (x_{j,k}^{g-1}, x_{j,k}^{g}]$, and $y_{j,k}^{0}$ represents if $x_{j,k}$ is positive. The real value variables $\lambda_{j,k}^{g}$ show the propotion of the distance between $x_{j,k}$ and $x_{j,k}^{g}$ over the length of the interval if $x_{j,k} \in (x_{j,k}^{g-1}, x_{j,k}^{g}]$ or $x_{j,k} \in (x_{j,k}^{g}, x_{j,k}^{g+1}]$.

Appendix II: Continuous improvement algorithm

CI is used for comparison in Section 6.2.1. The procedure is shown in Algorithm 3. The input includes the budget B^* and the stopping condition. For MP1, stopping condition is $B = B^*$. For MP2, the budget B^* is equal to $+\infty$, and the stopping condition is $TH \geq TH_0(1+\Delta TH^*)$. The input parameters $r,\ s_0,\ s_{min}$ are related to the step length of improvement, s, at each iteration. The step length s is initially set to be equal to s_0, and each time the selected machine shifted to a different position from the last iteration, the step length is reduced to $r(s-s_{min})+s_{min}$, where $r \leq 1$ and s_{min} is the defined minimal value of s. The procedure includes simulating the system (step 1), selecting the machine to be improved (Step 2), and improving the selected machine with a certain amount s (step 3 and 4). Lines 18–19 assure that the budget can be fully used in MP1. The procedure stops when the stopping condition is true, or no machine is selected to improve at step 2. The machine to be improved is selected as the machine whose downtime is with the highest sensitivity and whose reduction amount does not reach the upperbound $U_{j,k}$. The way to calculate the sensitivity of each machine $d_{j,k}$ in line 5 is from Chan and Schruben (2008), with the dual optimal solution from Algorithm 2.

In Section 6.2.1, CI algorithms with eight sets of different values of $s_0,\ s_{min},\ r$ are implemented, and their values can be found in Table 2.

Label	CI-HHH	CI-HHL	CI-HLH	CI-HLL
(s_0, s_{min}, r)	$(0.8, 0.1, 0.8)$	$(0.8, 0.1, 0.5)$	$(0.8, 0.05, 0.8)$	$(0.8, 0.05, 0.5)$
Label	CI-LHH	CI-LHL	CI-LLH	CI-LLL
(s_0, s_{min}, r)	$(0.2, 0.1, 0.8)$	$(0.2, 0.1, 0.5)$	$(0.2, 0.05, 0.8)$	$(0.2, 0.05, 0.5)$

Table 2.: Parameters of CI algorithm.

Algorithm 3 CI algorithm.

Input:
B^*, r, s_0, s_{min}, stopping condition.

Ensure:
Solution of MP1 or MP2: $x_{j,k}, y_{j,k}$.

$l \leftarrow 1, s \leftarrow s_0, B \leftarrow 0$
Call Algorithm 1 in the article for simulation.
while stopping condition is false **do**
  Get dual solution $\bar{u}_{i,j}$ with Algorithm 2 in the article.
  $(j,k)_l \leftarrow \arg\max_{(j,k)}\{d_{j,k} | x_{j,k} < U_{j,k}\}$, where $d_{j,k} \leftarrow |\sum_{i=1}^{N} \frac{\partial f_{j,k}(x_{j,k}, r_{i,j,k,q})}{\partial x_{j,k}} \bar{u}_{i,j}|$.
  if $(j,k)_l$ exists **then**
    if $y_{(j,k)_l} = 0$ **then**
      if $B^* - B > C^y$ **then**
        $y_{(j,k)_l} \leftarrow 1$.
      else
        $U_{(j,k)_l} \leftarrow 0$.
        Continue.
      end if
    end if
    if $(j,k)_l \neq (j,k)_{l-1}$ **then**
      $s \leftarrow r(s - s_{min}) + s_{min}$.
    else
      $B \leftarrow \sum_{j=1}^{m} \sum_{k=1}^{F_j} (C^x x_{j,k} + C^y y_{j,k})$.
      $x_{(j,k)_l} \leftarrow min\{x_{(j,k)_l} + s, U_{(j,k)_l}, x_{(j,k)_l} + \frac{B^* - B}{C^x}\}$.
      $l \leftarrow l + 1$.
    end if
  else
    Break.
  end if
  Call Algorithm 1 in the article for simulation.
end while

Appendix III: System parameters

Tables 3 to 6 indicate buffer space b_j, mean and Coefficient of Variation (CV) of cycle time ($\mathbf{P}$), mean of uptime ($\mathbf{T_{up}}$) and mean of downtime ($\mathbf{T_{down}}$) of the systems in the numerical experimen. In all experiments, processing time is generated from truncated normal distribution on interval (0, 2CT), and time to failure is generated from Weibull distribution with shape parameter $k = 2$. Downtime distributions are varied in the different sections. In sections 6.1 and 6.2, repair time follows right-skewed triangular distribution. The relationship among lower limit a, upper limit b and mode c is $b - c = 2(c - a)$, and the value of $c - a$ is reported in the tables. In section 6.3, repair time follows exponential distribution.

System ID	Stage j	b_j	CT		UT	DT	
			Mean	CV	Mean	Mean	c-a
A-5	1, 2, 4, 5	3	2	0.1	10	3	1
	3	3	2.2	0.1	10	4	1
B-5-I	1 - 4	3	2	0.1	10	3	1
	5	-	2.2	0.1	10	4	1
B-5-II	1	3	2	0.1	10	3	1
	2	4	2	0.1	10	3	1
	3	5	2	0.1	10	3	1
	4	6	2	0.1	10	3	1
	5	-	2.2	0.1	10	4	1
C-5	1, 3, 4	3	2	0.1	10	3	1
	2	3	2.15	0.1	10	3	1
	5	-	2	0.1	10	4	1
C-9	1 - 4, 6 - 8	3	2	0.1	10	3	2
	5	3	2.15	0.1	10	3	2
	9	-	2	0.1	10	4	2
D-5	1, 4, 5	3	2	0.1	10	3	1
	2	3	2.15	0.1	10	3	1
	3	3	2	0.1	10	4	1
E-5	1 - 5	3	2	0.1	10	3	1
E-9	1 - 9	3	2	0.1	10	3	1

Table 3.: Parameters of systems (section 6.1, 6.2.1 and 6.2.2).

		CT		UT	DT	
Stage j	b_j	Mean	CV	Mean	Mean	c-a
1	9	5.1	0.1	1716.78	29.69	9.90
2	12	5.1	0.1	3553.11	35.89	11.96
3	12	5.3	0.1	1346.28	34.52	11.51
4	9	5.3	0.1	638.68	44.4	14.80
5	6	3.8	0.1	3381.48	27.27	9.09
6	5	5.2	0.1	749.09	30.4	10.13
7	6	5.2	0.1	909.42	54.97	18.32
8	10	5	0.1	1027.90	33.98	11.33
9	9	3.5	0.1	-	-	-
10	30	5.1	0.1	442.59	27.75	9.25
11	6	5.5	0.1	1272.75	47.53	15.84
12	9	5.3	0.1	1921.29	45.23	15.08
13	9	4.7	0.1	6066.72	61.28	20.43
14	6	3.5	0.1	-	-	-
15	7	5	0.1	2178.95	35.43	11.81
16	8	5.1	0.1	2984.08	33.19	11.06
17	6	4.1	0.1	26621.56	214.69	71.56
18	6	4.9	0.1	2380.83	51.07	17.02
19	7	4.9	0.1	2407.41	51.64	17.21
20	31	4.9	0.1	639.55	75.83	25.28
21	128	6.1	0.1	221.61	45.39	15.13
22	128	5.1	0.1	5242.77	229.85	76.62
23	-	5.1	0.1	5242.77	229.85	76.62

Table 4.: Parameters of long serial line (section 6.2.3).

		CT		UT 1	DT 1		UT 2	DT 2	
Stage j	b_j	Mean	CV	Mean	Mean	c-a	Mean	Mean	c-a
1	60	2.65	0.1	1500	200	40	90	2	0.4
2	12	2.85	0.1	450	50	10	60	5	1
3	41	2.5	0.1	500	80	16	60	5	1
4	93	3	0.1	800	20	4	80	2	0.4
5	22	2.4	0.1	650	60	12	48	5	1
6	90	2.75	0.1	690	85	17	30	2	0.4
7	80	2.8	0.1	820	40	8	60	4	0.8
8	99	2.65	0.1	520	120	24	55	6	1.2
9	58	2.3	0.1	1200	80	16	30	1	0.2
10	-	2.7	0.1	900	35	7	85	3	0.6

Table 5.: Parameters of the system with multi-failure (section 6.2.4).

			CT		UT	DT
	Stage j	b_j	Mean	CV	Mean	Mean
DT-BN 1	1	4	1	0.2	17	3
	2	4	1	0.2	11	3
	3	4	1	0.2	17	3
	4	4	1	0.2	17	3
	5	4	1	0.2	11.5	3
	6	4	1	0.2	17	3
	7	-	1	0.2	17	3
DT-BN 2	1	100	3.49	0.5	300	40
	2	80	3	0.5	350	100
	3	150	2.5	0.5	140	65
	4	100	2.86	0.5	400	85
	5	100	2.92	0.5	360	120
	6	-	3.1	0.5	140	35
DT-BN 3	1	100	3.49	0.5	300	60
	2	80	3	0.5	350	100
	3	150	2.5	0.5	140	65
					800	130
	4	100	2.86	0.5	400	85
	5	100	2.92	0.5	360	120
	6	-	3.1	0.5	130	25
					150	40

Table 6.: Parameters of DT-BN systems (section 6.3).

Appendix IV: Proof of propositions

MP1-S

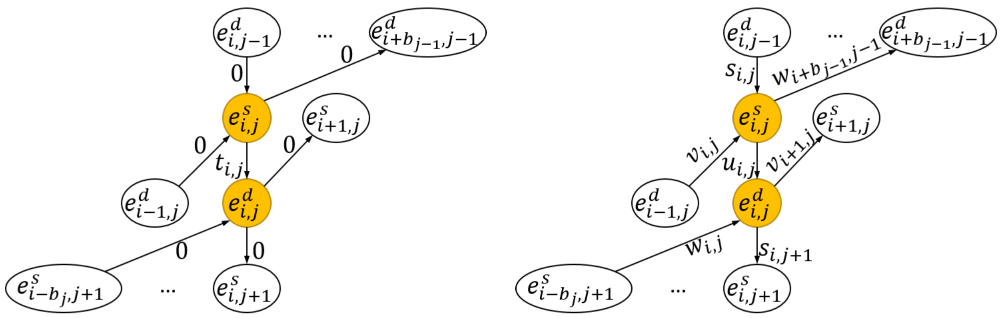
$$\begin{aligned}
&min\{z\} \\
&s.t. \\
&z = \frac{e^d_{N,M}}{N} && : \vartheta && (1)\\
&e^d_{i,j} - e^s_{i,j} \geq t_{i,j} && : u_{i,j} \quad \forall i = 1,...,N,\ j = 1,...,M && (2)\\
&e^s_{i,j} - e^d_{i-1,j} \geq 0 && : v_{i,j} \quad \forall i = 1,...,N,\ j = 1,...,M && (3)\\
&e^s_{i,j} - e^d_{i,j-1} \geq 0 && : s_{i,j} \quad \forall i = 1,...,N,\ j = 1,...,M && (4)\\
&e^d_{i,j} - e^s_{i-b_j,j+1} \geq 0 && : w_{i,j} \quad \forall i = 1,...,N,\ j = 1,...,M && (5)\\
&e^\xi_{i,j} \geq 0 && : \alpha^\xi_{i,j} \quad \forall \xi \in \{s,d\} i = 1,...,N,\ j = 1,...,M && (6)
\end{aligned}$$

MP1-S-Dual:

$$\begin{aligned}
&max\{\sum_{i=1}^{N}\sum_{j=1}^{M} t_{i,j}u_{i,j}\} \\
&s.t. \\
&s_{i,j} + v_{i,j} - u_{i,j} - w_{i+b_{j-1},j-1} = 0 && : e^s_{i,j} \quad j = 1,...,M,\ i = 1,...,N,\ (i,j) \neq (1,1) && (7)\\
&u_{i,j} + w_{i,j} - s_{i,j+1} - v_{i+1,j} = 0 && : e^d_{i,j} \quad j = 1,...,M,\ i = 1,...,N,\ (i,j) \neq (N,M) && (8)\\
&u_{N,M} - \frac{\vartheta}{N} = 0 && : e^d_{N,M} && (9)\\
&\alpha^s_{1,1} - u_{1,1} = 0 && : e^s_{1,1} && (10)\\
&\vartheta = 1 && && (11)\\
&\alpha^s_{1,1},\ s_{i,j},\ u_{i,j},\ v_{i,j},\ w_{i,j} \geq 0 && j = 1,...,M,\ i = 1,...,N
\end{aligned}$$

Proposition 4.1. MP1-S-Dual is to find the maximum weighted flow in the graph of ERG. The weights on arcs are equal to the time delays, and all the nodes $e^s_{i,j}$ and $e^d_{i,j}$ hold flows. The 'start' node is a source, and the 'end' node is a sink absorbing $\frac{\vartheta}{N}$ unit flow.

Proof. Figure 4(b) presents the same element as in Figure 4(a), with dual variables in MP1-S-Dual labeled on the arcs, instead of the time delay in MP1-S. For the node $e^s_{i,j}$, the balance of its flow, i.e., the difference between input flow and output flow, is equal to $s_{i,j} + v_{i,j} - u_{i,j} - w_{i+b_{j-1},j-1}$, the same as the lefthand side of constraint (7). Thus, (7) can be interpreted as



(a) Element in the ERG of serial production lines.

(b) Element of the dual subproblem.

Figure 4.: Elements in the ERG and of the dual subproblem.

nodes of starting events neither absorb nor create any flow. For the node $e^d_{i,j}$, the balance of its flow is equal to $u_{i,j} + w_{i,j} - s_{i,j+1} - v_{i+1,j}$, the same as the lefthand side of constraint (8). Thus, (8) represents that nodes of departure events neither absorb nor create any flow. (9) implies the flow balance of $e^d_{N,M}$, and $\frac{\vartheta}{N}$ can be regarded as the flow on the arc from $e^d_{N,M}$ to the end node. (10) implies the flow balance of $e^s_{1,1}$, and $\alpha^s_{i,j}$ can be regarded as the flow on the arc from start node to $e^s_{1,1}$.

Therefore, MP1-S-Dual can be interpreted as finding the maximum weighted flow in the network, where the weights are equal to the delay on the triggering arcs, and that all the nodes hold flows except the end node as a sink absorbing $\frac{\vartheta}{N}$ unit flow, and the start node as a source. □

Proposition 4.2. The dual solution using Algorithm 1 and Algorithm 2 is optimal solution of model MP1-S-Dual.

Proof. The proof uses duality theory. First, the proposed solutions are proved to be feasible solution of MP1-S and MP1-S-Dual, respectively. Then, the solutions are proved to be equal to each other.

Lines 10–14 in Algorithm 1 assure that constraints (2) and (5) are satisfied. Lines 5–9 in Algorithm 1 assure that constraints (3) and (4) are satisfied. Therefore, Algorithm 1 provides feasible solution $e^s_{i,j}, e^d_{i,j}$ of MP1-S. Line 17 asssures that constraint (1) is satisfied.

Lines 6–7 in Algorithm 2 imply that $u_{i,j}+w_{i,j} = (bu_{i,j}+bw_{i,j})(s_{i,j+1}+v_{i+1,j})$. Each event $e^d_{i,j}$ is with only one input flow, i.e., $bu_{i,j} + bw_{i,j} = 1$, so $u_{i,j} + w_{i,j} = s_{i,j+1} + v_{i+1,j}$, i.e., constraints (8) hold. Similarly, lines 4–5 in Algorithm 2 assure that constraints (7) hold. Line 1 assures that constraints (9) to (10) are satisfied. Therefore, Algorithm 2 provides feasible solution of MP1-S-Dual.

In the spanning tree represented by $bu_{i,j}, bv_{i,j}, bw_{i,j}, bs_{i,j}$, there is one and only one path from the start node to the end node. We denote the path as $\mathcal{P}$.

Lines 6, 8, 11, 13 in Algorithm 1 imply that each event occurring time $e^s_{i,j}$ or $e^d_{i,j}$ is calculated by adding a time delay, either 0 or $t_{i,j}$, to its triggering event occurring time. Following a triggering path, the occurring time of the last event on the path can be calculated as the sum of all the time delay on the path. The occurring time of event $e^d_{N,M}$ is equal to the sum of all the time delays on $\mathcal{P}$. Thus, providing the solution in Algorithm 1, the objective function of MP1-S is equal to $\frac{\text{accumulated time delay along } \mathcal{P}}{N}$.

Lines 4–7 in Algorithm 2 imply that the input flow of one node is equal to its output flow. Input flow of one node $e^s_{i,j}$ or $e^d_{i,j}$ is equal to sum of the flow absorbing by the leaves of its spanning tree. Since the end node is the only sink in the graph absorbing $\frac{1}{N}$ unit flow, the only arcs carrying non-zero flow are the arcs on $\mathcal{P}$. Providing the solution in Algorithm 2, since the weights on the arcs are equal to the time delays, the objective function of MP1-S-Dual is equal to $\frac{\text{accumulated time delay along } \mathcal{P}}{N}$.

The objective of MP1-S and of MP1-S-Dual are equal. According to strong duality, the feasible solutions provided by Algorithm 1 and Algorithm 2 are the optimal solutions. □

Appendix V: ERG of serial production line

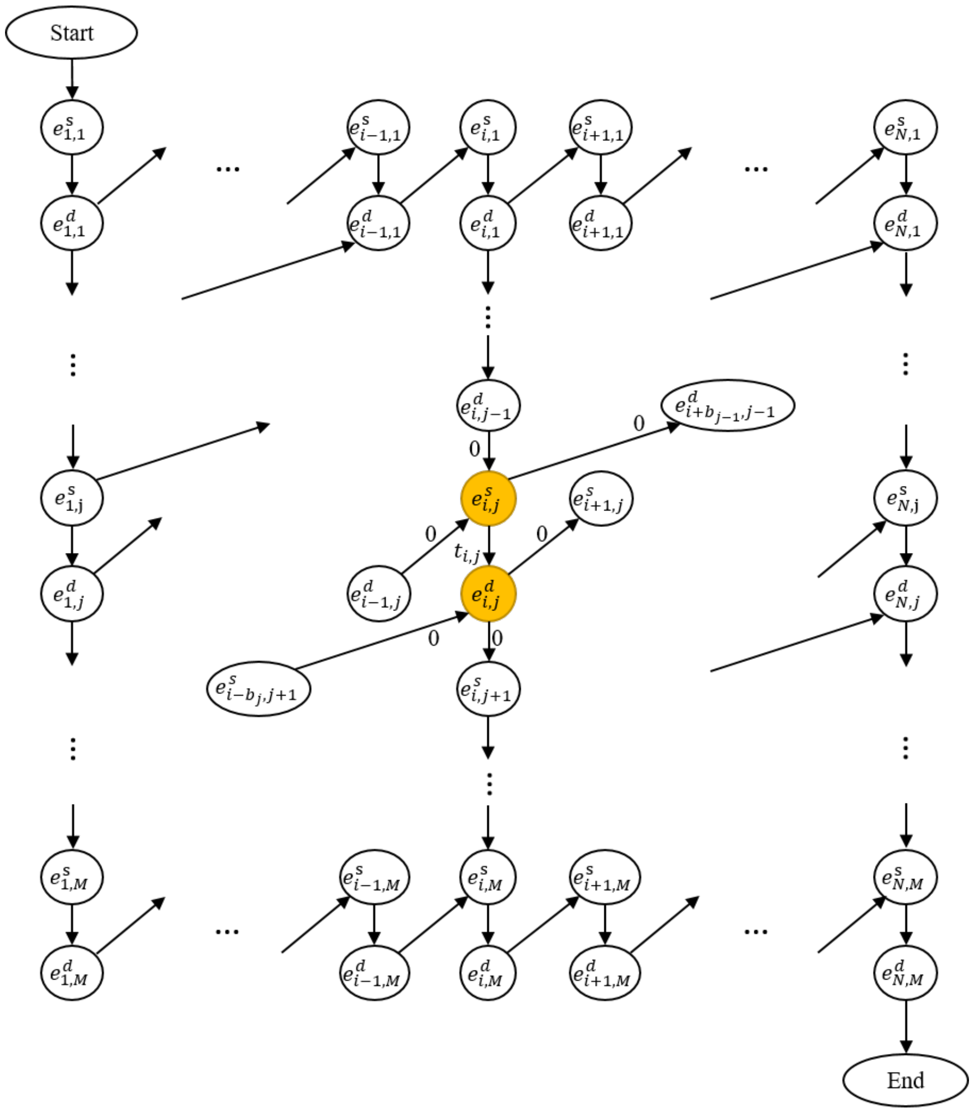


Figure 13.: Event relationship graph of serial production line.

Appendix VI: numerical results of section 6.2.2

System ID	Stage No.	Original	MP1-L x_j	MP1-H x_j	MP2-L x_j	MP2-H x_j
B-5-I	1	-	0	0	0	0
	2	-	0	0	0	0
	3	-	0	0.179	0	0
	4	-	0	0.521	0	0
	5	-	0.8	0.8	0.315	0.622
	TH	0.324	0.349	0.351	0.334	0.344
	ΔTH (%)	-	7.76%	8.34%	3.00%(*)	6.00%(*)
	Cost	-	90(*)	180 (*)	41.5	72.2
System ID	Stage No.	Original	MP1-L x_j	MP1-H x_j	MP2-L x_j	MP2-H x_j
B-5-II	1	-	0	0	0	0
	2	-	0	0	0	0
	3	-	0	0	0	0
	4	-	0	0.8	0	0
	5	-	0.8	0.8	0.306	0.595
	TH	0.325	0.351	0.351	0.334	0.344
	ΔTH (%)	-	8.12%	8.13%	3.00%(*)	6.00%(*)
	Cost	-	90(*)	180 (*)	40.6	69.5
System ID	Stage No.	Original	MP1-L x_j	MP1-H x_j	MP2-L x_j	MP2-H x_j
C-5	1	-	0	0	0	0
	2	-	0	0	0	0
	3	-	0.357	0.8	0.333	0.768
	4	-	0	0	0	0
	5	-	0.343	0.8	0.319	0.764
	TH	0.345	0.356	0.367	0.355	0.366
	ΔTH (%)	-	3.31%	6.31%	3.00%(*)	6.00%(*)
	Cost	-	90(*)	180 (*)	85.3	173.3
System ID	Stage No.	Original	MP1-L x_j	MP1-H x_j	MP2-L x_j	MP2-H x_j
D-5	1	-	0	0	0	0
	2	-	0.300	0.8	0.242	0.528
	3	-	0.400	0.8	0.319	0.692
	4	-	0	0	0	0
	5	-	0	0	0	0
	TH	0.343	0.355	0.367	0.353	0.363
	ΔTH (%)	-	3.52%	6.87%	3.00%(*)	6.00%(*)
	Cost	-	90(*)	180 (*)	76.0	142.0

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