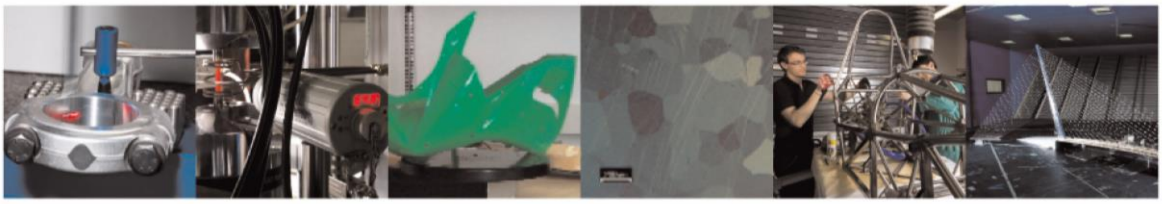

Manufacturing Signature for Tolerance Analysis

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1 Introduction

Tolerance analysis has a considerable weight in concurrent engineering, and it represents the best way to solve problem in order to ensure higher quality and lower costs. The need to assign dimensional and geometric tolerances to assembly components is due to the standardization of the production and to the correct working of the assembly. The dimensions and the tolerances of the assembly components combine, according to the assembly sequences, and generate the tolerance stack-up functions. Solving a tolerance stack-up function means to determine the nominal value and the tolerance range of a product function by combining the nominal values and the tolerance ranges assigned to the assembly components.

Many well-known approaches of the literature for tolerance analysis exist, even if they do not consider the actual surface due to a manufacturing process [1,2]. Many commercial CAT software packages was developed on the basis of those models, such as 3-DCS of Dimensional Control Systems®, VisVSA of Siemens® and so on [3,4].

None of the models proposed in the literature provide a complete and clear mechanism for handling all the requirements included in the tolerancing standards, and this limitation is reflected also in the available commercial CAT software. The main limitations include the following: no proper support for the appli-cation of the envelope rule and of the independence rule; cannot handle form tolerances (except for the vector loop model); no mechanisms for assigning probability density functions (PDFs) to model parameters starting from tolerances and considering toler-ance zone interactions; no proper representation of all the possible types of part couplings that include clearance [5].

A taxonomy was developed in a previous work to help evaluate, compare, and select tolerance analysis models, depending on the model characteristics and the type of application [6].

A widespread model of the literature is the *variational* that has been proposed by Martino and Gabriele [7], Boyer and Stewart [8], and Gupta and Turner [9]. The basic idea is to represent the variability of an assembly, due to tolerances and assembly constraints, through a parametric mathematical model. It models the dimensional and geometrical variations affecting a part by means of differential homogeneous transformation matrices.

The *variational* model of the literature considers the dimensional and the geometrical tolerances applied to some critical points (contact points among profiles belonging to coupled parts) on the surface of the assembly components. These points are gen-erally considered uncorrelated since the ideal surface is taken into account. Actually, every manufacturing process leaves on the surface a signature, i.e., a systematic pattern that characterizes all the features machined by that process.

In a previous work it was presented a comparison between the results obtained by applied a *variational* model with and without considering the correlation among the points of two circular profiles due to the manufacturing signature for a specific case study [10].

In this work, the comparison is deepened. The effect of a manufacturing process on solving a stack-up function is investigated for a circular profile obtained by turning. The analysis of toleran-ces was carried out by means of a new *variational* model that allows to simulate the form tolerance. This model was applied with and without considering the correlation among the points of the same circular profile due to the manufacturing signature. Seven different cases were considered. This work aims to be a first step toward the integration of the design and the manufacturing in a concurrent engineering approach.

The paper is organized as follow: in Sec. 2, the new variational model is presented. In Sec. 3, the case study is introduced. In Sec. 4, the tolerance stack-up function is solved by means of the variational model implemented in a MATLAB environment. In Sec. 5, the variational model is used to solve the stack-up function, once simulated the manufacturing signature of the circular profiles by an ARMAX model. In Sec. 6, a sensitivity analysis of the varia-tional model toward the manufacturing signature was developed by considering different values of the applied geometrical tolerances.

2 New Variational Model

Each functional requirement is schematized through an equation, which is usually called stack-up function, whose variables are the model parameters that are function of the dimensions and the tolerances assigned to the assembly components. It looks like

$$\mathrm{FR} = f(p_1, p_2, \ldots, p_n) \tag{1}$$

where FR is the considered functional requirement, $p_1,\ldots, p_n$ are the model parameters, and $f(p)$ is the stack-up function, that is usually not linear.

A functional requirement is usually a characteristic that relates two features. Its analytical expression is obtained by applying the equations of the *Euclidean geometry* to the features of the rigid parts that define the functional requirement or to the points of the features that define the functional requirement.

To define a stack-up function it is needed to follow a sequence of steps that allows to model each component's geometric variations due to the applied tolerances, the geometric variations between couples of components due to the assembling process

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and the functional requirements of the assembly (see Fig. 1). The first step creates a local model of tolerance analysis, the second a global model of tolerance analysis and the third creates the stack-up function of the global model. The whole sequence constitutes the frame of a tolerance analysis model. The first step aims to define the range of variation of the model's parameters of a feature from the tolerances assigned to each feature of each assembly component. This means to identify, for each applied tolerance of each feature of each component of the assembly, the ranges of rotation and translation of the barycenter of the nominal feature due to the applied tolerance. Those rotation and translation are the model's parameters, whose ranges are bounded by the ranges of the applied tolerance.

Typically, each feature has its local datum reference frame (DRF), while each component and the whole assembly have their own global DRF. In the nominal condition, the homogeneous transformation matrix (called **TN**) that allows passing from one DRF to another is known. When real features are machined, they depart from nominal. Assuming that real features maintain their nominal form (i.e., form deviations are neglected), the location of a real feature deviates from nominal; this deviation is expressed by parameters that constitute a differential homogeneous transformation matrix **DT**. Those parameters are associated to the translation or rotation of the nominal feature barycenter allowed by the applied tolerance. To pass from the global DRF of part i, R_i, to the local DRF of a feature j of part i, R_{ij}, it is enough to multiply the two matrices

$$\mathbf{T}_{Ri \to Rij} = \mathbf{TN}_{Ri \to Rij} \cdot \mathbf{DT}_{Rij} \quad (2)$$

where $\mathbf{T}_{Ri \to Rij}$ is the total transformation matrix to pass from the global DRF of part i to the local DRF of feature j of part i; $\mathbf{TN}_{Ri \to Rij}$ is the nominal transformation matrix to pass from the global DRF of part i to the local DRF of feature j of part i; and $\mathbf{DT}_{Rij}$ is the differential transformation matrix of feature j of part i.

If a feature may not be directly referred to the global DRF, it is reported to it through a chain of features. To calculate the total matrix it is enough to make the product of the single contributions.

The local model should schematize all the tolerance kinds, i.e., dimensional, form, orientation, location. To represent the form deviation through the model's parameters is a very critical aspect.

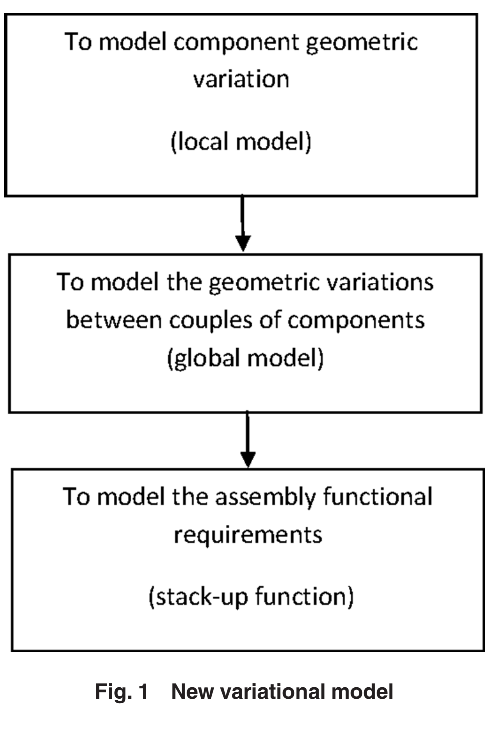

Fig. 1 New variational model

To consider the effect of the form deviation, the new model introduces a virtual transformation applied to the points of the surface on which a form tolerance is assigned. This virtual transformation was introduced by Chase and Magleby inside their *vector loop* model for tolerance analysis, as deeply discussed in Ref. [11]. The proposed model translates this concept to a *variational* model by associating a parameter p_f to the feature to which a form tolerance is applied. This parameter stands for the form deviation of the single points of the feature. It may be positive or negative; it is positive if it goes outside the material. It varies from point to point of the feature.

The second step aims to define the range of variation of the model's parameters from the variability of the coupled features during the assembly. The variability of the coupled features, by which the link among the parts is made, depends by the tolerances applied to the features themselves and by the assembly conditions.

Typically, the relative location of the parts is expressed by means of parameters which constitute the differential homogeneous transformation matrix **DA** (the transformation matrix is indicated by the letter A = assembly to distinguish it from the matrix **DT** that is for the part). The total transformation to pass from the global DRF of part i, R_i, to the global DRF of part l, R_l, is obtained simply by means

$$\begin{aligned}\mathbf{A}_{Ri \to Rl} &= \mathbf{AN}_{Ri \to Rl} \cdot \mathbf{DA}_{Ri \to Rl} \\ &= \mathbf{TN}_{Ri \to Rij} \cdot \mathbf{DT}_{Rij} \cdot \mathbf{DA}_{Rij \to Rlk} \cdot \mathbf{DT}^{-1}_{R_{lk}} \cdot \mathbf{TN}^{-1}_{R_l \to R_{lk}} \end{aligned} \quad (3)$$

where $\mathbf{A}_{Ri \to Rl}$ is the assembly matrix between part i and part l; $\mathbf{AN}_{Ri \to Rl}$ is the assembly matrix between part i and part l in nominal condition; $\mathbf{DA}_{Ri \to Rl}$ is the differential assembly matrix between part i and part l; $\mathbf{DA}_{Rij \to Rlk}$ is the differential assembly matrix between feature j of part i and feature k of part l; $\mathbf{TN}_{Ri \to Rij}$ is the nominal transformation matrix to pass from the global DRF of part i to the local DRF of feature j of part i; $\mathbf{DT}_{Rij}$ is the differential transformation matrix of feature j of part i; $\mathbf{DT}_{Rlk}$ is the differential transformation matrix of feature k of part l; and $\mathbf{TN}_{Rl \to Rlk}$ is the nominal transformation matrix to pass from the global DRF of part l to the local DRF of feature k of part l.

The differential assembly matrices $\mathbf{DA}_{Ri \to Rl}$ and $\mathbf{DA}_{Rij \to Rlk}$ are hard to evaluate, since they depend on both the tolerances applied to the features in contact and the assembly conditions.

The global model has to be able to schematize the joints with contact and the joints with clearance between the coupled features.

The third step is to derive the stack-up functions. This means that the *global model* has to be able to approach to the joints which give a linear structure of the FR equation (stack-up function) and to the joints which carry out to a complex structure of the FR equation (network function), as shown in Fig. 2.

Once the assembly parameters are known, all features can be expressed in the same global DRF of the assembly

$$\mathbf{X}_{ik} = \mathbf{AN}_{R_i \to R_l} \cdot \mathbf{DA}_{R_i \to R_l} \cdot \mathbf{TN}_{Rl \to Rlk} \cdot \mathbf{DT}_{Rlk} \cdot x_k \quad (4)$$

where $\mathbf{X}_{ik}$ and $\mathbf{x}_k$ are the vectors whose components are the coordinates of a generic point expressed in the global DRF of part i and in the local DRF of feature k.

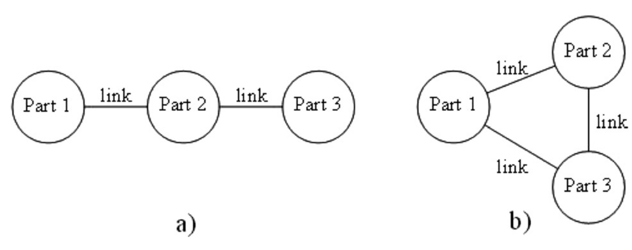

Fig. 2 (*a*) Linear stack-up function and (*b*) network stack-up function

Then, the equations of the functional requirements are defined by applying the analytical geometry.

Once modeled the stack-up functions, they may be solved by means of a worst case or a statistical approach [12]. To carry out a worst case approach, it is needed to define the worst configurations of the assembly (i.e., those configurations due to the cumulative effect of the smallest and the highest values of the tolerances assigned to the assembly components) that satisfy its assigned tolerances. This means to solve a problem of optimization (maximization and/or minimization) under constraints due to the assigned tolerances. Many are the methods developed by the literature to carry out a worst case approach (see Ref. [13]). To carry out a statistical approach, it is needed to translate each tolerance assigned to assembly components into one or more parameters of the stack-up function.

A PDF, that is function of both the manufacturing and the assembly processes, is assigned to each parameter. Being the definition of the relationship among the production and assembly processes and the PDF of the tolerance of the component strongly hard to estimate, the commonly used assumption is to adopt a Gaussian PDF. Moreover, a further assumption is to consider independent the parameters used to represent the variability of the features delimiting each dimensional tolerance. The variation of the FR is obtained by means of a Monte Carlo simulation technique [1,14]; it is usually calculated as ± three times the estimated standard deviation (three sigma paradigm of Ref. [12]).

3 Case Study

The case study is constituted by a box containing two circles, as shown in Fig. 3. The aim of this work is the measurement of the gap g between the second circle and the top side of the box as a function of the tolerances applied to the components.

Circular profiles machined by turning were studied and approximated by means of an ARMAX model [15].

This approach consists in reconstructing the circular profile by a set of evenly distributed points and then, this reconstructed pro-file, is sampled at an higher rate by resulting in a secondary set of sampled points which is used to estimate form error through a fitting criteria as the Least Square or the minimum zone methods. In their work, Colosimo and Pacella [16] showed that the manufacturing signature of the turning process was mainly affected by bilobe and three-lobe contours. This harmonic model was combined with a second-order autoregressive model of the noise. That is due to the correlations of the measurements of the machined profiles, since the data are obtained in similar process conditions and they are related to similar material properties. Combining the harmonic and the autoregressive parts, the parametric model of the identified process signature is given by

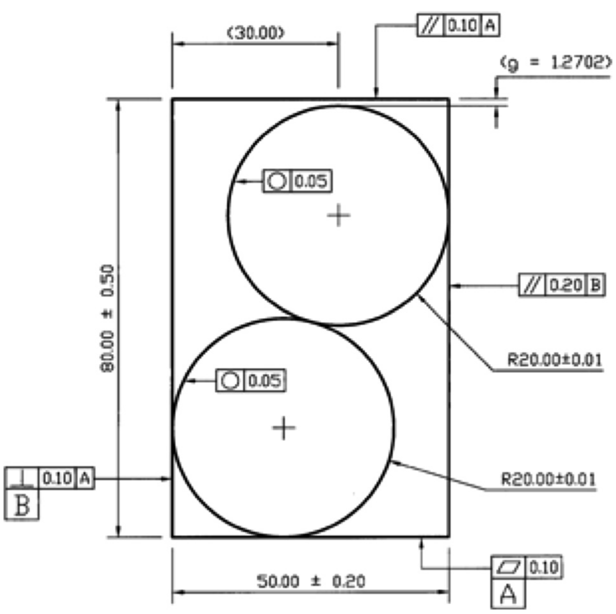

Fig. 3 Case study

$$Y_t = \sqrt{\frac{2}{N}} \sum_{i=2}^{3} [b_{2i-1}\cos(i \cdot t \cdot 2\pi/N) + b_{2i}\sin(i \cdot t \cdot 2\pi/N)] + \frac{1}{1 - a_1 B - a_2 B^2}\varepsilon_t \quad (5)$$

where $t = 1, 2, ..., N$ is the index of data points in the sampled profile, B is the backshift operator [17], and N is the number of equally spaced points that were measured on the profile.

For each index t, Y_t represents the radial distance (in millimeters) between the measured points and the LS substitute circle, that was measured at regular position $\vartheta_t = 2\pi/N$.

The signature model of Eq. (5) is a linear combination of two harmonic terms, the bilobe and the trilobe contours, plus a second-order autoregressive model of the noise. Each term of the first part of Eq. (5) is the ith harmonic characterized by i undula-tion per revolution. This model was applied to a case study consti-tuted by three parts, one fixed with planar surfaces and two circular profiles assembled on the fixed part. Instead considering the form tolerances applied to the two circular profiles, the ARMAX model was considered to simulate the real profile of the manufactured parts that, however, remains inside the form toler-ance zone. Therefore, the analysis of tolerances was carried out by means of the variational model shortly described in the previous paragraph. This model was applied with or without considering the correlation among the points of the same circular profile due to the manufacturing signature. Subsequently, six new cases were generated by changing the values of the applied geometrical tolerances by a scale factor (F). The variational model was applied to the new cases.

Both the dimensional and the geometrical tolerances shown in Fig. 3 were taken into account and the Envelope Principle was applied. The case study was solved through the statistical approach.

4 Simulation Without Signature

Once indicated with L_1, L_2, L_3, and L_4, the linear features of the box and with C_1 and C_2, the two circular parts as shown in Fig. 4, the assembly graph of Fig. 5 was built. It shows two joints of cylinder slider kind between the box and the circle 1 at point A and point B, one joint of parallel cylinder kind between the circle 1 and the circle 2 at point C, one joint of cylinder slider kind between the ball 2 and the box at point D and the measure to perform g. A DRF was assigned to each feature of each part and the whole assembly; all the global DRF's have the x-axis horizontal. The DRF of the box is also considered as the global DRF of the assembly. The homogeneous transformation matrices were evaluated for the feature L_1, L_2, L_3, and L_4 of the box and for the two circles C_1 and C_2, in order to express all the features in the same global DRF of the assembly.

Therefore, the assembly was created by imposing the assembly conditions that are shown in the assembly graph and the gap g was evaluated by means of the following analytical equation:

$$g = 1.270 + r_{z03}\Delta X_{13} - \Delta Y_{13} + 5r_{z03} - t_{y03} - r_2 - d_{EF} \quad (6)$$

where

$$\Delta X_{12} = \left[t_{y04} + r_1 + d_B + r_{z04}(40 - h)\right]/(1 + r_{z01}r_{z04}) \quad (7)$$

$$\Delta Y_{12} = r_{z01}\Delta X_{12} - 5r_{z01} + t_{y01} + r_1 + d_A = r_{z01}\Delta X_{12} + h \quad (8)$$

$$h = -5r_{z01} + t_{y01} + r_1 + d_A \quad (9)$$

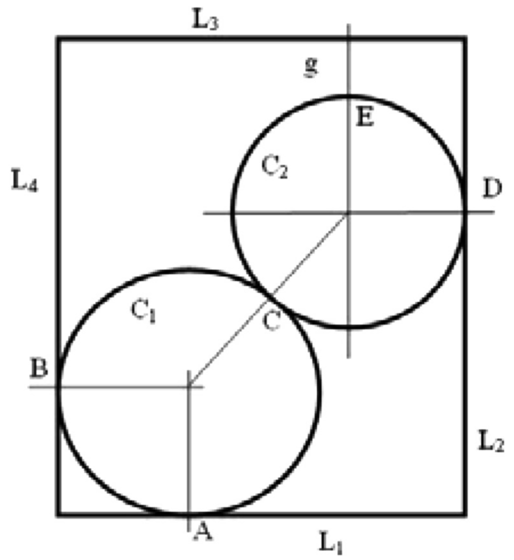

Fig. 4 Features of the parts

$$\Delta X_{13} = -r_{z02}\Delta Y_{13} - a \quad (10)$$

$$\Delta Y_{13} = b + \left(b^2 - c\right)^{1/2} \quad (11)$$

$$a = 18.730 r_{z02} + t_{y02} + r_2 + d \quad (12)$$

$$b = [r_{02}(10 - a - \Delta X_{12}) - 38.730 + \Delta Y_{12}]/\left(1 + r_{02}^2\right) \quad (13)$$

$$c = [(10 - a - \Delta X_{12})^2 + (38.730 - \Delta Y_{12})^2 - (40 + r_1 + r_2 + d_C)^2]/\left(1 + r_{02}^2\right) \quad (14)$$

with r_1 is the model parameter of the feature C_1, ΔX_{12}, and ΔY_{12} are the assembly parameters of the part C_1 on the box, r_2 is the model parameter of the feature C_2 and ΔX_{13} and ΔY_{13} are the as-sembly parameters of the part C_2 on the box. The t_{yi} parameters and the r_{zi} parameters are the translations and the rotations applied to the considered elements, which index is i, due to the tolerance values considered. For example t_{y03} is the translations of the ele-ment L3 measured in its DRF. The d_i are the model parameters due to the form variations applied to the circular profiles evaluated from the applied form tolerance values. For example d_{EF} is the pa-rameter evaluated summing d_E with d_F the two form parameters evaluated in the points E and in F.

The set of Eqs. (6)–(14) were solved by means of a statistical case approach by considering the model parameters as statistical variables following a Gaussian PDF. A Monte Carlo approach was used by 100.000 runs. The obtained value of the objective function g was 1.269 ± 0.524 mm.

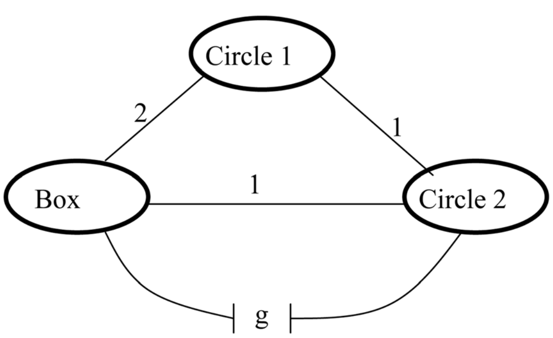

Fig. 5 Assembly graph

5 Simulation With Signature

The same mathematical model used in the previous paragraph was adopted for the current analysis with the only correction of the parameters which values were affected by the form tolerance of the circular profiles. In this way the model performed in the current analysis was the model referred in Eq. (6). Also in this case, g was the objective function and the function which parame-terize the positions of the centers of the two circular profile with respect to the boundary of the box is

$$R + r + d \quad (15)$$

where R is the nominal value of the radius of the circular profile, r is the model parameter due to the dimensional tolerance applied to the circular profiles and d is the model parameter due to the form tolerance applied to the circular profiles.

In this case, the values of dimensional and orientation toleran-ces are the same of the previous case, but the values of the form tolerance of the two circular profiles were substituted by the value of the form deviations computed by means of the manufacturing signature (i.e., by means of Eq. (5)).

The roundness of the two circles was simulated by the local val-ues of Y_t evaluated for the points on the circular profile. These val-ues were used to define the values of the parameters d_i used in the above mentioned variational model, where $i = a, b, c, d$ with a, b, c, and d are the contact points between the profiles ant the box. Since the manufacturing signature affects only the values of the d_i parameters, the value of r remains equal to $\pm$ 0.01 mm and R is equal to 20 mm.

In this case, the obtained value of the objective function g is 1.269 ± 0.526 mm that is very close to that obtained through the variational model without considering the manufacturing signa-ture (see previous paragraph).

A further analysis was carried out by considering the dimen-sional parameter r dependent by the manufacturing signature through the following relationship:

$$r = \frac{(D_1 + D_2)}{2} - D_n \quad (16)$$

where D_1 and D_2 are two orthogonal diameters, of the manufac-tured profile obtained by means of Eq. (5) and, thus, by summing the values of Y_t to the nominal diameter D_n that is equal to 40 mm. The obtained value of the objective function g is 1.263 ± 0.526 mm. It is very close to the value obtained without considering the manufacturing signature.

The values of the adopted ARMAX model parameters are showed in Table 1, while the values of the noise estimation ε_t were modeled by a Gaussian white noise with standard deviation equal to 0.287 μm. To simulate the profile by means of the ARMAX model was used a number of evenly distributed points equal to 1240. This number was chosen to keep points in corris-pondence of the contact points between circular profiles and box and between the two circular profiles, that one hinted at letter A, B, C, and D.

Table 1 ARMAX model parameters: (a) mean vector estimate (mm) and (b) covariance matrix estimate (mm^2)

	$\bar{b}_3$	$\bar{b}_4$	$\bar{b}_5$	$\bar{b}_6$	$\bar{a}_1$	$\bar{a}_2$
(a)	−0.0341	0.0313	0.0080	−0.0322	0.3714	0.2723
(b)	0.0004	−0.0002	0.0001	0	0.0001	0.0003
	−0.0002	0.0004	0.0001	0	0.0001	−0.0002
	0.0001	0.0001	0.0002	0	0.0001	0
	0	0	0	0.0003	0.0003	0.0003
	0.0001	0.0001	0.0001	0.0003	0.0072	0.0012
	0.0003	0.0002	0	0.0003	0.0012	0.0036

Table 2 Results of sensitivity analysis

Case study	F-factor	Circles form tolerance (mm)	g (mm)			
			With signature		Without signature	
			μ	3σ	μ	3σ
1	0.2	0.01	1.268	0.100	1.270	0.104
2	0.5	0.025	1.266	0.249	1.270	0.261
3	1	0.05	1.269	0.526	1.269	0.524
4	2	0.1	1.255	1.001	1.271	1.043
5	5	0.25	1.232	2.501	1.271	2.608
6	10	0.5	1.190	5.010	1.282	5.232
7	20	1	1.137	10.035	1.303	10.439

6 Sensitivity Analysis

The geometrical tolerances applied to the case study were changed by multiplying them of a scale factor (F), as shown in Table 2.

At first, two further cases were generated (case studies 1 and 2 in Table 2) by decreasing the form tolerance range. It was observed that decreasing the form deviation involves a decrease of the *g*-range due to signature, even if they remain near to those due to the model without signature.

Therefore, three further cases (case studies 4, 5, and 6) were produced by increasing the form tolerance range. The *g*-range due to the signature increases with the increase of the form deviation. The difference between the two models appears more evident. The *g*-range of the signature is smaller than that without signature; the mean value is smaller too. This is probably due to correlation among the points of the circle profile due to the signature.

A final case study was developed (7 in Table 2) to have an example of a rough process; it does not involve a control of form tolerance. The mean and the deviation of g due to the signature appear smaller than those due to the model without signature.

7 Conclusions

In this work, the problem to investigate the effects of considering the manufacturing signature in solving a tolerance stack-up function was tackled.

A case study involving three parts was defined and solved by means of a new variational model that allows to simulate the form tolerances. This model was applied with and without considering the correlation among the points of the same circular profile due to the manufacturing signature. A Monte Carlo approach was implemented with 100.000 runs of simulation.

Therefore, six further cases were generated by changing the applied geometrical tolerances of a scale factor.

The analysis of all the seven cases states that the application of the signature appears more evident with the increase of the range of the form tolerance. Moreover, the signature involves a mean value and a range of the requirement g smaller than those due to the simu-lations without signature. This is probably due to a negative correla-tion among the points of the circle profile due to the signature.

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