

Detailed simulation of dual-reflux pressure swing adsorption process

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Highlights

- Aspen Adsim[®] model for simulating realistic DR-PSA process scenarios is presented.
- 19 experimental runs reported by McIntyre et al. (2010) are simulated.
- Model predictions are in good agreement with experimental results.
- Effect of feed position on process performance is assessed via simulations.

Abstract

A model for the detailed simulation of dual-reflux pressure swing adsorption process developed in the frame of the commercial software Aspen Adsim[®] is presented. For validation purposes, simulations were performed and model predictions were compared with published experimental results. At cyclic steady-state, model predictions were found to be in good agreement with reported experimental results in terms of: (i) average ethane mole fraction in heavy product, (ii) average nitrogen mole fraction in light product, (iii) instantaneous heavy product composition profiles, and (iii) instantaneous column composition profiles. The predicted and experimental trends obtained by analyzing the effect of various operating parameters (light reflux flowrate, duration of feed/purge step, heavy product flowrate and mole fraction of heavy component in binary feed gas mixture) on process performance are also comparable. Overall, this simulation technique of dual-reflux pressure swing adsorption can serve as an effective tool for process design, cost reduction of laboratory and/or plant trials, and enhanced process understanding.

Keywords: Pressure swing adsorption; Dual-reflux; Mathematical modeling; Gas purification; Process simulation; Cyclic adsorption process

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1. Introduction

Since its inception more than half a century ago, the technology of pressure swing adsorption (PSA) has been widely studied and applied, becoming today the method of choice for the separation and/or purification of many gaseous mixtures. Some of its most popular industrial applications include hydrocarbon separation, hydrogen purification, air drying, and air separation. PSA systems employed for the processing of binary feed gas mixtures can be categorized into three main types: (i) stripping, (ii) rectifying and (iii) dual-reflux.

Stripping PSA systems are based on the Skarstrom cycle (Skarstrom, 1959) and are capable of producing only the light product (weakly adsorbed; in the following referred to as species *B*) at high purity, since the purity of the heavy product (strongly adsorbed; species *A*) is confined by thermodynamic constraints (Subramanian and Ritter, 1997). Rectifying PSA systems, developed by Diagne et al. (1994) and Ebner and Ritter (2002) and also known as enriching reflux PSA (Yoshida et al., 2003), have thermodynamic constraints on the purity of the light product, thus resulting in the capability of producing only the heavy product at high purities. In contrast, the purity of both the products is thermodynamically unconstrained in dual-reflux pressure swing adsorption (DR-PSA; Leavitt, 1992): consequently, DR-PSA processes are capable of achieving complete separation of binary feed gas mixture, thus producing two pure-component streams.

A typical DR-PSA unit comprises of a combined two-bed system with feed injection in a given position along the axis of the adsorption column. Such position (Z_F) divides each bed in two sections: ‘*Stripping Section*’ (*SS*) and the ‘*Rectifying Section*’ (*RS*). Two reflux streams (so the name DR-PSA), light reflux (*LR*, pure *B*), and heavy reflux (*HR*, pure *A*), are respectively injected at the *SS* and *RS* end of each column during constant pressure steps. Depending on the column operating pressure (high pressure, P_H , or low pressure, P_L) to which the binary feed gas mixture is supplied and the type of gas (*A* or *B*) with which the pressure swing is carried out, four different cycle configurations can be identified (Kearns and Webley, 2006):

(i) DR-PL-A: Feed to P_L and pressure swing with *A*;

61 (ii) DR-PL-B: Feed to P_L and pressure swing with B ;

62 (iii) DR-PH-A: Feed to P_H and pressure swing with A ;

63 (iv) DR-PH-B: Feed to P_H and pressure swing with B .

64
65 Gas separation processes based on DR-PSA have been studied experimentally in the literature. Diagne et al.
66 (1994, 1995a, 1995b) explored the application of this process to the CO_2 removal from air using zeolite 13X
67 as adsorbent. In these studies they demonstrated that feed gas containing 20% CO_2 can be concentrated to
68 values higher than 94% in the heavy product stream by proper selection of the operating conditions. More
69 recently, McIntyre et al. (2010) conducted experiments using wood-based activated carbons in DR-PL-A
70 configuration for the recovery and enrichment of dilute ethane from nitrogen. In this extensive study, they
71 performed 19 runs for 1,500 to 3,000 cycles each and analyzed the effect of various operating parameters on
72 process performance: light reflux flowrate, duration of feed/purge step, heavy product flowrate and feed
73 mole fraction of the heavy component. It was demonstrated that the average of these 19 runs with an ethane
74 feed concentration of 1.38 vol % gave an ethane enrichment of 45.8 times (63.2 vol %) and ethane recovery
75 of 84%, while producing N_2 at high purity (99.8 vol %) and recovery (>99%). These experimental studies
76 proved that both the light and heavy products can be obtained at high purities using DR-PSA systems.

77
78 Various modeling tools ranging from very simple to complex have been reported in the literature for the
79 design and optimization of PSA units (cf. Ruthven et al., 1994; Spoorthi et al., 2011; Thakur et al., 2011;
80 Sivakumar and Rao, 2011a, 2011b, 2012). The simplest modeling approach (usually indicated as
81 Equilibrium Theory) involves a large set of simplifying assumptions, such as instantaneous linear
82 equilibrium throughout the column, isothermal conditions, negligible mass transport resistances and axial
83 mixing, negligible pressure drop, and ideal gas behavior. The resulting equations have been solved by the
84 method of characteristics (Rhee et al., 1986) and its solution for conventional PSA systems was fully
85 detailed by Knaebel and Hill (1985). Equilibrium Theory was applied to DR-PSA systems by Ebner and
86 Ritter (2004), Kearns and Webley (2006) and most recently by Bhatt et al. (2013). At the opposite extreme,
87 the detailed PSA modeling approach requires much less assumptions, thus considering nonlinear adsorption
88 equilibrium along with transport limitations, pressure drop and temperature effects. Yang and Doong (1985)

89 were among the earliest to propose such a detailed modeling approach for conventional PSA systems and a
90 similar approach has been used for DR-PSA by Diagne et al. (1996) and Thakur et al. (2011). Various
91 configurations and/or process steps encountered in a PSA cycle need to be modeled via partial differential
92 equations (PDEs) linked by the equation of state and the thermodynamic properties of the gas and adsorbed
93 phases. Given the inherent complexity and iterative nature of PSA systems, the numerical solution of
94 detailed models is computationally demanding and time consuming.

95
96 In this work, we present a detailed model for the simulation of DR-PL-A processes developed in the frame
97 of the commercial software Aspen Adsim[®]. To validate the resulting model, all the 19 experimental runs
98 reported by McIntyre et al. (2010) and considering the separation of the binary mixture nitrogen-ethane have
99 been simulated. At cyclic steady-state, model predictions are compared with reported experimental results in
100 terms of: (i) average ethane mole fraction in heavy product, (ii) average nitrogen mole fraction in light
101 product, (iii) instantaneous heavy product composition profiles, and (iii) instantaneous column composition
102 profiles. Simulation results and reported experimental trends obtained by analyzing the effect of various
103 operating parameters (light reflux flowrate, duration of feed/purge step, heavy product flowrate and mole
104 fraction of heavy component in binary feed gas mixture) on process performance are compared. For
105 enhanced understanding of the process behavior, column composition profiles at the end of each process
106 cycle step are also provided for every run at CSS. Finally, since the feed position was fixed in all the
107 previous model calculations, additional simulations were performed to assess the effect of changing such
108 position on process performance.

109 110 **2. DR-PL-A cycle description**

111
112 The schematic diagram of the twin-bed DR-PL-A system under consideration in this work is depicted in Fig.
113 1. In this particular configuration, each of the two adsorption beds (*Bed – I* and *Bed – II*) undergoes a four
114 step cyclic process: two steps are simultaneously executed at constant pressure and remaining two at non-
115 constant pressure in both the beds. Note that only half-cycle is depicted in the figure, since the same steps
116 occur with the column numbers transposed. The feed injection position along the bed, defined in terms of

117 normalized axial coordinate (Z_F), divides each column into two sections; it is a dimensionless value, Z
118 being the axial coordinate normalized with respect to the column length ($Z = z/L_{bed}$). The section to the
119 left of the feed injection position ($Z < Z_F$) is termed as ‘*Stripping Section*’ or *SS* (material enriched in *B* is
120 either injected-in or pushed-out of the end of this section, $Z = 0$) and the section to the right of the feed
121 injection position ($Z > Z_F$) is termed as ‘*Rectifying Section*’ or *RS* (material enriched in *A* is either injected-
122 in or pushed-out of the end of this section, $Z = 1$).

123
124 Binary feed gas mixture with flowrate Q_F and composition y_F (mole fraction of *A* in feed gas mixture) is
125 supplied to *Bed – I* which is maintained at constant pressure equal to P_L during the feed step (*FE*);
126 simultaneously, *Bed – II* undergoes purge step (*PU*) while being maintained at constant P_H . Material
127 enriched in *A* is pushed out of the *Bed – I* undergoing *FE* at flowrate $Q_{H,out}$. It is compressed and portion
128 of $Q_{H,out}$ is taken out of the system as Heavy Product (*HP*) with flowrate Q_{HP} and the remaining portion is
129 supplied as Heavy Reflux (*HR*) to *Bed – II* undergoing *PU* at flowrate Q_{HR} . Material enriched in *B* exits
130 from *Bed – II* at flowrate $Q_{L,out}$, a portion of which is taken out of the system as Light Product (*LP*) at
131 flowrate Q_{LP} and the remaining quantity is supplied as Light Reflux (*LR*) at flowrate Q_{LR} to *Bed – I*. The
132 average mole fractions of *A* in light and heavy product are termed as \bar{y}_{LP} and \bar{y}_{HP} respectively. At the end of
133 the simultaneous *FE/PU*, the bed pressures need to be interchanged. This is accomplished by transferring
134 gas enriched in *A* from the rectifying end ($Z = 1$) of *Bed – II* (after compression) to the $Z = 1$ end of
135 *Bed – I*. Hence *Bed – II* undergoes blowdown step (*BD*) during which its pressure decreases from P_H to
136 P_L and simultaneously *Bed – I* undergoes pressurization step (*PR*) during which its pressure increases from
137 P_L to P_H . The total amounts (in kmol) of gas extracted from *Bed – II* and transferred to *Bed – I* during
138 these steps are indicated as N_{BD} and N_{PR} , respectively.

140 3. Model description and solution approach

141

142 As anticipated, the model aimed to the detailed simulation of the four-step DR-PL-A process described in
143 section 2 was developed in the frame of the commercial software Aspen Adsim[®]. The process cycle is

144 simulated by applying the so-called ‘Single Bed Approach’, thus following the behavior of one single
145 column out of the two beds working in parallel during the actual process. In order to reduce the
146 computational effort while ensuring reliable simulation results, the following assumptions are considered in
147 this work:

- 148 1. isothermal operation;
- 149 2. ideal gas behavior;
- 150 3. ideal plug-flow behavior without axial mixing;
- 151 4. lumped mass transport rates (constant mass transfer coefficient times a driving force in terms of
152 concentrations in adsorbed phase);
- 153 5. non-linear adsorption isotherms expressed as a function of partial pressures through the ‘Extended
154 Langmuir’ model;
- 155 6. pressure drop estimated via Ergun equation.

156 The model constitutive equations are fully detailed in Aspen Adsim[®] help menu; accordingly, they are
157 omitted here, with the exception of some selected terms which are summarized below (the meanings of all
158 variables and symbols used in this paper are provided in the final section Notations).

159

160 *Ideal gas behavior:*

$$161 \quad P y_i = R T c_i \quad (1)$$

162 where, P is the total pressure, y_i is the bulk gas-phase mole fraction of component i , R is the universal gas
163 constant, T is the system temperature and c_i is the bulk gas-phase concentration of component i .

164

165 *Material balances:*

166 Material flow pattern through the adsorption column is assumed to be ideal plug-flow, without axial mixing.

167 The mass balance for component i over a differential volume element, accounting for convection as well as
168 accumulation in both gas and solid phase is given by

$$169 \quad \frac{\partial(v_g c_i)}{\partial z} + \varepsilon_T \frac{\partial c_i}{\partial t} + \rho_B \frac{\partial \bar{q}_i}{\partial t} = 0 \quad (2)$$

170 where, v_g is the gas-phase superficial velocity, z is the axial co-ordinate, ε_T is the total bed voidage, t is the
 171 time coordinate, ρ_B is the adsorbent bulk density (i.e. the mass of the solid per unit volume of column) and
 172 \bar{q}_i is the particle-average concentration of species i in adsorbed phase per unit mass of solid.

173

174 *Mass transfer rate:*

175 The linear driving force (LDF) model was used to account for the resistances to mass transfer between the
 176 fluid and the porous media, given by

$$177 \frac{\partial \bar{q}_i}{\partial t} = k_{MTC_i}(q_i^* - \bar{q}_i) \quad (3)$$

178 where, q_i^* is the adsorbent loading of component i in equilibrium with the gas-phase composition and k_{MTC}
 179 is the lumped, effective mass transfer coefficient (*MTC*). This parameter is calculated by assuming that only
 180 the resistances to mass transfer in the external fluid film and in the macropores are significant:

$$181 \frac{1}{k_{MTC_i}} = \frac{r_p \bar{K}_{K,i}}{3k_{f,i}} + \frac{r_p^2 \bar{K}_{K,i}}{15\varepsilon_P D_{P,i}} \quad (4)$$

182 where r_p and ε_P are the radius and porosity of the adsorbent particle (P), respectively. $\bar{K}_{K,i}$ is the local
 183 Henry's coefficient obtained from the equilibrium isotherm as:

$$184 \bar{K}_{K,i} = RT \frac{\rho_B}{\varepsilon_i} \frac{\partial q_i^*}{\partial p_i} \quad (5)$$

185 where, ε_i is the interstitial (or external) porosity and p represents the partial pressure. The macropore
 186 diffusion coefficient (D_P) is in turn computed from the following equation:

$$187 D_{P,i}^{-1} = \tau(D_{K,i}^{-1} + D_{M,i}^{-1}) \quad (6)$$

188 where, τ is the adsorbent tortuosity factor, D_M is the constant molecular diffusion coefficient estimated by
 189 Aspen Adsim[®] properties database, and the Knudsen diffusion coefficient (D_K) is estimated using the
 190 following equation:

$$191 D_{K,i} = 97r_{P,mac} \left(\frac{T}{M_{W,i}} \right)^{0.5} \quad (7)$$

192 where $r_{P,mac}$ is the macropore radius and $M_{W,i}$ is the molecular weight of the component. The film
 193 resistance coefficient ($k_{f,i}$) is computed from the component Sherwood (Sh_i), Schmidt (Sc_i) and Reynolds
 194 (Re) number, using the following relationships:

$$Sc_i = \frac{\mu_g}{D_{M,i}\rho_g M_W} \quad (8)$$

$$Re = \frac{(2r_p M_W \rho_g v_g)}{\mu_g} \quad (9)$$

$$Sh_i = 2 + 1.1 Sc_i^{1/3} Re_i^{0.6} \quad (10)$$

$$k_{f,i} = \frac{(Sh_i D_{M,i})}{2r_p} \quad (11)$$

where, μ_g is the dynamic gas viscosity and ρ_g is the molar gas-phase density.

Momentum balance:

Pressure drop along the axial coordinate (valid for both laminar and turbulent flows) is estimated by Ergun's equation (Ergun, 1952). Pressure drop estimates depend on the flow direction of the bulk gas during different steps of the process cycle. Accordingly, the $(\partial P/\partial z)$ estimate of Eq. (12) should be considered to be negative during: Blowdown (*BD*) and Feed (*FE*); and positive during Pressurization (*PR*) and Purge (*PU*) steps.

$$\frac{\partial P}{\partial z} = \pm \left[\mu v_g \frac{0.15(1 - \varepsilon_i)^2}{(2r_p \psi)^2 \varepsilon_i^3} + 1.75 \times 10^{-3} M_W \rho_g v_g^2 \frac{(1 - \varepsilon_i)}{2r_p \psi \varepsilon_i^3} \right] \quad (12)$$

where ψ is the shape factor of the adsorbent particles.

Equilibrium isotherm:

The adsorption isotherm of the gaseous mixture is predicted from pure component isotherms by the so-called Extended Langmuir model. Namely, the adsorbed moles of component i per unit mass of adsorbent at equilibrium (q_i^*) are given by:

$$q_i^* = [(IP_{1,i})(p_i)] / \left[1 + \sum_{k=1}^2 (IP_{2,k})(p_k) \right] \quad (13)$$

219 where $IP_{1,i}$ and $IP_{2,i}$ are the Langmuir isotherm parameters for the pure component i and p represents the
220 gas partial pressure.

221

222 A qualitative representation of the considered DR-PL-A process simulation flowsheet is shown in Fig. 2. As
223 anticipated, the process cycle is simulated by applying the so-called ‘Single Bed Approach’ until the
224 achievement of cyclic steady-state conditions (*CSS*). The evolution of concentration, pressure and velocity
225 profiles in the simulated column is followed step by step throughout the duration of the entire separation
226 cycle, whereas the interplay with the second bed is accounted for through the introduction of ‘interaction
227 modules’ (*I*): this way, all the properties of the selected leaving stream are recorded and then reproduced as
228 input stream (as a first-in-first-out buffer profile) when required in the frame of the process cycle. As per the
229 description of twin-bed DR-PL-A system elucidated in section 2, both the adsorbent beds undergo the same
230 steps in a given cycle and are assumed to be identical (i.e., with same length, feed position, cross-sectional
231 area (A_{bed}), kind and amount of adsorbent, particle size, interstitial porosity (ε_i), particle porosity (ε_p),
232 solid (ρ_s) and bulk densities (ρ_B)). The selected computational scheme, based on the simulation of the
233 complete cycle of one single bed, significantly improves the computational speed since it reduces the total
234 number of equations to be solved for achieving the final results. Moreover, the ‘Single Bed Approach’
235 retains the accuracy of multiple bed simulation provided the transfer of information through the previously
236 mentioned interaction modules is accurate enough.

237

238 In accordance with the twin-bed DR-PL-A system depicted in Fig. 1, the simulated single column is
239 sketched in Fig. 2 as a series of two fixed-beds, Stripping Section (*SS*) and Rectifying Section (*RS*). The
240 respective lengths of these two sections define the position of the feed injection (Z_F) with respect to the
241 whole column. Since a single gas stream can enter or leave a fixed bed in Aspen Adsim[®], the input and
242 output streams of *RS* and *SS* are connected to the so-called ‘gas tank void’ modules. These tanks are set to
243 be of negligible volume and serve as connectors for multiple streams entering and exiting the beds.
244 Specifically, the tank connected to the *SS* end ($Z = 0$) is termed as ‘*SS* Tank’, the one connected to the *RS*
245 end ($Z = 1$) ‘*RS* Tank’ and the one in-between the two beds ‘Feed Tank’.

246

247 The flowrates (Q) of light reflux (LR), feed (F), heavy reflux (HR), blowdown (BD), heavy product (HP)
248 and light product (LP) are respectively regulated through the valves V_{LR} , V_F , V_{HR} , V_{BD} , V_{HP} and V_{LP} . The
249 extent of opening (or simply the position) of all of these valves can be set as required by the specific step
250 scenario. When open, all these valves ensure a specified flowrate with the exception of V_{LP} and V_{HR} , whose
251 flowrate is governed by PID controllers. The controller that governs Q_{LP} via V_{LP} is termed as light product
252 flowrate controller ($LPFC$) and the one that governs Q_{HR} via V_{HR} is termed as heavy reflux flowrate
253 controller ($HRFC$). Since light product is released during PU and heavy reflux is released during FE , the P_H
254 and P_L set points (input quantities) are respectively maintained by $LPFC$ and $HRFC$ by flowrate
255 manipulation of the specific streams.

256

257 The solution sequence of the module is governed by the ‘Cycle Organizer’ which allows to create the steps
258 and to manage all the variables and the working conditions that occur in the cyclic process. Inside such
259 organizer it is possible to create any number of steps, define the step termination event (like the duration of a
260 constant pressure step or the end pressure of a variable pressure step) and manipulate the flowsheet variables
261 for a given step (such as the closing of the valves, opening of the valves at a specific flowrate, etc.).
262 Furthermore, the ‘Cycle Organizer’ allows handling all the interaction modules which appear in the
263 flowsheet making possible the simulation of the entire cycle.

264

265 **4. Model adaptation to experimental apparatus and procedures**

266

267 As already mentioned, model validation is carried out by simulating the experimental data by McIntyre et al.
268 (2010). Namely, 19 experiments in DR-PL-A configuration for the recovery and enrichment of dilute ethane
269 (C_2H_6) in nitrogen (N_2) using MeadWestvaco BAX-1500[®] activated carbon have been considered. The pure
270 component adsorption isotherms of C_2H_6 and N_2 on the selected activated carbon were provided by
271 McIntyre et al. (2002). In the operating range of the experiments, C_2H_6 is adsorbed more than N_2 :

272 accordingly, C_2H_6 is henceforth referred to as heavy component (A) and y_F refers to its mole fraction in the
273 binary feed gas mixture.

274

275 Model parameter values, kept constant while simulating all the runs, are listed in Table 1, whereas the values
276 of the operating parameters that varied during the simulation of individual runs are listed in Table 2. To
277 maintain consistency, the simulated Run numbers used in this work are identical to the ones used by
278 McIntyre et al. (2010); hence, Table 2 in this work is identical to Table 2 in the paper by McIntyre et al.
279 (2010).

280

281 About the values in Table 1, length and diameter of each column (SS and RS) were set according to the
282 description of the experimental apparatus provided by McIntyre et al., 2010. The amount of adsorbent
283 (M_{ads}) in each of the columns was also stated, thus allowing the evaluation of the adsorbent bed volume
284 (V_{bed}) and bulk density (ρ_B). The values of average length and radius of adsorbent pellets mentioned by
285 Tolles et al. (2009) were utilized to calculate the radius (r_p) of equivalent spherical particles ($\psi = 1$).
286 Mesoporous as well as macroporous volume ($V_{meso+macro}$) along with maximum macropore radius
287 ($r_{p,mac}$) of the adsorbent, mentioned by Wilhelm et al. (2005) were assumed. The average tortuosity (τ)
288 value mentioned by Ruthven et al. (1994) for activated carbons was utilized in the model calculations. The
289 interstitial (or external) porosity (ε_i) provided by Hou et al. (2010) was assumed. This estimated value of ε_i
290 also matches the overall bulk porosity range for densely packed beds with cylindrical pellets, mentioned by
291 Zhang et al., 2006. This information was utilized to estimate the value of the adsorbent particle porosity (ε_p)
292 as follows:

293

$$294 \quad \varepsilon_p = \frac{M_{ads}(V_{meso+macro})}{V_{bed}(1 - \varepsilon_i)} = \frac{\rho_B(V_{meso+macro})}{(1 - \varepsilon_i)} \quad (14)$$

295

296 The experimental apparatus was using a pump for pressurization and depressurization. Instead of using a
297 pump module in the simulation (which parameters were not provided in the corresponding paper), the values

298 of all the molar flowrates were input for any given step, and the material flow direction was governed by
299 setting the following information in the cycle organizer:

- 300 (i) pressure values of the corresponding module at the entry/exit point of the streams;
- 301 (ii) close (✘) or open (✔) valves at designated and/ or controlled flowrates and;
- 302 (iii) whether an interaction module is nonfunctional (✘) or recording information (ℝ) or feeding back the
303 recorded information (ℱ).

304 The specific implementation of these input values for each experimental run will be detailed in the following
305 section (cf. Table 3).

306

307 The P_H and P_L set points (Table 1) used in *LPFC* and *HRFC*, respectively, have been kept constant for all
308 simulations and identical to the approximate values provided by McIntyre et al. (2010) for all experimental
309 runs. In the same paper, only the durations of *PR* and *BD* steps (10 seconds for all the runs) were provided
310 without the mention of the total amount of material pushed out of one column during *BD* and then supplied
311 to the second column during *PR*, thus preventing the evaluation of the actual profile (or time evolution) of
312 flowrate. However, the complete pressure profile of both the columns was provided for the base case (Run #
313 1) and it was mentioned that the pressure profile for all other runs is similar. Based on this analogy, the
314 average pressure at the end of the *BD* step of both the columns was assumed equal to the value $P_{BD,end}$ in
315 Table 1. Therefore, a specific value of the depressurization flowrate Q_{BD} being pushed out of the column
316 during step *BD* was evaluated in order to reduce the column pressure down to $P_{BD,end}$ at the end of the step.
317 The resulting value (cf. Table 1) was recorded in *RS* Tank (event driven step) and the amount of material
318 accumulated during *BD* was then supplied back to the column during *PR*. This arrangement slightly
319 increased the cycle time but provided a reasonable reproduction of the experimental pressure profiles.

320

321 According to McIntyre et al. (2010), the first experimental run (Run # 1, the base case) was initiated with the
322 columns filled with pure N_2 . After *CSS* was achieved, a process condition was changed, and a new run
323 commenced from the periodic state of the previous run; this same approach was applied to the complete
324 series of experiments. Accordingly, the same procedure was applied in modeling: the simulation of Run # 1
325 was initiated using pure N_2 in all the bed, tank and interaction modules, while all simulation runs thereafter

326 were initiated using the concentration profiles predicted for the previous run at *CSS* following the same
327 order of the 19 experimental runs reported by McIntyre et al. (2010).

328
329 As mentioned in the original paper, temperature oscillations of around 25 K were observed for all the
330 reported experimental runs. This temperature variation is lower than those typical of conventional PSA
331 systems and is explained by the limited pressure ratio P_H/P_L along with the low concentration of the most
332 adsorbable component in the feed, less than 1.4% on molar basis. Accordingly, the simulations were carried
333 out at constant temperature equal to 298 K, using model parameter values estimated at the same temperature.

334
335 All 19 simulations were carried out for batch intervals of 1,500 cycles. In order to assess the actual
336 achievement of *CSS* conditions, once the time invariance of the cycle-average compositions was reached the
337 component material balances were checked after every batch and steady-state conditions were considered to
338 be established when the resulting error was less than 5% for C_2H_6 and less than 2% for N_2 , similar to the
339 criterion mentioned by McIntyre et al. (2010). At *CSS*, the average N_2 material balance error for all the runs
340 was about 0.03% whereas the same assessment for C_2H_6 was about 2.5%. The larger error in C_2H_6 material
341 balance was due to the very low concentration of C_2H_6 in the feed stream. If *CSS* was not established, a
342 further batch of 1,500 simulation cycles was carried out and the checking procedure was repeated. Usually
343 *CSS* was achieved sooner than anticipated in most of the simulations (say, around 800 cycles); however, we
344 decided to run the same number of cycles as reported by the experimenters to maintain consistency and also
345 to check the *CSS* robustness. Depending on the cycle duration, the overall computational time ranged from 3
346 days to 2 weeks on Intel® Xenon® X5650, 2.66GHz, 64-bit processor.

348 **5. Base case simulation**

349
350 This section provides a detailed description of the approach used for simulating Run # 1 (base case); the
351 same approach was used for simulating all other runs. As anticipated, all the parameter values stated in
352 Table 1 were used. A four step DR-PL-A cycle was formulated in the cycle organizer (*BD*, *FE*, *PR* and *PU*).
353 The simulation of *BD* step was executed until the desired pressure value $P_{BD,end}$ was recorded in *RS* Tank

354 (event driven step), whereas FE was executed for the time stated in Table 2 (time driven step). Of course,
355 the execution times of the parallel steps PR and PU were equivalent to that of BD and FE , respectively.
356 Once the execution methodology of these four steps was defined, the *modus operandi* of the various valves
357 and interaction modules during each step in the cycle organizer was defined as shown in Table 3.

358

359 The cycle starts with blowdown (BD) step and the column is depressurized from P_H to $P_{BD,end}$. The gas
360 mixture is withdrawn from $Z = 1$ end of the column at Q_{BD} (cf. Table 1) via V_{BD} and the evolutions of
361 concentration, pressure and flow profiles are recorded in the pressure interaction module (PI). Then, the
362 feed step (FE) is simulated for the time t_{FE} mentioned in Table 2. Light reflux material is fed to the column
363 at constant flowrate Q_{LR} (Table 2) via the light reflux interaction module (LRI); simultaneously, the binary
364 feed gas mixture with composition y_F (Table 2) is fed to the column at fixed flowrate Q_F (Table 2) via V_F .
365 Material withdrawn from $Z = 1$ end of the column during this step is divided into two streams and gets
366 collected by: (i) heavy product module at constant flowrate (Q_{HP} in Table 2) via V_{HP} and (ii) heavy reflux
367 interaction module (HRI) at flowrate Q_{HR} controlled by $HRFC$ via V_{HR} in order to maintain the outlet
368 pressure equal to the set point P_L (Table 1). Note that the material withdrawn from $Z = 1$ end of the column
369 during this step is flowing to the Heavy Product module as first priority (by default simulation settings),
370 while the remaining material is sent to HRI . This way, the heavy product flowrate (set equivalent to the
371 corresponding experimental value) is consistently maintained in the simulation. Next, the pressurization step
372 (PR) is simulated. Pressure interaction module (PI) feeds back a stream with the profiles recorded during
373 BD at the flowrate Q_{PR} (equivalent to Q_{BD}).

374

375 At the high pressures resulting from PR , the purge step (PU) is executed. The heavy reflux (Q_{HR}) is pushed
376 by HRI (recorded during FE) inside the column. Material withdrawn from $Z = 0$ end of the column during
377 this step is divided into two streams and gets collected by: (i) Light product module at flowrate Q_{LP}
378 controlled by $LPFC$ via V_{LP} in order to maintain the pressure at the set point P_H (Table 1) and (ii) Light
379 reflux interaction module (LRI) at constant flowrate (Q_{LR} in Table 2) via V_{LR} . Similar to step FE , in order to
380 assure light reflux flowrate equal to the experimental one, the material withdrawn from $Z = 0$ end of the

381 column during this step is flowing to *LRI* with first priority (by default simulation settings) and then the
382 remaining material is sent to the light product module.

383

384 Once established *CSS* conditions, model predictions were compared with reported experimental results in
385 terms of: (i) average C_2H_6 mole fraction in heavy product (\bar{y}_{HP}); (ii) average mole fraction of N_2 in light
386 product ($1 - \bar{y}_{LP}$); (iii) instantaneous heavy product composition (y_{HP} : C_2H_6 mole fraction in heavy
387 product) profiles during *FE* step and; (iii) instantaneous column composition profiles (in terms of C_2H_6
388 mole fraction: $y_{C_2H_6}$) at the end of *PU* step.

389

390 Pressure profile and flow pattern of various streams during one cycle at *CSS* are depicted in Fig. 3a) and 3b),
391 respectively. The model estimated blowdown and pressurization times are normalized with respect to the
392 one reported by McIntyre et al. (2010). The cycle time (t_{cycle}) is identical for both figures: 3a) and 3b).
393 Since negligible pressure drop was observed across the simulated column length, the pressure profile of *SS*
394 Tank is representative of the entire column. For comparative analysis, actual experimental pressure profile
395 of both the columns (reported in their Fig. 3 by McIntyre et al., 2010; measured near $Z = 0$ end of each
396 column) is also plotted in the same figure. Note that the pressure profiles during a complete cycle (*BD*, *FE*,
397 *PR* and *PU*) occurring in *each* of the experimental columns are plotted in Fig. 3a). Discrepancy between the
398 experimental and simulation pressure profile can be ascribed to the simplistic model assumptions coupled
399 with the fact that the pump module was not utilized in the simulation. In general, the experimental and
400 calculated pressure profiles are comparable and the close achievement of P_L and P_H set points (Table 1) is
401 apparent from Fig. 3a). As explained above, Q_{HR} is the variable manipulated by *HRFC* (via V_{HR}) to ensure
402 pressure equal to P_L during the feed step: since the step starts at $P_{BD,end}$ (which is higher than the set point,
403 $P_{BD,end} > P_L$), valve V_{HR} is fully open and the corresponding flow (heavy reflux) reaches its maximum limit
404 (the bias of *HRFC*) and remains equal to such value until the set point pressure is established at *RS* Tank.
405 Conversely, *PU* starts at a pressure value lower than the set point P_H : therefore, *LPFC* restricts Q_{LP} by
406 closing the valve V_{LP} until the set point pressure is established at *SS* Tank and then manipulates the same

407 flow to maintain the pressure constant and equal to P_H . Note that this second interval corresponds to non-
408 zero and non-constant Q_{LP} in Fig. 3b).

409
410 While simulating all the available experimental runs, a reasonable match between simulation and
411 experimental results at *CSS* could be obtained when a pressure profile very similar to the one shown in Fig.
412 3a) was qualitatively established. As explained above, the pressure profile was mainly determined by fine-
413 tuning both the controllers, *HRFC* and *LPFC*. Moreover, after establishing *CSS* conditions, the consistency
414 between experimental and simulation results was checked by mass balance calculations, as elaborated in the
415 next section.

417 6. Results and discussion

418
419 In this section, a comparative analysis between simulation and reported experimental results at *CSS* is
420 presented for all the available experimental conditions. Namely, the effect on process performance of the
421 following process parameters is explored:

- 422
- 423 a) light reflux flowrate (Q_{LR}): Run # 1 to 6;
 - 424 b) feed or purge step time ($t_{FE/PU}$): Run # 7, 8, 1 and 9 to 12;
 - 425 c) heavy product flowrate (Q_{HP}): Run # 13, 1, 14 and 15;
 - 426 d) ethane mole fraction (y_F) in binary feed gas mixture: Run # 16, 17, 1, 18 and 19.
- 427

428 In order to maintain consistency, the maximum and minimum limits of both the axes for each case are held
429 constant for ease of comparison. In all cases, 1,500 to 3,000 cycles were simulated to achieve *CSS*: besides
430 being identical to the typical experimental values, the cyclic steady state nature of the simulation results was
431 demonstrated by fulfilling the following constraints:

- 432
- 433 (i) material balance error of the component, less than 5% for C_2H_6 and 0.1% for N_2 (values respectively
434 identical and much more stringent than those reported by the experimenters);

- 435 (ii) achievement of both experimental set point values of pressure, P_H and P_L ;
- 436 (iii) pressure profile and flow pattern of various streams qualitatively similar to those shown in Fig. 3 for the
- 437 base case and;
- 438 (iv) bed composition profile at the end of PU and the composition profile of heavy product during FE for
- 439 each run were very similar to the ones reported by McIntyre et al., 2010.

440

441 The experimental flowrates Q_F , Q_{HP} , Q_{LR} (cf. Table 2) and the pressure values P_H , P_L (cf. Table 1) were set

442 and/or maintained in all simulations. On the other hand, the flowrate Q_{LP} was not fixed in the experimental

443 setup, but controlled in the model simulations by $LPFC$ via V_{LP} in order to maintain the set point pressure

444 P_H . Regardless of this dissimilarity, a very good match was found between the light product flowrate (Q_{LP})

445 recorded in the experiments and predicted by the model in all examined cases. This agreement represents a

446 meaningful model validation in terms of the remaining flowrate Q_{HR} and of the internal recycle loop.

447 Simulated values of heavy reflux flowrates (Q_{HR}) as a function of operating parameters are also presented.

448 Since the specific values of Q_{HR} were not mentioned by McIntyre et al. (2010), simulated data is not

449 compared with experimental results for this specific parameter. Moreover, for enhanced understanding of

450 the process behavior, column composition profiles (in terms of C_2H_6 mole fraction: $y_{C_2H_6}$) at the end of each

451 process cycle step are also provided for every run at CSS .

452

453 **Effect of the light reflux flowrate (Q_{LR}): Run # 1 to 6**

454

455 Following the procedure described in the previous section, Runs # 1 to 6 were simulated to assess the effect

456 of changing the light reflux flowrate (Q_{LR}) on process performance when keeping constant all the remaining

457 conditions (cf. Table 2). The values of the operating parameters used while simulating these runs are shown

458 in the same table, whereas the simulation results are plotted and compared with the experimental ones in

459 Fig. 4 and 5.

460

461 Q_F and Q_{HP} were kept practically constant as Q_{LR} was increased from Run # 1 to 6. Since practically

462 constant Q_{LP} was also observed in these runs, the only obvious flowrate being manipulated by the system is

463 Q_{HR} , as depicted in Fig. 4c) and also indicated by McIntyre et al. (2010). Therefore, Q_{HR} increased with
464 increase in Q_{LR} that, in turn, caused an increase in the flowrates of the internal recycle loop.

465
466 In agreement with the remarks reported by McIntyre et al. (2010), two competing effects come into play in
467 this case: (i) improved bed regeneration during FE due to the increase in Q_{LR} (manifested by the column
468 composition profiles at the beginning and end of FE in Fig. 6) and (ii) deeper C_2H_6 penetration into the
469 column due to the increase in Q_{HR} (evident from Fig. 5c and 5d). At lower Q_{LR} values, the first effect is
470 dominant, thus causing an increase in \bar{y}_{HP} (Fig. 4a) and $(1 - \bar{y}_{LP})$ (Fig. 4b). On the contrary, C_2H_6
471 penetration into the column (Fig. 5c and 5d) becomes dominant at higher Q_{LR} values, resulting in decline of
472 \bar{y}_{HP} . The initial increment in Q_{LR} results in an increase in light product purity $(1 - \bar{y}_{LP})$, however, such
473 parameter practically remains unaffected by further increment in Q_{LR} values. These practically constant light
474 product purities imply that the deeper C_2H_6 penetration into the column due to the increase in Q_{HR} was not
475 enough to significantly affect the column composition profiles at $Z = 0$ end (depicted in Fig. 6). It is also
476 evident from Fig. 6 that the increase in internal recycle loop (due to increase in both Q_{LR} and Q_{HR}) from Run
477 # 1 to 6 increases the volume of adsorbent involved in the actual adsorption and desorption process. This
478 combination of effects, result in the trends depicted in Fig. 4a), 4b) and, 5, in good agreement with the
479 experimental observations.

480
481 The dip in some of the column composition profiles (in terms of C_2H_6 mole fraction: $y_{C_2H_6}$) at the *end* of
482 high pressure PU step *beyond* the feed position ($Z = 0.5$) reported by the experimenters (depicted in Fig. 5c
483 and some other runs, to be investigated later on) are noteworthy. McIntyre et al. (2010) reported that such
484 perturbations result from the mixing of dilute ethane feed stream with an ethane rich gas inside the column
485 during the low pressure feed step (FE). The simulation results also show such perturbations at the *end* of FE
486 (as depicted in Fig. 6) at the feed position. As the cycle proceeds, the experimenters report that such
487 perturbations appeared in the column composition profiles (for some of the runs) at the *end* of the PU step
488 *beyond* the feed position ($Z = 0.5$), while in the simulations such perturbations were only observed
489 specifically at the feed position. The model behavior seems more physically consistent: considering that
490 during PR and PU steps the flow is reversed (that is, from $Z = 1$ towards $Z = 0$), the perturbation is expected

491 to move towards $Z = 0$, that is, it should either appear also *before* and not only *beyond* the feed position
492 ($Z = 0.5$) or be reabsorbed.

493 494 **Effect of the feed/purge step duration ($t_{FE/PU}$): Run # 7, 8, 1 and 9 to 12**

495
496 Simulations for Run # 7, 8, 1 and 9 to 12 were performed to assess the effect of increasing feed/purge step
497 duration ($t_{FE/PU}$) on process performance. Again, all the remaining model parameters remained practically
498 constant (cf. Table 2), whereas the simulation results are compared to the experimental ones in Fig. 7 and 8.
499 Column composition profiles at the end of each process cycle step are provided for every run at CSS in Fig.
500 9.

501
502 According to McIntyre et al. (2010), the increase in $t_{FE/PU}$ had minimal effect on C_2H_6 uptake by the
503 adsorbent due to the very low concentration of C_2H_6 in the feed. However, heavy and light recycle flowrates
504 were allowed to persist longer with increasing duration of the feed/purge step (evident in Fig. 9 by the
505 increases in the volume of adsorbent involved in the actual adsorption and desorption process with increase
506 in $t_{FE/PU}$): this might explain the initial surge in \bar{y}_{HP} and $(1 - \bar{y}_{LP})$ with increase in $t_{FE/PU}$. After this initial
507 increase, the two competitive effects of bed regeneration (due to increase in light recycle time: evident from
508 the column composition profiles at the beginning and end of *FE* depicted in Fig. 9) and C_2H_6 penetration in
509 the column (due to increase in heavy recycle time: evident from Fig. 8c and 8d and the column composition
510 profiles at the beginning and end of *PU* depicted in Fig. 9) cancel each other, resulting in the stable trends
511 observed in Fig. 7a) and 7b).

512
513 Notice that the increments in $t_{FE/PU}$ resulted in decline of Q_{HR} (Fig. 7c). In the original paper, it was
514 mentioned that the pressure profile for all the runs was similar to the one provided for the base case (Run #
515 1); the simulation results confirmed this expectation. Table 2 shows that all operating parameters remained
516 nearly constant as $t_{FE/PU}$ increased from Run # 7, 8, 1 and 9 to 12. Therefore, when increasing $t_{FE/PU}$ in
517 these runs: the system managed to attain and maintain the designated pressure set points by decreasing Q_{HR} .

518

519 The unusual column composition profiles for Run # 7 (depicted in Fig. 9a) are noteworthy, since only in this
520 run the heavy component concentration in $Z = 1$ at the *FE* start is lower than that at the *FE* end. In this run,
521 the short duration of low pressure *FE* (desorption step: since the pressure reduces from $P_{BD,end}$ to P_L , cf.
522 Table 1) did not allow for enough heavy product withdrawal from the bed. This phenomenon can be further
523 verified from the instantaneous heavy product composition profile during *FE* (depicted in Fig. 8a and 8b).
524 On the other hand, the short duration of the high pressure *PU* (adsorption step, where the pressure still
525 increases, cf. Fig. 3) was not enough to adsorb the incoming ethane rich heavy reflux. These combined
526 effects led to the unusual column composition profiles for Run # 7.

527

528 **Effect of the heavy product flowrate (Q_{HP}): Run # 13, 1, 14 and 15**

529

530 Simulations for Run # 13, 1, 14 and 15 were performed to assess the effect of increasing the heavy product
531 flowrate (Q_{HP}) on process performance. The simulation results are plotted and compared with the
532 experimental ones in Fig. 10 and 11 whereas, column composition profiles at the end of each process cycle
533 step are depicted for every run at *CSS* in Fig. 12. In these runs, Q_{LR} holds identical values and the process is
534 always able to attain the same P_H and P_L set points (Table 1). Since other operating parameters practically
535 remained constant during these runs (Table 2), the decline of Q_{HR} with increase in Q_{HP} is apparent from Fig.
536 10c. This Q_{HR} trend is in agreement with the experimental observations leading to reduced C_2H_6 penetration
537 in the column (Fig. 11c and 11d) that, in turn, caused declining \bar{y}_{HP} trend (Fig. 10a) and rising $(1 - \bar{y}_{LP})$
538 trend (Fig. 10b). Reduced Q_{HR} also lowers the volume of adsorbent involved in the major adsorption and
539 desorption activity (as depicted in Fig. 12). The experimental and predicted heavy product composition
540 profile trends (depicted in Fig. 11a and 11b) are also in agreement with one another.

541

542 **Effect of ethane mole fraction (y_F) in the feed mixture: Run # 16, 17, 1, 18 and 19**

543

544 Simulations for Run # 16, 17, 1, 18 and 19 were performed to assess the effect of change in ethane mole
545 fraction (y_F) in the binary feed gas mixture on process performance. The simulation results are compared

546 with those reported by McIntyre et al. (2010) in Fig. 13 and 14. Column composition profiles at the end of
547 each process cycle step are provided for every run at *CSS* in Fig. 15. The parameters remaining practically
548 constant in these runs are: Q_F , Q_{HP} , Q_{LR} and $t_{FE/PU}$ (Table 2). Increase in y_F resulted in an increased C_2H_6
549 (heavy component or strongly adsorbed species) uptake by the adsorbent. In this scenario, the system
550 managed to attain the same P_L set point (Table 1) by increasing Q_{HR} (Fig. 13c). This combined effect of
551 increased C_2H_6 uptake and deeper C_2H_6 penetration in the column (due to increase in Q_{HR} : evident from Fig.
552 14c and 14d) lead to an increase in heavy product purity (Fig. 13a) and decrease in light product purity (Fig.
553 13b). The experimental and predicted instantaneous heavy product composition profile trends (depicted in
554 Fig. 14a and 14b) are also in agreement with one another. It can be observed that the volume of adsorbent
555 involved in the actual adsorption and desorption process increases with increase in y_F .

556 557 **Effect of dimensionless feed position (Z_F)**

558
559 In all the 19 runs reported by McIntyre et al. (2010), the dimensionless feed position was fixed ($Z_F = 0.5$).
560 Once the model was validated by comparative analysis of simulation and experimental results at *CSS*,
561 additional simulations were performed to assess the effect of change in the dimensionless feed position (Z_F)
562 on process performance. The results of these simulations are presented in Fig. 16, 17 and 18. Note that,
563 except Z_F , the values of all other operating parameters were identical to those of the base case (Run #1).
564 Since the total bed length (L_{bed}) was also held constant, by changing the dimensionless feed position we
565 actually change the lengths of the column sections. Such changes impact on the process performance in non-
566 trivial way, as explained below.

567
568 Notably, process performance remains unaffected for $Z_F \leq 0.5$. This can be clearly understood by
569 examining the base case column composition profile reported by McIntyre et al. (2010) at the end of high
570 pressure purge (*PU*) step: it is nearly equal to the feed gas composition for $Z \leq 0.4$. According to the DR-
571 PL-A cycle description mentioned in section 2, after *PU* the bed is depressurized (*BD*) by removing gas
572 from the rectifying end ($Z = 1$), and then the feed step (*FE*) starts. The base case simulation revealed that
573 the constant composition plateau at y_F moves further right (towards $Z = 1$ end) during *BD* and extends till

574 $Z \cong 0.5$ at the start of FE (Fig. 18c). Since ethane concentration in the feed and column do not differ
575 significantly till this dimensionless position in the column, the process performance remains unaffected for
576 $Z_F \leq 0.5$. This can be clearly understood by examining the instantaneous heavy product composition
577 profiles (Fig. 17a) and column composition profiles (Fig. 17b and 18a, 18b, 18c and 18d) for $Z_F \leq 0.5$.

578
579 The picture changes when Z_F value beyond 0.5 is considered: N_2 concentration in the rectifying section
580 (RS) increases (as evident from Fig. 17b, and 18e). Nitrogen, being the light component, desorbs much
581 quickly than ethane: then, the time required by the system to attain the low pressure (P_L) set point (during
582 FE) reduces. Once P_L is achieved by the system, $HRFC$ closes V_{HR} (thereby ceasing Q_{HR}). Hence, although
583 Q_F , Q_{HP} and Q_{LR} (equivalent to Run # 1) are constant, the decrease in Q_{HR} (Fig. 16c) hampers the internal
584 recycle loop for Z_F value beyond 0.5. This causes the decline of \bar{y}_{HP} and $(1 - \bar{y}_{LP})$ depicted in Fig. 16a)
585 and 16b), respectively.

587 7. Conclusions

588
589 A model developed in the frame of the commercial software Aspen Adsim[®] for the detailed simulation of
590 dual-reflux pressure swing adsorption process (DR-PSA) is presented. Model intricacies needed for the
591 detailed simulation of DR-PSA process are also elaborated. Even though the model can be readily applied
592 for simulating different DR-PSA configurations, we focused exclusively on DR-PL-A configuration (feed to
593 P_L and pressure swing with A). Simulations were performed after model adaptation to the experimental
594 apparatus and procedures reported by McIntyre et al. (2010) for recovery and enrichment of dilute ethane
595 from nitrogen. Detailed description of the approach used for simulating a reference case (the so-called base
596 case, Run # 1) is provided.

597
598 In accordance with the reported experimental modus operandi, 19 simulations were performed to analyze the
599 effects of different operating parameters on process performance: (i) light reflux flowrate (Q_{LR}); (ii) feed or
600 purge step time ($t_{FE/PU}$); (iii) heavy product flowrate (Q_{HP}) and (iv) ethane mole fraction (y_F) in the feed
601 mixture. At CSS , model predictions were found to be in good agreement with reported experimental results

602 in terms of: (i) average C_2H_6 mole fraction in heavy product (\bar{y}_{HP}) and; (ii) average nitrogen mole fraction
603 in light product ($1 - \bar{y}_{LP}$). Pressure profile, C_2H_6 mole fraction profile in the heavy product during *FE* and
604 C_2H_6 mole fraction profile in the column at the end of *PU* were also found to be qualitatively similar to the
605 experimental ones. For enhanced understanding of the process behavior, column composition profiles at the
606 end of each process cycle step are also provided for every run at *CSS*. Finally, since the feed position was
607 fixed in all the 19 runs reported by McIntyre et al. (2010), additional simulations were performed to assess
608 the effect of change in feed position on process performance.

609
610
611 Overall, the model exhibited reliability and versatility (it can be applied to different unit arrangements as
612 well as DR-PSA process configurations) thereby serving as an effective tool for process design, cost
613 diminution of laboratory and/or plant trails, and enhanced process understanding.

A	strongly adsorbed species, heavy product/ component
A_{bed}	cross-sectional area of the adsorption column, m^2
B	weakly adsorbed species, light product/ component
BD	Blowdown step
$Bed - I$	Adsorption column depicted in Fig.1
$Bed - II$	Adsorption column depicted in Fig.1
c_i	Bulk gas-phase concentration of component i , $kmol/m^3$
CSS	Cyclic steady state
D_{column}	Diameter of each adsorption column, m
D_K	Knudsen diffusion coefficient, m^2/s
D_M	Molecular diffusion coefficient, m^2/s
D_P	Macropore diffusion coefficient, m^2/s
DR-PH-A	dual-reflux pressure swing adsorption system with feed to high pressure bed and pressure swing using heavy gas
DR-PH-B	dual-reflux pressure swing adsorption system with feed to high pressure bed and pressure swing using light gas
DR-PL-A	dual-reflux pressure swing adsorption system with feed to low pressure bed and pressure swing using heavy gas
DR-PL-B	dual-reflux pressure swing adsorption system with feed to low pressure bed and pressure swing using light gas
DR-PSA	dual-reflux pressure swing adsorption
\mathbb{F}	represents that the interaction module is feeding back the recorded information
FE	Feed step
HP	Heavy product
HR	Heavy reflux
$HRFC$	Heavy reflux flowrate controller
HRI	Heavy reflux interaction
I	Interaction module
IP_1	First isotherm parameter, $mol/(kg.kPa)$
IP_2	Second isotherm parameter, l/kPa
$\bar{K}_{K,i}$	Local Henry's coefficient obtained from equilibrium isotherms, <i>dimensionless</i>
k_{MTC}	Lumped, effective mass transfer coefficient, l/s
k_f	Film resistance coefficient, l/s
L_{bed}	Length of each adsorption column, m
LP	Light product
$LPFC$	Light product flowrate controller
LR	Light reflux
LRI	Light reflux interaction
L_{RS}	Length of rectifying section of the column, m
L_{SS}	Length of stripping section of the column, m
M_{ads}	Adsorbent weight, kg
M_W	Molecular weight, $kg/kmol$
N	Number of moles, $kmol$
N_{BD}	Total number of moles of gas released during Blowdown step, $kmol$
N_{PR}	Total number of moles of gas fed to the column during Pressurization step, $kmol$

P	Pressure, total pressure, final pressure, kPa
p	Partial pressure, kPa
$P_{BD,end}$	Pressure at the end of blowdown step, kPa
PDE	Partial differential equation
P_H	High pressure, kPa
PI	Pressure interaction
PID	Proportional-Integral-Derivative
P_L	Low pressure, kPa
PR	Pressurization step
PSA	Pressure swing adsorption
PU	Purge step
Q	Flowrate, $sccm$
Q_{BD}	Flowrate of gas released during blowdown step, $sccm$
$Q_{BD/PR}$	Flowrate of gas released during blowdown step or flowrate of gas fed to the column during pressurization step, $sccm$
Q_F	Flowrate of feed gas, $sccm$
$Q_{H,out}$	Flowrate of gas released from the low pressure column during feed step, $sccm$
Q_{HP}	Flowrate of heavy product, $sccm$
Q_{HR}	Flowrate of heavy reflux, $sccm$
\bar{q}_i	Particle-average concentration of species i in adsorbed phase per unit mass of solid, $kmol/kg$
q_i^*	Adsorbent loading of component i which is in equilibrium with the gas-phase composition, $kmol/kg$
$Q_{L,out}$	Flowrate of gas released from the column during purge step, $sccm$
Q_{LP}	Flowrate of light product, $sccm$
Q_{LR}	Flowrate of light reflux, $sccm$
Q_{PR}	Flowrate of gas fed to the column during pressurization step, $sccm$
R	Universal gas constant, $(kPa.m^3)/(kmol.K)$
\mathbb{R}	represents that the interaction module is recording information
Re	Reynolds number, <i>dimensionless</i>
r_p	Radius of adsorbent particle, mm
$r_{p,mac}$	Radius of macropores in adsorbent particle, m
RS	Rectifying section
s	represents time in seconds
sec	represents time in seconds
$sccm$	represents flowrate in standard cubic centimeters per minute, $std\ cm^3/min$
Sh	Sherwood number, <i>dimensionless</i>
Sc	Schmid number, <i>dimensionless</i>
SS	Stripping section
T	Temperature, K
t	Time, s
t_{cycle}	Time of one DR-PS-A cycle, s
t_{FE}	Feed step duration, s
$t_{FE/PU}$	Feed or purge step duration, s
t_{PU}	Purge step duration, s
V_{bed}	Adsorbent bed volume, m^3

V_F	represents the valve through which feed material flows
v_g	Gas-phase superficial velocity, <i>m/s</i>
$V_{meso+macro}$	Mesoporous and macroporous volumes, <i>m³/kg</i>
y	Mole fraction of heavy component in any stream, final composition (in terms of mole fraction of heavy component), specific concentration value, <i>dimensionless</i>
$y_{C_2H_6}$	Mole fraction of C_2H_6 , <i>dimensionless</i>
\bar{y}	Average mole fraction of heavy component in any stream, <i>dimensionless</i>
y_i	Bulk gas-phase mole fraction of component <i>i</i> , <i>dimensionless</i>
y_F	Mole fraction of heavy component in binary feed gas mixture, <i>dimensionless</i>
\bar{y}_{HP}	Average mole fraction of heavy component in heavy product stream, <i>dimensionless</i>
y_{HP}	Mole fraction of heavy component in heavy product stream, <i>dimensionless</i>
\bar{y}_{LP}	Average mole fraction of heavy component in light product stream, <i>dimensionless</i>
$(1 - \bar{y}_{LP})$	Average mole fraction of light component in light product stream, <i>dimensionless</i>
Z	axial co-ordinate normalized with respect to column length, <i>dimensionless</i>
z	position along the length of the adsorption column, axial co-ordinate, <i>m</i>
$Z = 0$	Stripping section end of the column, light material is either injected-in or is released at this position during the process, <i>dimensionless</i>
$Z = 1$	Rectifying section end of the column, heavy material is either injected-in or is released at this position during the process, <i>dimensionless</i>
Z_F	Feed injection position along the length of the adsorption column, <i>dimensionless</i>

618

619

620 *Greek letters*

621

ε_i	Interstitial (or external) porosity of the adsorbent, <i>dimensionless</i>
ε_p	Adsorbent particle porosity, <i>dimensionless</i>
ε_T	Total bed voidage, <i>dimensionless</i>
ψ	Shape factor of adsorbent particle, <i>dimensionless</i>
ρ	Density, <i>kg/m³</i>
ρ_B	Bulk density of adsorbent, <i>kg/m³</i>
ρ_g	Molar gas-phase density, <i>kmol/m³</i>
ρ_S	Solid density of adsorbent, <i>kg/m³</i>
τ	Tortuosity of adsorbent particle, <i>dimensionless</i>
μ	Dynamic viscosity, <i>kg/(m.s)</i>

622

623

624 *Symbols*

625

✘	represents closed valve or nonfunctional interaction
✓	represents open valve at designated or controlled flowrate

626

627

628

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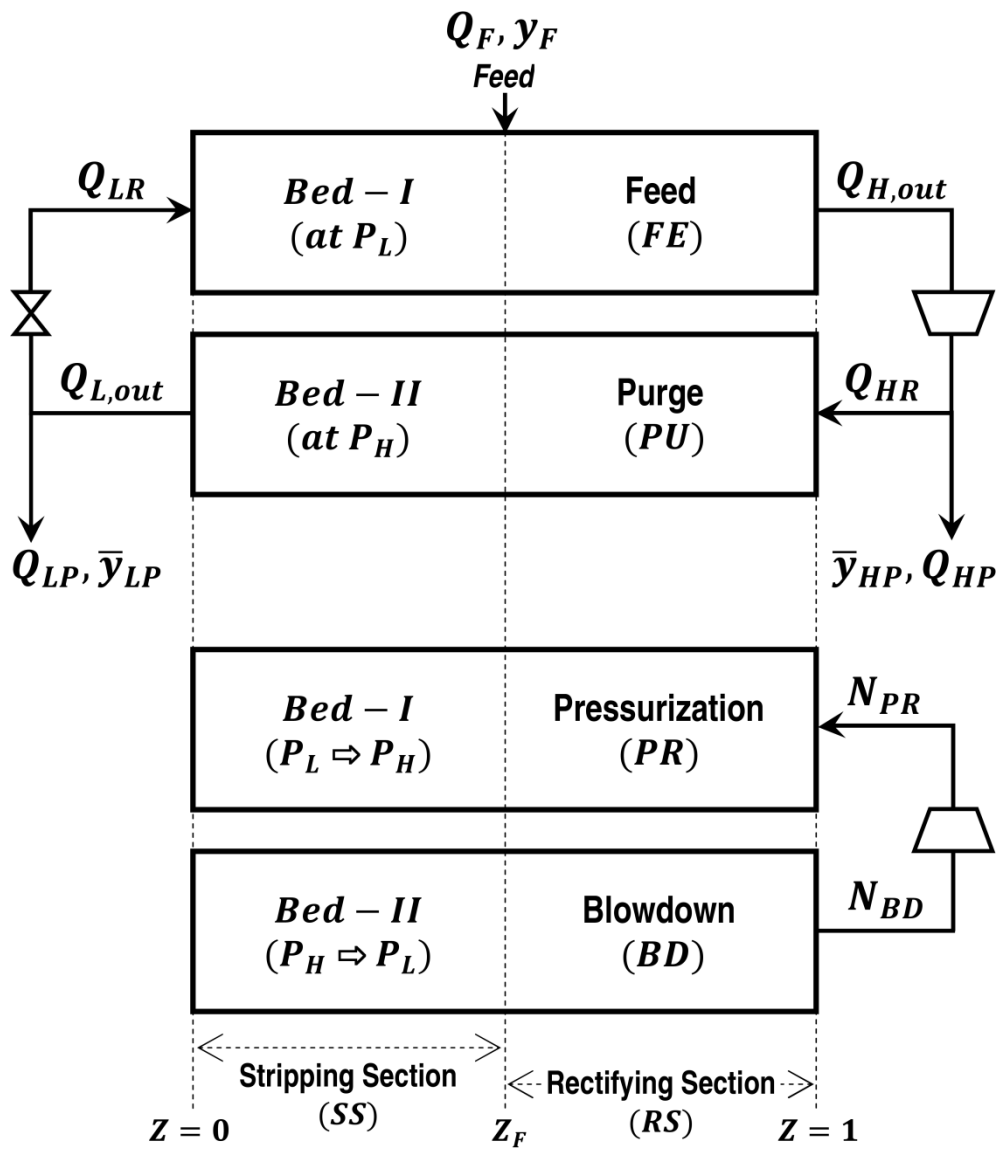


Fig. 1. DR-PL-A cycle steps and flows.

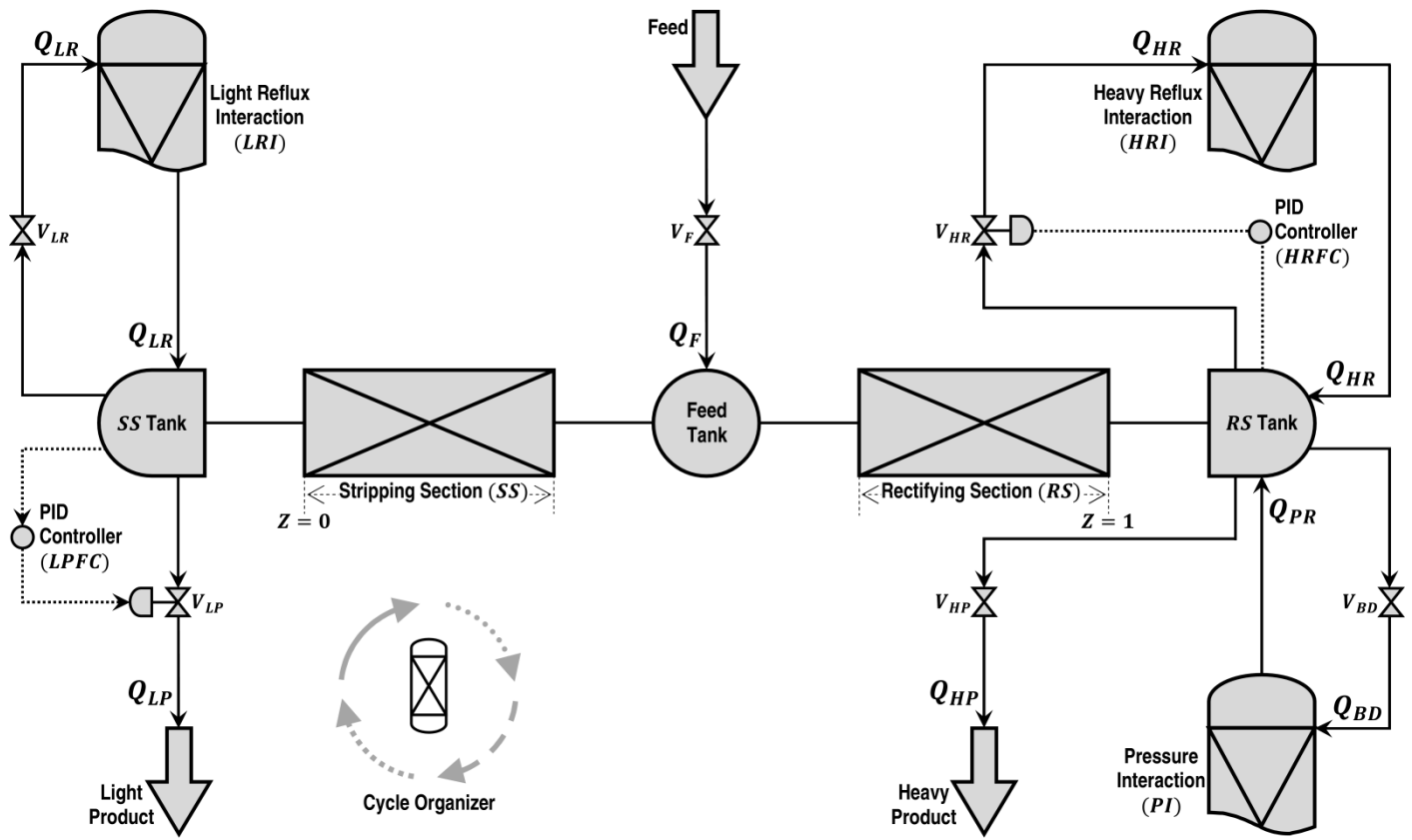
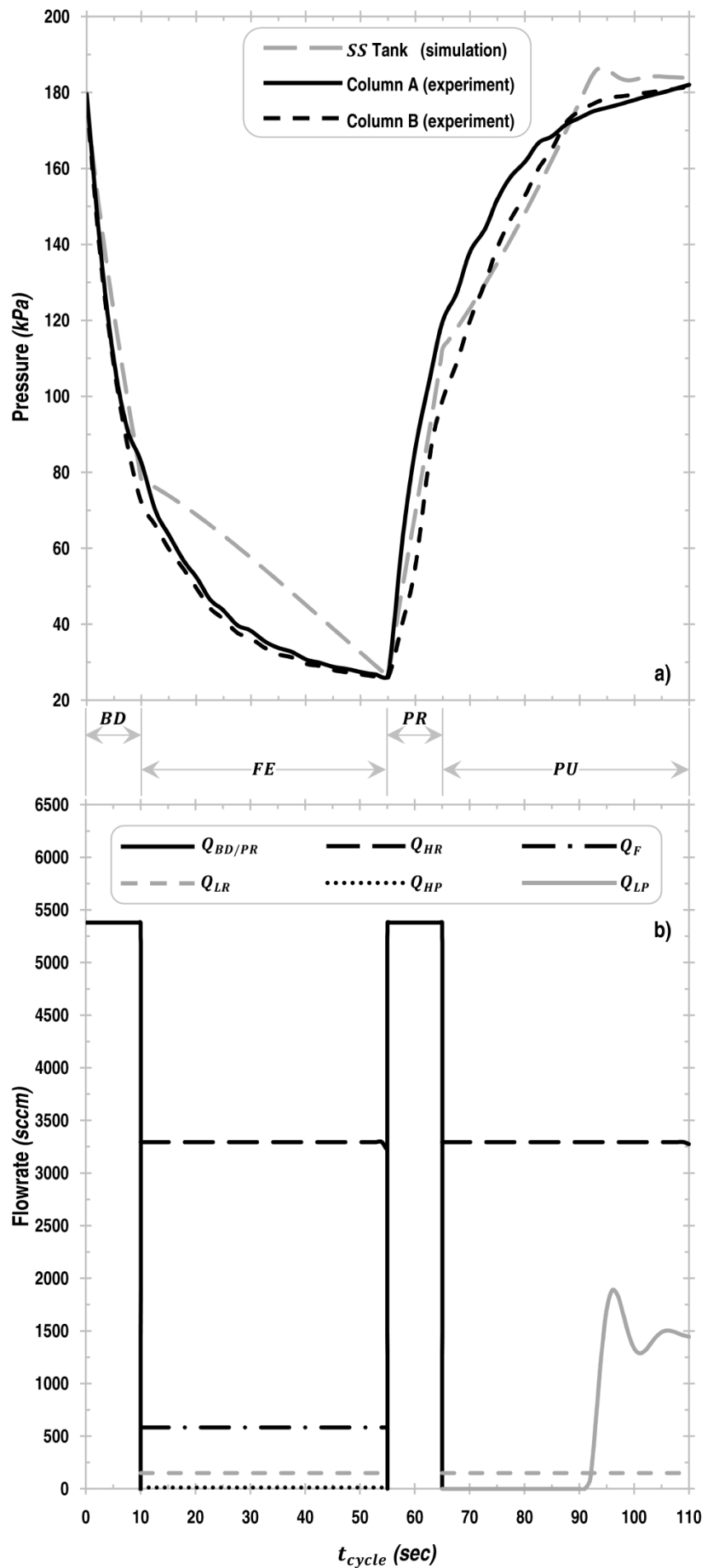


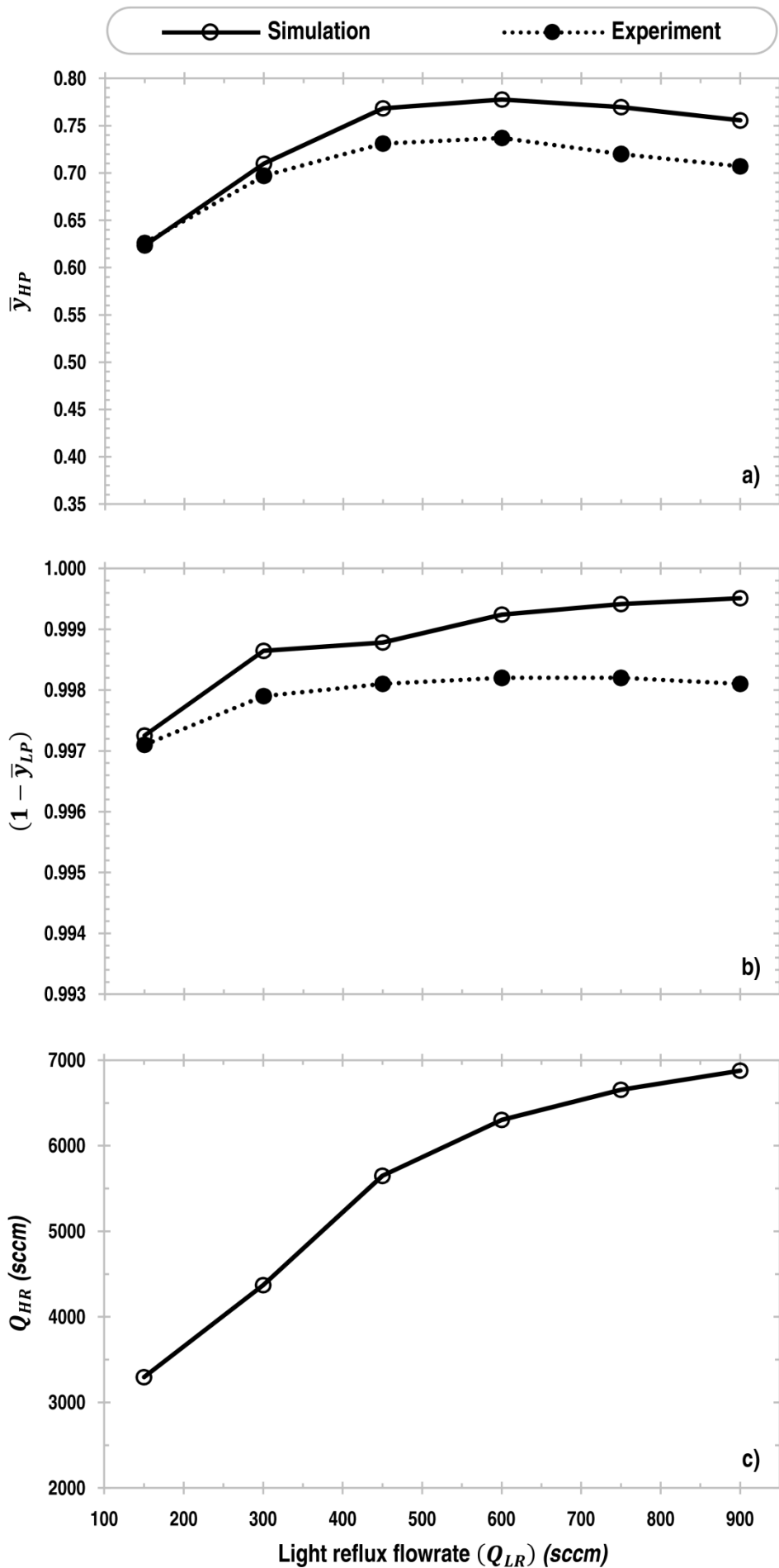
Fig. 2. Schematic representation of DR-PL-A process simulation flowsheet.



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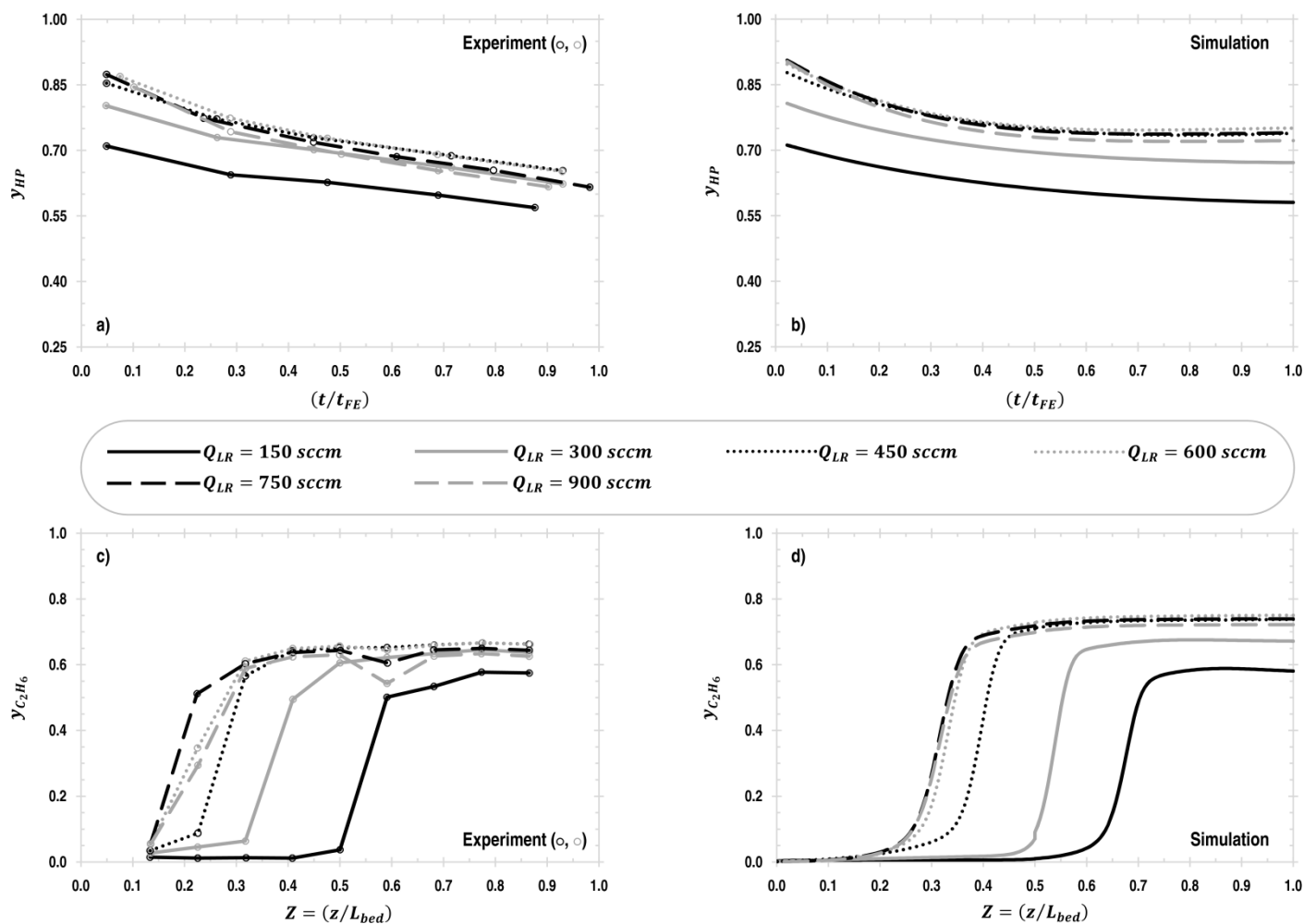
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Fig. 3. Base case (Run # 1) results at *CSS* during one DR-PL-A cycle. a) Simulation and experimental pressure profiles and b) simulated flow pattern of various streams (*FE*: Feed; *BD*: Blowdown; *PR*: Pressurization and; *PU*: Purge step).

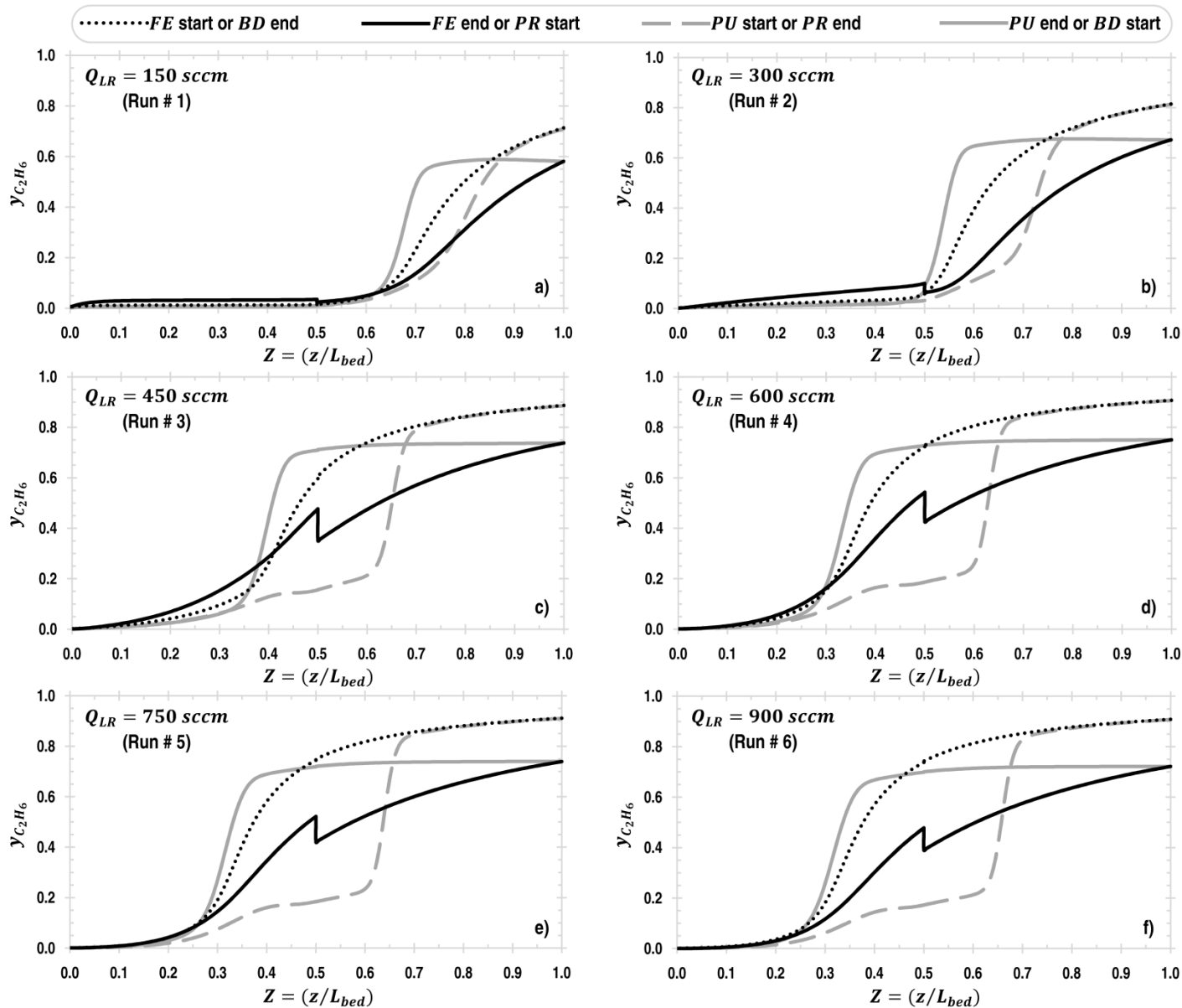


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712 Fig. 4. Simulation and/or experimental (McIntyre et al., 2010) results at *CSS* for Run # 1 to 6 showing the effect of light reflux flowrate
 713 (Q_{LR}) on: a) average C_2H_6 mole fraction in heavy product (\bar{y}_{HP}); b) average mole fraction of N_2 in light product ($1 - \bar{y}_{LP}$) and; c)
 714 heavy reflux flowrates (Q_{HR}).



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717 Fig. 5. Results at cyclic steady state (CSS) for Run # 1 to 6 showing the effect of light reflux flowrate (Q_{LR}) on: a) ethane mole
718 fraction profile in heavy product (y_{HP}) measured by the experimenters (McIntyre et al., 2010) during feed step (FE); b) model
719 predicted ethane mole fraction profile in heavy product (y_{HP}) during feed step (FE); c) ethane mole fraction profile in the column
720 ($y_{C_2H_6}$) at the end of high pressure purge step (PU) measured by the experimenters (McIntyre et al., 2010) and; d) model predicted
721 ethane mole fraction profile in the column ($y_{C_2H_6}$) at the end of high pressure purge step (PU).

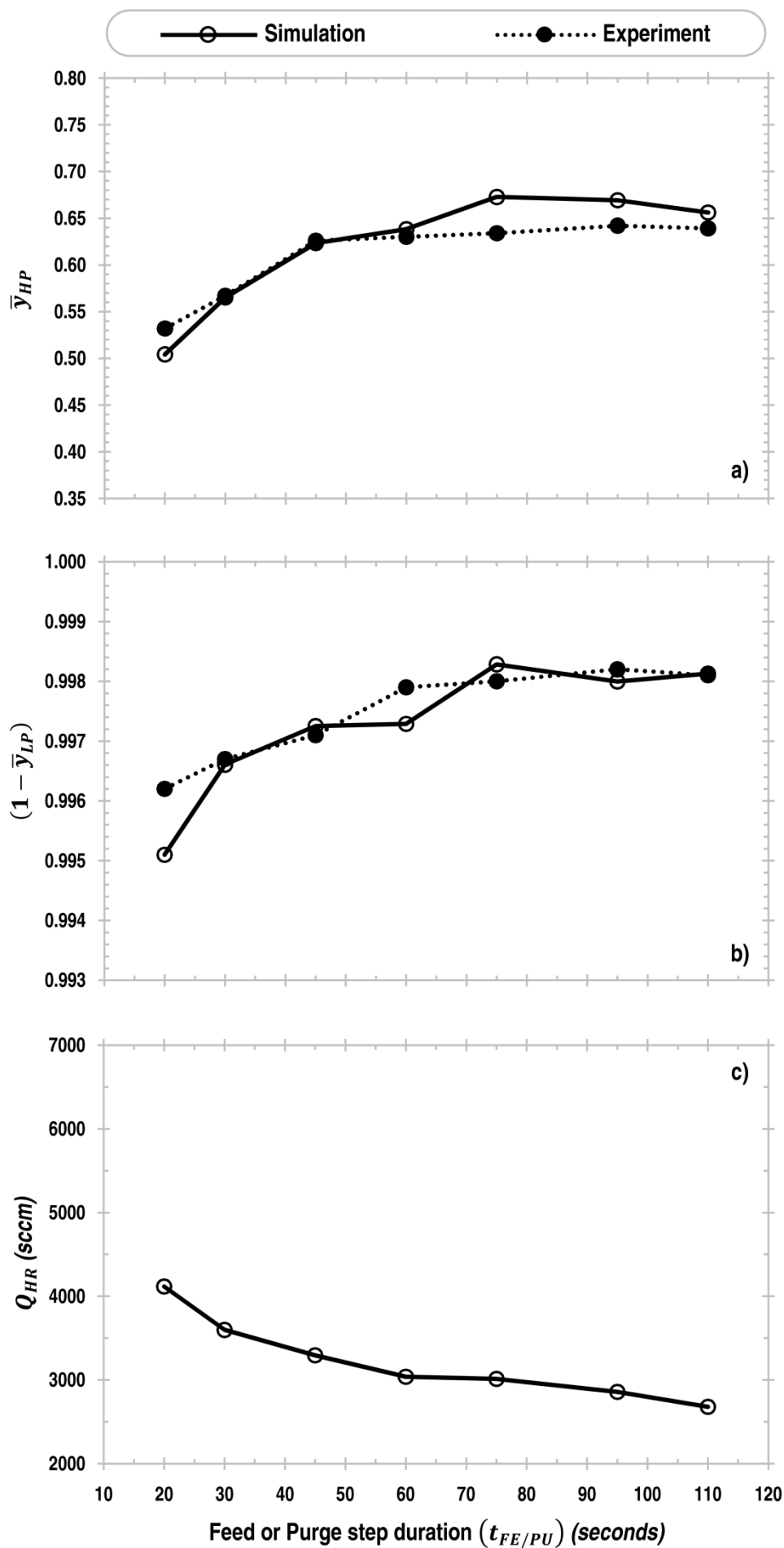


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735 Fig. 6. Model predicted cyclic steady state (CSS) column composition profiles at the *beginning* (start) and *end* of each DR-PL-A cycle
 736 step for Run # 1 to 6 (*FE*: Feed; *BD*: Blowdown; *PR*: Pressurization and; *PU*: Purge step).

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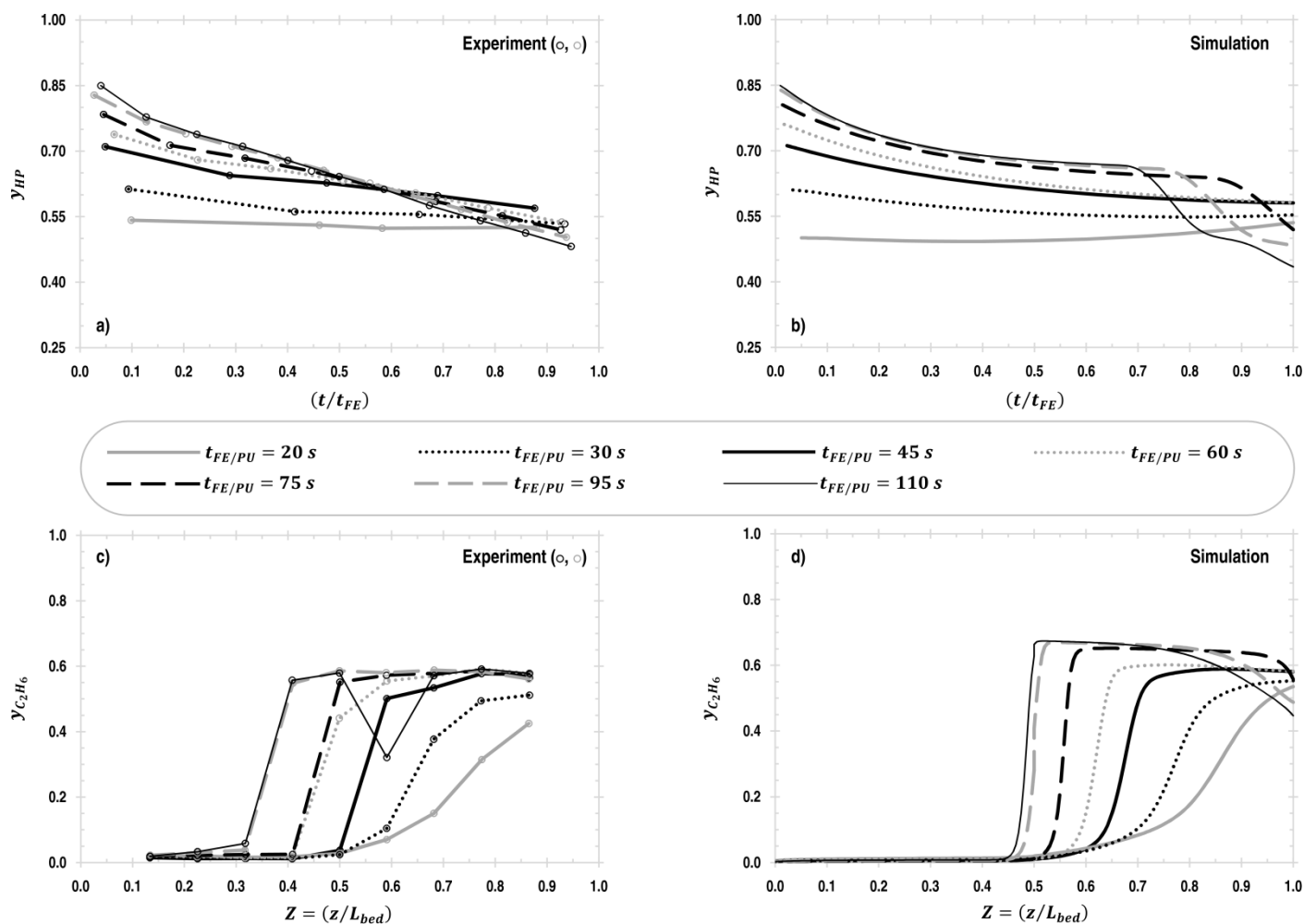
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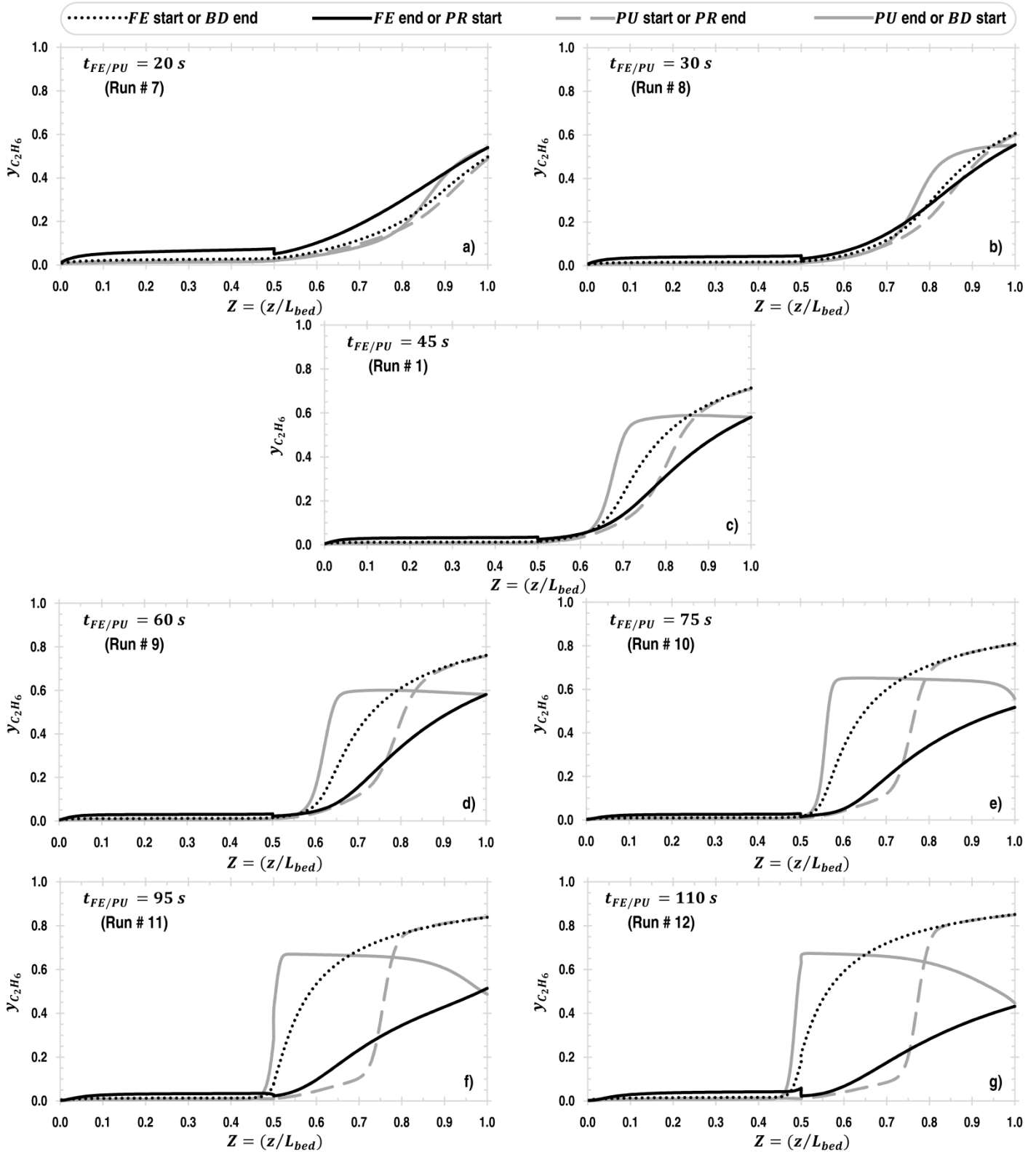
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Fig. 7. Simulation and/or experimental (McIntyre et al., 2010) results at CSS for Run # 7, 8, 1 and 9 to 12; showing the effect of feed or purge step duration ($t_{FE/PU}$) on: a) average C_2H_6 mole fraction in heavy product (\bar{y}_{HP}); b) average mole fraction of N_2 in light product ($1 - \bar{y}_{LP}$) and; c) heavy reflux flowrates (Q_{HR}).



745 Fig. 8. Results at cyclic steady state (CSS) for Run # 7, 8, 1 and 9 to 12; showing the effect of feed or purge step duration ($t_{FE/PU}$)
 746 on: a) ethane mole fraction profile in heavy product (y_{HP}) measured by the experimenters (McIntyre et al., 2010) during feed step
 747 (FE); b) model predicted ethane mole fraction profile in heavy product (y_{HP}) during feed step (FE); c) ethane mole fraction profile
 748 in the column ($y_{C_2H_6}$) at the end of high pressure purge step (PU) measured by the experimenters (McIntyre et al., 2010) and;
 749 d) model predicted ethane mole fraction profile in the column ($y_{C_2H_6}$) at the end of high pressure purge step (PU).

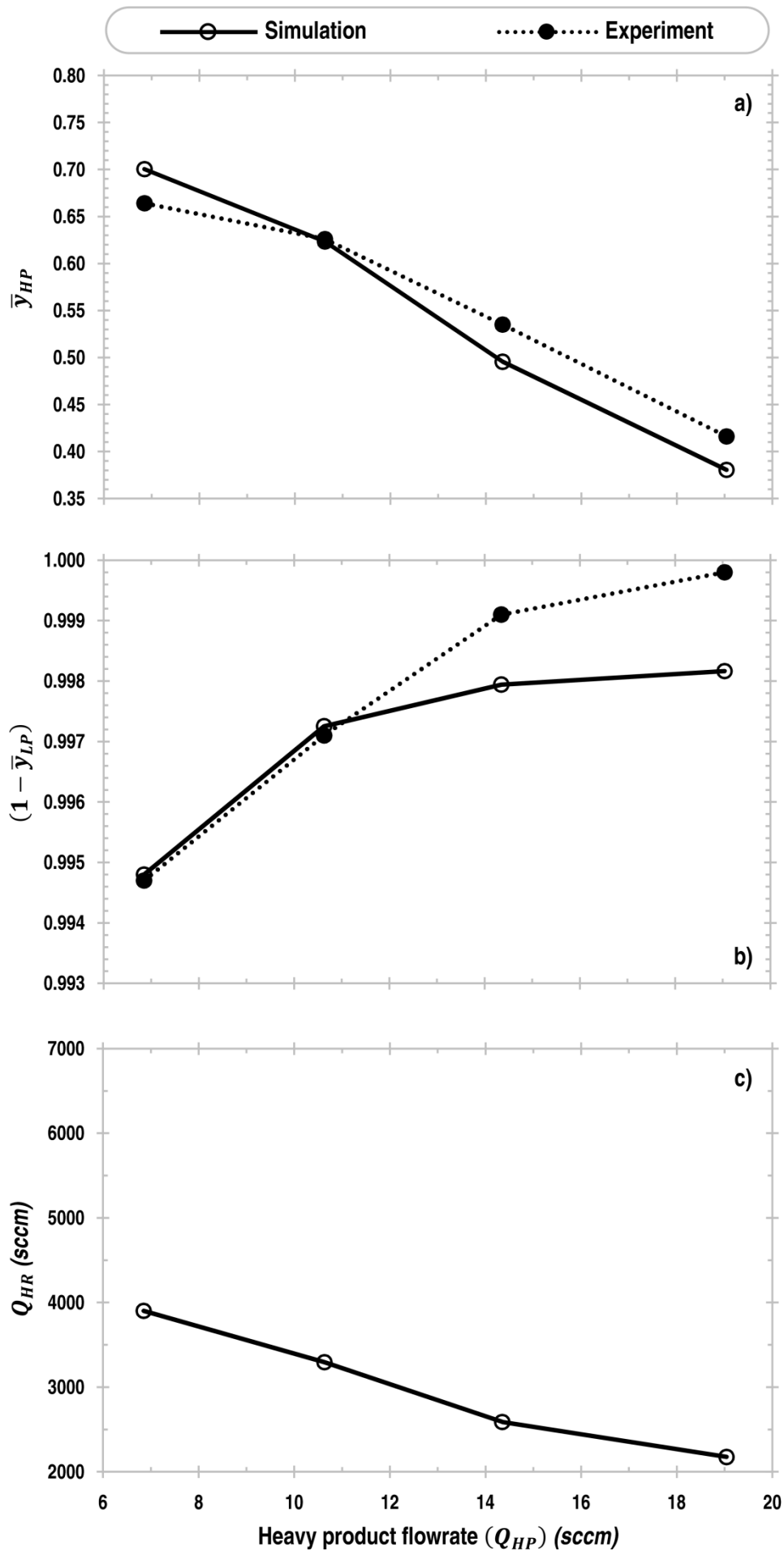


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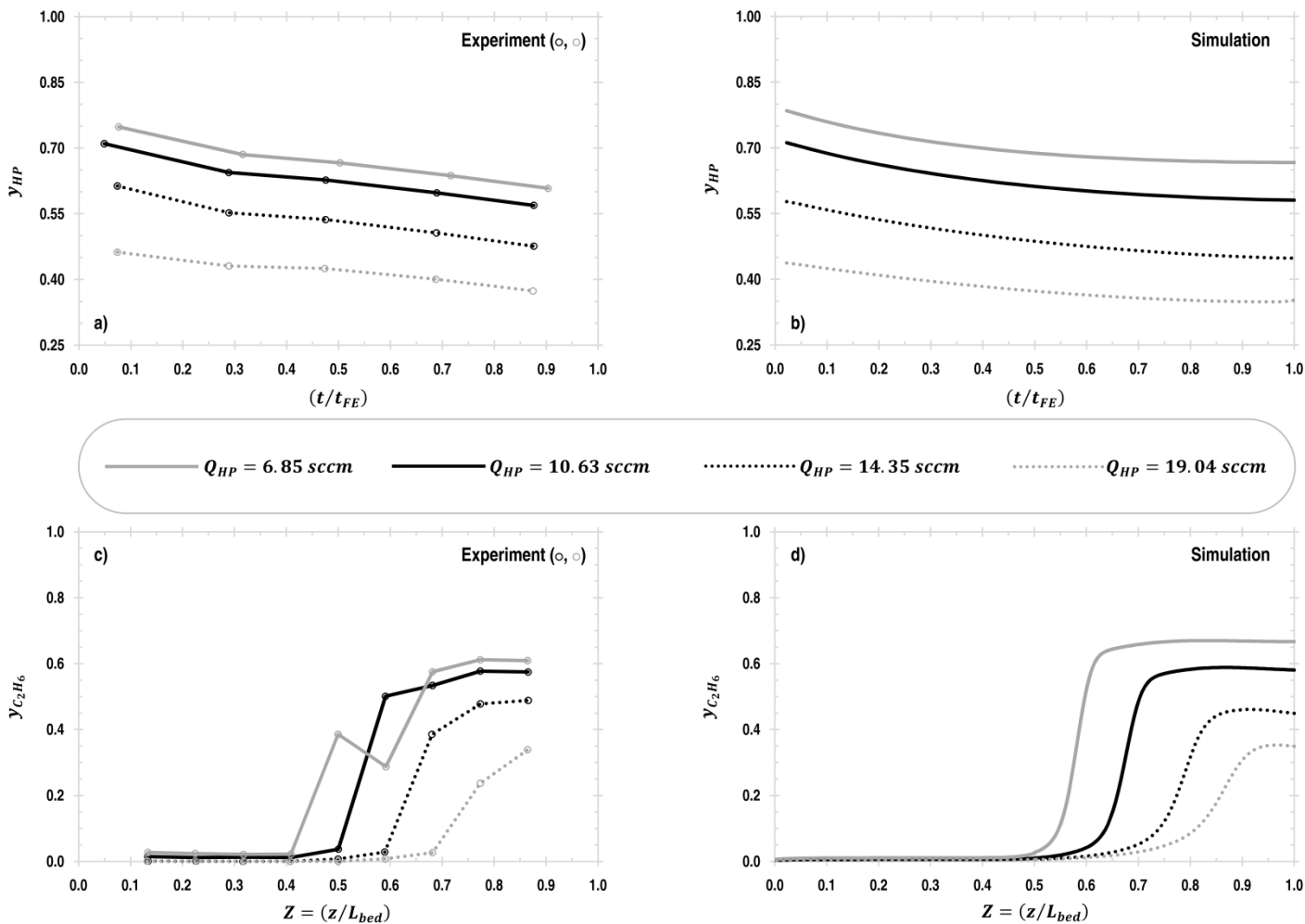
764 Fig. 9. Model predicted cyclic steady state (CSS) column composition profiles at the *beginning* (start) and *end* of each DR-PL-A cycle
 765 step for Run # 7, 8, 1 and 9 to 12 (FE: Feed; BD: Blowdown; PR: Pressurization and; PU: Purge step).

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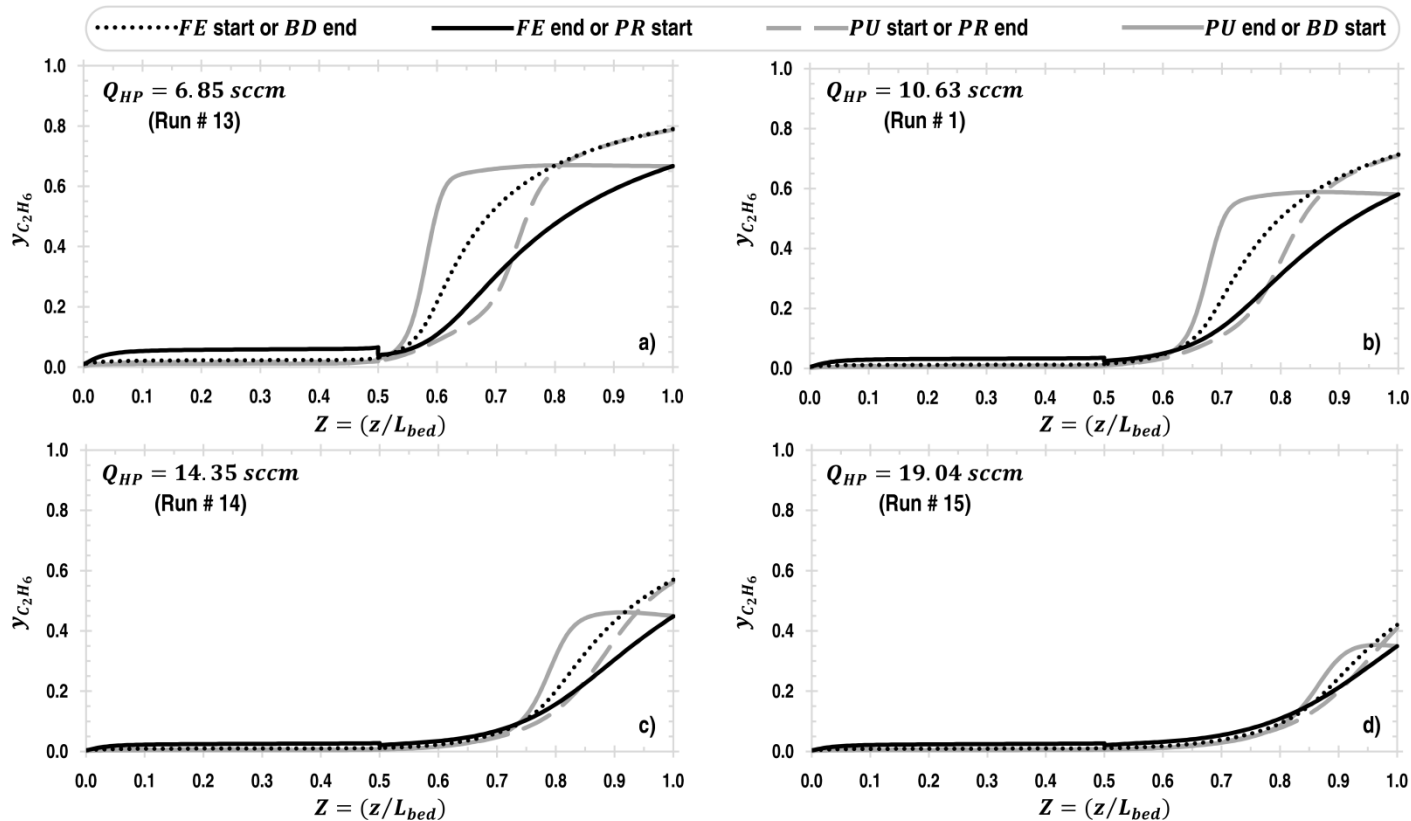


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768 Fig. 10. Simulation and/or experimental (McIntyre et al., 2010) results at CSS for Run # 13, 1, 14 and 15 showing the effect of heavy
 769 product flowrate (Q_{HP}) on: a) average C_2H_6 mole fraction in heavy product (\bar{y}_{HP}); b) average mole fraction of N_2 in light product
 770 ($1 - \bar{y}_{LP}$) and; c) heavy reflux flowrates (Q_{HR}).



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773 Fig. 11. Results at cyclic steady state (CSS) for Run # 13, 14 and 15 showing the effect of heavy product flowrate (Q_{HP}) on: a)
774 ethane mole fraction profile in heavy product (y_{HP}) measured by the experimenters (McIntyre et al., 2010) during feed step (FE); b)
775 model predicted ethane mole fraction profile in heavy product (y_{HP}) during feed step (FE); c) ethane mole fraction profile in the
776 column ($y_{C_2H_6}$) at the end of high pressure purge step (PU) measured by the experimenters (McIntyre et al., 2010) and; d) model
777 predicted ethane mole fraction profile in the column ($y_{C_2H_6}$) at the end of high pressure purge step (PU).



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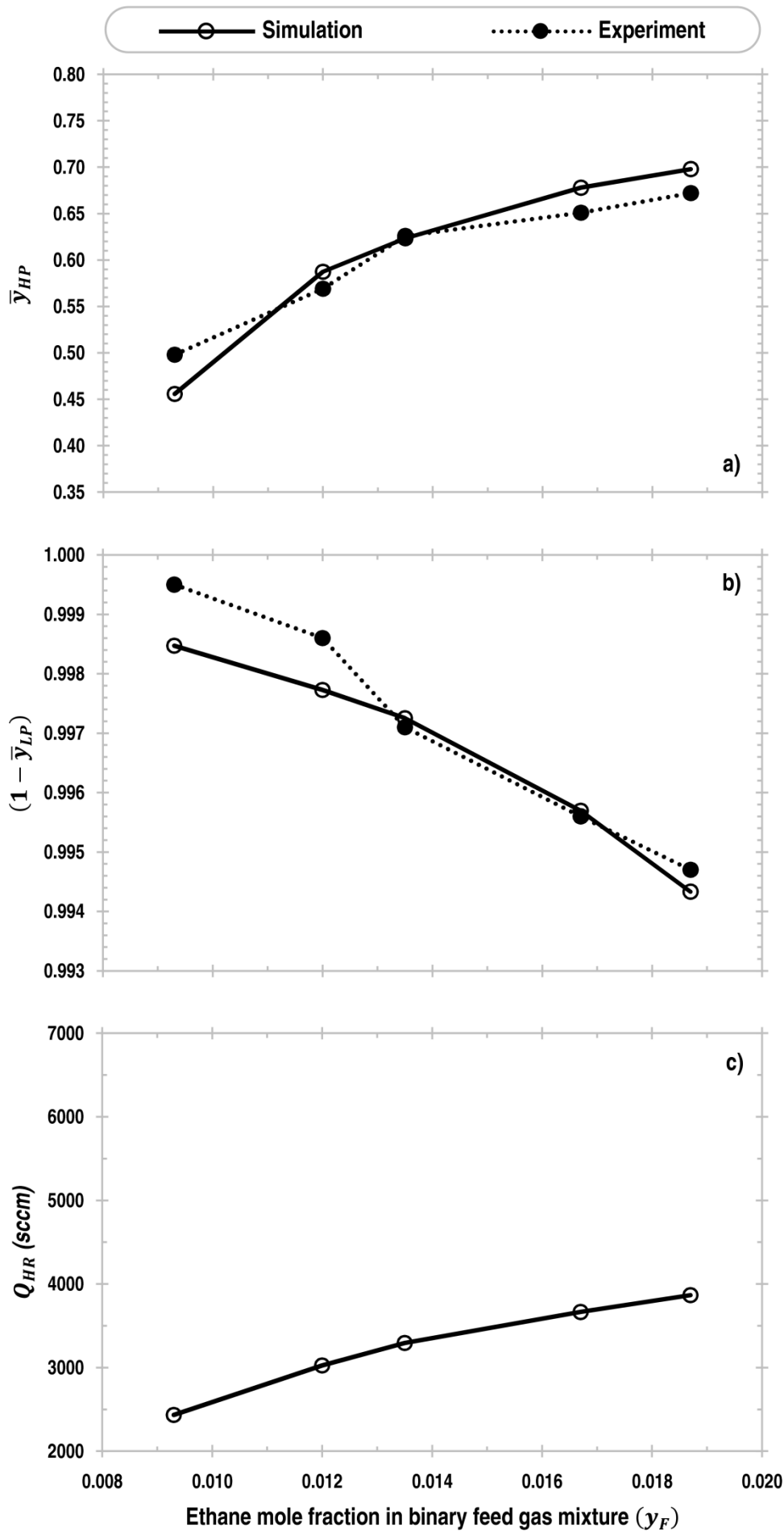
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Fig. 12. Model predicted cyclic steady state (CSS) column composition profiles at the *beginning* (start) and *end* of each DR-PL-A cycle step for Run # 13, 1, 14 and 15 (*FE*: Feed; *BD*: Blowdown; *PR*: Pressurization and; *PU*: Purge step).



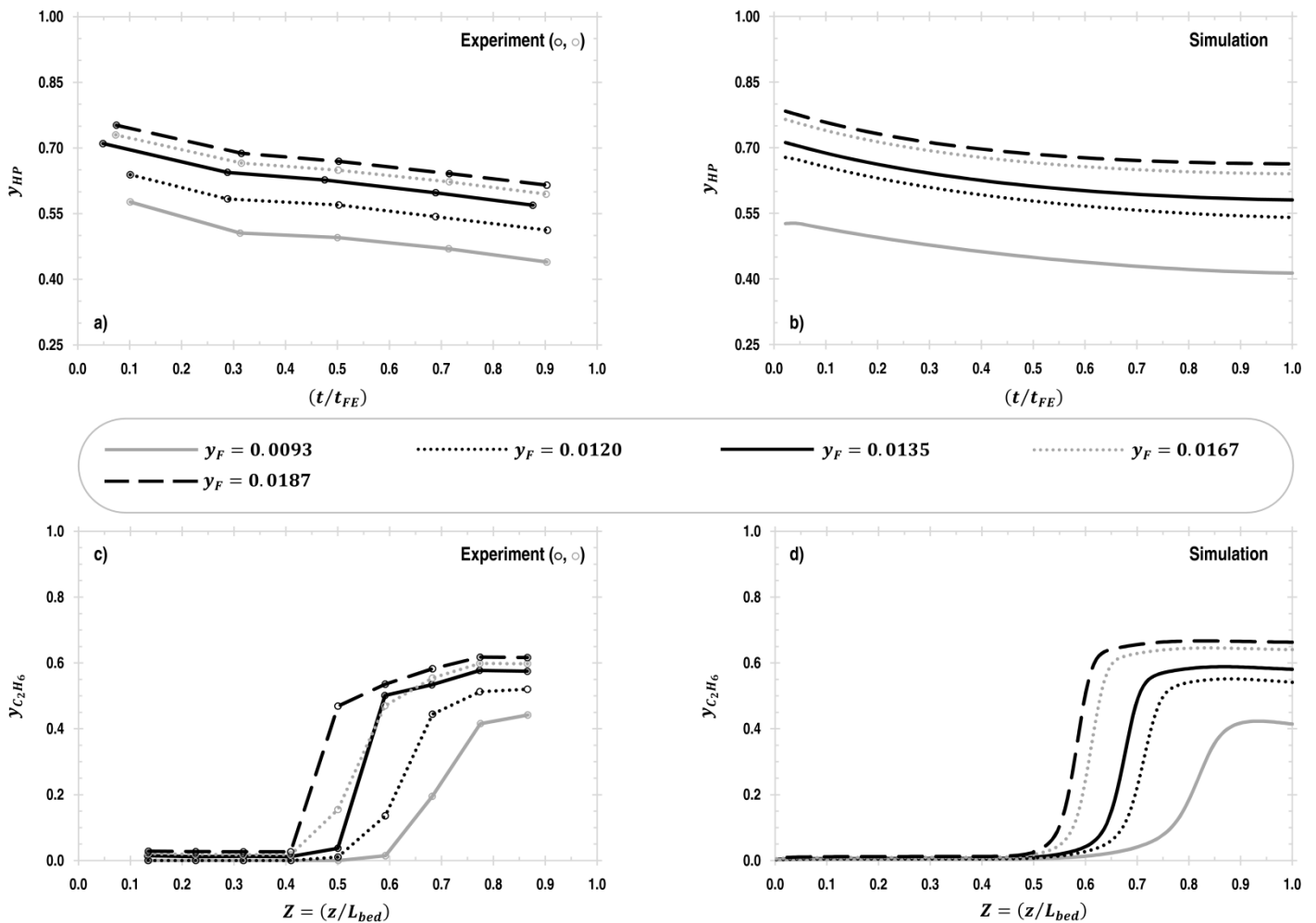
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Fig. 13. Simulation and/or experimental (McIntyre et al., 2010) results at CSS for Run # 16, 17, 1, 18 and 19 showing the effect of ethane mole fraction (y_F) in binary feed gas mixture on: a) average C_2H_6 mole fraction in heavy product (\bar{y}_{HP}); b) average mole fraction of N_2 in light product ($1 - \bar{y}_{LP}$) and; c) heavy reflux flowrates (Q_{HR}).



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800 Fig. 14. Results at cyclic steady state (CSS) for Run # 16, 17, 1, 18 and 19 showing the effect of ethane mole fraction (y_F) in binary
 801 feed gas mixture on: a) ethane mole fraction profile in heavy product (y_{HP}) measured by the experimenters (McIntyre et al., 2010)
 802 during feed step (FE); b) model predicted ethane mole fraction profile in heavy product (y_{HP}) during feed step (FE); c) ethane mole
 803 fraction profile in the column ($y_{C_2H_6}$) at the end of high pressure purge step (PU) measured by the experimenters (McIntyre et al.,
 804 2010) and; d) model predicted ethane mole fraction profile in the column ($y_{C_2H_6}$) at the end of high pressure purge step (PU).

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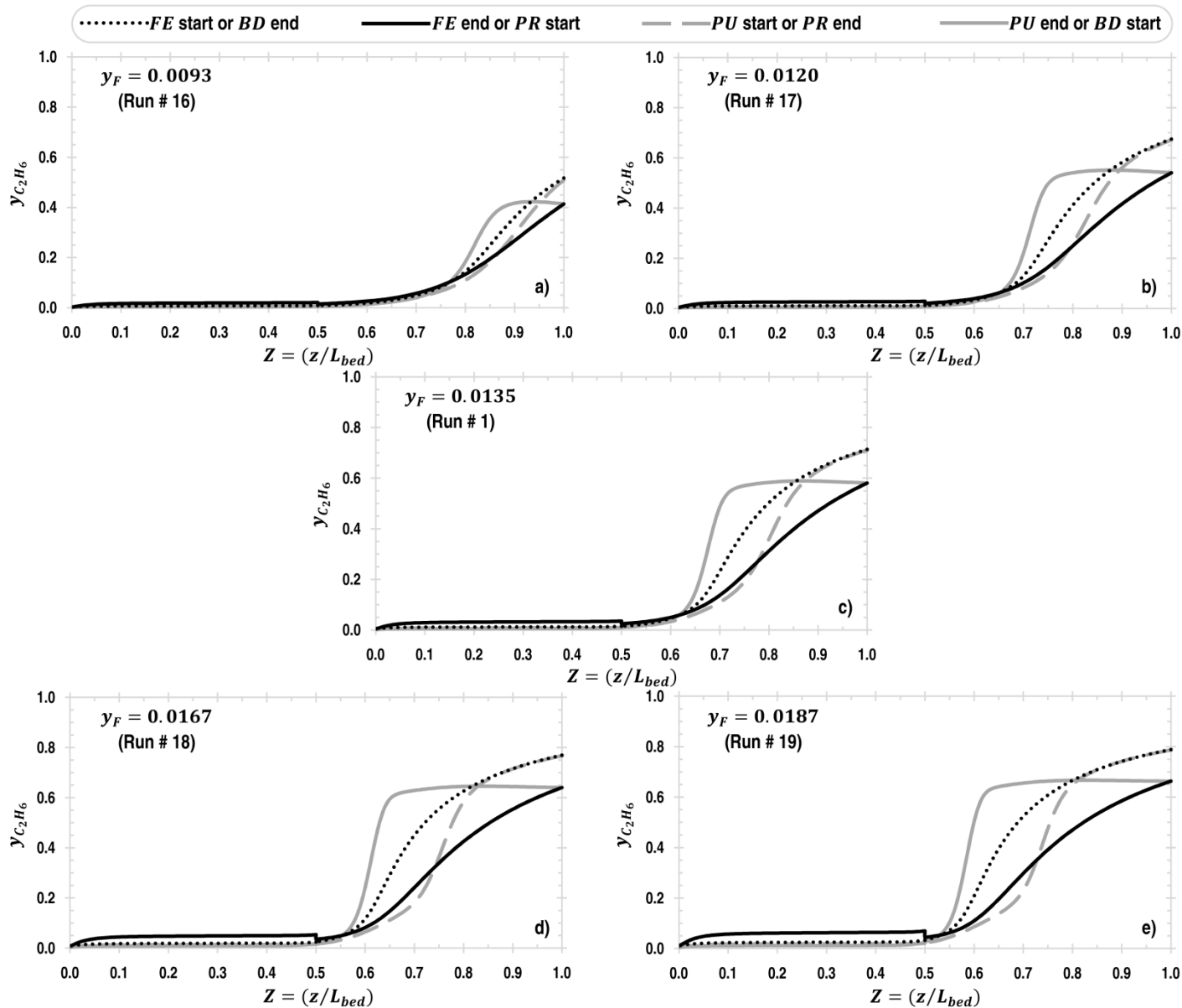
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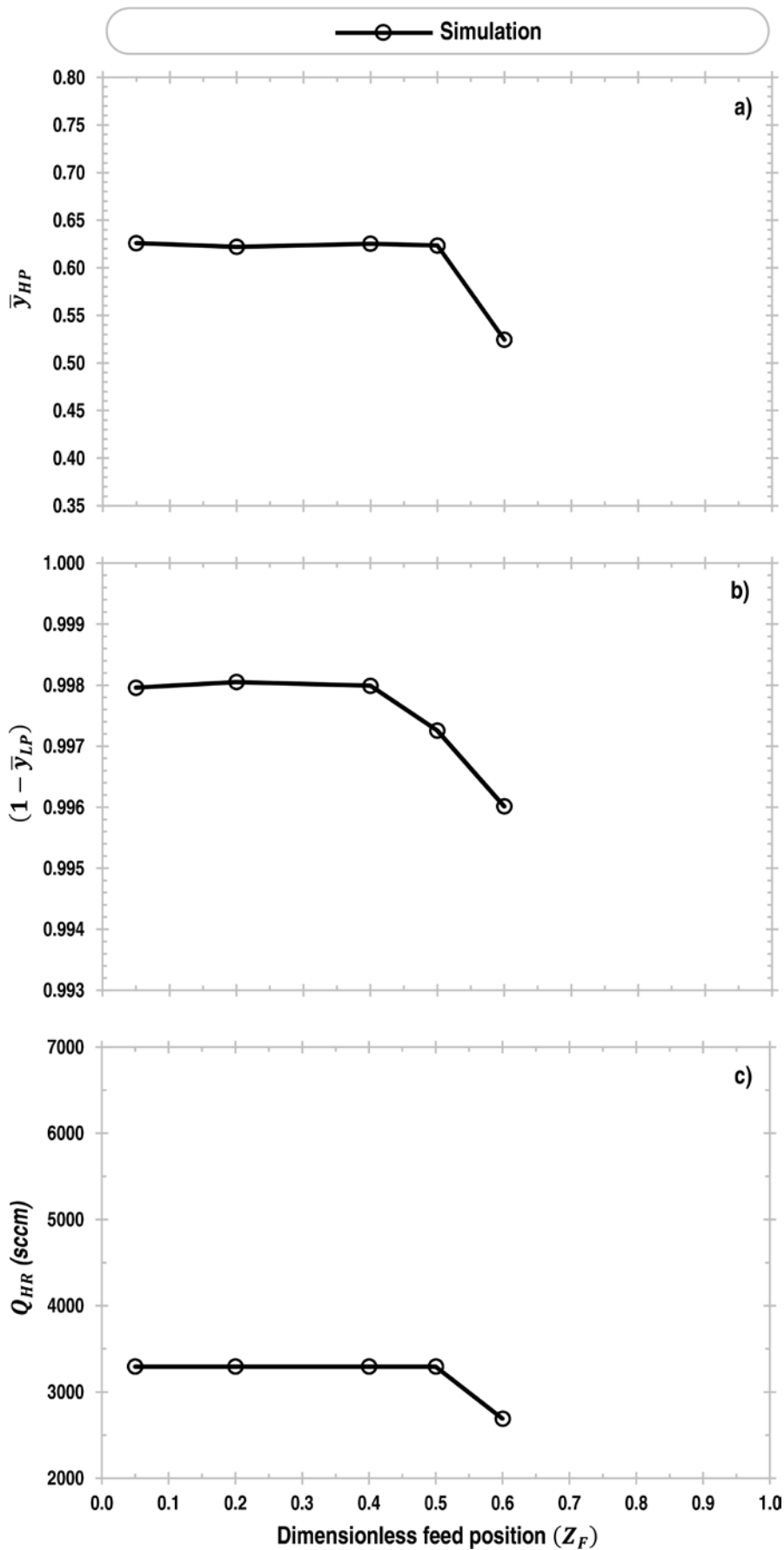
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Fig. 15. Model predicted cyclic steady state (*CSS*) column composition profiles at the *beginning* (start) and *end* of each DR-PL-A cycle step for Run # 16, 17, 1, 18 and 19 (*FE*: Feed; *BD*: Blowdown; *PR*: Pressurization and; *PU*: Purge step).



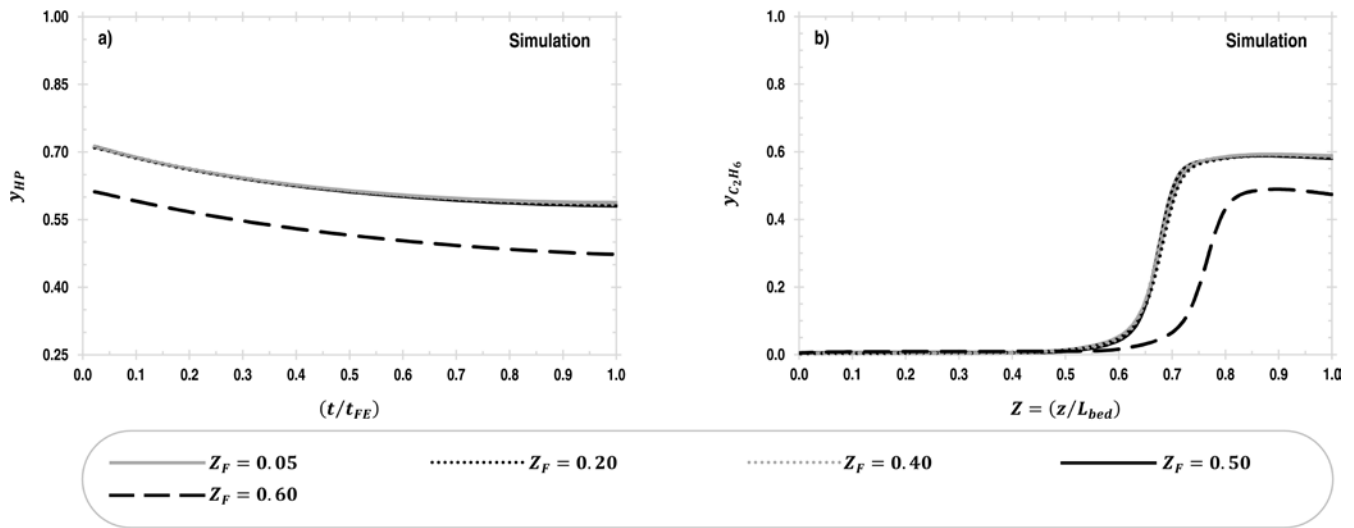
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Fig. 16. Simulation results at CSS showing the effect of dimensionless feed position (Z_F) on: a) average C_2H_6 mole fraction in heavy product (\bar{y}_{HP}); b) average mole fraction of N_2 in light product ($1 - \bar{y}_{LP}$) and; c) heavy reflux flowrates (Q_{HR}).



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826 Fig. 17. Model predicted results at cyclic steady state (CSS) showing the effect of dimensionless feed position (Z_F) on: a) ethane
 827 mole fraction profile in heavy product (y_{HP}) during feed step (FE) and; b) ethane mole fraction profile in the column ($y_{C_2H_6}$) at the
 828 end of high pressure purge step (PU).

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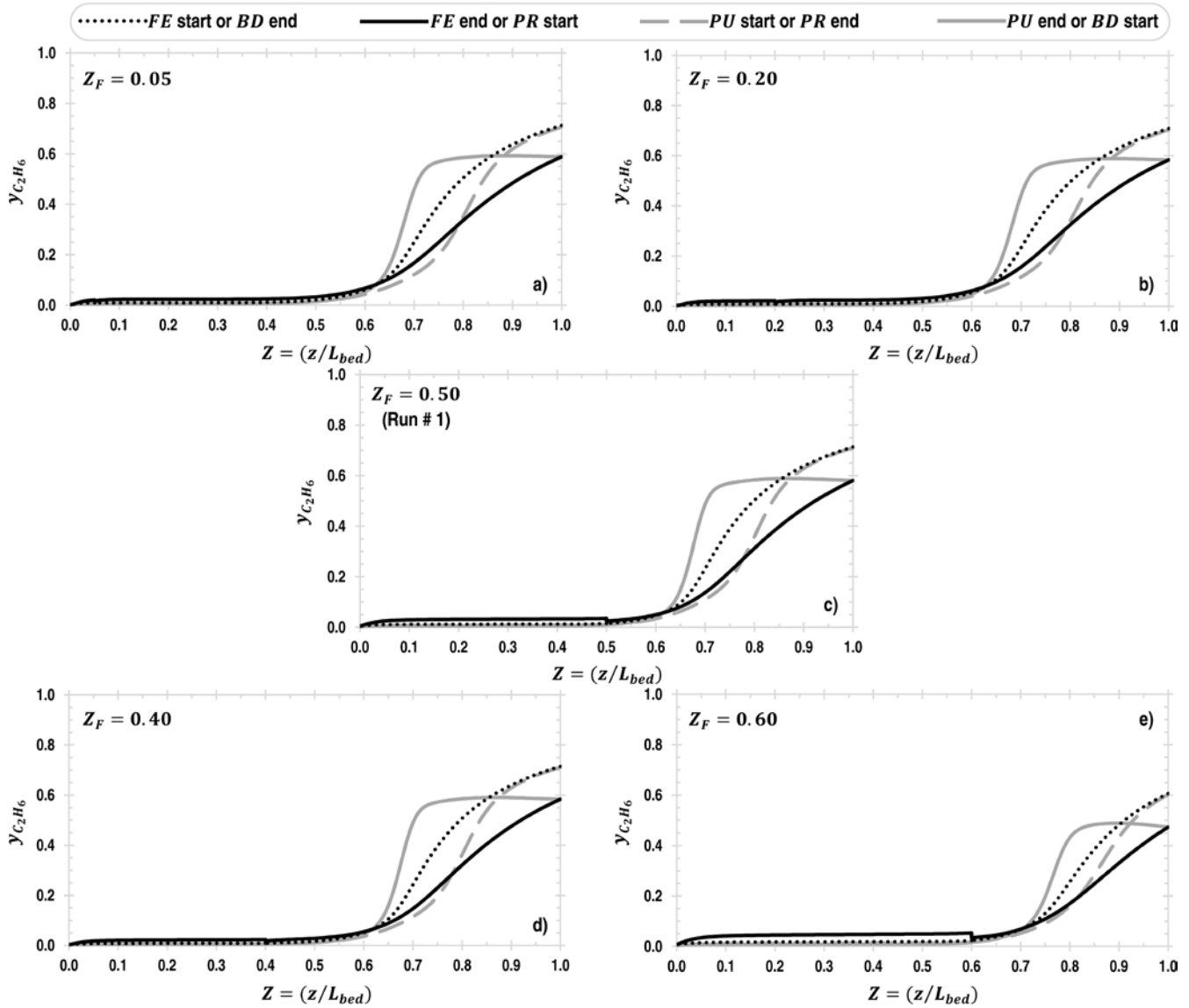
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Fig. 18. Model predicted cyclic steady state (CSS) column composition profiles at the *beginning* (start) and *end* of each DR-PL-A cycle step (*FE*: Feed; *BD*: Blowdown; *PR*: Pressurization and; *PU*: Purge step).

Design parameters			
Length of <i>SS</i>	L_{SS}	0.435	<i>m</i>
Length of <i>RS</i>	L_{RS}	0.435	<i>m</i>
Diameter of each column	D_{column}	0.024	<i>m</i>
Interstitial porosity	ε_i	0.32	
Operating parameters			
Temperature	T	298.15	<i>K</i>
High pressure set point in <i>LPFC</i>	P_H	184	<i>kPa</i>
Low pressure set point in <i>HRFC</i>	P_L	26	<i>kPa</i>
Depressurization flowrate	Q_{BD}	5379.35	<i>sccm</i>
<i>BD</i> end pressure	$P_{BD,end}$	78.75	<i>kPa</i>
Adsorbent properties			
Radius of adsorbent particle	r_p	1.31	<i>mm</i>
Particle porosity	ε_p	0.38	
Bulk density	ρ_B	284.88	<i>kg/m³</i>
Shape factor	ψ	1	
Tortuosity	τ	4	
Macropore radius	$r_{p,mac}$	0.15	μm
Isotherm data at 298.15 K			
First isotherm parameter for C_2H_6	IP_{1,C_2H_6}	9.14×10^{-2}	<i>mol/(kg.kPa)</i>
Second isotherm parameter for C_2H_6	IP_{2,C_2H_6}	1.14×10^{-2}	<i>l/kPa</i>
First isotherm parameter for N_2	IP_{1,N_2}	3.48×10^{-3}	<i>mol/(kg.kPa)</i>
Second isotherm parameter for N_2	IP_{2,N_2}	1.25×10^{-3}	<i>l/kPa</i>

Table 1. Parameter values used for all simulations.

Run #	y_F	Q_F (sccm)	Q_{HP} (sccm)	Q_{LR} (sccm)	t_{FE} or t_{PU} (s)
1 (Base Case)	0.0135	583.6	10.63	150	45
2	0.0139	572.1	9.97	300	45
3	0.0138	585.1	9.79	450	45
4	0.0136	580.3	9.81	600	45
5	0.0137	580.8	9.83	750	45
6	0.0136	583.9	9.89	900	45
7	0.0135	586.3	10.83	150	20
8	0.0135	584.6	10.51	150	30
9	0.0130	601.3	10.34	150	60
10	0.0139	573.3	10.28	150	75
11	0.0139	575.0	10.27	150	95
12	0.0141	567.2	10.29	150	110
13	0.0136	578.8	6.85	150	45
14	0.0135	587.9	14.35	150	45
15	0.0138	577.1	19.04	150	45
16	0.0093	575.6	10.40	150	45
17	0.0120	576.4	10.14	150	45
18	0.0167	581.8	10.45	150	45
19	0.0187	587.5	10.75	150	45

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887 Table 2. Values of the operating parameters used in Run # 1 (Base Case) to Run # 19. Parameters in bold italics in a column indicate
888 the group of runs used to study the effect of that parameter on process performance. This table is the identical to Table 2 reported by
889 McIntyre et al. (2010).

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Step	V_{LR}	V_F	V_{HR}	V_{BD}	V_{HP}	V_{LP}	LRI	HRI	PI
BD	x	x	x	✓	x	x	x	x	ℝ
FE	x	✓	✓	x	✓	x	F	ℝ	x
PR	x	x	x	x	x	x	x	x	F
PU	✓	x	x	x	x	✓	ℝ	F	x

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Table 3. Modus operandi of various valves and interaction models during different steps in the cycle organizer. 'x' represents closed valve or nonfunctional interaction; '✓' represents open valve at designated or controlled flowrate; 'ℝ' represents that the interaction model is recording information and 'F' represents that the interaction model is feeding back the recorded information.