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## Comparison of SIF solutions for cracks under rotating bending and their impact upon propagation lifetime of railway axles

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### Abstract

The stress intensity factor (SIF) is a crucial input parameter for the definition of the inspection intervals based on the damage tolerance approach. In the present work the applicability and precision of existing analytical stress intensity factor solutions for the cracks in railway axle geometries, subjected to rotary bending and residual stresses is discussed, by comparison with a reference set of solutions obtained from finite element (FE) analyses. Both the SIFs and the crack shape evolution are considered, comparing the predicted crack shape growth, from FE and analytical solutions, with a series of experimental data from the literature. Finally, the effect of the different approximations for the propagation lifetime and non-destructive tests (NDT) reliability of railway axles is discussed.

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**Keywords:** railway axles; stress intensity factor comparison; residual stresses; residual lifetime

### 1. Introduction

Railway axles are designed to have an infinite life-time. Even if this is accepted as adequate, the fact remains that occasional failures have been and are observed in service. Hillmansen and Smith [1] recently cited 37 failures during a period of 27 years (1975 - 2002) for a total number of 170 000 axles circulating in UK. Similar figures have been observed all around Europe in other references [2]. The typical failure positions are the press- fits for wheels, gears, and brakes or the axle body close to notches and transitions[2]. Such failures always occur as fatigue crack propagation whose nucleation can be due to different causes. In the case of railway axles, the presence of widespread corrosion or the possible damage due to the ballast impacts [3] may constitute such causes.

This kind of failures is usually tackled by employing the 'damage tolerance' methodology, whose philosophy consists in determining the most opportune inspection interval given the 'probability of detection' (POD) of the

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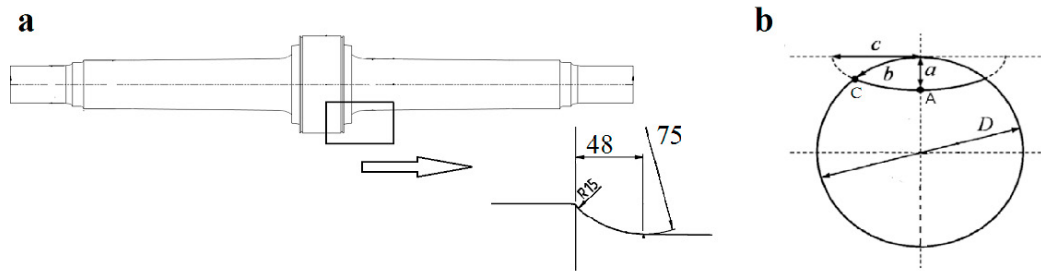


Fig. 1. geometry of the axle: a) dimensions at the T-notch b) crack configuration

adopted non-destructive testing (NDT) technique or, alternatively, in defining the needed NDT specifications given a programmed inspection interval. The knowledge of several factors is essential for an accurate calculation regarding damage tolerance analysis including: Initial crack dimensions, the acting load spectra during the service life of the axle, the crack growth behavior of the adopted steel grade, which describes fatigue crack propagation rate in each cycle and evaluation of stress intensity factor along the crack front.

In order to have a precise prediction of crack propagation rate and consequently residual lifetime, it is necessary to have an accurate estimation of driving forces on the cracked component  $\Delta K$ . Usually FE analyses are performed to calculate the SIFs along the crack front, which gives precise results, but the computational effort, due to the fact that several crack configurations need to be modeled, is very high. Moreover, the FE methodology is not flexible: apart from the analyzed cracks and geometries, the results cannot be easily extended.

Besides the FE analyses for K factors, there is also a need for less expensive analytical solutions, which allow at least an approximate estimation of the parameter. Currently there is a wide range of analytical SIF solutions available in literature [4], involving different loading conditions, crack location, crack shape and cracked component shape. The SIF solutions are mainly evaluated adopting two kinds of approaches. One based on the FE modeling of the cracked component, which results from FE evaluations, considering several crack shapes and dimensions, are usually interpolated or used for generating a set of equations in order to obtain an analytical solution for the SIF of a developing crack, the other one onto an analytical approach adopting the so-called weight function [5]. The weight function depends only on geometrical and boundary conditions, so by determining the weight function for a given geometry it is possible to predict the SIF for any stress field acting on the crack plane for the same geometry.

The analytical SIF solutions were adopted for the T-notch and axle body of two different railway axles as representative for freight and high speed passenger train applications and the results were compared with the obtained FE solution, then the impact of estimated SIFs was investigated on residual lifetime and crack shape evaluation, however for the sake of brevity only the corresponding analysis for the freight axle is presented in this paper. Four important aspects regarding damage tolerance analysis of railway axles were considered in this research as follows: the stress intensity factor prediction, rotary bending and residual stress, the choice of the initial crack shape and load spectra.

## 2. Finite element analysis

The FE analysis was carried out on the adopted full-scale specimens, shown in Fig. 1a, specially designed according to relevant standards [6], for the three point rotary bending facility available in Lucchini's R&D laboratories. Fatigue cracks in railway axles tend to have semi-elliptical shapes (see Fig. 1b). For analyzing crack propagation, it is necessary to perform separate analyses for the surface point and for the deepest point in the crack front, because it will allow the crack to grow in depth to length ratio as it does in real case. Since there were no specific analytical SIF solutions available for the given geometry, several crack configurations were considered in the assessment locations. In particular five different crack depths  $a$  (1, 3, 5, 7 and 9 mm) and five aspect ratios  $a/c$  (0.2, 0.4, 0.6, 0.8, 1.0), with  $c$  being the semi-surface length, were considered in the present study.

SIFs were obtained on the basis of J-integral determination using the method of virtual crack extension and domain integrals. As is shown in Fig. 2 the most suitable approach for this purpose is to have a reasonably refined and structural mesh in the global model, where the stresses had to be carefully measured, and then defining a sub-model for the local

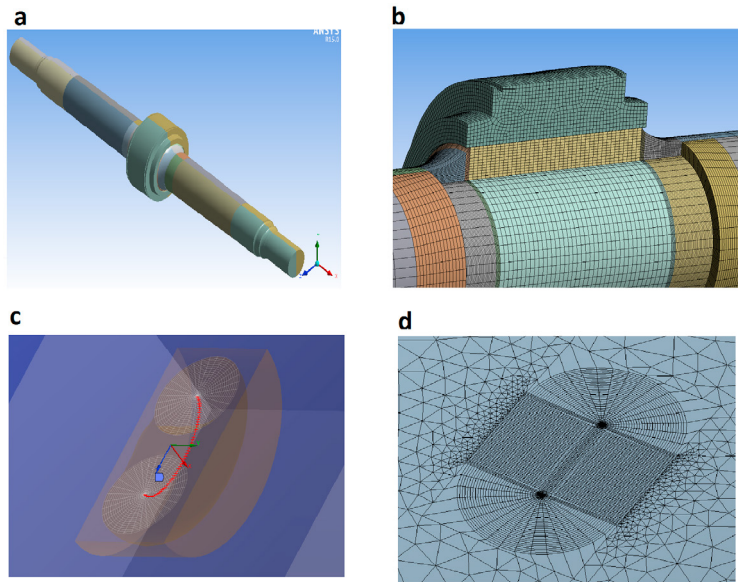


Fig. 2. Finite element modeling: (a) global geometry; (b) mesh refinement in the critical location; (c) inserting the crack in the sub-model; (d) refined structural mesh around the crack

of interest. The material was idealized as a linear-elastic steel, having Young's modulus  $E = 210$  GPa and Poisson's ratio  $\nu = 0.3$ . The implemented model is represented in Fig. 2. Axle and auxiliaries were modeled separately, and joined through a surface to surface coupling, including the interference fit option, for taking into account the stresses from press-fit; the mean value of the interference, and 0.3 as friction coefficient, were adopted. Two separate steps were adopted for the application of the loads: during the first step, only the non-linear contact with interference fit option was applied, then, in the second step, the pure bending in the notch were accomplished by applying a linear pressure on the axle cross-section. The non-linearity effect of press-fit simulation on the longitudinal stress path across the thickness in the crack plane was investigated by applying the press-fit interference and bending moment simultaneously and compared with results first procedure. The results of Fig. 3c demonstrates that, there is no significant difference between the two procedures in terms of stress paths, however, there is a huge difference from the point of view of computation time, in which clearly speaks in favor of, considering the bending loading and press-fit effect separately in the analysis.

As it can be seen in Fig. 3a, due to the press-fit, the wheel squeeze the axle under its seat and the surrounding regions stretches, as a consequence, a long with the applied bending stress the probability of developing a crack increases in the geometrical transition. In particular case, when the geometry transition is severe the press-fit effect become dominant and acts as a mean stress and it is not negligible. Madia et al. [7] presented set of equations based on the Carpinteri [8] solution, already taking into account the rotary bending, by introducing the mean contribution to the SIFs at the deepest and surface points of a crack front.

The J-integral was obtained for eight contours at the crack front, it can be seen in Fig. 4 that a stabilized SIF is obtained from the second contour upwards. The path independence of the J-integral is an index of the good quality and reliability of the mesh refinement. To calculate the boundary correction factor  $f$ , the SIF was scaled to the maximum nominal stress  $\sigma_N$  in the minimal cross section with the diameter  $d$  of the uncracked shaft. For the deepest point of the crack front A, the calculated energy release rate can directly used for obtaining the stress intensity factor. however  $\frac{1}{\sqrt{r}}$  singularity of the stress state is not full-field in general for surface cracks in the near surface domain (point C). Due to this boundary layer effect, the classical SIF is not accurate. In order to calculate SIFs at the surface point a normalized distance of 0.05 was considered.

Table 3.1 summarizes the evaluated SIF values and corresponding shape factors for 25 different crack configurations, by considering the rotary bending effect and residual stresses induced by press-fit at assessment location.

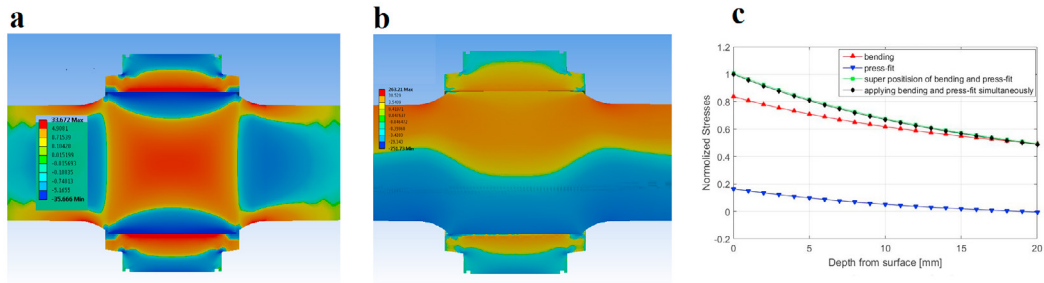


Fig. 3. stress analysis in notch section a)longitudinal stresses induced by press-fit; b) stress field taking into account both press-fit and bending; c)calculated in-depth profiles.

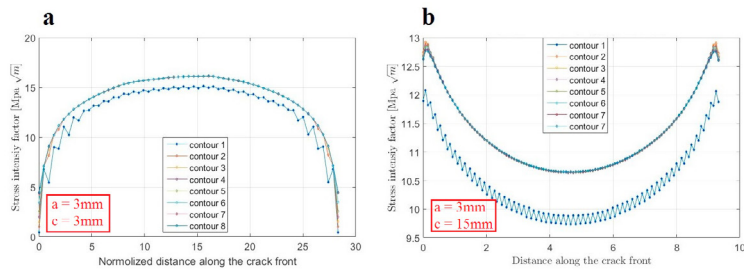


Fig. 4. KI values along the crack front for eight contour for two particular crack configurations at T-notch

Table 1. Shape factors ( $k/(\sigma_{nom} \sqrt{\pi a})$ ) for applied nominal bending moment of  $\sigma_N$  at  $\theta = 0^\circ$  and  $90^\circ$  and the SIF value due to the press fit. The reference interference value referring to the axle radius is  $\gamma_{ref} = 125\mu m$ .

crack depth a (mm)	aspect ratio (a/c)	Deepest point of the crack (A)		Surface point, a crack (C)		
		shape factor at $0^\circ$ for bending	SIF value for press-fit ( $Mpa \sqrt{m}$ )	shape factor at $0^\circ$ for bending	shape factor at $90^\circ$ for bending	SIF value for press-fit ( $Mpa \sqrt{m}$ )
1	0.2	1.175	0.992	0.651	0.029	0.53
1	0.4	1.063	0.885	0.755	0.015	0.607
1	0.6	0.946	0.825	0.856	0.01	0.731
1	0.8	0.856	0.732	0.882	0.007	0.781
1	1	0.76	0.638	0.882	0.006	0.762
3	0.2	1.073	1.452	0.594	0.079	0.846
3	0.4	1.025	1.363	0.75	0.047	1.091
3	0.6	0.914	1.213	0.834	0.032	1.214
3	0.8	0.815	1.065	0.869	0.023	1.246
3	1	0.728	0.948	0.876	0.018	1.259
5	0.2	1.025	1.629	0.539	0.114	1.04
5	0.4	0.951	1.553	0.715	0.076	1.367
5	0.6	0.862	1.373	0.809	0.053	1.517
5	0.8	0.768	1.223	0.848	0.039	1.597
5	1	0.687	1.083	0.856	0.03	1.606
7	0.2	0.956	1.631	0.516	0.143	1.193
7	0.4	0.902	1.571	0.691	0.103	1.537
7	0.6	0.808	1.428	0.783	0.072	1.687
7	0.8	0.726	1.234	0.827	0.054	1.808
7	1	0.65	1.064	0.838	0.041	1.787
9	0.2	0.887	1.601	0.503	0.166	1.339
9	0.4	0.842	1.522	0.66	0.127	1.669
9	0.6	0.765	1.361	0.756	0.091	1.874
9	0.8	0.69	1.196	0.807	0.068	1.969
9	1	0.62	1.053	0.821	0.053	1.995

### 3. Analytical SIF solutions

The precise estimation of the SIF, is so related to an accurate evaluation of the shape function  $f$ , given the shape and dimension of the crack relatively to the component. There is no existence weight function to directly cover the case of cracked railway axle. What is usually done is considering a solution similar to geometry and loading condition. There are widely available weight function solutions for various crack components under different stress state. In the present study the weight function solutions present by Shiratori [9], Wang and Lambert [10], Pommeir [11] and Varfolomeev [12] is adopted for SIF calculation in notch axle. For SIF evaluation in axle body, the adopted solutions are the ones

presented by: Carpinteri[8], Raju-Newman[13] and Levan-Royer [14], which developed for the case of round bar subjected to the bending load.

For the SIFs evaluation, the bending longitudinal stress profile along the perspective crack plan were obtained by dedicating FE analysis, which is discussed in section 2, and suitably interpolated for the application of the SIF solutions. The same procedure were applied for the press-fit induced residual stresses. It is worth to remark that, the stress distribution is assumed to be of a polynomial form in all the employed weight functions, except the Wang-Lambert weight function, which is more flexible in term of curve fitting. SIF values were independently determined for the two stress condition at the deepest point (A) and the free surface point (C) of an hypothetical crack, in particular the ones that were considered during the FE analysis, and then superimposed. The comparison between predicted SIF values with FE analysis result for bending and press-fit loading conditions are demonstrated in the Fig 5 and 6 respectively.

The results of Figs. 5-6 reveal that, for the semi-elliptical surface cracks inserted in the T-notch, the estimated SIFs at the deepest point and surface point by the weight function presented by Wang-Lambert gives the best approximation

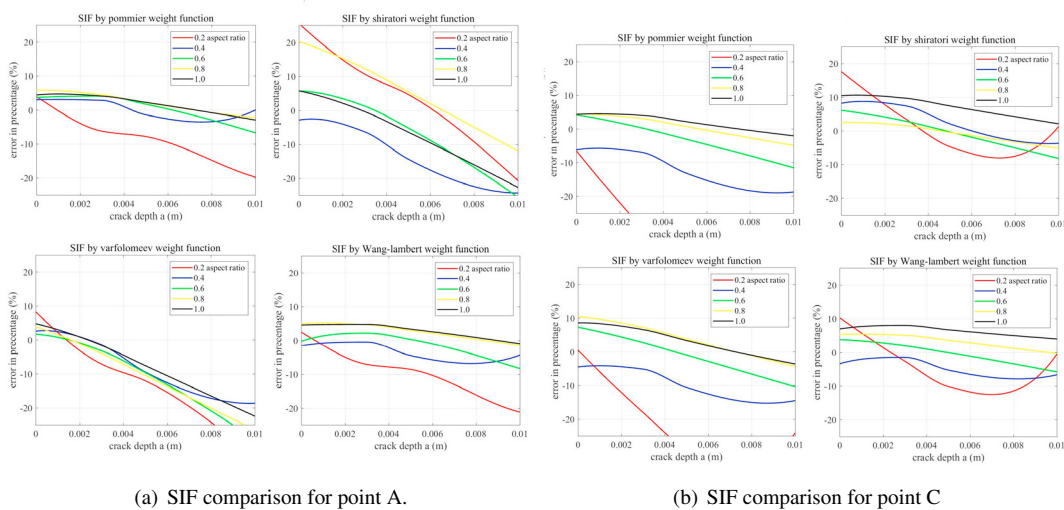


Fig. 5. SIF comparison ( $\frac{K_{FE}-K_{AS}}{K_{FE}} \times 100$ ) between analytical solutions with FE results for rotary bending stress state

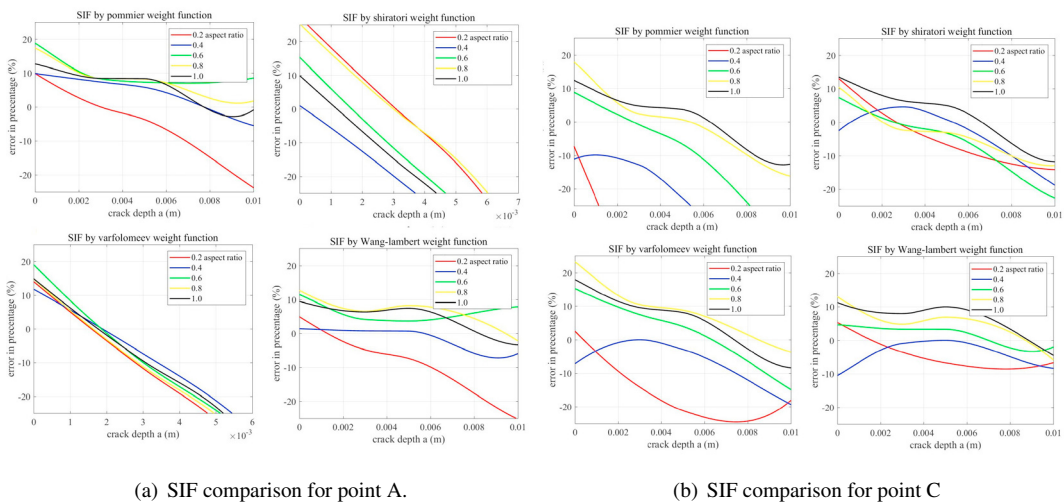


Fig. 6. SIF comparison( $\frac{K_{FE}-K_{AS}}{K_{FE}} \times 100$ ) between analytical solutions with FE analysis for press-fit induces stress state



for the bending stress state as well as for the press-fit induced stress state. The precision of these estimations are within the range of 10% for both points. It should be noted that, due to the stress singularity in the intersecting point of crack front with the free surface of the axle, for the lower aspect ratio the accuracy of predicted SIFs at point C decreases for all the solutions. Moreover, when the cracks tends to have semi-circular shape the predicted SIFs by Pommier weight function provide relatively better accuracy at both points. For mechanically short cracks, where the crack depth is between 1mm and 3mm the predicted SIFs by Varfolomeev weight function, are quite precise in the deepest point, however the error raises at point C when the aspect ratio decreases and crack tends to be flatter.

#### 4. Crack propagation analysis

Crack propagation analyses focused on the influence of estimated SIFs by the analytical approaches, on residual lifetime and crack shape evolution under realistic load spectra, derived from typical in-service load spectra available in the literature [15] and representative of about 57000 km of service. The continuous load spectra and its discretization as load blocks are shown, normalized, in Fig. 7a.

The fatigue crack growth algorithm plays a crucial role in life prediction. At any stage of crack propagation the crack geometry and aspect ratio,  $a_i/c_i$  and  $a_i/d$ , were evaluated as then input parameters for the next step. The values for  $K$ ,  $\Delta K$  and load ratio  $R$  at points A and C were then determined by the analytical approaches present in this study. For determination of the crack growth rate per cycle ( $da/dN - \Delta K$  curve), the so-called NASGRO [16] equation were used. The fatigue crack growth properties of A1N, axle material, were extensively studied at [17], and the regarding parameters and more detail about application of NASGRO equation can be find there.

The results of crack propagation analyses for two different crack configurations located in the T-notch under load spectra, is demonstrated in Fig. 8. It can be seen that, FE solution estimates the shortest residual lifetime for all the crack configurations, and the adopted solution by Wang-Lambert weight function gives the closest result to the FE prediction. The error in residual lifetime estimation raises significantly when the evaluated SIF for majority of the load blocks are in the vicinity of the threshold value. This is due to the fact that, the threshold value determines the number of damaging loading cycles from the load spectrum, as a result in the case of SIF underestimation, these load blocks will not contribute to the crack propagation, which will lead to overestimation in residual lifetime prediction or vice versa. It is worth to mention that, 8% load spectra reduction in FE solution, covers all the possible errors in estimating the residual lifetime by using analytical SIF solutions. From the crack shape evaluation point of view, the results of Wang-Lambert solution match with the FE results. For a same load spectra regardless of initial crack shape, the estimated crack shape by Varfolomeev and pommier tends to reach semi-circular shape, while in the Wang-Lambert suggest a semi-elliptical shape before fracture, as it does in the real case.

In particular case, in order to make comparison between the estimated crack shape evolution with available experimental results in the literature [18], a simulation were carried out for a fictitious crack by the depth of  $a = 0.94mm$  and crack length of  $c = 1.21mm$ , located in the smooth part of a small scale axle with diameter  $d = 55mm$  made of A1N material and subjected to the constant bending moment of  $S_{nom} = 260Mpa$ .

The results in the Fig. 9 shows that, despite the fact that, the accuracy of estimated SIF by adopting Raju-Newman and Carpinteri solutions are lower than the Wang-Lambert method in compare with FE analysis but, at the early stages

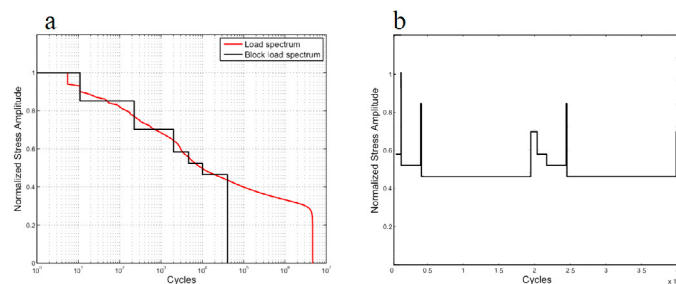


Fig. 7. applied load spectra derived from in-service load [15]; a) Normalized block load spectrum; a) applied block load sequence

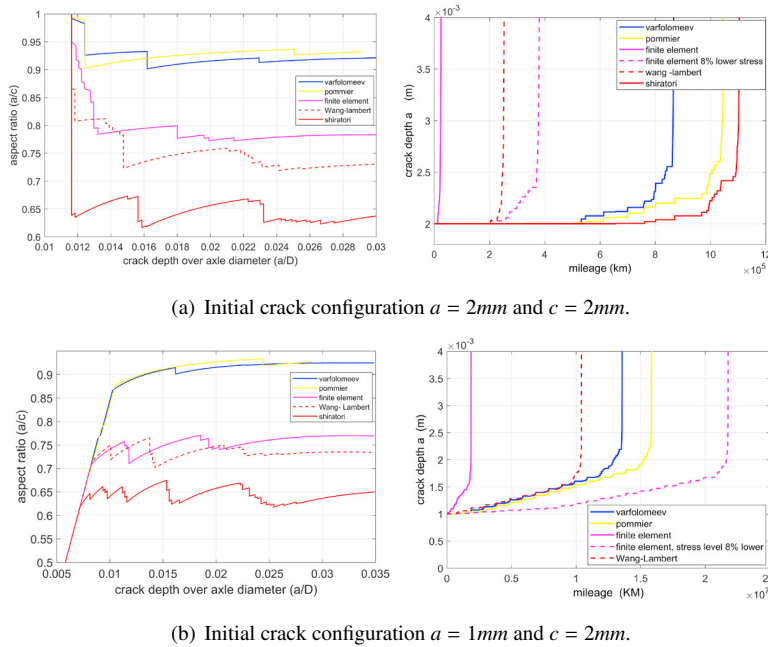


Fig. 8. Crack shape evaluation and corresponding residual lifetime estimation under applied load spectra

of crack propagation, the crack shape evolution obtained by these solutions is conformed with the experimental results, however, by the crack extension it tends to follow the shape, which is predicted by FE and Wang-Lambert solutions.

The crack propagation simulation. It should be noted that, since the impact of compression residual stress induced by the manufacturing process in outer surface of the axle, were neglected in this simulation, it can be concluded that the effect of this stress field is dominant in the crack shape evolution, specially in the early stages of short crack propagation. It is also worth to remark that, the influence of SIF error on estimating residual lifetime for the cracks subjected to constant load amplitude is not pronounced.

### 5. Conclusion

The precision of the available analytical SIF solutions for the semi-elliptical surface cracks, applicable in the geometrical transitions and axle body of railway axle, were investigated. Three dimensional finite element analysis have been conducted to calculate the exact value of stress intensity factor for wide range of crack configurations

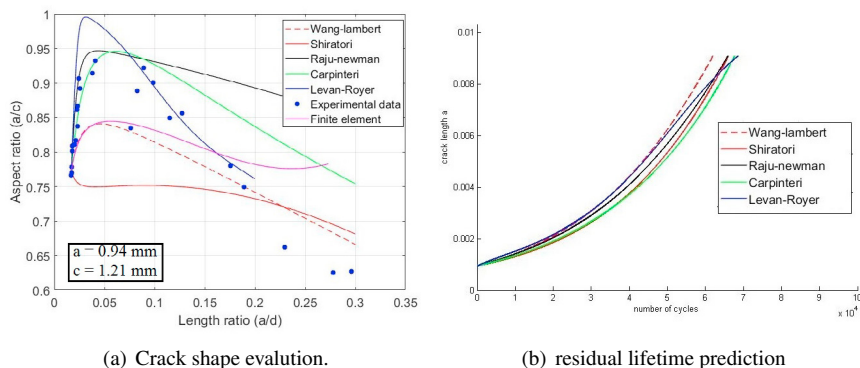


Fig. 9. Crack shape evaluation comparison between numerical prediction and experimental results

as reference solution. The stress intensity factor were estimated applying analytical solutions for two different non-linear stress state (bending, press-fit). Comparing the results with finite element solutions revealed that Wang-Lambert weight function gives the best approximation where the maximum difference was found to be less than 5% for the deepest point and 8% for the surface point.

The impact of stress intensity factor approximation in residual lifetime prediction were investigated through series of crack propagation simulations considering realistic load spectra and a comparison with the available experimental results in the literature. The error in residual lifetime estimation raises significantly when the evaluated SIF for majority of the load blocks are in the vicinity of the threshold value. It is worth to mention that, 8% load spectra reduction in FE solution, covers all the possible errors in estimating the residual lifetime by using analytical SIF solutions. Also a good agreement between the prediction and experimental results, was found with respect to crack shape evolution.

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