Straight running – stability analysis with a driving simulator

dott-ing Danilo BIGGIO
Politecnico di Milano, now with Ferrari GES

dr-ing Fabio DELLA ROSSA
Politecnico di Milano

dott-ing Marco FAINELLO
Danisi Engineering

prof dr-ing Giampiero MASTINU
Politecnico di Milano

Abstract

The straight running of the system composed by a car plus driver is studied. Straight running is an important case study for analysing stability. Despite the lateral slip angles of the tyres are small, the system is highly non linear, due essentially to the driver action. Following the simple model of McRuer, later developed by Mistschke and revised by many other authors, we have developed a mathematical model of a car plus driver. The dynamic behaviour of the mathematical model has shown the presence of limit cycles generated by so called Hopf-bifurcations. The mathematical model predicts that, despite the understeering vehicle is globally stable, the driver can make the whole system (car plus driver) unstable. This occurs in case an external disturbance is sufficiently strong. If the external disturbance is small, the understeering vehicle plus driver remains stable. There is a speed above which the understeering car plus driver is unstable, usually such a speed is much greater than the maximum speed of the car on high grip surface. The statements introduced above have been validated by employing the driving simulator of Danisi Engineering, Nichelino, Italy. We experimentally saw that limit cycles do exist and that the driver can make the understeering vehicle model of the simulator quite unstable. We were able to validate the mathematical model by including two humans in the driving loop. One driver was a professional driver, the other one was a novice. The same non linear behaviours were highlighted for the two drivers, however, the amplitudes of the limit cycles and the ability of controlling the car were higher for the professional driver. A question arises whether an electronic power steering (EPS) may reduce or cancel instability. The answer is that there are a number of possible solutions for ESP to counteract the effect of unstable limit cycle.

Introduction

The paper focuses on straigh running of cars on an even or slightly uneven road. In particular stability is studied.

Common drivers feels that a car, running straight ahead, may become unstable if a disturbance of sucient level occurs. To describe accurately such an intuitive fact, an in-depth mathematical reasoning is needed, which addressed in the paper.

Academic books deal with the stability of cars referring to linearized models [10, 25]. We introduce a linear model of a car providing analytical formuale giving the front and rear lateral slips, the yaw angle and the steering angle [38] which is needed to run straight ahead. Road vehicles run straight with a side-slip angle, which is due to some inherent tyre characteristics, namely ply-steer and conicity. Additionally, road banking acts to deviate sideways the straight motion of the vehicle. Normally drivers, to run straight, have to slightly steer and to apply a torque at the steering wheel. The influence of tire ply-steer and conicity on straight motion have been dealt with by Lindenmuth [39] who indicated the existence of a relationship between steering wheel rotation during straight running and tyre conicity and ply-steer . Topping [40] stated that vehicle pull or steering wheel rotation is originated by tyres. A sound influence on Ply-steer Residual Aligning Torque (PRAT) by tire tread was highlighted experimentally by Matyja [41]. The mentioned tyre effects are just the very basic issues referring to vehicle stability since they influence the linearization of the equations of motion at steady state running.

The linearization of the equations of motion is the very preliminary step for stability analysis. Actually, studying stability by considering a limited disturbance is misleading. In other words, classical academic hypotheses on straight running stability of cars cannot provide any prediction if the disturbance is strong. We resorted to Bifurcation theory to allow a proper understanding of the phenomenon, as shown in [8, 6, 5, 7, 18, 26, 27, 34]. In the paper we provide an early experimental substantiation of bifurcation theory referring to straight running stability of cars.

In the literature, a number of papers have been written on bifurcations of road vehicle models [8, 3, 13, 15, 17, 22, 30, 31, 35, 36]. We focus here on the influence of an actual driver in the loop.

The paper is structured as follows. At first the linearized expressions of relevant kinematics characteristics of straight running of cars are given. Then the nonlinear model of a simple car and driver is introduced to provide an easy dealing with bifurcation theory. Then actual experiments at Danisi driving simulator are given and interpreted accoring with bifurcation theory. The behaviours of a novice driver and that of a professional driver are compared. The possibility of solving stability problems by an Electronic Power Steering system (EPS) are discussed.

1 Steady-state straight ahead running

In Fig.1 a simple model of a car is depicted. Geometric parameters and the forces acting at the tyres are reported.

Let us consider tyre caracteristics, which read

$$F_{yF} = C^{*}_{eff,F}\alpha_F + F_{y0,F} \tag{1.1}$$

$$F_{yR} = C^{*}_{eff,R}\alpha_R + F_{y0,R} \tag{1.2}$$

$$M_{zF} = -C_{M\alpha,F}\alpha_F + M_{z0,F} \qquad (1.3)$$

$$M_{zR} = -C_{M\alpha,R}\alpha_R + M_{z0,R} \qquad (1.4)$$

where $C^*_{eff,F}$ and $C^*_{eff,F}$ are the effective axle cornering stiffnesses [24, 27], front or rear, respectively. The offsets due to ply-steer and conicity are $F_{y0,F}$, $F_{y0,R}$, $M_{z0,F}$, $M_{z0,R}$.

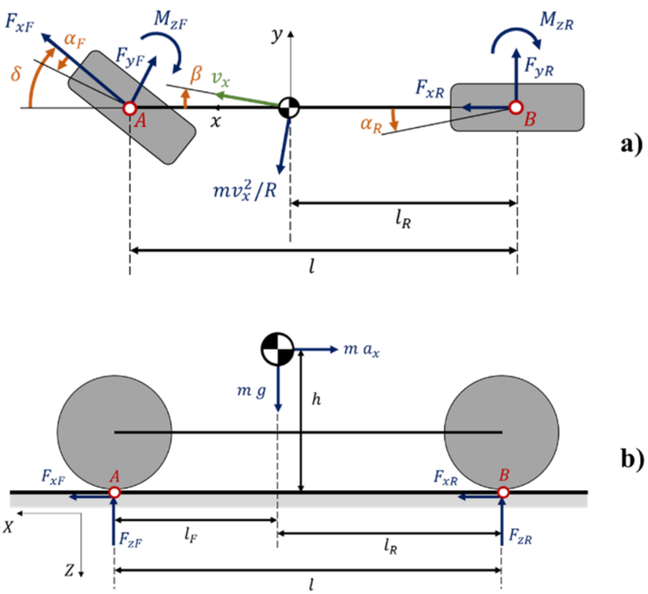


Fig.1. Simple model of a car, a): top view, b): lateral view

After [39], it is possible to derive analytically the lateral slips at the front and rear axle, α_F and α_R, respectively. Such lateral slips are given as funcion of lateral acceleration (divided by gravity) $v_x^2/R\,g = a_y/g$, yielding to Eqs.1.5-1.10

$$\alpha_F = K_{\alpha F} \cdot \left(\frac{a_y}{g} - \varphi_r\right) + \alpha_{F,st} \qquad (1.5)$$

$$K_{\alpha F} = \frac{lF_{zF}C^*_{eff,R} + C_{M\alpha,R}mg}{lC^*_{eff,F}C^*_{eff,R} - C_{M\alpha,F}C^*_{eff,R} + C_{M\alpha,R}C^*_{eff,F}} \approx \frac{F_{zF}}{C^*_{eff,F}} \qquad (1.6)$$

$$\alpha_{F,st} = \frac{-C^*_{eff,R}\left(M_{z0,F}+M_{z0,R}+lF_{y0,F}\right)-C_{M\alpha,R}\left(F_{y0,F}+F_{y0,R}\right)}{lC^*_{eff,F}C^*_{eff,R}-C_{M\alpha,F}C^*_{eff,R}+C_{M\alpha,R}C^*_{eff,F}} \approx -\frac{F_{y0,F}}{C^*_{eff,F}} \quad (1.7)$$

$$\alpha_R = K_{\alpha R} \cdot \left(\frac{a_y}{g} - \varphi_r\right) + \alpha_{R,st} \quad (1.8)$$

$$K_{\alpha R} = \frac{lF_{zR}C^*_{eff,F} - C_{M\alpha,F}mg}{lC^*_{eff,F}C^*_{eff,R} - C_{M\alpha,F}C^*_{eff,R} + C_{M\alpha,R}C^*_{eff,F}} \approx \frac{F_{zR}}{C^*_{eff,R}} \quad (1.9)$$

$$\alpha_{R,st} = \frac{C^*_{eff,F}\left(M_{z0,F}+M_{z0,R}-lF_{y0,R}\right)+C_{M\alpha,F}\left(F_{y0,F}+F_{y0,R}\right)}{lC^*_{eff,F}C^*_{eff,R}-C_{M\alpha,F}C^*_{eff,R}+C_{M\alpha,R}C^*_{eff,F}} \approx -\frac{F_{y0,R}}{C^*_{eff,R}} \quad (1.10)$$

Given the analytical expressions of α_F and α_R, the vehicle side-slip angle β

$$\beta = -\alpha_R + l_R/R \quad (1.11)$$

and the steering angle δ

$$\delta = l/R - \alpha_F + \alpha_R \quad (1.12)$$

can be derived [24, 25, 27].

In [39] the validation of the above formulae is discussed with satisfactory results.

2 Models of vehicles and driver model

Let us now introduce the models that have been used to assess the nonlinear behaviour of car and driver. We introduce a simple car model and a simple driver model to deal with Bifurcation theory easily. Then we use a full nonlinear model to be implementented into the driving simulator and use it with a humand driver in the loop.

2.1 Simple car and simple driver model

The equations of motions of the system in Fig. 2 read

$$\left\{\begin{array}{rcl} \dot{v} & = & \frac{1}{m}(F_{y_f}+F_{y_r})-ur \\ \dot{r} & = & \frac{1}{J}(aF_{y_f}-bF_{y_r}) \\ \dot{\delta} & = & \frac{1}{\tau}(-\delta+ke+k_d\dot{e}) \\ \dot{y}_G & = & u\sin(\psi)+v\cos(\psi) \\ \dot{\psi} & = & r \end{array}\right. \quad (2.1)$$

where the symbols are reported in Fig.2.

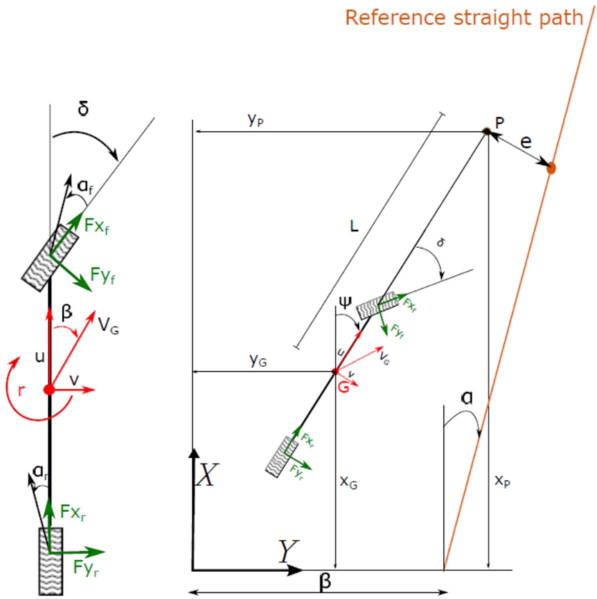


Fig.2. Simple vehicle and simple driver model.

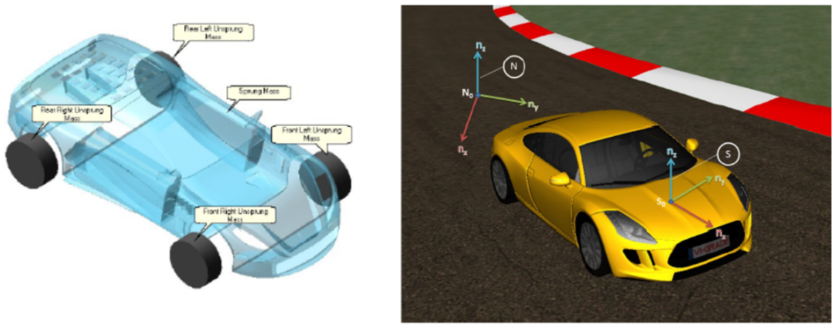
The tyre characteristic is nonlinear, the vehicle is understeering. The tyre data are reported in Tab.1.

Tab.1 Data of Magic Formulae of tyre characteristics

	B	C	E	D
Front	6.92	2.34	0.83	9493.94
Rear	10.31	2.30	1.02	9805.56

2.2 Complex car model

In order to let the human driver at the driving simulator interact with a close-to-reality car model, the VI-Grade model of a car has been introduced. The same tyre characteristics and chassis parameters of the simple model introduced in the previous section were adopted. Basically, the complex car model is a 14 degrees of freedom multi-body model, in which the bodies are the vehicle chassis and the four wheels. The car body has six degrees of freedom and each wheel has two degrees of freedom, one corresponding to the rolling rotation and one corresponding to the relative displacement between the wheel and the car body. Two reference systems are used, one global, fixed with respect to the ground and one local, fixed with the vehicle, with the origin fixed at the mid point of the front axle, the longitudinal axis is parallel to the vehicle centreline, the normal axis is orthogonal to the ground and directed upwards. The lateral axis constitutes a right-hand reference system with the other mentioned axes. The reference systems are represented in Figure 3. The directions of both the lateral and vertical axes of the simple car with simple driver model are reversed with respect to the corresponding axes of the complex car model. The results will be reported using the reference systems of the simple car with simple driver model.



(a) 14 dof vehicle model

(b) Reference systems

Fig.3. VI-Grade Complex model and reference systems.

3 Nonlinear behaviour of simple car model with simple driver model – Bifurcations

Let us focus on the simple car model with simple driver model. We consider Eq. (2.1). We do not integrate the equations of motions since this would be tedious. We are interested in the limit value that a disturbance may reach to induce a spin of a car initially running straight ahead. We derive the bifircation diagram by a proper continuation alorithm [42,43]. By applying Matcont [42,43], the bifurcation diagram in Fig.4 is found. In Fig. 4 (left figure) we report a three-dimensional projection of the bifurcation diagram of the simple car model with simple driver model. The mathematical model is described by Eq. (2.1) and has 5 state variables. In Fig.4, solid blue lines indicate equilibria which are stable. They could be either steady-state or limit cycles. Dashed red lines indicate unstable equilibria or unstable limit cycles. We see that, increasing the forward speed, a bifurcation speed can be encountered. At longitudinal speeds lower than the bifurcation one, an unstable limit cycle exists in the state-space. The unstable limit cycle repulses the state space trajectories. The motion of the vehicle can be repulsed towards the straight running equilibrium or can diverge, depending on the initial state-space condition. In Fig. 4 (right figure) the maximum lateral error with respect to the reference path of the unstable limit cycle has been reported. The amplitude increases by decreasing the speed, so at lower speeds the perturbation able to make the response of the vehicle unstable has to be very high, as common sense suggests. The lateral position disturbance or the yaw angle disturbance are initial states for the model.

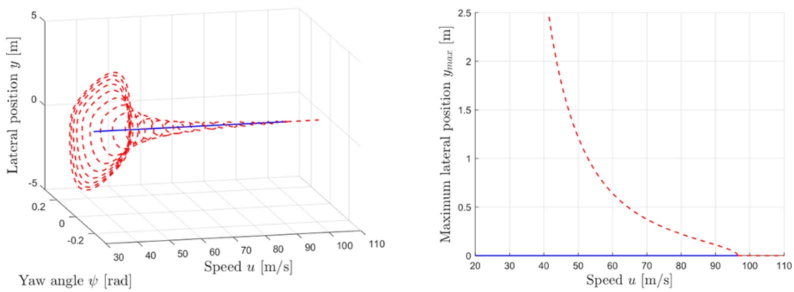


Fig.4. Bifurcation diagram.

4 Straight running experiments at the driving simulator

We checked whether the theoretical results reported in Fig.4 are close to reality. Actually the combination of the simple vehicle model with the simple driver model may provide false information. The driving simulator at Danisi Engineering (located in Nichelino, Italy) has been used (Fig.5). The complex model described in Sect. 2.2 has been employed.

The experimental tests at the driving simulator were perfomed by simulating the straight running of a car. The car was subject to a lateral force acting at a certain distance from the centre of gravity. Such disturbance corresponds to initial states of the model in Sect.2.1. Namely lateral position disturbance and/or yaw angle disturbance are a consequence of the lateral force applied at the car body.

gianpiero.mastinu@polimi.it

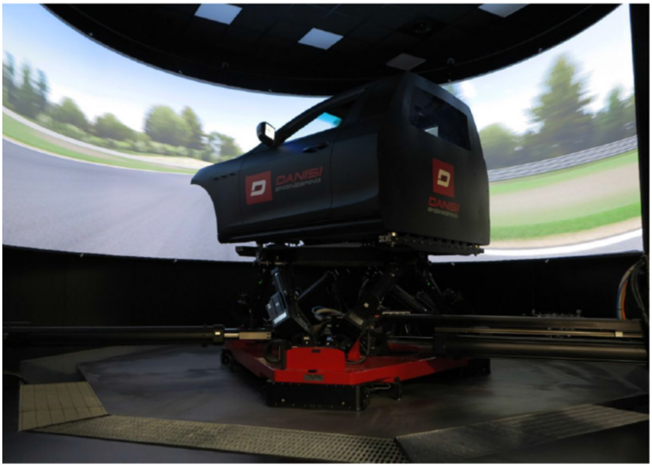


Fig. 5. Danisi Driving Simulator

Similar results with respect to the ones shown in Fig.4 have been found at the Danisi Driving Simulator, the results are reported in Fig.6. Fig.6 is to be compared with Fig.4. The stable or unstable responses of the vehicle and driver are reported, as function of the disturbance parameter, namely the moment arm of the lateral force applied at the vehicle body as described above. The force is 10000N.

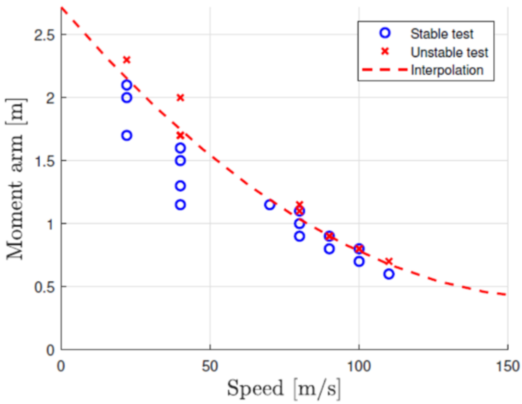


Fig. 6. Stable and unstable vehicle and driver motion as recored at the Danisi Driving Simulator, corresponding to the bifurcation diagram in Fig.4.

gianpiero.mastinu@polimi.it

We see that the level of disturbance is crucial to define a stable or an unstable behaviour. We can state that qualitativeli the simple car model with the simple drive model do predict what is experienced in a driving simulator. The question whether the driving simulator is close to reality is a topic for further studies, generally the outomes form a driving simulator are trusted as reasonable. We notice that testing stability at high speed may be quite unpractical and even impossible due to the safety issues that are implied.

5 Comparison of experienced driver with novice driver

While performing experiments at the driving simulator we did compare an experienced driver with a novice driver. This was made to avoid a bias. We did want to obtain results as general as possible. To make the control more tricky, we made the vehicle oversteering by switching the front with the rear tyres (see Tab.1).

In Fig.7 the driving activity of the novice driver is reported. The speed is increased up to more than 100m/s. At t=45s and speed=47m/s the bifurcation speed is reached. The system composed by car and driver becomes unsteady and undergoes into a chaotic motion. We see that the lateral displacement is very big. The frequency content of the steering angle is concentrated in the neighbourhood of 0.4Hz. The driver is not able to suppress the chaotic motion.

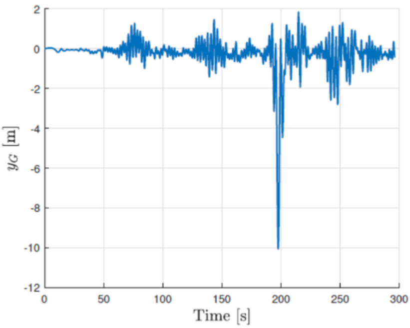
In Fig.8 the driving activity of the experienced driver is reported. Again, the speed is increased up to more than 100m/s. At t=120s and speed=47m/s the bifurcation speed is reached. Also the experienced driver is not able to suppress the chaotic motion that arises.

Let us compare the driving activity of the experienced driver with the one of the novice driver. We notice that the experienced driver may control the steering wheel angle not only in the range nearby 0.4 Hw but also in the range from 1Hz to 1.5Hz. This changes the time history but the amplitudes of both lateral position and yaw rate are still quite dangerous.

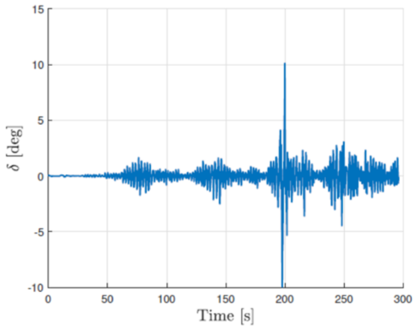
6 Role of EPS on stability of straight running car

We address the problem of avoiding instability by employing an EPS in the driving loop. We only address the problem of exixtence of a solutions. We cannot provide here a solution that would take a proper study to be developed. We just suggest a future topic for research.

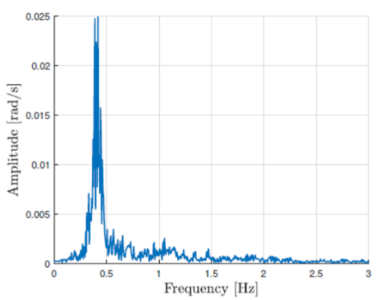
The theoretical hypothesis, still to be confirmed, is that the EPS, in case of an hard disturbance, recognizes the occurrence and takes over the driver control. In [44] such control transfer was studied. With this basic assumpion, in Eq.(2.1) two alternative or combined controls could be added.



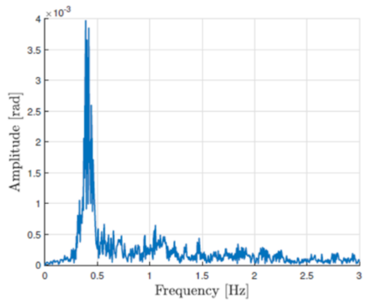
(a) Lateral position



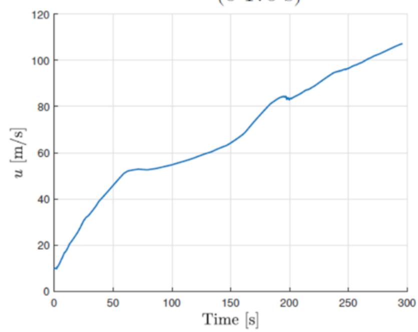
(b) Steering angle



(c) Frequency spectrum: Yaw rate (0-170 s)



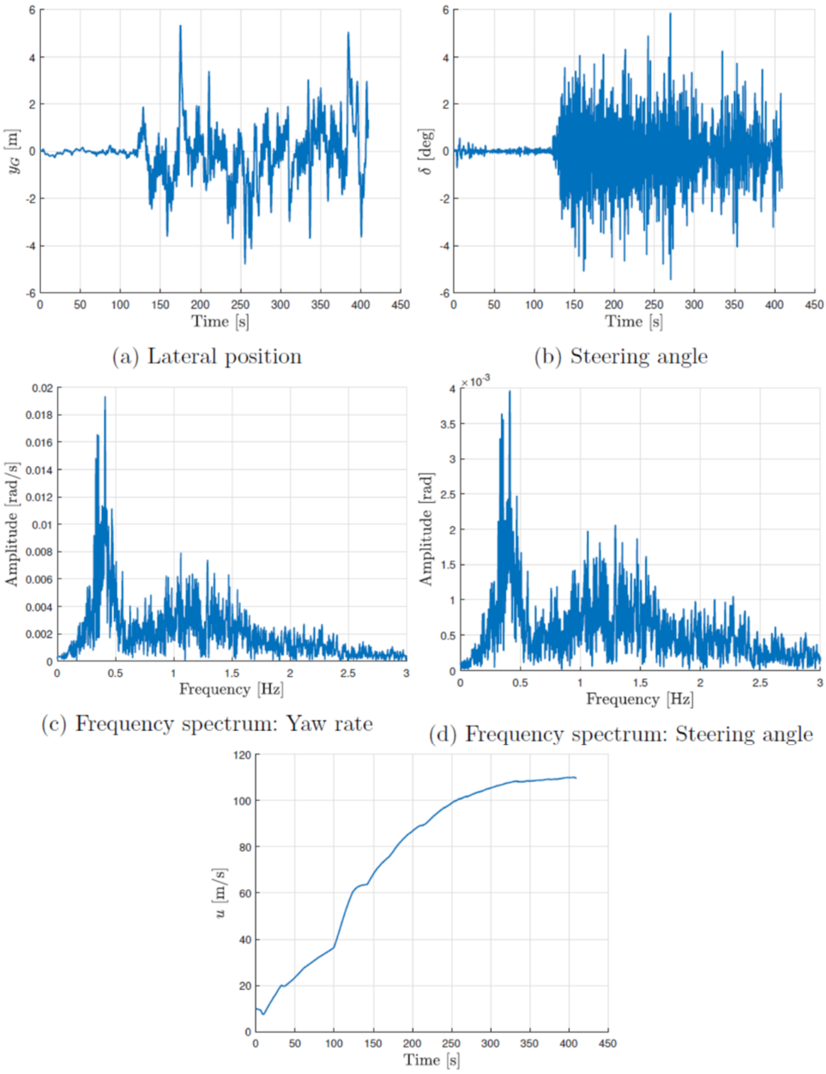
(d) Frequency spectrum: Steering angle (0-170 s)



(e) Longitudinal speed

Fig.7. Novice driver with an oversteering vehicle running straight, driving simulator results. The speed is increased up to more than 100m/s. At t=45s and speed=47m/s the bifurcation speed is reached. The system car and driver become unsteady undergoing into a chaotic motion.

gianpiero.mastinu@polimi.it



(a) Lateral position

(b) Steering angle

(c) Frequency spectrum: Yaw rate

(d) Frequency spectrum: Steering angle

(e) Longitudinal speed

Fig.8. Experienced driver with an oversteering vehicle running straight, driving simulator results. The speed is increased up to more than 100m/s. At t=120s and speed=47m/s the bifurcation speed is reached. The system car and driver becomes unsteady undergoing into a chaotic motion.

gianpiero.mastinu@polimi.it

The first control aims at destroying the unstable limit cycle nearby the critical or bifurcation speed. The normal form of the bifurcation, in case of a subcritical Hopf, reads

$$\dot{x_1} = px_1 - x_2 + x1(x_1^2 + x_2^2)$$
$$\dot{x_2} = x_1 + px_2 + x_2(x_1^2 + x_2^2)$$

in which x_i (i=1,2) and p are proper combinations of system's states and system's parameters, respectively.

By introducing non linear control terms in Eq.7.1 that make vanish the non linear part, namely

$$x1(x_1^2 + x_2^2)$$
$$x_2(x_1^2 + x_2^2)$$

we could inherently destroy the limit cycle. This is obviuosly a very preliminary hint for a future research.

The seconnd control may be more easy to be implemented and refers to adding a more strong derivative (damping) coefficient to the steering control in Eq. 2.1, in other words we increase the value of the parameter k_d in Eq.2.1.

In Fig. 9 the bifurcation or critical speed is plotted as function of both the driver's delay time and the parameter k_d. For normal driver's lag time (0,14÷0,20, [25]) we see that increasing k_d make the the bifurcation or critical speed increase. This implies also a considrable increase of the amplitude of the unstable limit cycle, which corresponds to improved stability.

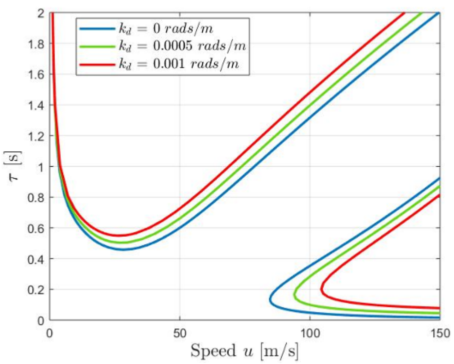


Fig. 9. Bifurcation or critical speed as function of the driver's delay time τ and of the steering control derivative coefficient k_d . Such a coefficient may be associated not only to the driver action but also to ESP.

7 Conclusion

Bifurcation theory is able to describe the the stabiliy of the system vehicle plus driver. Actually, by said theory, we can explain why a car looses stability due to a disturbance of sufficient extent. A mathematical modelling of the driver's behaviour is still missing, so the matching between bifurcation theory (i.e. simulation by nonlinear models) and actual driving simulator results cannot be given in a quantitative sense. Despite this inherent limitation, we highlighted that the source of instability is related to a disturbance the car is subjected. This sheds some light on a number of issues related to active safety and on the design of future autonomus vehicles. We have investigated the influence of driver's experience on the ability to drive after the bifurcation speed has been reached. We discovered that, despite the experienced driver is more effective than the novice one in controlling the steering angle, still the chaotic motion, and thus instability, could not be supressed.

It seems that the crucial problem of straight running stability could be studied at the driving simulator. Actual experiments seem too dangerous to be perfomed on a track. We also tried to answer the question whether or not EPS could solve the problem of stability. We just addressed the exixtence of a solution. Providing an actual solution is out of the scope of this paper. There are two possible control schemes that could be used, either single or combined, to fight against instability. One could be effective nearby the critical or bifurcation speed, the other could be effective at any speed. The development of such controls is here proposed for further studies.

Literature

[1] Danisi engineering, via ippolito nievo 62, nichelino (to) { italy. www.danisieng.com.

[2] Vi-grade: Driving simulation | dynamic driving simulator. www.vi-grade.com/en/products/vi-carrealtime/.

[3] C. Bobier-Tiu, C. Beal, J. Kegelman, R. Hindiyeh, and J. Gerdes. Vehicle control synthesis using phase portraits of planar dynamics. Vehicle System Dynamics, 2018.

[4] B. Catino, S. Santini, and M. Di Bernardo. Mcs adaptive control of vehicle dynamics: an application of bifurcation techniques to control system design. In 42nd IEEE International Conference on Decision and Control, volume 3, pages 2252-2257. IEEE, 2003.

[5] F. Della Rossa, M. Gobbi, G. Mastinu, C. Piccardi, and G. Previati. Bifurcation analysis of a car and driver model. Vehicle System Dynamics, 52(sup1):142-156, 2014.

[6] F. Della Rossa and G. Mastinu. Analysis of the lateral dynamics of a vehicle and driver model running straight ahead. Nonlinear Dynamics, 92(1):97-106, 2018.

[7] F. Della Rossa and G. Mastinu. Straight ahead running of a nonlinear car and driver model-new nonlinear behaviours highlighted. Vehicle system dynamics, 56(5):753-768, 2018.

[8] F. Della Rossa, G. Mastinu, and C. Piccardi. Bifurcation analysis of an automobile model negotiating a curve. Vehicle System Dynamics, 50(10):1539-1562, 2012.

[9] A. Dhooge, W. Govaerts, and Y. A. Kuznetsov. Matcont: a matlab package for numerical bifurcation analysis of odes. ACM Transactions on Mathematical Software (TOMS), 29(2):141-164, 2003.

[10] T. D. Gillespie. Fundamentals of vehicle dynamics. Technical report, SAE, 1992.

[11] J. Guckenheimer and P. Holmes. Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields. Springer, 1983.

[12] K. Guo and H. Guan. Modelling of driver/vehicle directional control system. Vehicle System Dynamics, 22(3-4):141-184, 1993.

[13] Z. Hao, L. Xian-sheng, S. Shu-ming, L. Hong-fei, G. Rachel, and L. Li. Phase plane analysis for vehicle handling and stability. International journal of computational intelligence Systems, 4(6):1179-1186, 2011.

[14] H. Hu, Z. Gao, Z. Yu, and Y. Sun. An experimental driving simulator study of unintentional lane departure. Advances in Mechanical Engineering, 9(10):1687814017726290, 2017.

[15] H. Hu and Z. Wu. Stability and hopf bifurcation of four-wheel-steering vehicles involving driver's delay. Nonlinear Dynamics, 22(4):361-374, 2000.

[16] Y. A. Kuznetsov. Elements of applied bifurcation theory, volume 112. Springer Science & Business Media, 2013.

[17] D.-C. Liaw, H.-H. Chiang, and T.-T. Lee. Elucidating vehicle lateral dynamics using a bifurcation analysis. IEEE Transactions on Intelligent Transportation Systems, 8(2):195-207, 2007.

[18] Z. Liu and G. Payre. Global bifurcation analysis of a nonlinear road vehicle system. Journal of Computational and Nonlinear Dynamics, 2(4):308{315, 2007.

[19] Z. Liu, G. Payre, and P. Bourassa. Nonlinear oscillations and chaotic motions in a road vehicle system with driver steering control. Nonlinear Dynamics, 9(3):281-304, 1996.

[20] Z. Liu, G. Payre, and P. Bourassa. Stability and oscillations in a time-delayed vehicle system with driver control. Nonlinear Dynamics, 35(2):159-173, 2004.

[21] C. C. Macadam. Understanding and modeling the human driver. Vehicle System Dynamics, 40(1-3):101-134, 2003.

[22] G. Mastinu, F. Della Rossa, and C. Piccardi. Nonlinear dynamics of a road vehicle running into a curve. In Applications of Chaos and Nonlinear Dynamics in Science and Engineering-Vol. 2, pages 125-153. Springer, 2012.

[23] G. Mastinu, A. Lattuada, and G. Matrascia. Straight-ahead running of road vehicles analytical formulae including full tyre characteristics. Vehicle System Dynamics, in press, 2018.

[24] G. Mastinu and M. Ploechl. Road and off-road vehicle system dynamics handbook. CRC press, 2014.

[25] M. Mitschke and H. Wallentowitz. Dynamik der kraftfahrzeuge, volume 4. Springer, 1972.

[26] E. Ono, S. Hosoe, H. D. Tuan, and S. Doi. Bifurcation in vehicle dynamics and robust front wheel steering control. IEEE transactions on control systems technology, 6(3):412-420, 1998.

[27] H. Pacejka. Tire and vehicle dynamics. Elsevier, 2005.

[28] M. Ploechl and J. Edelmann. Driver models in automobile dynamics application. Vehicle System Dynamics, 45(7-8):699-741, 2007.

[29] G. Reymond, A. Kemeny, J. Droulez, and A. Berthoz. Role of lateral acceleration in curve driving: Driver model and experiments on a real vehicle and a driving simulator. Human factors, 43(3):483-495, 2001.

[30] S. Shen, J. Wang, P. Shi, and G. Premier. Nonlinear dynamics and stability analysis of vehicle plane motions. Vehicle System Dynamics, 45(1):15-35, 2007.

[31] S. Shi, L. Li, X. Wang, H. Liu, and Y. Wang. Analysis of the vehicle driving stability region based on the bifurcation of the driving torque and the steering angle. Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering, 231(7):984-998, 2017.

[32] S. H. Strogatz. Nonlinear Dynamics and Chaos with Student Solutions Manual: With Applications to Physics, Biology, Chemistry, and Engineering. CRC Press, 2018.

[33] S. Tousi, A. Bajaj, and W. Soedel. Finite disturbance directional stability of vehicles with human pilot considering nonlinear cornering behavior. Vehicle System Dynamics, 20(1):2155, 1991.

[34] H. True. On the theory of nonlinear dynamics and its applications in vehicle systems dynamics. Vehicle System Dynamics, 31(5-6):393-421, 1999.

[35] X. Wang and S. Shi. Analysis of vehicle steering and driving bifurcation characteristics. Mathematical Problems in Engineering, 2015, 2015.

[36] X. Wang, S. Shi, L. Liu, and L. Jin. Analysis of driving mode e
ect on vehicle stability. International journal of automotive technology, 14(3):363-373, 2013.

[37] D. H. Weir. Models for steering control of motor vehicles. In Proceedings, Fourth Annual NASA-University Conference on Manual Control, 1968, pages 135-169. US Government Printing Office 1968

[38] A Lattuada, G Mastinu, G Matrascia, Tire Ply-Steer, Conicity and Rolling Resistance – Analytical Formulae for Accurate Assessment of Vehicle Performance during Straight Running, Technical Paper 2019-01-1237 ISSN 0148-7191, DOI: https://doi.org/10.4271/2019-01-1237, Published April 2, 2019 by SAE International in United States

[39] Lindenmuth BE. Tire conicity and ply steer effects on vehicle performance. SAE Technical Paper 740074. 1974

[40] Topping RW. Tire induced steering pull. SAE Technical Paper 750406. 1975

[41] Matyja FE. Steering pull and residual aligning torque. Tire Science and Technology. 1987 Jul-Sep;15(3):207–240

[42] Y. A. Kuznetsov.Elements of applied bifurcation theory, volume 112. Springer Science &Business Media, 2013

[43] A. Dhooge, W. Govaerts, and Y. A. Kuznetsov. Matcont: a matlab package for numerical bifurcation analysis of odes. ACM Transactions on Mathematical Software (TOMS),29(2):141–164, 2003.

[44] K Yoshimoto, X Tang, Research of Driver Assistance System for Recovering Vehicle Stability from Unstable States, SAE Technical Paper2001-01-1276, DOI: https://doi.org/10.4271/2001-01-1276

gianpiero.mastinu@polimi.it