Extracting fault classification rules from fuzzy clustering

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ABSTRACT: The practical applicability of a diagnostic tool demands the physical transparency of the underlying models, for the interpretation of the relationships between the involved variables, for direct inspection and for validation. In this work, a methodology is developed for transforming an opaque, fuzzy clusteringbased classification model into a fuzzy logic model based on transparent linguistic rules. This is obtained by introducing appropriate constraints onto the fuzzy input partitioning interface. The methodological approach is applied to a diagnostic task concerning the classification of simulated nuclear transients in the feedwater system of a Boiling Water Reactor.

1 INTRODUCTION

The detection and classification of anomalies and faults in nuclear components are important tasks for their safe and efficient operation. Conceptually, the basis for performing these tasks is that different system faults initiate different patterns of evolution of the interested variables, measured by properly located sensors. The diagnostic problem then becomes one of pattern classification, i.e. association of the different patterns of evolution to the different classes of system faults.

Extensive research is being carried out in the investigation of fuzzy clustering techniques for pattern classification (Klawonn F., Kruse R., 1995), (Botzheim J. et al., 2001), (Zio E., Baraldi P., 2005). These techniques have proven effective also in fault classification tasks (Yang M., 1993), (Dunn J., 1974), (Gustafson D.E., Kessel, W.C., 1979), (Zio E., Baraldi P., 2005), (Reifman J., 1997), (Goddu G. et al., 1998) but often remain "black boxes" as to the interpretation of the physical relationships underpinning the classification. On the other hand, the applicability in practice of a diagnostic tool requires the physical transparency of the underlying models, for the interpretation of the relationships between the involved variables, for direct inspection and for validation.

In the present work, a method for manipulating clusters of labelled data to extract transparent and easily interpretable classification rules is proposed. The first stage of the development of the rule-based classification model amounts to finding clusters corresponding to different types of fault. This is done by processing pre-classified, labelled 'training' data by means of a supervised evolutionary possibilistic clustering scheme developed by some of the authors (Zio E., Baraldi P., 2005). Then, a fuzzy rule-based model is built by optimally partitioning the range of each input (the so called universe of discourse, UOD) to reflect the previously obtained clusters, with each fuzzy cluster inducing a fuzzy classification rule. The Fuzzy Sets (FSs) making up a rule corresponding to a fuzzy cluster are obtained by projecting the cluster onto the individual onedimensional coordinate axis of the involved variables (Klawonn F., Kruse R., 1995), (Sugeno M., Yasukawa T., 1993).

Section 2 sets the terminology and framework of fuzzy reasoning (Klir G.J., Yuan B., 1995). Section 3 illustrates the membership functions' (MFs) properties and semantic constraints which are introduced to achieve a transparent model and the pruning process introduced to "clean" the model. Section 4 reports the application of the approach to the classification of simulated nuclear transients in the feedwater system of a Boiling Water Reactor (BWR). A discussion concerning the advantages and limitations of the proposed approach is provided in the last Section.

2 FUZZY REASONING

The two key elements of fuzzy reasoning are the Fuzzy Rule Base (FRB) (or Knowledge Base, KB) and the fuzzy inference engine. The former consists of a set of *R if-then* rules. The generic *j*-th fuzzy rule, j = 1, 2, ..., R, is made up of a number of *ante-cedent* and *consequent* linguistic statements, suitably related by fuzzy *connections*:

$$R_{j}: if (x_{1} is X_{1j}) and \dots and (x_{n} is X_{nj}) then (y_{1} is Y_{1j}) and \dots and (y_{m} is Y_{mj})$$
(1)

The linguistic variables x_p , p = 1,...,n, are the antecedents, represented in terms of the FSs X_{pj} of the UOD U_{x_p} , with MFs $\mu_{X_{pj}}(x_p)$. The linguistic variables y_q , q = 1,...,m, are the consequents, represented by the FSs Y_{qj} of the UOD U_{y_q} , with MFs $\mu_{Y_{qj}}(y_q)$. The connective operator *and* links two fuzzy concepts and it is generally implemented by means of a *t-norm*, typically the minimum operator or the algebraic product. The rules of the FRB are joined by the connective *else*, generally implemented by means of an *s-norm*, typically the maximum operator (Klir G.J., Yuan B., 1995), (Zadeh, L.A., 1965).

The fuzzy inference engine receives the (linguistic) variables which constitute the Fact, viz.,

Fact: x_1 is X'_1 and ... and x_n is X'_n

where X'_p is a FS on the UOD U_{x_p} of the *p*-th linguistic input variable x_p , and compares it with the antecedents of the rules in the FRB to arrive at the Conclusion, viz.,

Conclusion: y_1 is Y'_1 and ... and y_m is Y'_m

where Y'_q is a FS on the UOD U_{y_q} of the q-th output variable y_q .

In the case of fault classification, the fuzzy inference engine i) receives as Fact the n values of the monitored variables, possibly fuzzyfied to account for measurement imprecision, ii) computes the 'strength' with which each of the R rules in the FRB is activated by the incoming input Fact and iii) properly combines the consequents of the rules, weighed by their respective strengths, to determine the output memberships to the different fault classes (Klir G.J., Yuan B., 1995).

3 OBTAINING A TRANSPARENT FUZZY RULE-BASED MODEL

In (Zio E., Baraldi P., 2005) a classifier based on fuzzy clustering has been proposed. The resulting classification model remains a 'black box', due to the difficulties of describing and interpreting the multi-dimensional FSs of the identified clusters in terms of rules antecedents.

The method here propounded to extract a transparent, rule-format FRB from the obtained fuzzy clusters proceeds according to the following 3 steps:

1. projection of the n-dimensional fuzzy clusters into n mono-dimensional FSs. According to the clustering classification algorithm presented in (Zio E., Baraldi P., 2005), each *n*dimensional training pattern \vec{x}_k , k = 1,...,N, is possibilistically classified by its membership $\mu_i(\vec{x}_k)$ to each class i = 1,...,c. This produces *c* clusters represented by an equal number of *n*-dimensional FSs, each of which can be projected onto the input variables as follows (Babuska R., Verbuggen H.B., 1995):

i. the mono-dimensional MFs of the antecedents FSs are generated by pointwise projection of the membership value $\mu_i(\bar{x}_k)$ onto the antecedent variables UODs (Klawonn F., Kruse R., 1995), (Babuska R., Verbuggen H.B., 1995), (Castellano G. et al., 2003), (Nelles O. et al., 1999) (Figure 1).



Figure 1. Projections of one 2-dimensional cluster onto the UODs of 2 antecedents x_1 and x_2 (abscissa: antecedent values; ordinate: membership value of the generic k-th pattern x_k to the cluster projection, k = 1, ..., N)

ii. the resulting non-convex MFs are transformed into convex MFs (Figure 2). To do this, starting from the smallest value of the antecedent x_p , only the membership of those values that have a higher membership than the previous one are kept, until the maximum membership value is reached (Sugeno M., Yasukawa T., 1993). Then, the same procedure is applied starting from the highest value of the antecedent, until the maximum MF is reached.



Figure 2. Approximation of the cluster projections of Figure 1 into convex non typical FSs

iii. the convex FSs are approximated by linear interpolation to MFs of trapezoidal shape (Figure 3). Before performing the linear interpolation, all membership values under a threshold (chosen to be 0.1 in the present work) are rounded off to 0 and analogously all membership values above an upper threshold (chosen to be 0.9 in the present work) are rounded off to 1.



Figure 3. The trapezoidal FSs corresponding to the cluster projections of Figure 2

By so doing, the *n*-dimensional FS X_i representing the *i*-th cluster is transformed into a fuzzy proposition of the kind:

if
$$(x_1 \text{ is } X_{1i})$$
 and ... and $(x_n \text{ is } X_{ni})$ (2)

where X_{pi} is the projection of cluster *i* onto the *p*-th input variable, *i* = 1, 2, ..., *c*, *p* = 1, 2, ..., *n*. Obviously, the method is approximate and some information on the cluster is inevitably lost in the projection, due to the decomposition error arising from projecting the multi-dimensional FS into its mono-dimensional constituents (Babuska R., Verbuggen H.B., 1995). On the other hand, it enables expressing the FRB in a form with a clear and interpretable semantic meaning.

2. enforcement of appropriate semantic constraints on the obtained FSs.

To achieve the physical interpretability of the model, semantic constraints are imposed to the FSs obtained in the previous step in an attempt to obtain an "op-timal" interface (De Oliviera J.V., 1999). This is sought through the procedure described below in which each of the FSs modifications required is actually carried out only if the classification performance on the training data is not significantly decreased.

i.<u>Pruning of FSs covering a large portion of the</u> <u>UOD</u>

Some FSs projections can turn out to be covering great portions of the variables UODs, adding little specific information to the model and overshadowing more focused FSs (Figure 4). Such sets can be removed from the antecedents of the rules (Babuska R., 1999).

The criterion for elimination of the FSs widely covering the UOD U_{x_p} is (Song B.G. et al., 1993):

$$\beta_o l_{X_{pi}} \ge U_{x_p}; p = 1, ..., n; i = 1, ..., c$$
 (3)

where $l_{X_{pi}}$ is the width at half height of the *i*-th FS X_{pi} of variable x_p and $\beta_o \ge 1$ is the so-called overlap parameter. The larger is the value of β_o , the more severe is the pruning criterion.



Figure 4. Overlapping MFs obtained from the clusters projection. The thick solid line in the left Figure denotes the FS to be pruned

The pruning of a FS modifies only the rules in which the FS appears as antecedent. The modification amounts to canceling from the antecedents the one corresponding to the eliminated FS.

ii. Addition of FS "nearly zero"

If the training data do not contain realizations from the class of no faults (stationary state), there is no cluster representing such situation and correspondingly no antecedents FSs and no rules. In this case, a new triangular FS called "nearly zero" is forced in the partition of the UOD U_{x_p} of each variable x_p . The new FS is centered in 0 and the zero-membership vertices are arbitrarily chosen equal to ± 0.1 of the minimum and maximum of the UOD U_{x_p} of the antecedent variable x_p , respectively. A rule tailored to stationary conditions can then be added to the FRB.

iii. Annihilation of narrow FSs

In order to avoid the overlapping among pairs of linguistic terms and the possible consequent semantic inconsistencies, it is necessary to have sufficiently distinct FSs. If a FS X_{pj} is too narrow (Figure 5), its contribution is too specific and model transparency is somehow lost. Annihilation of FS X_{pj} is performed if there is a FS X_{pi} for which the following criterion is satisfied (Babuska R., 1999):

$$l_{X_{pi}} \mu_{X_{pi}} \left(\frac{z_1 + z_2 + z_3 + z_4}{4} \right) \ge \beta_a l_{X_{pj}};$$

$$i = 1, ..., c; \ j = 1, ..., c; \ i \neq j$$
(4)

where $l_{X_{pi}}$ and $l_{X_{pj}}$ are the half-height widths of the FSs X_{pi} and X_{pj} of the same input variable x_p , $\beta_a \ge 1$ is the annihilation parameter and z_s , s = 1, 2, 3, 4, stands for the input variable values corresponding to the four vertices of a trapezoidal MF (Botzheim J. et al., 2001), (Juang C.F. et al., 1999), (Lin C.T., Lee C.S.G., 1994). The larger is the value of β_a , the more severe is the annihilation criterion (Botzheim J. et al., 2001), (Song B.G. et al., 1993).



Figure 5. Annihilation of a narrow FS (arrow)

The FRB is appropriately modified by replacing the canceled FS X_{pi} with the FS X_{pi} .

iv. Fusion of similar FSs

If two FSs describing the same variable are sufficiently overlapped, then they should be fused into a single FS because similar (Botzheim J. et al., 2001), (Song B.G. et al., 1993). Appropriate measures can be used in order to asses the pairwise similarity of the FSs in the FRB.

The similarity measure Ω of the two FSs X_{pi}

and X_{pi} here adopted is given by the ratio be-

tween the intersection and the union of their two areas (Salmeri M. et al, 2000):

$$\Omega(X_{pi}, X_{pj}) = \frac{|X_{pi} \cap X_{pj}|}{|X_{pi} \cup X_{pj}|} = \frac{|X_{pi} \cap X_{pj}|}{|X_{pi}| + |X_{pj}| - |X_{pi} \cap X_{pj}|}$$
(5)

If the value of Ω is higher than a pre-established threshold, the two FSs are deemed similar and they are fused (Figure 6). The four parameters of the new, fused trapezoidal MF will be:

$$z_{fus,s} = \frac{z_i l_{X_{pi}} + z_j l_{X_{pj}}}{l_{X_{pi}} + l_{X_{pj}}}; \ s = 1, 2, 3, 4$$
(6)

where $z_{fus,s}$ stands for the input variable values corresponding to the four vertices of the trapezoidal MF (Botzheim J. et al., 2001), (Juang C.F. et al., 1999), (Lin C.T., Lee C.S.G., 1994) resulting from the fusion and $l_{X_{pi}}$, $l_{X_{pj}}$ are the half-height widths of the FSs X_{pi} and X_{pj} , respectively.



Figure 6. Fusion of two similar FSs (arrows)

3. generation of the fuzzy rules.

The implementation of the previous steps 1 and 2 leads to the generation of a FRB formed by *c* rules, one for each physical class. The antecedent part of the rules is given by eq. (2), while, with respect to the set up of the consequent part, a discrete output variable y_q is associated to each class, q = 1,...,c. Each output variable is described by two linguistic labels {*YES*, *NO*}, with corresponding singletons FSs Y_q^{NO} and Y_q^{YES} .

Then, in the consequent part of the fuzzy rule representing the *i*-th class, all the output variables y_q , $q \neq i$, appear labelled with the FS Y_q^{NO} , except the *i*-th output variable y_i , representing the *i*-th class, which is labelled with Y_q^{YES} :

if
$$(x_1 \text{ is } X_{1i})$$
 and ... and $(x_n \text{ is } X_{ni})$ then
 $(y_1 \text{ is } Y_1^{NO})$ and ... $(y_i \text{ is } Y_i^{YES})$... and $(y_c \text{ is } Y_c^{NO})^{(7)}$

This form of the consequents allows a possibilistic classification which provides the degree of membership of a pattern to each class and thus an easier handling of multiple faults (Castellano G. et al., 2003).

The c fuzzy logic rules derived from the identified clusters constitute the FRB of the classification model. On the basis of these rules, the possibilistic classification of the generic pattern \vec{x}' of the values of the monitored variables is performed by a Mamdani Fuzzy Inference Engine leading to the y_1 is Y_1 ' and ... and y_c is Y_c ' fuzzy conclusion where Y_q' , q = 1, ..., c, is the discrete output FS of the variable y_q , constituted by the two values of membership or non-membership to class q. Figure 7 shows an example of output FSs for a given input pattern to be classified into one of three possible classes: the pattern most possibly belongs to class 3 (with degree 0.95) but it could possibly belong also to class 1 (with degree 0.7) and 2 (with degree 0.3).



Figure 7. Example of a possibilistic classification into 3 classes.

4 CASE STUDY: CLASSIFICATION OF TRANSIENTS IN THE FEEDWATER SYSTEM OF A BWR

4.1 Problem statement

The identification of a predefined set of faults in a Boiling Water Reactor (BWR) is considered. Transients corresponding to the faults have been simulated by the HAMBO simulator of the Forsmark 3 BWR plant in Sweden (Puska E., Noemann S., 2002).

The considered faults occur in the section of the feedwater system where the feedwater is preheated from 169 °C to 214 °C in two parallel lines of highpressure preheaters while going from the feedwater tank to the reactor. Process experts have identified a set of 18 faults that are generally hard to detect for an operator and that produce efficiency losses if undetected (Roverso D., 2003). The c = 6 faults regarding line 1 are here considered as the classes to be distinguished by the classification. These are numbered F1-F5 and F7, coherently with the original numbering (Puska E., Noemann S., 2002).

For each type of fault, the patterns to be used for building the classification model have been constructed by simulating transients with the plant at 80% of full power, taking values every 6 seconds from $t_{in} = 80s$ to $t_{fin} = 200s$. Among the 363 monitored signals, only n = 5

Among the 363 monitored signals, only n=5 signals have been chosen for the transient classification using the feature selection algorithm proposed in (Zio E. et al., 2005): position level of control valve for preheater EA1 (PLV), temperature of drain 4 before valve VB3 (T1), water level of tank TD1 (WL), feedwater temperature after preheater EA2 (T2) and feedwater temperature after preheater EB2 (T3).

4.2 Application and results

The objective is that of using the available preclassified patterns for building a classifier based on fuzzy clustering and then extracting from it a set of transparent and accurate diagnostic rules for classifying the feedwater system faults. 80% of the available patterns have been used for building the classifier and the remaining 20% for testing its accuracy.

The application of the evolutionary algorithm for optimizing the possibilistic clustering model described in (Zio E., Baraldi P., 2005) leads to 6 clusters, each one corresponding to a different type of fault. These are translated into a possibilistic clustering classifier, based on a FRB in which the multidimensional input FSs correspond to the multidimensional fuzzy clusters. With respect to the final class assignment of an incoming pattern \vec{x} ' $(x_1, x_2, ..., x_n)$ starting from the inferred output FSs Y_1, Y_2, \dots, Y_c the pattern is possibilistically assigned to all the classes whose corresponding output y_q , q = 1, 2, ..., c, has the FS $Y_q^{'}$ with membership value to the linguistic label $\{YES\}$ larger than a threshold γ (here chosen equal to 0.6). Figure 8 shows an example of possibilistic classification of a pattern. If none of the membership grades to the label $\{YES\}$ is larger than γ , then the pattern is labeled 'atypical'. If more than one membership grade is larger than γ , then the pattern is labeled 'ambiguous'.



Figure 8. Class assignment

Projecting the multi-dimensional clusters onto the UODs of the five antecedents corresponding to the five input signals, the FSs reported in Figure 9 are obtained. With this partition of the n=5 mono-dimensional antecedents UODs, a new fuzzy rule-based classification model is built, based on a FRB with rules of the form of eq. (7) and equal in number to the fault classes. Comparing these results with those obtained directly from the multi-dimensional input FSs representing the clusters, a minor deterioration of the classification performance is observed: one pattern previously correctly classified is now

found atypical. This minor decrease in the performance is due to the loss of information following the projection of the multi-dimensional FSs into their mono-dimensional constituents.



Figure 9. Projection of the six clusters onto the input signals

Applying the transparency constraints of Section 3 for obtaining an optimal partition of the UODs U_{x_n}

of the input variables x_p , p = 1,...,5, the FSs in Figure 10 are obtained.



Figure 10. Final partition of the inputs UOD

Application of the steps 2 and 3 of the procedure illustrated in Section 3 results in a more transparent FRB without decreasing the classification performance of the multi-dimensional cluster (Table 1). In particular, all the test patterns are now correctly classified, except one pattern characterized by the first input variable x_1 with a value out of the range

of the training patterns. This pattern is correctly labeled as atypical by the FRB of the classification model.

To appreciate the transparency of the seven rules obtained after the last step of the proposed approach, Table 2 reports the resulting FRB.

			1		
1	Type of FRB	Correct [%]	Error [%]	Am- biguous [%]	Atypi- cal [*] [%]
Mu	lti-dimensional input FSs	96	0	0	4
nensional s after:	Projection	92	0	0	8
Mono-din input FS	Rule-based classifier	96	0	0	4

Table 1. Classification performances

* Including the patterns with an input variable out of range.

Rule	- IF	PLV	T1	WL	T2	Т3	THEN	F1	F2	F3	F4	F5	F7	
1		Open	Colder	Higher	Nearly equal	Very cold		Yes	No	No	No	No	No	
2		Partially open	Colder	Lower	Colder	Hotter		No	Yes	No	No	No	No	
3		Very closed	Colder	Lower	Nearly equal	Colder		No	No	Yes	No	No	No	
4		Partially closed	Hotter	Lower	Hotter	Very hot		No	No	No	Yes	No	No	
5		More closed	Colder	Lower	Nearly equal	Colder		No	No	No	No	Yes	No	
6			Closed	-	Lower	-	Hotter		No	No	No	No	No	Yes
7		Partially open	Colder	Lower	Nearly equal	Colder		No	No	No	No	No	No	

Table 2. The Table of rules of the FRB

5 CONCLUSIONS

An innovative approach to building a transparent fuzzy logic model for pattern classification has been propounded for tackling fault diagnosis tasks. Differently from other classification techniques, the proposed approach for mining transparent fuzzy rules from data emphasizes the linguistic interpretability of the acquired knowledge, which is a fundamental requirement for the application of diagnostic tools in safety-critical fields like nuclear engineering.

Starting from an a priori available partition of labeled patterns in different classes, an evolutionary clustering algorithm is applied to find an optimal Mahalanobis metric for each cluster. Each cluster induces a classification rule. Mono-dimensional FSs corresponding to a given cluster are then obtained by projection onto each input variable domain. These are optimally combined into a transparent and physically interpretable representation of the input/output relations in terms of fuzzy rules, while maintaining accuracy in the classification.

As output, the model provides the possibilistic membership grades to the different classes, thus explicitly accounting for the ambiguities of the classification problem inherent in its characterizing input features.

The methodology has been successfully applied to a test case regarding the classification of simulated transients of the feedwater system of a Boiling Water Reactor. The results obtained are satisfactory in terms of both classification accuracy and model interpretability.

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