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DEVELOPMENT AND APPLICATION OF AUTOMATED OMA ALGORITHMS

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ABSTRACT

Within the contest of vibration-based monitoring, the paper summarizes the conceptual development of automated procedures of modal parameter estimation (MPE) and modal tracking (MT). The performance of the proposed algorithms is demonstrated using data collected both in single data-sets and over a period of continuous monitoring. The MPE process, based on the automated interpretation of the stabilization diagram associated to any parametric identification methods, is exemplified by using the SSI-Cov technique and consists of three key steps: (1) filtering a high number of spurious poles in the stabilization diagram (2) clustering process and (3) improving the accuracy of the estimates. Eventually, the mean modal parameters are computed for each clustered mode. The MT task is based on the definition of: (a) a pre-selected list of baseline modes with adaptive thresholds and (b) a dynamic reference list of modes associated to fixed thresholds.

Keywords: Automated OMA, Masonry tower, Modal parameter estimation, Modal tracking

1. INTRODUCTION

The development of efficient strategies of Structural Health Monitoring (SHM), aimed at assessing the health condition of structural systems, is still a research topic of utmost importance and is receiving increasing attention from different disciplines. Among the SHM approaches, one of the most popular is based on Operational Modal Analysis (OMA) techniques and the use of modal parameters as damage-sensitive features [1-3]. Moreover, the technological advances and the consequent easier installation of dynamic monitoring systems on both large infrastructures [1-2], [3-6] and Cultural Heritage structures [3], [6-10] have favored the studies on vibration-based SHM.

In vibration monitoring, it is essential to combine state-of-art sensors and robust algorithms in order to manage the huge amount of continuously collected data and to automatically identify the modal parameters at pre-selected time intervals. Among the different OMA methods available in the literature, the FDD [11] is very popular in the application to the time series collected in a single test (single set-up or multiple set-ups) but it is less used within continuous monitoring projects because its difficulty to be automated. On the contrary, SSI procedures [12] are widely used for automated OMA because the SSI techniques are suited to be automated and are capable to identifying weakly excited and closely space modes [13-15].

The focus of the present paper is a series of robust tools that allow fully automated OMA avoiding any user interaction and mimicking the choices of an expert user during the modal identification task. As usual, the implemented approach consists of two different procedures: (a) the automated Modal Parameter Estimation (MPE) procedure, aimed at extracting the modal parameters from the stabilization diagram [12-15] of each single data-set; (b) the Modal Tracking (MT) procedure developed to automatically provide the evolution in time of the dynamic characteristics.

The MPE task consists of three key steps, which are subsequently applied in order to: (1) identify and remove a high number of spurious poles, (2) estimate the modal parameters through a clustering procedure and (3) improve the accuracy of the obtained estimates through the application of simple statistical tools. It should be noticed that the noise modes removal and the clustering are based on previously developed algorithms [13-15], whereas the post-processing, aimed at improving the accuracy of the modal estimates, is one distinctive aspect of the proposed procedures.
The automated MT is mainly performed by: (i) exploiting the consistency of the modal parameters, (ii) adopting a reference list of structural modes and (iii) enduring the correct tracking with self-adaptable thresholds. It is worth noting that the definition and the appropriate management of the self-adaptive thresholds characterizes the proposed MT algorithm.

The organization of this paper is as follows. The developed MPE and MT procedures are presented in section 2 and both the cleaning action exerted by each step of the MPE tool and the distinctive aspects characterizing the proposed MT approach are discussed. Section 3 exemplifies the application of the MPE algorithm to a single dataset obtained during the continuous monitoring of the *Gabbia* tower [3], [16] in Mantua, Italy. In section 4, the implemented procedures have been applied to a evaluate the time evolution of the resonant frequencies of the same tower over a period of six months and validation of the results is demonstrated through the comparison with the modal frequencies obtained by manual application of the commercial software ARTeMIS [17].

2. THE PROPOSED AUTOMATED OMA TOOLS

2.1. Modal Parameters Estimation

The procedure implemented to automatically identify the modal parameters consists of a series of subsequent tasks applied to the stabilization diagram [12-15] obtained from a single dataset. In the presented applications, the vibration data measured in operational conditions have been processed using the covariance-driven Stochastic Subspace Identification (SSI-Cov) method [12]. As previously stated, the MPE approach is composed by three steps:

1) Pre-filtering, i.e. the removal of certainly spurious poles that are detected by applying three single-mode validation criteria. As usual [12-15], validation criteria considering the physical consistency of damping ratios and the complexity of mode shape components have been adopted;
2) Clustering, i.e. the process of detecting and grouping all the poles of the stabilization diagram sharing the same characteristics in terms of modal parameters. This step ideally corresponds to the inspection of the stabilization diagram, carried out by an expert user in a manual approach, to identify the alignments of stable poles;
3) Post-processing, i.e. the removal of possible replications of the structural modes and outliers in order to conceivably increase the accuracy of the modal estimates.

The pre-filtering and clustering tasks are exemplified in Fig. 1 and Fig. 2, respectively.

Figure 1a schematically shows the results obtained by applying the SSI method for different model orders (from 4 to 30). Each pole is defined in terms of eigenvalue and eigenvector, with the eigenvalue being associated to modal frequency and damping ratio and the eigenvector being used to estimate the corresponding mode shape. Therefore, possible measures of the mode shape complexity are the Modal Phase Collinearity [14-15] (MPC, which provides an estimate of the correlation between the imaginary and real part of the mode shape components and tends to unity for real modes) and the Mean Phase Deviation [14-15] (MPD, which measures the phase angle of the mode shape vector in the complex plane and tends to 0 for real modes). The action exerted by the pre-filtering step is illustrated in Figs. 1b-d. As shown in Fig. 1b, discrimination between certainly spurious poles and the physical ones is firstly performed by removing the poles associated to negative damping ratio or high damping (i.e., damping exceeding a 10% threshold, which seems a conservative value for civil engineering structures under ambient or operational excitations). Subsequently and shown in Figs. 1c-d, the complexity of the mode shapes is checked by using both the MPC and the MPD validation criteria, as the complexity measure given by the two indices is not completely equivalent [15]. This choice aims to guarantee the removal of a large amount of spurious poles.

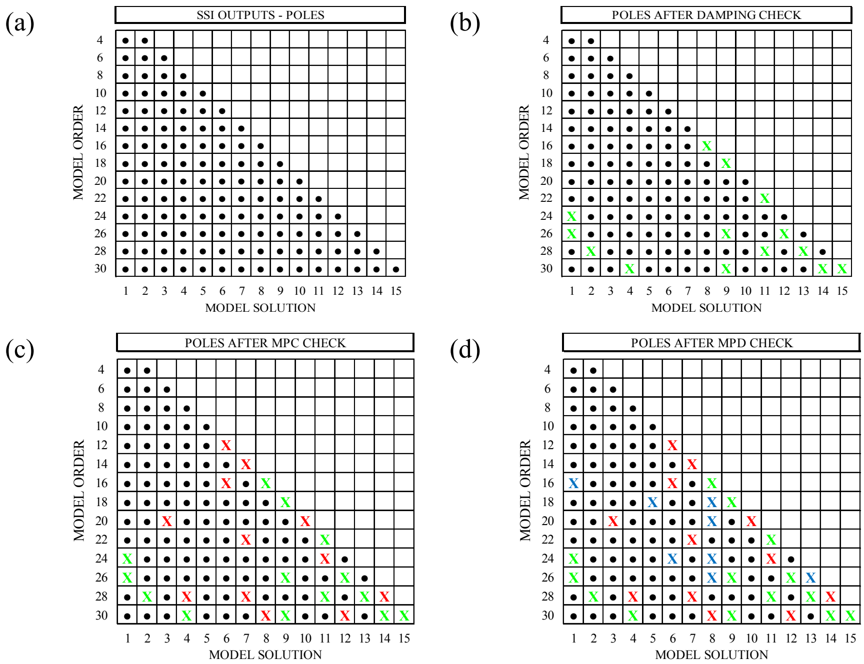

Figure 1. Action exerted by the pre-filtering: (a) SSI output; (b) after the damping check; (c) after the MPC check and (d) after the MPD check (the "X" mark indicates the removed poles).

After cleaning the stabilization diagram from noise and spurious poles, a clustering procedure is used to group all the poles that share same characteristics in terms of modal parameters. In the context of OMA, the most popular way to group stable poles is to measure the distance of all pairs of estimated poles: this operation can be performed because stable poles are generally grouped in high density areas whereas noise modes are much more scattered. According to [12-13], the distance between pair of estimates is calculated checking the similarity between natural frequencies and mode shapes and adopting a hierarchical clustering approach. This approach turned out to be very effective for modal identification of recent constructions using monitoring system with diffused sensors: in such instances, the generation of a high quantity of noise modes is avoided and an easier detection of the structural ones. On the other hand, applying the same strategy to ancient buildings and masonry constructions, in which the monitoring process is usually carried out using a limited number of sensors, could not be always straightforward. For this reason, the metric described in [15] is herein adopted and the clustering approach is based on the definition of one reference point inside each cluster; in other words, the clustering process herein presented uses the concepts of reference pole and fixed reject distance (see e.g. [13], [15]) to group all those poles that share the same characteristics in terms of modal parameters (i.e., natural frequencies and mode shapes). The metric used in the clustering procedure is the following:

$$d_{i,ref} = \frac{|f_{i,ref} - f_{i,j}|}{f_{i,ref}} + 1 - \frac{|\varphi_{i,ref}^{H} \cdot \varphi_{i,j}|^{2}}{\left[(\varphi_{i,ref}^{H} \cdot \varphi_{i,ref}) \cdot (\varphi_{i,j}^{H} \cdot \varphi_{i,j})\right]} \quad (1)$$

where:

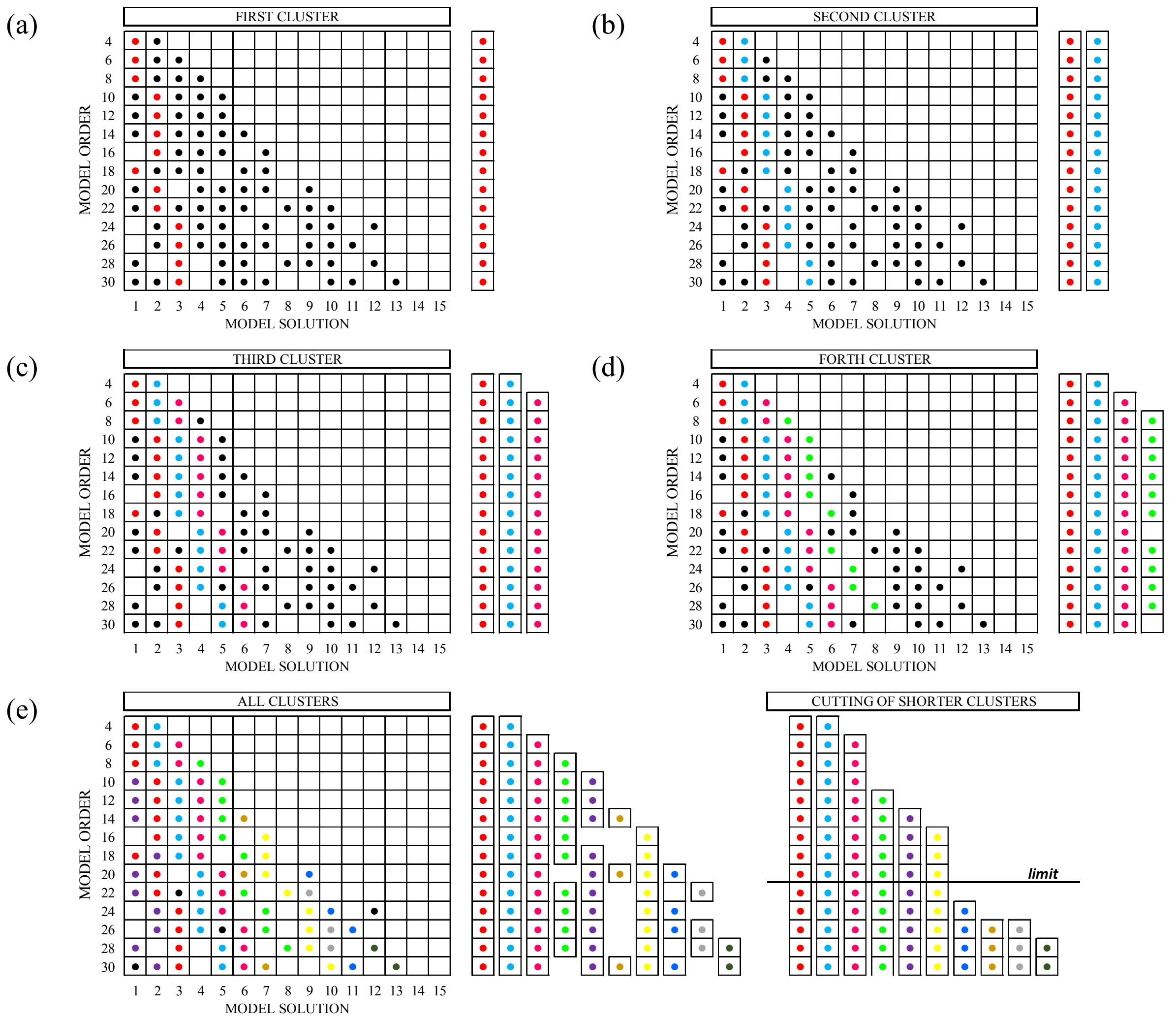

Figure 2. Schematic representation of the clustering process: (a)-(d) construction of the first clusters; (e) allocation of all poles in the clusters and cutting of the shorter clusters.

- $d_{i,ref}$ represents the inter-cluster distance, defined as the distance between each candidate pole and the reference pole of the cluster;
- $f_{i,ref}$ and $\varphi_{i,ref}$ are the mean frequency and mean mode shape of the reference pole, which are updated with the icreased dimension of the cluster;
- $f_{i,j}$ and $\varphi_{i,j}$ are the modal parameters corresponding to the current pole..

The procedure is repeated in order to scan all available poles and it is stopped only when all poles have been scanned and grouped into clusters. The clustering procedure consists of the following steps (Fig. 2):

1) Starting from the lowest model order, the reference point of the first cluster is associated to the pole with the lowest natural frequency;
2) The similarity between the reference point and the poles obtained for increased orders is evaluated using eq. (1) and if the distance does not exceed a pre-selected inter-cluster threshold, the current pole is kept on;
3) Detection of the closest pole among the previously detected ones;
4) Selection and inclusion of the closest pole into the cluster;
5) Definition of the new reference cluster point (in terms of mean natural frequency and mean modal shape) using all poles present into the cluster;
6) Repetition of the points 2)-5) until the poles belonging to the highest model order are reached.

A schematic representation of the main steps constituting the clustering procedure is shown in Fig. 2 When the highest model order is reached, the first cluster is completed (Fig. 2a) and all poles already assigned to the cluster are no more considered. Subsequently, the above steps 1)-6) are repeated in the same way to group poles with same characteristics into another representative cluster (Figs. 2b-d). When all poles have been considered and grouped, as it is customary, the shorter clusters (i.e., the clusters containig a number of elements lower than one third of those present in the largest cluster) are considered as noise modes and deleted (Fig. 2e).

In the end of the clustering procedure, only the clusters standing above the limit are not removed and saved for the next subroutine. Moreover, the mean values in terms of natural frequencies, mode shapes and modal damping ratios become the set of reference modes used in the subsequent post-processing, aimed at checking the results provided by the clustering process and at improving the accuracy of the previous modal estimates by applying simple statistical rules. The post-processing is composed by three different *checks*:

a) The first check is a new clustering process, which is now agglomerative and scans the previously clustered poles. The reference points associated to each previously defined cluster are used as centroids and the inter-cluster distance is now defined by the real distribution of poles inside each previously defined cluster. The present check is performed to amend possible inaccuracies (such as lost of poles belonging to the cluster or, conversely, outliers fallen in the cluster) occurred at the beginning of the cluster generation and to conceivably reduce the dependence of the identified modes on user parameters (i.e. inter-cluster threshold) that needs to be tuned before the identification. The new clustering process often results in slight changes of the estimated mean values, with the accuracy improvement being highlighted by the decrease of standard deviations. It is further noticed that, since the reference values have been already defined, the computational cost of the second clustering is drastically reduced;
b) The second check is introduced to remove any possible replication of structural modes;
c) Finally, possible outliers affecting the damping estimates are detected and removed by applying a simple statistical tool based on the box-plot rule.

The final outputs of the MPE procedure are the mean values of the modal estimates (mean natural frequency, mean mode shape and median modal damping ratio) and the geometric mean value extracted considering the distribution of the MPC and MPD values associated to each pole of the cluster.

2.2. Modal Tracking

Within the context of OMA-based SHM, the tracking process deserves a special attention because it directly provides the input for the statistical tools adopted to detect any novelty or abnormal change in the investigated system [1-3], [9-10]. based-OMA damage detection strategies. Despite the MT represents the very initial step of any OMA-based strategy of SHM, a few strategies have been developed in the literature for the full automation of this phase. A common approach [13] adopted to link the currently identified modes to a baseline list is based on similarity of the modal parameters in terms of natural frequency and mode shape (using MAC index).

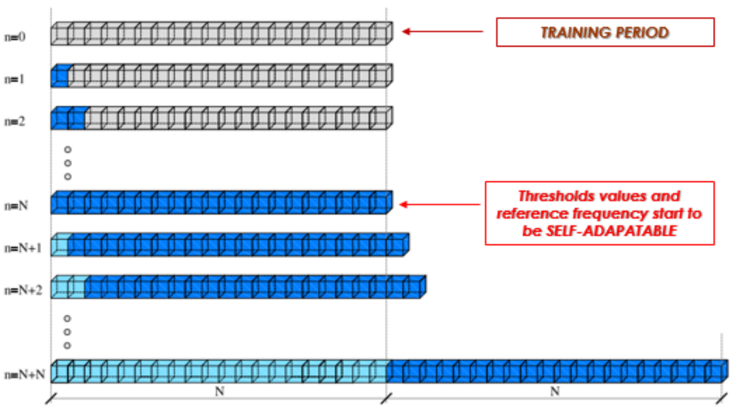

Figure 3. Schematic of the Modal Tracking procedure: training period (grey cubes) and tracking window used for the definition of adaptable thresholds (blue cubes).

The main improvements of the MT procedure herein proposed can be summarized in four aspects: (a) exploiting the consistency of the modal parameters over time, (b) adopting a short training period to make the process fully automated; (c) using a dynamic set of reference modes together with self-adaptable thresholds in order to account for the variations associated to the thermal effects. It should be noticed that the training period plays an important role because it is used to define the adaptive reject thresholds and the complete automation of the process. Figure 3 shows a graphical scheme to better understand the main feature of the developed MT strategy As shown in Fig. 3, the procedure works in two different stages: in the training period (grey slots) and after the raining period (blues slots).

During the training period, the current modal estimates are linked to a pre-selected list of reference modes if two conditions are satisfied. The two conditions involve the computation of the euclidean distance, defined in term of both frequency and MAC variation, between the current modal estimates and two sets of reference modes: the pre-selected list of baseline modes and a dynamic list of modes, which is formed and continuously updated after each MPE. As usual, the current modal estimates are retained (i.e., are linked to the reference ones) if all the distances are lower than pre-defined tolerances. It is worth noting that the dynamic references allow a correct tracking also when the tracking zones between different modes overlap (even if this performance is more evident when training phase is expired).

The training period expires when a limited number, say *N*, of modal parameters estimates has been completed. During this period: (a) a dynamic set of reference modes has been formed and (b) adaptive thresholds have been defined:

$$d^{f}_{i-ref,j} = \sqrt{std\left(d^{f}_{i-ref,j}\right)} \qquad d^{MAC}_{i-ref,j} = \sqrt{std\left(d^{MAC}_{i-ref,j}\right)} \tag{2}$$

where $std(d^{f}_{i,ref\text{-}j})$ and $std(d^{MAC}_{i,ref\text{-}j})$ are the standard deviation of the distance – computed in terms of natural frequency and MAC, respectively – between the *j-th* estimates ($j=1\ldots N$) related to *i-th* reference mode, collected into each representative group.

Herein after, the *tracking window* (see blue cubes in Fig. 3) shifts one position when each new dataset of modal estimates is considered and the MT process is again performed by checking the distances between the current modal estimates and the two series of reference modes (the baseline list and the dynamic one). The first check (with the distances being computed with respect to the static reference modes) is carried out by using the adaptive thresholds (2), whereas the second one (with the distances being computed with respect to the dynamic reference modes) refers to pre-selected thresholds. At each shift of the tracking window, both the thresholds (2) and the dynamic reference modes are updated by using the last available *N* sets of linked modes.

The proposed MT approach has the objective of modifying the reject thresholds (2) in order to cover the normal fluctuation of the modal parameters but also possible high variations induced by any environmental phenomena. The reliability of the MT approach will be highlighted by the results obtained for the case study presented in sections 3-4, where the resonant frequency of one (local) mode changes dramatically between the winter and the summer period. As it will be shown, the algorithm does not suffer any significant failure and automatically performs the correct tracking of the detected modes using a short period of training. Another important advantage obtained adopting this strategy is referred to the computational cost, that it is quite limited due to the minimal quantity of allocated memory continuously used in the definition of the adaptive distance thresholds.

3. APPLICATION TO A SINGLE DATASET

The validation of the developed tools is herein exemplified using the data collected during the continuos dynamic monitoring of the *Gabbia* tower in Mantua, Italy [3], [16]. The investigated tower (Fig. 4), with its 54.0 m height, is the tallest tower in Mantua. The tower was erected in the 13th century by the Bonacolsi family (i.e. the Lords governing Mantua at that time) for defensive purposes.

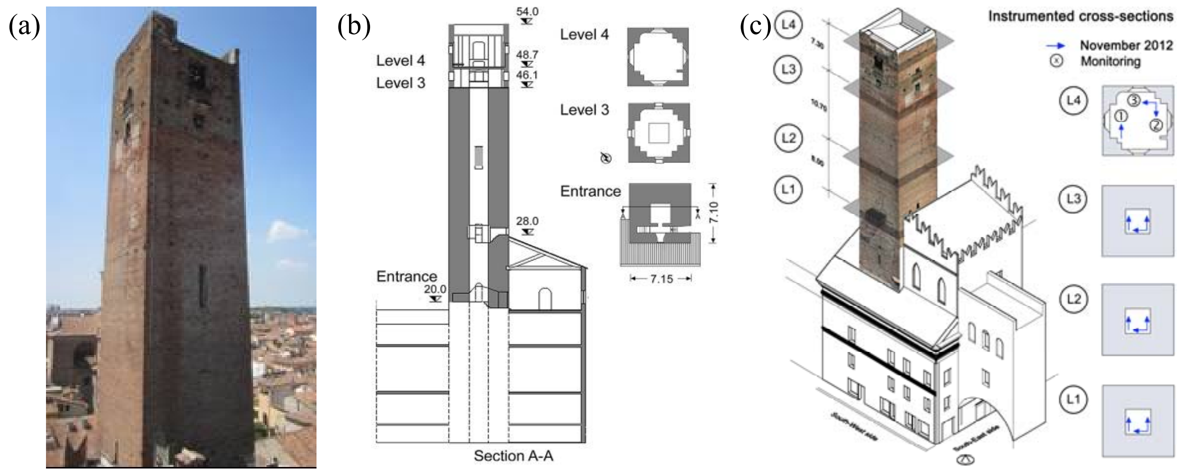

Figure 4. (a) View of the *Gabbia* tower (Mantua, Italy); (b) Sections of the tower (dimensions in m); (c) Instrumented cross-sections and layout of the accelerometers during the preliminary tests (November 2012) and the continuous dynamic monitoring.

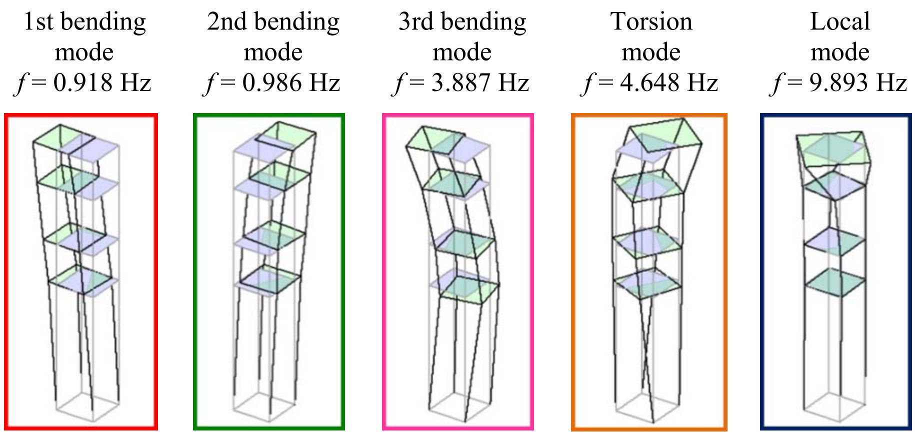

Figure 5. The *Gabbia* tower: vibration modes identified in the preliminary dynamic test [4] (SSI-Data, 27/11/2012).

The structure is built in solid brick masonry and the load-bearing walls are about 2.4 m thick, except in the upper levels, where the walls thickness decreases to about 0.7 m. As shown in Fig. 4, the tower is nowadays part of an important palace, whose load-bearing walls seem to be not effectively connected to the tower, whereas various vaults and floors of the palace are directly supported by the tower. While the main part of the building, below the height of about 46.0 m from the ground level, did not exhibit any evident structural damage (with the materials being only affected by superficial decay), the upper part of the tower turned out to be in a poor state of preservation [4].

An ambient vibration test, preparatory to the continuous dynamic monitoring of the tower, was performed on November 27th, 2012 (Fig. 4c) [4], [16]. The modal identification was performed considering time windows of 3600 s and applying the datadriven Stochastic Subspace Identification algorithm (SSI-Data) [12] available in the commercial software ARTeMIS [17]. Figure 5 summarizes the identified dynamic characteristics of the tower [4] and allowed to confirm that the installation of only 3 accelerometers in the upper available level is in principle sufficient to identify the vibration modes that are normally excited at the low amplitude of ambient vibration detected in the structure. In addition, one local mode was identified at 9.89 Hz (Fig. 5) and involved torsion of the upper part of the tower: the presence of a local mode provided further evidence of the structural effect of the change in the masonry quality and morphology (including un-toothed opening infillings and discontinuities) observed in the upper part of the tower during the visual inspection.

Figure 6 summarizes the results of the application of the newly developed MPE procedure to one single dataset recorded on 17/12/2012, during the continuous monitoring of the tower. The analyzed dataset includes three time series of 3600 s.

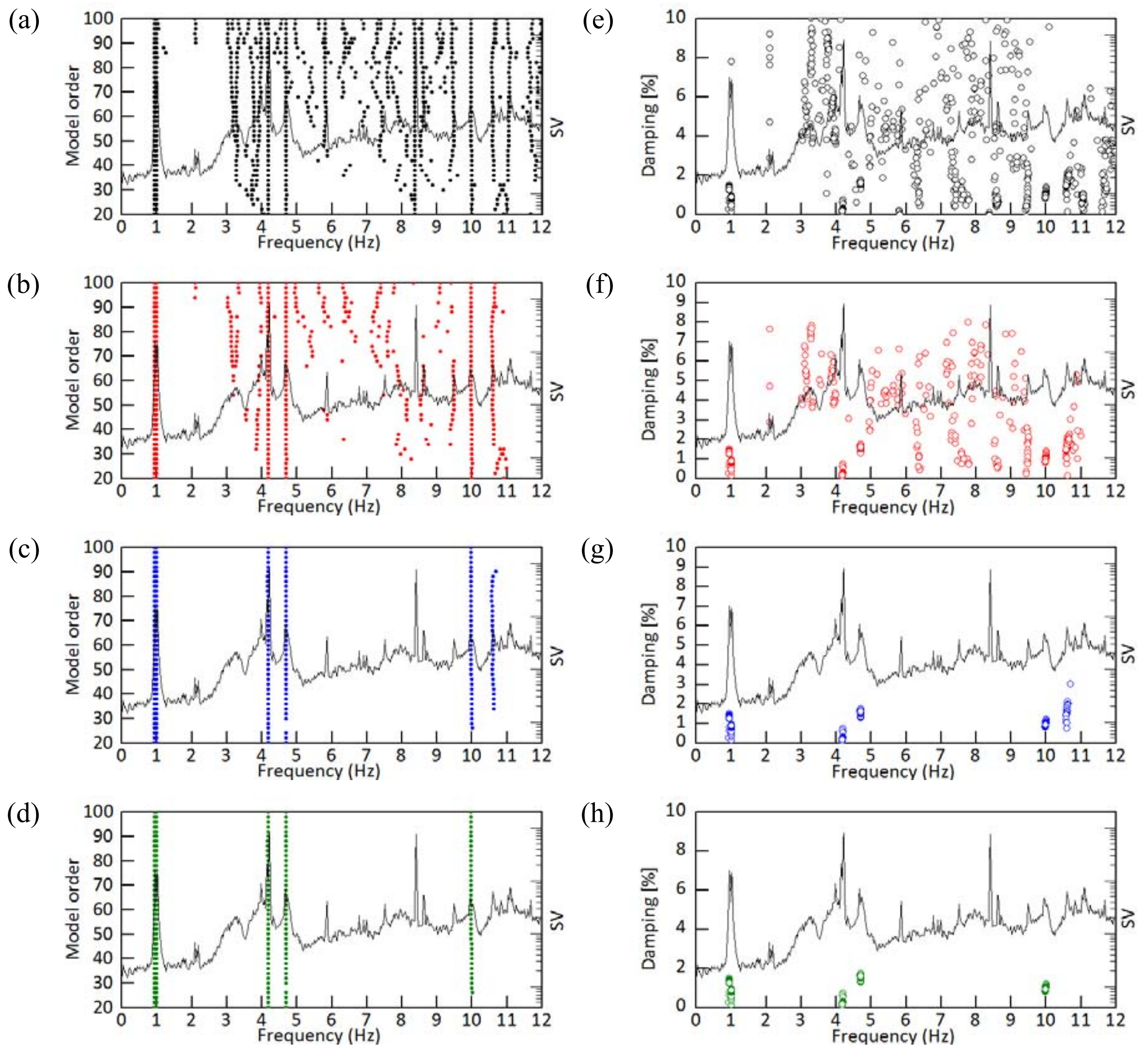

Figure 6. Stabilization diagrams frequency-damping plots: (a) after the SSI-COV run; (b) after the pre-filtering (check on damping ratios and modal complexity); (c) after clustering; (d) after the post-processing to improve accuracy (final results).

Figure 6 shows the typical cleaning action exerted by the different steps of the MPE procedure on the stabilization diagrams. In more details: (a) Fig. 6a shows the SSI outputs obtained for increasing model order; (b) Fig. 6b shows the results obtained after the pre-filtering step; (c) Fig. 6c shows the stable alignments obtained by the clustering process and (d) Fig. 6d contains the final alignments of stable poles corresponding to physical modes performed after the removal of a replicating mode and some outliers provided in the previous step. All plots in Fig. 6 also show the first Singular Value (SV) line of the spectral matrix, which is the mode indication function used in the FDD method [11] to highlight the resonant frequencies.

Figure 6 highlights the robust performance provided by the MPE procedure, with all modes of the tower (Fig. 5) being clearly detected, notwithstanding the low level of measured acceleration (which is testified by the large number of spurious poles in Fig. 6a as well as by the inspection of the first SV line) and the limited number of available sensors.

4. APPLICATION TO CONTINUOUS MONITORING

In order to demonstrate the reliability of the developed tools in the application to dynamic monitoring, six months of data collected on the *Gabbia* tower were considered and the automatically identified natural frequencies are compared with the modal estimates obtained by manual runs [16] of the SSI-data method available in the commercial software ARTeMIS [17]. The investigated monitoring period spans from 12/12/2012 to 30/06/2013 and includes 4651 1-hour datasets of three acceleration time series.

Figures 7a and 7b shows the variations of the modal frequencies versus time, obtained from the manual and the automatic approach, respectively. Inspection of Figs. 7a and 7b provides clear evidence of the correspondance between the results. Moreover, the time evolution of the local mode (described by the blue line in Fig. 7) confirms the robustness of the implemented MT strategy when modal frequencies exhibit significant change.

It is worth noting that the dark red vertical line drawn in Fig. 7b indicates the end of the training period, that was set equal to 4 days (96 estimates). During this period, to ensure the tracking of the modes the rejection distances for all natural frequencies and MAC values were set equal to $d_{\mathrm{fi,max}}$=0.20 Hz and $d_{\mathrm{MACi,max}}$=0.30, respectively.

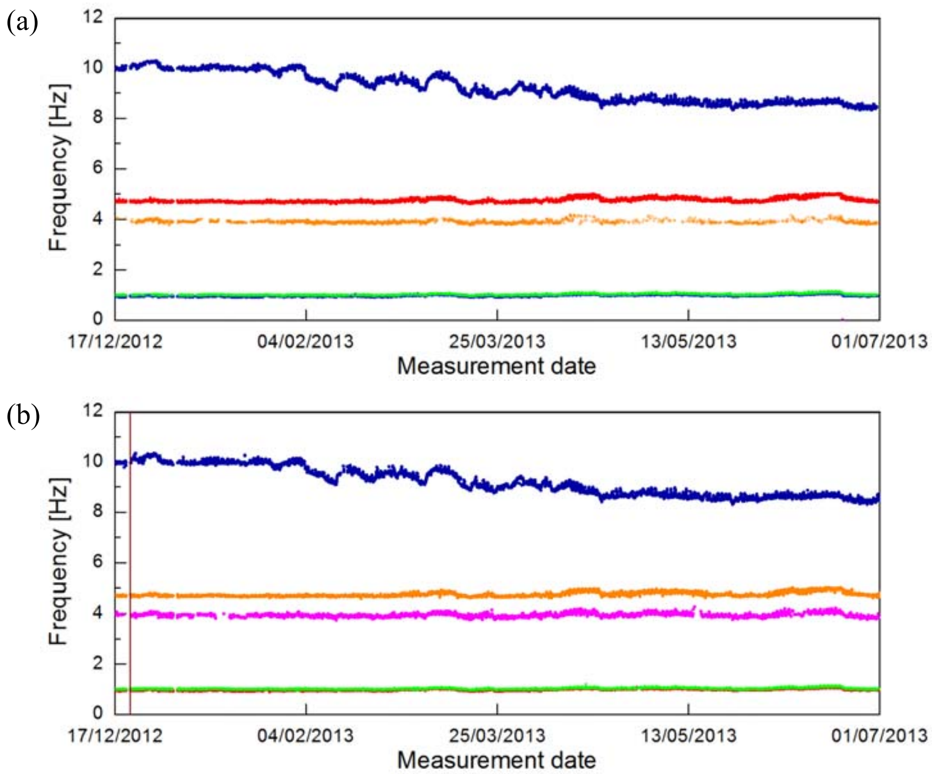

Figure 7. Time evolution of the identified natural frequencies from 17/12/2012 to 30/06/2013: (a) Manual results; (b) Automatically identified results.

In order to better exemplify the efficiency of the developed tools, more details on the evolution of the natural frequencies and the variation of the mode shapes (using the MAC index), related to six months of continuous monitoring, are given. Figure 8 shows the automatically identified modal frequencies and the evolution of the corresponding adaptive thresholds for: (a) the first bending mode, (b) the torsion modes and (c) the local mode. It is worth highlighting as after a short period of training, the continuous updating of the adaptive thresholds enables a correct tracking and prevents the failure of the automatic process (avoiding any intervention of the user and conceivably limiting the comparison of outliers). Similarly to the frequency thresholds also the MAC threshold is updated during the tracking phase, as shown in Fig. 9 for the same modes considered in the previous Fig. 8.

In conclusion, Table 1 compares the performance of manual and fully automated. In more details, the first column of Table 1 identies each vibration mode, the second and third column reports the success rate associated to manually and automatically extracted frequencies, respectively; in the last column, the percentage of commonly identified values – within a frequency tolerance lower than 0.02 Hz – is reported.

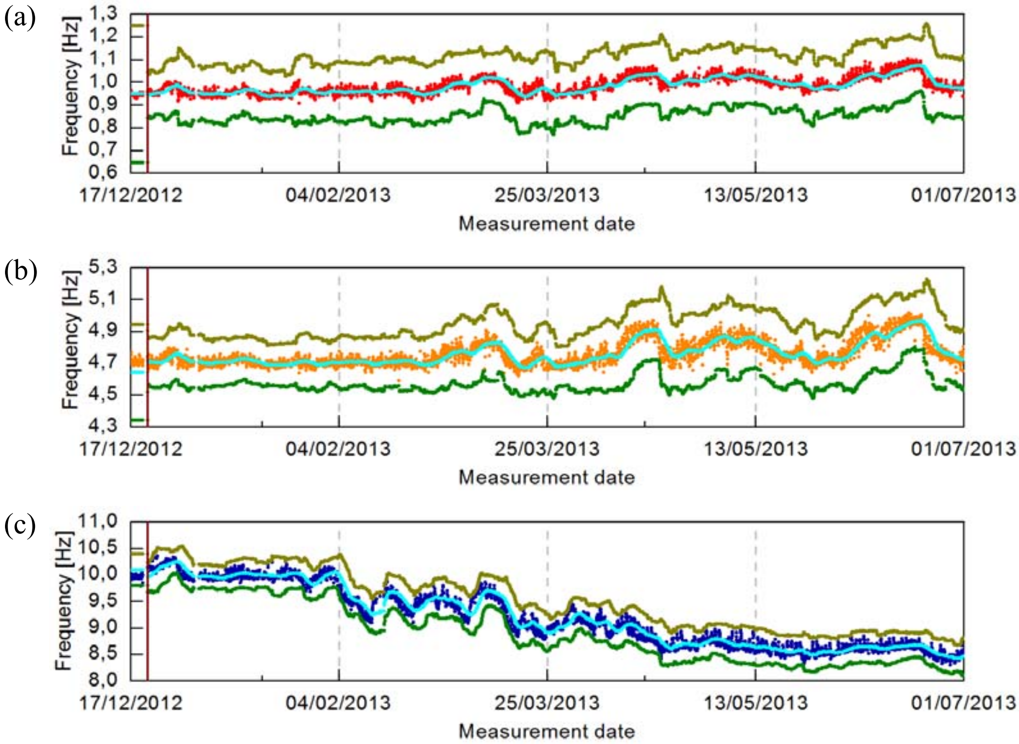

Figure 8. Time evolution (from 17/12/2012 to 30/06/2013) of the identied natural frequency and the corresponding adaptive threshold: (a) First bending mode; (b) Torsion mode; (c) Local mode.

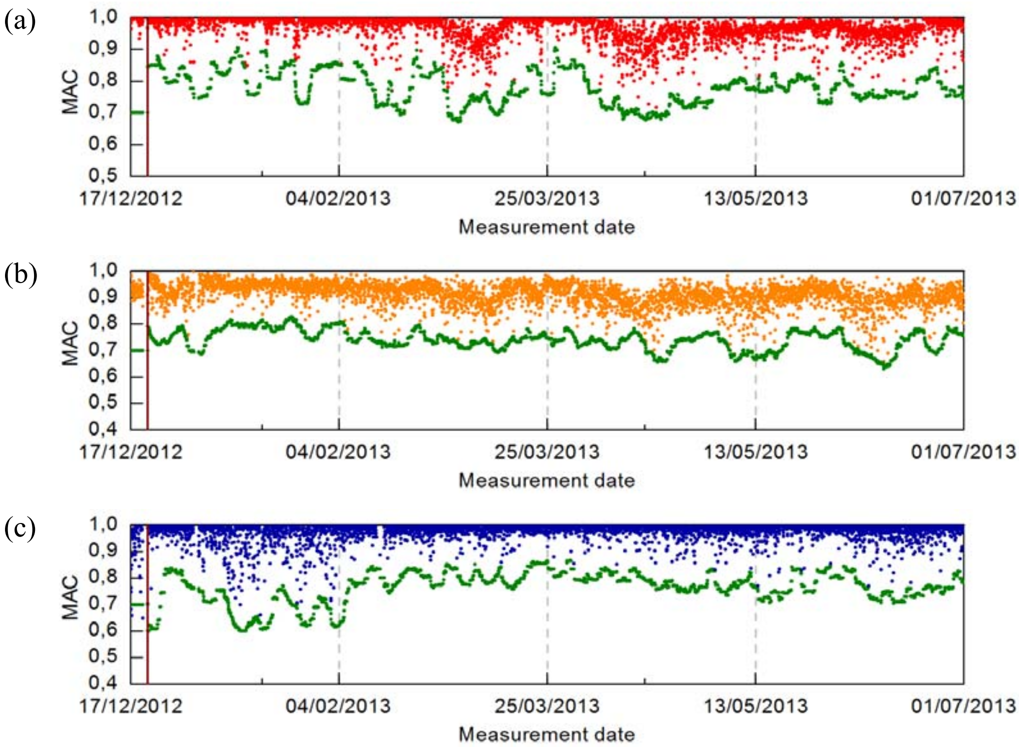

Figure 9. Time evolution (from 17/12/2012 to 30/06/2013) of the MAC value and the corresponding adaptive threshold: (a) First bending mode; (b) Torsion mode; (c) Local mode.

Table 1. Comparison between manually and automatically identified natural frequencies.

Mode Type	Manual identification rate [%]	Automated identification rate [%]	Correspondences with Δf<0.02 Hz [%]
1st bending	80.97	95.16	95.43
2nd bending	80.56	88.11	94.22
3rd bending	31.25	69.98	47.21
Torsion	76.53	93.18	91.06
Local	78.89	91.96	92.19

5. CONCLUSIONS

An automated modal identification procedure, based on parametric methods and the construction of stabilization diagrams, has been presented in the paper. The developed approach is based on the combination of two different procedures: (a) the MPE algorithm, that automatically extracts the modal parameters from a single recorded dataset and (b) the MT algorithm used to evaluate the evolution in time of the natural frequencies and the mode shapes. The validation of the developed tools was exemplified using real data continuously collected during the dynamic monitoring of the historic *Gabbia* tower (Mantua, Italy) [4]. [16].

The application of the proposed algorithms allowed to accurately identifying and tracking the evolution in time of 5 normal modes of the tower; in addition, accurate tracking of the structural modes turns out to be obtaoned even in the case of high variations induced the environmental conditions.

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