A Machine-Learning Based Epistemic Modeling Framework for Textile Antenna Design

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***Abstract*—A novel machine-learning based framework to evaluate the effect of design parameters affected by epistemic uncertainty on the performance of textile antennas is presented. In particular, epistemic variations are characterized in the framework of possibility theory, which is combined with Bayesian optimization to accurately and efficiently perform uncertainty quantification. A suitable application example validates the proposed method.**

***Index Terms*—Bayesian optimization, epistemic uncertainty, fuzzy variables, Gaussian process, textile antenna.**

I. Introduction

Uncertainty quantification (UQ) problems for antenna designs are usually defined in a statistical framework: the design parameters under uncertainty effects are regarded as random variables with specific distributions [1]–[5]. If the distribution of the random variables is known in terms of, for example, their joint probability density function, several techniques can be adopted to estimate the effect of stochastic variations of design parameters on the antenna's performance. These include sampling-based methods such as Monte Carlo analysis [6], which requires a large number of simulations or measurements of the antenna under study, or stochastic spectral methods such as Polynomial Chaos expansion [7]–[9], which builds a suitable stochastic surrogate model describing the variations of the antenna's performance.

The accuracy of statistical approaches heavily depends on the estimation of the distribution (i.e. probability density function) of the random parameters considered, which is a complex problem. One possibility is to choose *a priori* the distribution of the parameters. For example, the uniform probability density function is generally adopted to represent a complete lack of knowledge, except for lower and upper limits inferred from engineering insight. However, there is no evidence that other distributions (e.g. Gaussian) may not be more suitable to describe the UQ problem at hand. Additionally, a probabilistic framework is not always applicable for all types of UQ problems. For example, the geometry and material characteristics of textile antennas heavily depend on their operating conditions: while worn, a textile antenna will bend and the substrate will be compressed according to the movements of the wearer, changing the antenna radiation characteristics [10]. Rather than assuming that several geometrical and electrical parameters of the antenna are unknown because their values follow a specific distribution over an interval, it is more realistic to assume that the uncertainty stems from an inherent lack of knowledge about the antenna's operating conditions. In this situation, the design parameters are regarded as epistemic variables and the UQ problem can be formalized in terms of *possibility theory* [11].

In this contribution, for the first time the theory of fuzzy variables (FVs) is combined with a machine-learning based approach, namely Bayesian optimization (BO) [12], to efficiently propagate the epistemic uncertainty on the performance of the antenna under study. The proposed methodology allows one to solve UQ problems for textile antenna design, as the number of full-wave simulations needed to evaluate the performance of the antenna under study is minimized. A suitable case study, being a dual polarized textile patch antenna whose substrate parameters are affected by epistemic uncertainty, validates the proposed method.

The manuscript is organized as follows. In Section II, the relevant features of possibility theory and their application to epistemic UQ problems are discussed. Section III introduces the proposed BO-based approach and describes its application to solve epistemic UQ problems. A representative numerical example is presented in Section IV, while conclusions are drawn in Section V.

II. Problem Formulation

An overview of the relevant properties of epistemic variables in the framework of possibility theory is given in Section II-A, while the definition of epistemic UQ problems is discussed in Section II-B. For a complete reference to possibility theory and epistemic UQ problems the reader is referred to [11], [13]–[16].

A. Possibility theory and epistemic variables

In the framework of possibility theory, an epistemic variable (i.e. a design parameter) x is represented by a possibility distribution (PD) $\pi(x)$, defined by:

$$\pi : \mathbb{R} \rightarrow [0,1], \exists\, x \in \mathbb{R} : \pi(x) = 1. \quad (1)$$

While the concept of probability represents the frequency of occurrence of an event, the PD indicates *the degree of confidence* that a specific event is possible or not. Hence,

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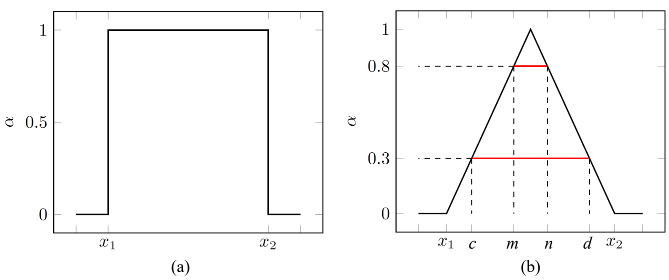

Fig. 1: (a) A uniform PD with support $[x_1, x_2]$. (b) A triangular PD and two of its α-cuts (red lines) at different α levels.

the set $[0, 1]$ contains the plausibility scores of x, where 0 corresponds to an impossible value, and 1 corresponds to a perfectly possible value. For example, let us consider the rectangular (or uniform) PD indicated in Fig. 1(a). All the values of x in the interval $[x_1, x_2]$ are perfectly possible, since the PD is always equal to 1 in such interval, while all the others are impossible, since $\pi(x) = 0$ for $x \notin [x_1, x_2]$. It is important to remark that a uniform probability distribution function defined in $[x_1, x_2]$ indicates that all values are equally probable (they have the same probability of occurrence), while a rectangular PD indicates that all values are possible, without any assumption on how often each value occurs. Hence, uniform PDs are particularly suitable to represent total ignorance [13], [17] (where no information on the variation of the design parameters is available).

Different types of PDs can be defined according to the level of knowledge available for the epistemic variable x: for example, the triangular PD of Fig. 2(b) indicates that a higher degree of confidence is assigned to one value, where $\pi(x) = 1$, that gradually decreases for all other values. Fixing a confidence threshold $\alpha \in [0, 1]$ delimits an interval in the PD. For example, in Fig. 2(b) $\alpha = 0.3$ identifies the interval $[c, d]$, while $\alpha = 0.8$ leads to $[m, n]$. Note that any PD is fully identified by the knowledge of the intervals corresponding to all the possible values of α. These intervals will be referred to as α-cuts in the rest of the contribution.

Finally, the possibility Π and the necessity N functions of an event $A \in \mathbb{R}$ are defined as

$$\Pi(A) = \sup_{x \in A} \pi(x); \quad \mathrm{N}(A) = 1 - \sup_{x \notin A} \pi(x). \tag{2}$$

The measures Π and N can be interpreted in a probabilistic sense: for a family of probability measures $P(A)$, the relation $N(A) \leq P(A) \leq \Pi(A)$ holds [16], [18].

B. Epistemic UQ problems in antenna design

Denote the quantity of interest of the antenna under study (i.e. its input reflection coefficient or, in general, its scattering parameters) as $f(\mathbf{x})$, where $\mathbf{x}$ is a vector collecting the design parameters subject to uncertainty, which are regarded as epistemic variables. The goal is to estimate the possibility distribution (PD) of the objective function and, consequently, the corresponding possibility Π and necessity N measures. Indeed, Π and N form the upper and lower bounds, respectively, for all possible cumulative distribution functions (cdfs) of $f(\mathbf{x})$ [16]. Hence, in the framework of possibility theory it is possible to estimate, based on the experience, the boundaries that delimit the actual distribution of f, but not the specific distribution of f, since no probabilistic information about the variables $\mathbf{x}$ is available.

In order to reach this goal, this letter adopts the theory of FVs [19]. The interested reader is referred to [14], [15] for a thorough discussion on the application of the FVs' mathematical formalism in the framework of possibility theory. First, the interval of possibility values $[0, 1]$ is divided into a finite set of N_α α-cuts. Each value α delineates a domain $\mathbf{\Omega}_\alpha$, which is bounded in each dimension by the interval of the corresponding α-cut of the pertinent epistemic variable in the vector $\mathbf{x}$. Next, for each α-cut, the minimum and maximum of the objective function are computed for $\mathbf{x} \in \mathbf{\Omega}_\alpha$. Finally, the minimum $\min_\alpha(f)$ and the maximum $\max_\alpha(f)$ of the objective function for all the N_α α-cuts define the desired $\pi(f)$. Once the PD of the objective function is computed, the corresponding possibility and necessity functions can be obtained using (2).

Hence, a series of N_α minimization and maximization problems must be solved. The standard "brute force" approach is to adopt a dense sampling of $\mathbf{x} \in \mathbf{\Omega}_\alpha$ and to evaluate $f(\mathbf{x})$ for all the chosen samples in order to estimate the minimum and maximum. This approach is computationally expensive, since full wave simulations are often required to estimate the objective functions (such as scattering parameters), and it can offer only limited accuracy if the objective function is non-smooth in $\mathbf{\Omega}_\alpha$. These issues become especially relevant when the number of epistemic variables increases. In order to overcome these limitations, a machine-learning based framework is proposed in Section III.

III. PROPOSED METHODOLOGY

As discussed in Section II-B, solving epistemic UQ problems can be formulated in the framework of possibility theory as solving N_α minimization and maximization problems in a bounded domain. Regarding FVs, one interesting property of the α-cuts is that they always define nested intervals [19], [20], independently of the specific PD considered. Denote by α_0 and α_{N_α} the α-cuts associated with possibility 0 and 1, respectively; then, the following holds: $\mathbf{\Omega}_{\alpha_i} \subseteq \mathbf{\Omega}_{\alpha_{i-1}}$ for $i = 1, \ldots, N_\alpha$. This property can be easily verified by inspecting Figs. 1(a) and 1(b). Hence, $2N_\alpha$ optimization (being N_α minimization and N_α maximization) problems can be suitably defined on a bounded domain that progressively decreases (starting from α_0 until α_{N_α}) or increases (starting from α_{N_α} until α_0).

To solve this task, we adopt a machine-learning based approach called BO, detailed in Section III-A and III-B. Indeed, BO offers greater efficiency in terms of the number of function evaluations compared to aforementioned "brute force" approaches based on a dense sampling of $\mathbf{x} \in \mathbf{\Omega}_\alpha$. BO is also more advantageous compared to gradient based optimization methods as it provides a global optimum rather than a local one. Furthermore, it is a model-based strategy, leading to an efficient optimization process. In this work, we modify the standard formulation of BO in order to efficiently solve epistemic UQ problems: first, by leveraging on the property that α-cuts always define nested intervals, the optimization problems for each α-cut considered are not solved independently, but the accuracy of the computed solution is improved by iteratively refining the BO surrogate model. Second, the BO algorithm is suitably modified in order to solve only N_α optimization problems, rather than $2N_\alpha$ minimization and maximization problems. The proposed methodology is described in the following sections.

A. *Brief overview of BO*

BO is a sequential strategy to solve a global minimization (or maximization) problem defined as follows:

$$\min_{\mathbf{x} \in \mathrm{X} \subset \mathbb{R}^D} f(\mathbf{x}), \tag{3}$$

where D is the number of design parameters $\mathbf{x}$. BO is particularly effective when the objective function $f(\mathbf{x})$ is computationally expensive to evaluate and has a non-smooth behavior with respect to $\mathbf{x}$, leading to the presence of several local optima. This is quite common in electromagnetic problems, where full-wave analyses are typically employed to estimate output quantities, such as the scattering parameters. A *direct* optimization of $f(\mathbf{x})$, e.g. using a gradient method [21], requires several evaluations (through simulations or measurements) of the objective function for different values of the parameters $\mathbf{x}$. The computational cost of this operation can be very high, since $f(\mathbf{x})$ is expensive to evaluate. Hence, the basic idea of BO is to progressively construct and optimize a surrogate model instead, which is significantly more efficient to evaluate than the objective function. However, contrary to other surrogate-based optimization strategies, its surrogate model in BO is *stochastic*, not deterministic. In particular, the model uncertainty can be used to define a suitable sampling strategy, which is called acquisition function in the BO framework.

A flowchart of the BO algorithm is depicted in Fig. 2. First, the objective function $f(\mathbf{x})$ is calculated for an initial set of the design parameters $[\mathbf{x}_\mathrm{k}]_{\mathrm{k}=1}^{\mathrm{K}} \in \mathrm{X}$ (chosen e.g. according to a Latin hypercube). Then, a stochastic surrogate model of $f(\mathbf{x})$ is computed based on the data acquired so far. This model is used by the acquisition function to determine the location of the candidate optimum, which is then evaluated by a new (expensive) simulation of the objective function. If none of the stopping criteria is met, the surrogate model is updated. Hence, each additional simulation refines the surrogate model, increasing the probability of finding the solution to the optimization problem (3).

In this work, Gaussian processes (GPs) [22] are chosen as a stochastic surrogate model, because of its analytic inference, accuracy and modeling power. In particular, the Matérn (5/2) was chosen as GP kernel, due to its capability to model a wide class of functions (including non-differentiable ones).

As a sampling strategy, different acquisition functions can be adopted in BO, such as the Probability of Improvement [23] and Expected Improvement (EI) [24]. In particular, EI was chosen this work, which is defined as

$$\mathrm{E}\left[\mathrm{I}(\mathbf{x})\right] = \mathrm{E}\left[\max\{0, f_{\min} - y\}\right] \tag{4}$$

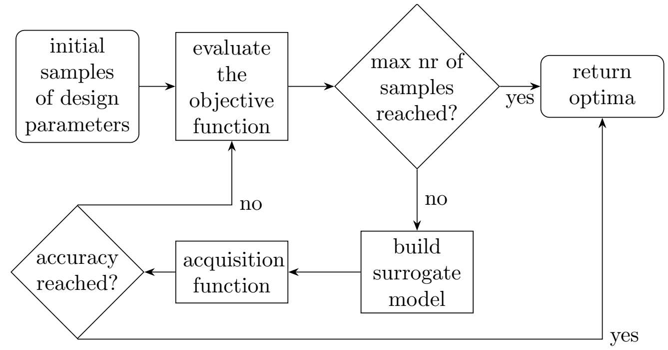

Fig. 2: Flowchart of the BO algorithm.

where E is the expectation operator, $I(\mathbf{x})$ is a suitable measure of improvement defined at the point $\mathbf{x}$, $f_{\min}$ is the current evaluated minimum of the objective function and y is the prediction of the GP surrogate model at point $\mathbf{x}$. Since y is a Gaussian random variable, the expectation in (4) can be calculated analytically.

Finally, the training of the GP model parameters and the BO have been performed via *GPyOpt*, a Bayesian optimization package in Python [25]. A complete description of the BO properties is given in [26]–[28].

B. *BO for Epistemic UQ problems in antenna design*

As described in Section II-B, it is essential to compute both the minimum and the maximum of the objective function in order to estimate a PD. For this purpose, the acquisition function (4) is modified as follows:

$$\mathrm{EI}_{\mathrm{mm}}(\mathbf{x}) = \max\{\mathrm{E}\left[\max\{0, f_{\min} - y\}\right], \mathrm{E}\left[\max\{0, y - f_{\max}\}\right]\} \tag{5}$$

This approach is especially efficient, since the candidate points in the space of the design parameters with higher potential for a better optimum of either kind (minimum or maximum) are evaluated. Hence, the proposed optimization strategy is capable of finding both optima with the minimal number of evaluations of the objective function $f(\mathbf{x})$.

Now, a single optimization problem must be solved for each of the N_α nested intervals $\boldsymbol{\Omega}_{\alpha_i}$. The procedure adopted in this work is described as follows: first, BO is performed at the top α level ($\alpha = 1$), using a small portion of the total computational budget. Then, the optimization for all the consecutive α levels is carried out by making use of the samples already evaluated for the "upper" α levels, while only few additional samples are evaluated for each subsequent α level. However, in case a better optimum is found in the current α level, the optimum for previous levels can be updated accordingly, whenever possible. Hence, the GP surrogate model computed for α_{N_α} is sequentially refined for each α-cut until α_0 is reached. Note that this strategy is applicable because the intervals $\boldsymbol{\Omega}_{\alpha_i}$ are nested.

IV. Application Example

The proposed approach is applied to the dual-polarized textile patch antenna presented in [29]. This antenna operates in the $[2.4, 2.4835]$ GHz ISM band, with a nominal substrate

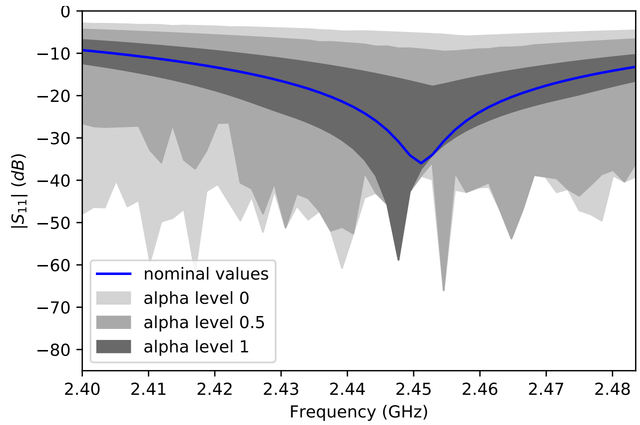

Fig. 3: Bounds of the magnitude of S_{11} (dB) obtained with the proposed BO-based method at different α levels.

height $h = 3.94\,\text{mm}$ and permittivity $\epsilon_r = 1.53$. Both parameters are regarded as epistemic variables defined in the support $[3.44, 4.44]\,\text{mm}$ and $[1.43, 1.63]$, respectively. In particular, a uniform PD is chosen for the height and a triangular one for the relative permittivity, in order to demonstrate that the proposed approach is general and independent of the specific PDs assigned to the epistemic variables. The goal is to estimate the uncertainty of the antenna scattering parameters for 50 frequency samples in the $[2.4, 2.4835]\,\text{GHz}$ band. The Momentum electromagnetic field simulator of Advanced Design System (ADS) [30] is adopted to estimate the scattering parameters as a function of the frequency and the epistemic variables. For each frequency point, 36 α-cuts ranging from possibility level 0 to 1 are considered.

A computational budget of 11 (h, ϵ_r) samples is assigned to the BO for $\alpha = 1$, while a budget of 2 additional samples is given for each following α level, for a total maximum computational budget of 81 (h, ϵ_r) samples. Figure 3 shows the estimated PD at different α levels over the considered frequency range. The shades illustrate the minima and the maxima of all possible values of $|S_{11}|$ computed by the proposed BO-based method at different α levels.

Next, the results of the proposed technique are compared to the grid search (GS) method described in Section II-B for two different numbers of sample points. The corresponding Π and N measures are presented in Figs. 4 and 5. In particular, first, a uniform grid of 9×9 (h, ϵ_r) samples is allocated for the GS approach. Clearly, for the same computational budget the BO offers greater accuracy than the approach based on GS. However, in this particular case the BO is more expensive compared to the GS: the first requires 16 minutes per single frequency point to estimate the PDs, while the second requires 13 minutes. The 3 minutes difference is due to the GP model building process and the optimization performed in the BO framework. Second, in order to achieve a comparable accuracy, a uniform grid of 51×51 (h, ϵ_r) samples is adopted for the GS method, in which case the elapsed time is 8 hours and 11 minutes per single frequency point. The results show that, compared to GS, in order to reach the same accuracy, the proposed method requires less computational resources.

Finally, as an additional illustration, the parameters h and ϵ_r are now considered as tradional uniform random variables with support [3.44, 4.44] and [1.43, 1.63], respectively, and the corresponding cdf, computed with 10000 (h, ϵ_r) samples, is shown in Figs. 4 and 5 for $|S_{11}|$ and $|S_{21}|$. As indicated in Section II-A, the cdf is always in the domain defined by the possibility and necessity functions. The calculations have been performed on a computer with 8 GB and 8 cores Intel(R) Core(TM) i7-2600 CPU @ 3.40GHz.

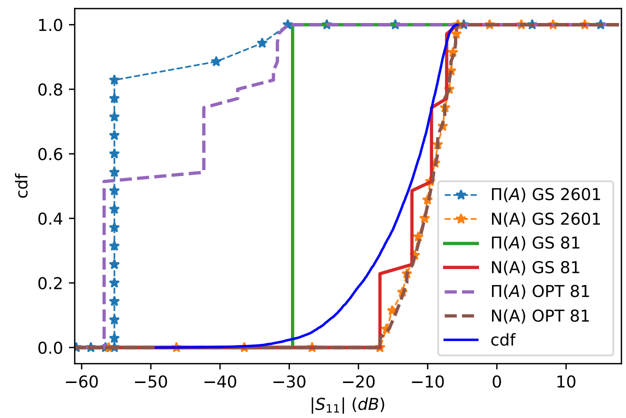

Fig. 4: Possibility and Necessity functions of the magnitude of S_{11} (dB) at 2.4557 GHz estimated with the GS and the proposed BO-based method for different number of samples.

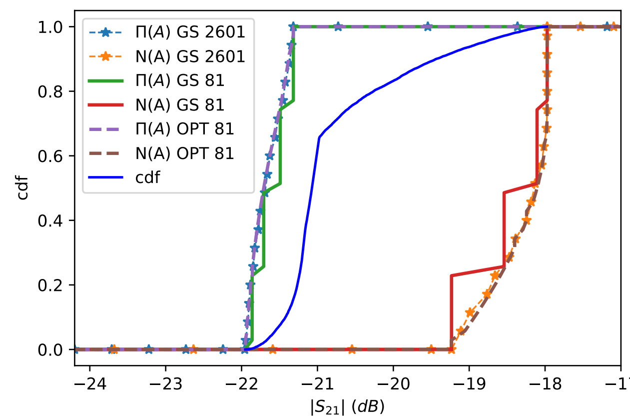

Fig. 5: Possibility and Necessity Functions of the magnitude of S_{21} (dB) at 2.4557 GHz estimated with the GS and the proposed BO-based method for different number of samples.

V. Conclusion

A novel machine-learning based modeling framework is presented in this contribution to solve epistemic UQ problems. The proposed approach characterizes epistemic variations using possibility theory, and leverages on the theory of fuzzy sets combined with a suitable BO-based approach to solve UQ problems in antenna design. Contrary to stochastic approaches, no characterization of the parameters' variability in a probabilistic sense is needed. Owing to the flexibility of BO, the proposed technique can be applied to a large variety of problems, leading to an accurate characterization of non-smooth behaviors with respect to the epistemic parameters. A suitable application example validates the performance of the proposed method.

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