Introduction

Data on hydrocarbon reservoir attributes (e.g., permeability, porosity) are only available at a set of sparse locations, thus resulting (at best) in an incomplete knowledge of spatial heterogeneity of the system. This lack of information propagates to uncertainty in our evaluations of reservoir performance and of the resulting oil recovery. A variety of studies framed in the context of a stochastic approach have been performed for single phase fluid flow and transport (e.g., Dagan, 1989; Gelhar, 1993; Dagan and Neuman, 1997; Zhang and Winter 1999; Sanchez-Vila et al., 2006; Guadagnini et al., 2018 and references therein). Here, we consider a two-phase flow setting taking place in a randomly heterogeneous (correlated) permeability field and study competitive effects on fractional flow due to (*a*) viscous and (*b*) gravity forces through a suite of detailed computational experiments. With reference to these types of problems, Zhang and Tchelepi (1999) consider immiscible two-phase displacement as described through the Buckley-Leverett approach to estimate saturation and the associated uncertainty in random media. These authors ground their study on an extension of (statistical) moment equations of single phase fully saturated subsurface flow (see, e.g., Guadagnini and Neuman, 1999; Zhang, 2002; Ye et al., 2004; Zhang and Lu, 2004) to two-phase flow. This approach entails approximating otherwise exact moment equations through a perturbative technique. Examples of studies of heterogeneity effects within the context of the Buckley-Leverett approach with focus on saturation front displacement are illustrated by Noetinger et al. (2006) and Teodorovich et al. (2011). To the best of our knowledge, an assessment of the feedback between viscous and gravity forces and random spatial heterogeneity of permeability fields is still unexplored. This is precisely the aim of our study, which is set in a numerical Monte Carlo (MC) context and is targeted to characterize oil recovery estimates under uncertainty for a one-dimensional waterflooding scenario.

Problem formulation

In the absence of source terms, mass conservation of water in a two-phase flow setting reads:

$$\phi \frac{\partial S_w}{\partial t} + \nabla \cdot \mathbf{q_w} = 0 \tag{1}$$

Here, ϕ denotes porosity, S_w is water saturation, t representing time; the water flux vector, $\mathbf{q_w}$, is driven by the action of three processes: (*a*) viscous forces, (*b*) gravitational segregation, and (*c*) capillary forces. Introducing fractional flow of water, f_w, as the ratio of water flow to total (oil and water) flow, one obtains (e.g., Blunt, 2017):

$$q_w = \frac{\lambda_w}{\lambda_t} q_t + K \frac{\lambda_w \lambda_o}{\lambda_t} (\rho_w - \rho_o) g_x + K \frac{\lambda_w \lambda_o}{\lambda_t} \frac{dP_c}{dS_w} \frac{\partial S_w}{\partial x} \equiv f_w q_t \tag{2}$$

Here, $\lambda_j = K_{r,j} / \mu_j$ is mobility ($K_{r,j}$ and μ_j being relative permeability and viscosity of water (*j* = *w*) or oil (*j* = *o*), respectively); $\lambda_t = \lambda_w + \lambda_o$ is total mobility; q_t is total flow; K is absolute permeability; ρ_j (*j* = *w*, *o*) denotes fluid density; P_c is capillary pressure; and g_x represents the component of gravity along the flow direction (i.e., $g_x = g \sin\theta$ for flow tilted at an angle θ (evaluated counter-clockwise) from the horizontal).

In a one-dimensional setting Eq. (1) can be rewritten as:

$$\phi \frac{\partial S_w}{\partial t} + q_t \frac{\partial f_w}{\partial x} = 0 \tag{3}$$

where *x* denotes a spatial coordinate. In the absence of capillary forces, f_w can be expressed as:

$$f_w = \frac{1 + K_{ro} N_{gv}}{1 + \mu_w K_{ro} / \mu_o K_{rw}} \tag{4}$$

where the gravity term, N_{gv}, being expressed as:

$$N_{gv} = \frac{K(\rho_w - \rho_o) g_x}{\mu_o q_t} \tag{5}$$

Methodological Approach

Equation (3) is solved numerically via a Finite Difference approach in the presence of a spatially heterogeneous random permeability field. Taylor series expansion is used to approximate the derivatives of the state variables, high order terms in the expansion being considered to improve the accuracy of the solution (Ferziger and Peric, 2002). The latter is assessed through a comparison against the available Buckley-Leverett analytical solution (Buckley and Leverett, 1942) associated with a uniform permeability field. A grid convergence analysis is performed, considering as a benchmark the available analytical solution in the presence of viscous displacement corresponding to a waterflooding scenario controlled by an imposed water injection rate. We tackle random spatial variability of permeability in a numerical MC framework. The latter entails generating a collection of random realizations of a spatially correlated permeability field with given statistics. We consider the natural logarithm of permeability, $Y = \ln(K)$, as a second-order stationary spatial random process with given mean, $\langle Y \rangle$, and zero-mean fluctuation, $Y' = Y - \langle Y \rangle$. The latter is modeled as a Gaussian random field characterized by an isotropic exponential covariance. Porosity is considered as a deterministic constant in our study.

Showcase Scenario

We study a one-dimensional waterflooding scenario developing in a porous medium initially fully saturated with oil, water and oil being modeled as immiscible fluids. We consider constant injection rate of water at one end of the system, oil production being monitored at the downstream boundary. We analyze various scenarios, starting from a horizontal system (i.e., θ = 0 deg) and systematically assessing the effects of -90 deg $\leq \theta \leq 90$ deg.
As an example, Figure 1 depicts the dependence of oil recovery on θ (positive when counter-clockwise from the horizontal-line), thus providing a visual depiction of the impact of gravity effects on oil recovery. Results are obtained for constant viscous effects (i.e., N_{gv} (5) is varied solely through θ) and a uniform absolute permeability of 100 millidarcy (corresponding to $\langle Y \rangle$ = 4.6), and are depicted in terms of oil recovery at water breakthrough time (i.e., when the first breakthrough of the injected water is recorded) and Final Oil Recovery (i.e. oil recovery after 1.5 pore volume of water injection). Downhill flows (i.e., θ < 0) are associated with the highest recoveries.

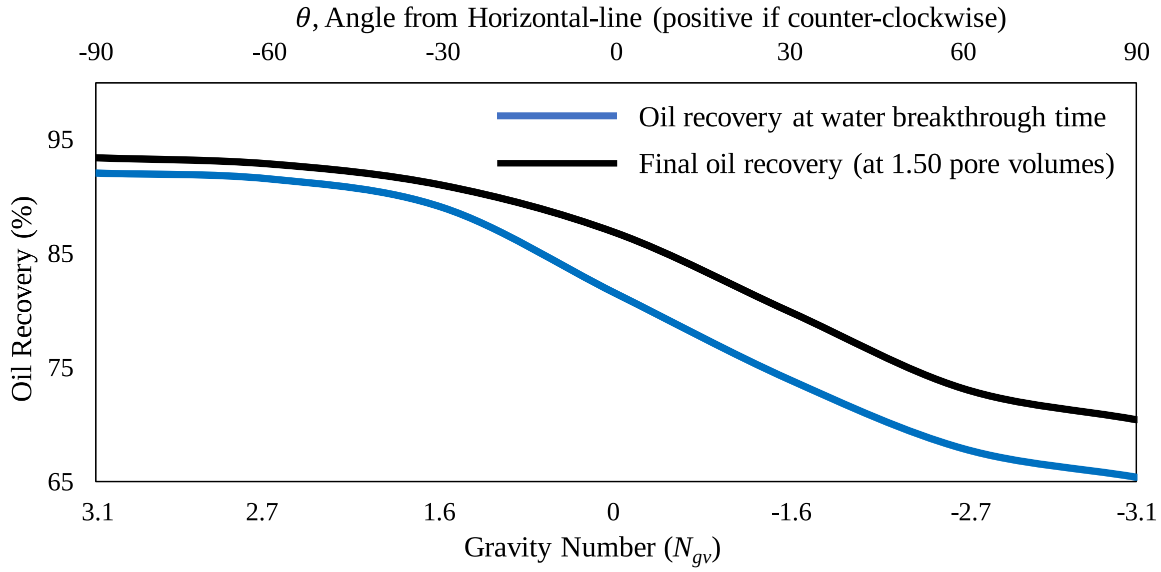

Figure 1 *Gravity effects on oil recovery (at water breakthrough time, blue, and after 1.5 pore volumes, black) of a two-phase flow in a homogeneous one-dimensional domain. Results are in terms of the gravity number N_{gv} (5) and angle θ according to which the domain is tilted from the horizontal.*

Results and discussion

Our numerical MC study is based on the generation of N = 10,000 realizations of Y. In our examples, the variance of Y is taken as σ_Y^2 = 0.01, correlation scale of Y being λ_Y = 0.2 L (where L is equal to unity, in consistent units, a total of M = 100 generation nodes per correlation scale being employed). Figure 2 depicts the empirical probability density function (*pdf*) obtained for the final oil recovery when flow is horizontal (i.e., N_{gv} = 0.0; or θ = 0). The corresponding Gaussian density associated with the same mean and variance is also shown for comparison.

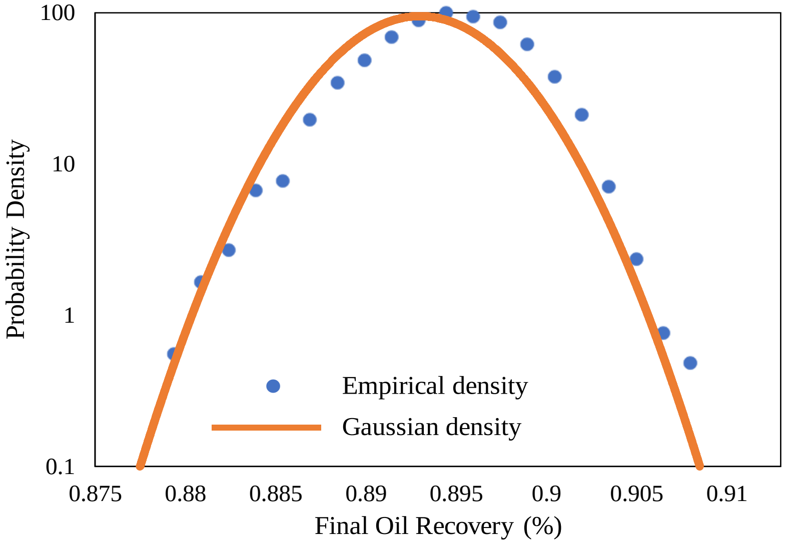

Figure 2 *Sample (i.e., based on N = 10,000 MC realizations) probability density function of final oil recovery; The Gaussian density associated with the same mean and variance is also shown.*

Figure 3 depicts the dependence of oil recovery variance on θ, N_{gv} and σ_Y^2.

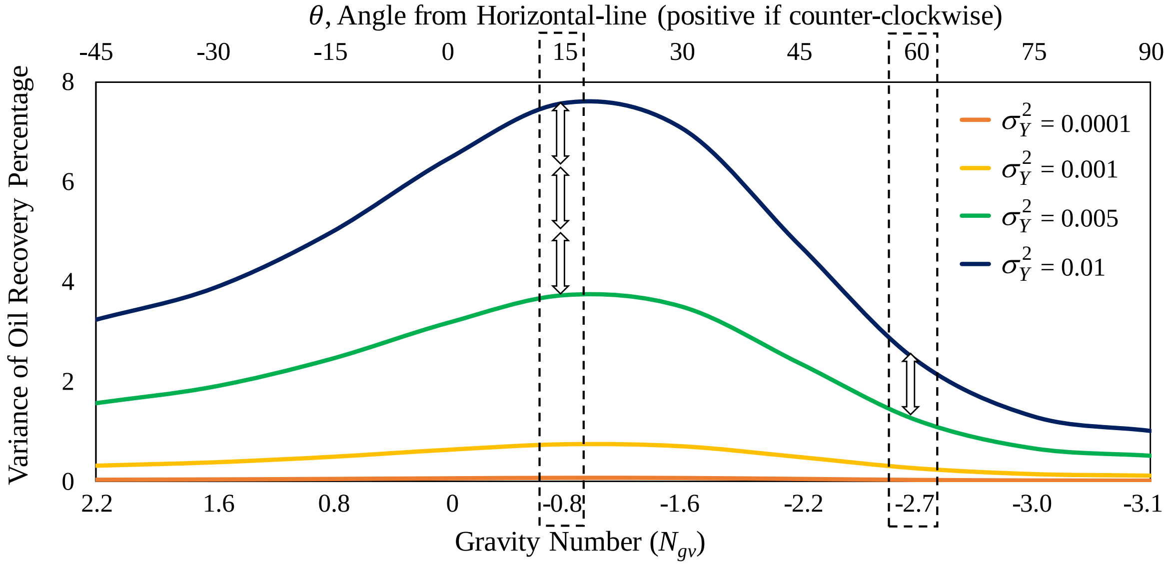

Figure 3 *Effect of gravity on variance of oil recovery. Results are in terms of the gravity number N_{gv} (5) and angle θ according to which the domain is tilted from the horizontal.*

Comparison of the results depicted in Figure 3 indicates that increasing the spatial variability of permeability yields an enhanced uncertainty (as expressed in terms of variance) in oil recovery. The lowest values of variance are observed for vertical flows (i.e., θ = 90). This behavior is consistent with the observation that gravity effects are largest for these settings, thus resulting in a collection of realizations where oil recovery fractions for a given time tend to be consistently small for θ = 90. We further note that the largest values of variance of oil recovery fraction are observed for intermediate values of θ, where viscous and gravity forces have a joint effect on the system dynamics. The dotted boxes in the figure show that increasing log-permeability variance from 0.005 to 0.01 (with keeping a constant $\langle Y \rangle$), the resulting uncertainty is three times larger at θ = 15 deg than at θ = 60 deg. Note that oil recovery uncertainty changes also as a function of fluid properties, e.g. in terms of mobility ratio (not shown).

Conclusions

Our work leads to the following major conclusions:

1. Uncertainty in the spatial distribution of permeability propagates to final oil recovery fractions in a way that depends on the feedback between gravity and viscous forces driving the system.
2. Uncertainty of final oil fraction recovered (as rendered in terms of variance) is smallest for vertical flows, consistent with the observation that the absolute value of the gravity number is largest in such scenarios and is dominant in controlling the flow dynamics.
3. Variance of final oil fraction recovered tends to be higher when there is competition between the effects of gravity and viscous forces, the latter being influenced by the strength of the spatial variability of permeability.

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