Symbol Error Rate Analysis of OFDM System with CFO over TWDP Fading Channel

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Abstract In this manuscript, an exact symbol error rate (SER) analysis of orthogonal frequency division multiplexing (OFDM) system is presented in the presence of carrier frequency offset (CFO) over two wave with diffuse power (TWDP) fading channel. Both Binary Phase Shift Keying (BPSK) and Quadrature Phase Shift Keying (QPSK) modulation techniques are considered in this study over TWDP channel which contains Rayleigh, Rician and two ray fading as its special cases. The results are validated by means of Monte Carlo simulations and also verified from the benchmark results available in the literature.

Keywords Carrier frequency offset (CFO) · error probability analysis · two waves with diffuse power (TWDP) fading channel.

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1 Introduction

The orthogonal frequency division multiplexing (OFDM) has been embraced as the transmission technique for 5^{th} generation of the cellular system due to its several advantageous features, such as simple channel estimation, low-complexity equalization, efficient hardware implementation, and easy combination with multiple-input multiple-output transmission [1]. However, some problems still exist in OFDM systems. One of the main issues of OFDM is its sensitivity to synchronization errors, especially carrier frequency offset (CFO) at the receiver. This CFO generates intercarrier interference (ICI) which demolishes the orthogonality between subcarriers and results in the degradation of the performance of OFDM system [2].

The degradation in performance due to ICI can be calculated either from the probability of error or from the signal-to-interference plus noise ratio (SINR). However, the probability of error is considered as a more accurate metric than SINR because it provides a more insightful evaluation of the degradation. In the literature, several studies are available which take into account the effect of either CFO or symbol timing offset (STO) or both while calculating the probability of error of OFDM system [3]- [8]. Two different approaches have been used in these studies. In first approach, bit error rate (BER) analysis has been done by approximating the ICI as a Gaussian process [3]- [4]. This approach is valid only for low values of SNR [4]. On the contrary, in another approach, the exact SER expressions of OFDM system in the presence of either CFO or STO are obtained using characteristic function and Beaulieu series [5]- [8]. However, in most of the studies, the performance analysis is done considering Rayleigh and Rician fading channels only. In the available literature, closed form expressions of symbol error rate (SER) of both Binary Phase Shift Keying (BPSK) and Quadrature Phase Shift Keying (QPSK) OFDM system with CFO is not existing over two wave with diffuse power (TWDP) fading channel.

The TWDP channel describes the small scale fading in presence of two dominant multipath components called as specular components and multiple diffused scatter components [9]. Two physical parameters of the wireless channel are used to characterize TWDP channel:

- The first parameter is the ratio between the average of two specular components and the diffuse power defined by $K = \frac{T_1^2+T_2^2}{2\sigma^2}$, where T_1 and T_2 denotes the magnitudes of the specular components and $2\sigma^2$ represents the power of the diffuse component.
- The second parameter gives the comparative strength of the two specular waves and is defined as $\Delta = \frac{2T_1T_2}{T_1^2+T_2^2}$.

The PDF of the TWDP fading channel reduces to the well known Rayleigh and Rician PDFs for the two special cases when $K = 0$ and $K \neq 0$, $\Delta = 0$ respectively. Moreover, for very high values of K and $\Delta = 1$ the resulting PDF reduces to two ray fading in which the two multipath components are of equal weights. Hence, TWDP PDF can be visualized as a generalized fading to better represent the real world fading scenarios [10].

Therefore, an attempt is made in this letter using the similar approach as given in [6] to analyse the SER performance of BPSK and QPSK OFDM system with CFO over frequency selective TWDP fading channel. The results from the proposed expression are verified from the results available in literature by substituting different values of the TWDP fading parameters K and $\varDelta$.

The paper is organized as follows. Section 2 describes the model of the OFDM system. In Section 3 SER analysis is given. Comparison of theoretical results with simulation results is given in Section 4. Section 5 gives the concluding remarks of the paper.

2 OFDM System Model

In this paper, it is assumed that the channel coefficients remain constant for the complete duration of the OFDM symbol. The channel coefficients h_i, $i = 1, 2, \cdots, L$ are modeled as complex Gaussian random variables with zero mean and variances $\sigma_{h_i}^2 = 1/L$. The received signal on the kth sub-carrier is given by

$$y_k = \tau_k X_k S_1 + \sum_{m=1, m\neq k}^{N} \tau_m S_{m-k+1} X_m + n_k, \quad k = 1, 2, \ldots, N, \tag{1}$$

where, X_k denotes the k^{th} symbol drawn from a given complex constellation, $[\tau_1, \tau_2, \cdots, \tau_N]^T = F_L h$, $h = (h_1, h_2, \cdots, h_L)^T$ represents the vector of channel coefficients and F_L denotes the first L columns of the Discrete Fourier Transform matrix, N indicates the number of sub-carriers and n_k is the kth sample of an independent and identically distributed sequence of zero mean complex Gaussian noise where the real and imaginary components have equal variance σ_R^2. The ICI coefficient S_k is given as [6]-

$$S_k = \frac{\sin\left(\pi\left[k-1+\epsilon\right]\right) \exp\left(j\pi\left(1-\frac{1}{N}\right)\left(K-1-\epsilon\right)\right)}{N \sin\left(\frac{\sin(\pi[k-1+\epsilon])}{N}\right)}, \tag{2}$$

where, ϵ is the CFO normalized to sub-carrier spacing defined as $\epsilon = \delta_f \times T_u$, T_u is the useful period of one OFDM symbol and δ_f is the CFO. The absolute value of the complex fading coefficient $|\tau|$ has a TWDP PDF given by [9]-

$$\begin{aligned} p_{|\tau|}\left(|\tau|\right) =& \frac{2|\tau|}{C_{\tau_1\tau_1}} \exp\left(-K - \frac{|\tau|^2}{C_{\tau_1\tau_1}}\right) \\ & \sum_{i=1}^{L} C_i \left(\frac{1}{2}\exp(\beta_i K) I_0\left(\frac{\tau}{\sqrt{C_{\tau_1\tau_1}/2}}\sqrt{2K(1-\beta_i)}\right)\right) + \\ & \left(\frac{1}{2}\exp(-\beta_i K) I_0\left(\frac{\tau}{\sqrt{C_{\tau_1\tau_1}/2}}\sqrt{2K(1+\beta_i)}\right)\right), \end{aligned} \tag{3}$$

where, $L\left(\geq \frac{K\varDelta}{2}\right)$ represents the order of the PDF, C_i is the parameter containing L coefficients whose values are stated in [11, Table II], $\beta_i = \varDelta \cos\left(\frac{\pi(i-1)}{2L-1}\right)$, $C_{\tau_1\tau_1}$ is the covariance of $p_{|\tau|}\left(|\tau|\right)$ as defined in [6, (27)] and I_0 is the zeroth-order modified Bessel function of the first kind [12].

3 SER Analysis

In this section, the SER analytical expression for BPSK and QPSK based OFDM techniques are derived.

3.1 BPSK

In BPSK modulation, symbol X_k is drawn from the set $\{\pm 1\}$. It is assumed that the symbol on the first sub-carrier is $X_1 = 1$, which must be equalized by multiplying the received signal with the conjugate of τ_1, represented as $\bar{\tau_1}$ to form the decision variable $\Re(\bar{\tau_1} r_1)$. With approach similar to [6], the conditional bit error probability for BPSK modulation can be defined as-

$$P_s(\xi|\tau_1) = \frac{1}{2^{N-1}} \sum_{k=1}^{2^{N-2}} Q\left(\frac{\tau_1[\Re(S_1)+a_k]}{\sigma\sqrt{1+\frac{b_k}{2\sigma^2}}}\right) + Q\left(\frac{\tau_1[\Re(S_1)-a_k]}{\sigma\sqrt{1+\frac{b_k}{2\sigma^2}}}\right). \quad (4)$$

where, $a_k = \Re\left(P_k^T C_{\tau_1\tau_1}\right)$ and $b_k = P_k^T\left(C_{\tau\tau} - C_{\tau\tau_1} C_{\tau\tau_1}^H]\right)\bar{P}_k$ with diag$(\cdot)$, $(\cdot)^T$ and $(\cdot)^H$ being the diagonal matrix, transpose and Hermitian operations, respectively. $P_k = e_k$ diag$(S(1), S(2), \cdots, S(N-1))$ with e_k as a vector of length $(N-1)$ containing the binary representation of the number $(2^{N-1}-k)$, where $0s$ are replaced by $-1s$ [6]. Now, the unconditional probability of symbol error is evaluated by averaging (4) over the PDF of the TWDP fading channel (3) as-

$$P_s(\xi) = \int_0^\infty P_s(\xi|\tau_1) p_{|\tau_1|}(|\tau_1|)\, d\tau_1 \quad (5)$$

After observing the derivation given in appendix I, the final analytical expression of the SER is presented in (6), where $\gamma = E_s/N_0$ is the SNR in which E_s is the average energy per symbol and $N_0 = 2\sigma_R^2$ is the power spectral density of complex AWGN.

3.2 QPSK

In QPSK modulation, the symbol X_k is drawn from the set $\{\pm 1 \pm j\}$. As done for BPSK, it is assumed that the symbol transmitted on the first sub-carrier is $X_1 = 1 + j$. The approach of [7] has been followed to derive the conditional SER expression given by-

$$P_s(\xi|\tau_1) = 1 - \frac{1}{2^{2N-2}} \sum_{k=1}^{2^{N-2}} \sum_{n=1}^{2^{N-2}} \sum_{m=1}^{4}$$
$$Q\left(\frac{-\tau_1(S_A + \mu_{k,n}[1,m])}{\sqrt{\gamma C_{\tau_1\tau_1}/2}\sqrt{1+\frac{V_{k,n}[m]}{\gamma C_{\tau_1\tau_1}}}}\right) Q\left(\frac{-\tau_1(S_B + \mu_{k,n}[2,m])}{\sqrt{\gamma C_{\tau_1\tau_1}/2}\sqrt{1+\frac{V_{k,n}[m]}{\gamma C_{\tau_1\tau_1}}}}\right). \quad (7)$$

where, $S_A = (\Re(S_i)\Im(S_i))^T$, $S_B = (\Im(S_i) - \Re(S_i))^T$, for $i = 1, 2, \cdots N$, $\mu_{k,n}[l,m]$ is the $(l,m)^{th}$ entry of 2×4 matrix $N = ((N_A + N_B)(-N_A - N_B)(N_A - N_B)(-N_A +$

$$P_s(\xi) = \frac{1}{2^{N-1}} \sum_{k=1}^{2^{N-2}} \sum_{i=1}^{L} \frac{C_i}{2} \left[e^{(-K+\beta_i K)} \sum_{l1=0}^{\infty} \frac{[K(1-\beta_i)]^{l1}}{l1!} + e^{(-K-\beta_i K)} \sum_{l1=0}^{\infty} \frac{[K(1+\beta_i)]^{l1}}{l1!} \right]$$

$$\left\{ \left[\frac{1}{2} \left(1 - \sqrt{\frac{\frac{C_{\tau_1\tau_1}\gamma[\Re(S_1)+a_k]^2}{C_{\tau_1\tau_1}+b_k}}{1+\gamma\left(\frac{C_{\tau_1\tau_1}[\Re(S_1)+a_k]^2}{C_{\tau_1\tau_1}+b_k}\right)}} \right) \right]^{l1+1} \sum_{l2=0}^{l1} \binom{l1+l2}{l2} \right.$$

$$\left[1 - \frac{1}{2} \left(1 - \sqrt{\frac{\frac{C_{\tau_1\tau_1}\gamma[\Re(S_1)+a_k]^2}{C_{\tau_1\tau_1}+b_k}}{1+\gamma\left(\frac{C_{\tau_1\tau_1}[\Re(S_1)+a_k]^2}{C_{\tau_1\tau_1}+b_k}\right)}} \right) \right]^{l2} + \left[\frac{1}{2} \left(1 - \sqrt{\frac{\frac{C_{\tau_1\tau_1}\gamma[\Re(S_1)-a_k]^2}{C_{\tau_1\tau_1}+b_k}}{1+\gamma\left(\frac{C_{\tau_1\tau_1}[\Re(S_1)-a_k]^2}{C_{\tau_1\tau_1}+b_k}\right)}} \right) \right]^{l1+1}$$

$$\left. \sum_{l2=0}^{l1} \binom{l1+l2}{l2} \left[1 - \frac{1}{2} \left(1 - \sqrt{\frac{\frac{C_{\tau_1\tau_1}\gamma[\Re(S_1)-a_k]^2}{C_{\tau_1\tau_1}+b_k}}{1+\gamma\left(\frac{C_{\tau_1\tau_1}[\Re(S_1)-a_k]^2}{C_{\tau_1\tau_1}+b_k}\right)}} \right) \right]^{l2} \right\} \quad (6)$$

$N_B))$ with $N_A = C^{-1}_{\tau_1\tau_1}(\Re(P_K^T C_{\tau\tau_1})\Im(P_K^T C_{\tau\tau_1}))^T$, $N_B = C^{-1}_{\tau_1\tau_1}(\Im(P_K^T C_{\tau\tau_1}) - \Re(P_K^T C_{\tau\tau_1}))^T$ and $V_{k,n}[m]$ is the mth element of 1×4 matrix $V = (V_1 V_2 V_3 V_4)$ with $V_m = P_i C_{\tau\tau_1} P_i^T$.

The unconditional SER can now be derived by averaging (7) over the fading channel PDF. The final SER expression for the QPSK modulation is given in (8). The derivation is given in Appendix II.

4 Simulation Results

The SER expressions derived in previous section for both BPSK and QPSK based OFDM system are numerically evaluated and verified with Monte Carlo simulations for different values of TWDP fading channel parameter (*i.e.*, K and Δ) and CFO (*i.e.*, $\epsilon = 0.01$, 0.1 and 0.2). To calculate both analytical as well as simulation results, the number of subcarriers N is assumed to be 16 with cyclic prefix of length $N/4$ and 3 tap fading channel ($L = 3$). It is observed that while calculating the ASER from the derived expressions, 50 terms of l_1 are sufficient to achieve an acceptable accuracy. Any further increase in the number of terms does not affect the results.

Fig. 1 and Fig. 2 shows the SER versus E_s/N_0 plots of both BPSK and QPSK based OFDM system for $K = 0$ and 10, respectively. It is clearly visible from these plot that the results from derived expressions convert to the Rayleigh and Rician fading case at $K = 0$ and 10 respectively and hence validates the derived expression. The effect of varying CFO and modulation scheme (BPSK/QPSK) can also be observed from these plots.

The SER performance of BPSK is better than QPSK for a fixed value of CFO and K, as expected. Similarly, the SER performance degrades as CFO increases for a given modulation scheme and channel parameter (*i.e.*, K and Δ). Figure 3 shows the SER plots of both BPSK and QPSK OFDM with varying channel parameters K and Δ for a fixed value of CFO (*i.e.*, $\epsilon = 0.1$). It is evident from this figure that for a fixed value of $K(= 10)$, the SER performance degrades as Δ increases from

$$P_s(\xi) = 1 - \frac{1}{2^{2N-2}} \sum_{k=1}^{2^{N-2}} \sum_{n=1}^{2^{N-2}} \sum_{m=1}^{4} \Bigg[\frac{1}{2} \sum_{i=1}^{L} C_i \Bigg[\exp(-K + \beta_i K) \sum_{l_1=0}^{\infty} \frac{1}{l_1!} \left(K(1-\beta_i)\right)^{l_1} +$$

$$\exp(-K - \beta_i K) \sum_{l_1=0}^{\infty} \frac{1}{l_1!} \left(K(1+\beta_i)\right)^{l_1} \Bigg]\Bigg] \Bigg[\pi \Bigg(\frac{\frac{\pi}{2} - \arctan\left(\frac{A_2}{A_1}\right)}{\pi} - \frac{1}{\pi} D_1$$

$$\Bigg\{ \left(\frac{\pi}{2} + \tan^{-1} D_2\right) \sum_{k_1=0}^{m_1-1} \left(\frac{2k_1!}{k_1!k_1!}\right) \frac{1}{(4(1+c_1))^{k_1}} + \sin(\tan^{-1} D2) \sum_{k_1=1}^{m_1-1} \sum_{i_1=1}^{k_1} \frac{T_{i_1 k_1}}{(1+c_1)^{k_1}}$$

$$\Big[\cos(\tan^{-1} D_2)\Big]^{2(k_1-i_1)+1} \Bigg\}\Bigg) + \pi \Bigg(\frac{\arctan\left(\frac{A_2}{A_1}\right)}{\pi} - \frac{1}{\pi} D_3 \Bigg\{ \left(\frac{\pi}{2} + \tan^{-1} D_4\right)$$

$$\sum_{k_2=0}^{m_2-1} \left(\frac{2k_2!}{k_2!k_2!}\right) \frac{1}{(4(1+c_2))^{k_2}} + \sin(\tan^{-1} D4) \sum_{k_2=1}^{m_2-1} \sum_{i_2=1}^{k_2} \frac{T_{i_2 k_2}}{(1+c_2)^{k_2}}$$

$$\Big[\cos(\tan^{-1} D_4)\Big]^{2(k_2-i_2)+1} \Bigg\}\Bigg)\Bigg]$$

$$\text{where, } D_1 = \sqrt{\frac{c_1}{1+c_1}} \operatorname{sgn}\left(\frac{\pi}{2} - \tan^{-1}\left(\frac{A_2}{A_1}\right)\right), D_2 = -D1 \cot\left(\frac{\pi}{2} - \tan^{-1}\left(\frac{A_2}{A_1}\right)\right),$$

$$D_3 = \sqrt{\frac{c_2}{1+c_2}} \operatorname{sgn}\left(\tan^{-1}\left(\frac{A_2}{A_1}\right)\right), D_4 = -D2 \cot\left(\tan^{-1}\left(\frac{A_2}{A_1}\right)\right),$$

$$c_1 = \frac{A_1^2 \gamma C_{\tau_1 \tau_1}}{2}, c_2 = \frac{A_2^2 \gamma C_{\tau_1 \tau_1}}{2}. \tag{8}$$

0 to 1. This effect shows that the severity of fading increases for higher values of Δ. Similarly, for a fixed value of Δ ($\Delta = 0$), the SER performance improves as K increases from 0 to 10. This is expected because the SER performance of Rician channel is found to outperform the performance of Rayleigh channel. This is because of the presence of specular component in Rician channel.

5 Conclusion

In this letter, the expressions of SER for BPSK and QPSK OFDM system under the effect of CFO are derived over frequency selective TWDP fading channel. The derived analytical results have been verified using Monte Carlo simulation for different values of TWDP fading channel parameter (*i.e.*, K and Δ) and also from the benchmark results available in the previous published literature. As expected, the results show that the performance degrades as CFO increases for different values of K and Δ.

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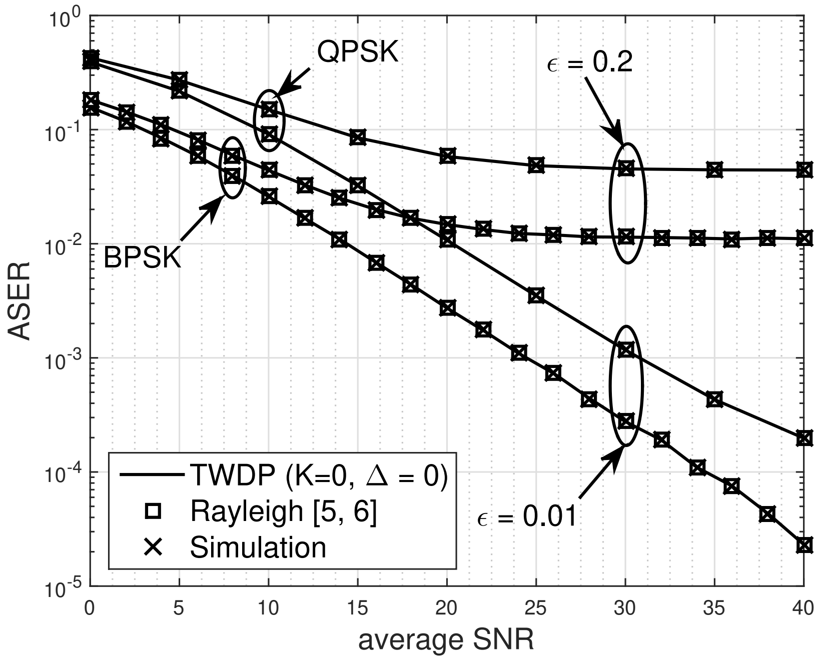

Fig. 1 SER of BPSK OFDM transmission over the TWDP fading channel at different values of CFO with $N = 16$.

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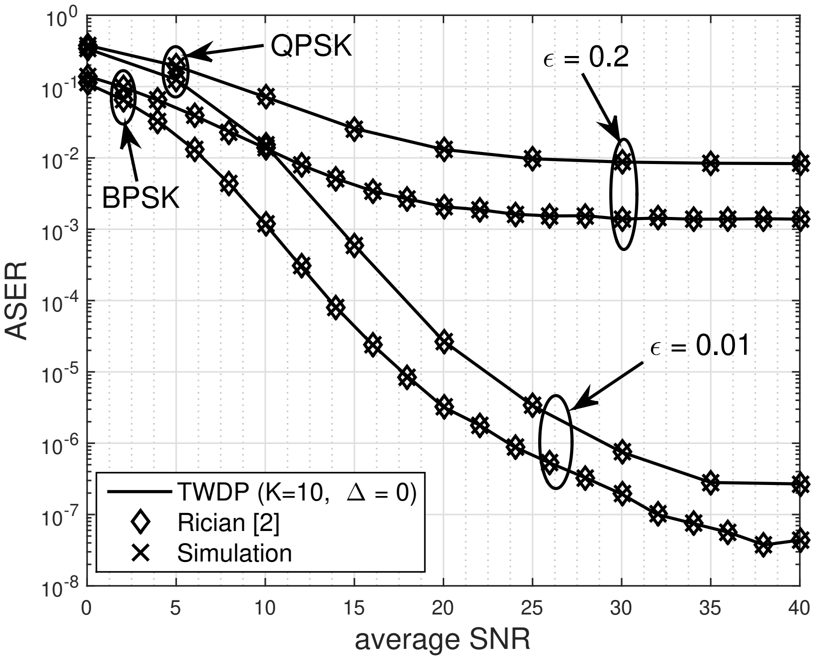

Fig. 2 SER of QPSK OFDM transmission over the TWDP fading channel at different values of CFO with $N = 16$.

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Appendix-I

Analytical expression for BPSK

The integral (5) can be simplified by using the definition of Q function given in [13] and expressed as a sum of four similar terms-

$$P_s(\xi) = \frac{1}{2^{N-1}} \sum_{k=1}^{2^{N-2}} \left[I_1 + I_2 + I_3 + I_4 \right], \tag{I.1}$$

Here, the computation of I_1 is considered as an example where, I_1 is defined as-

$$I_1 = \frac{1}{\pi} \int_0^{\pi/2} \sum_{i=1}^{L} C_i \frac{\exp(-K)}{C_{\tau_1\tau_1}} \frac{\exp(\beta_i K)}{2} \int_0^{\infty} 2\tau \exp\left(\frac{-\tau^2}{C_{\tau_1\tau_1}}\right)$$

$$\exp\left(\frac{-\tau^2 [\Re(S_1) + a_k]^2}{(C_{\tau_1\tau_1} + b_k)\sin^2\psi}\right) I_0 \left(\tau \sqrt{\frac{4K(1-\beta_i)}{C_{\tau_1\tau_1}}}\right) d\tau d\psi. \tag{I.2}$$

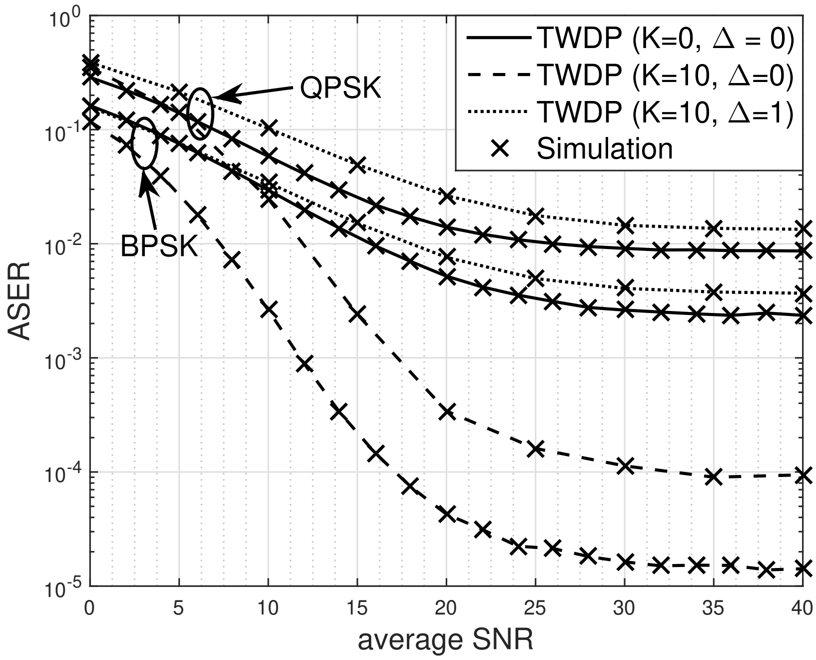

Fig. 3 SER of QPSK OFDM transmission over the TWDP fading channel at different values of CFO with $N = 16$.

After some mathematical adjustments and using the identity [12, (6.61)], the above expression can be rewritten as

$$
\begin{aligned}
I_1 = \frac{1}{\pi} \int_0^{\pi/2} \sum_{i=1}^{L} \frac{C_i}{2} \exp(-K + \beta_i K) \\
\left[\exp\left(\frac{K(1-\beta_i)\sin^2\psi(C_{\tau_1\tau_1} + b_k)}{C_{\tau_1\tau_1}[\Re(S_1) + a_k]^2 + \sin^2\psi(C_{\tau_1\tau_1} + b_k)} \right) \right] \\
\left(\frac{\sin^2\psi(C_{\tau_1\tau_1} + b_k)}{C_{\tau_1\tau_1}[\Re(S_1) + a_k]^2 + \sin^2\psi(C_{\tau_1\tau_1} + b_k)} \right).
\end{aligned} \tag{I.3}
$$

By using Maclaurin series for $\exp(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ and by using [12], the integral in (I.3) can be solved as

$$I_1 = \sum_{i=1}^{L} \frac{C_i}{2} \left[\exp(-K + \beta_i K) \sum_{l1=0}^{\infty} \frac{[K(1-\beta_i)]^{l1}}{l1!} \right]$$
$$\left[\frac{1}{2} \left(1 - \sqrt{\frac{\frac{C_{\tau_1\tau_1}[\Re(S_1)+a_k]^2}{C_{\tau_1\tau_1}+b_k}}{1 + \frac{C_{\tau_1\tau_1}[\Re(S_1)+a_k]^2}{C_{\tau_1\tau_1}+b_k}}} \right) \right]^{l1+1}$$
$$\sum_{l2=0}^{l1} \binom{l1+l2}{l2} \left[1 - \frac{1}{2} \left(1 - \sqrt{\frac{\frac{C_{\tau_1\tau_1}[\Re(S_1)+a_k]^2}{C_{\tau_1\tau_1}+b_k}}{1 + \frac{C_{\tau_1\tau_1}[\Re(S_1)+a_k]^2}{C_{\tau_1\tau_1}+b_k}}} \right) \right]^{l2} . \tag{I.4}$$

Similarly, the integrals I_2, I_3 and I_4 can be solved and combined to form the complete SER expression given in (6).

Appendix-II

Analytical expression for QPSK

The unconditional SER for QPSK is determined as

$$P_s(\xi) = \int_0^{\infty} 1 - \frac{1}{2^{2N-2}} \sum_{k=1}^{2^{N-2}} \sum_{n=1}^{2^{N-2}} \sum_{m=1}^{4} Q\left(-\tau A_1\right) Q\left(-\tau A_2\right) f_\tau(\tau) d\tau, \tag{II.1}$$

where $A_1 = \frac{(S_A + \mu_{k,n}[1,m])}{\sqrt{\gamma C_{\tau_1\tau_1}/2}\sqrt{1+\frac{V_{k,n}[m]}{\gamma C_{\tau_1\tau_1}}}}$, $A_2 = \frac{(S_B + \mu_{k,n}[2,m])}{\sqrt{\gamma C_{\tau_1\tau_1}/2}\sqrt{1+\frac{V_{k,n}[m]}{\gamma C_{\tau_1\tau_1}}}}$. By using the identity of product of two Q functions [7], and substituting the value of Q function from [13], $f_\tau(\tau)$ from (3); the above expression (II.1) can be simplified as sum of four different integral terms-

$$P_s(\xi) = 1 - \frac{1}{2^{N-1}} \sum_{k=1}^{2^{2N-2}} \sum_{n=1}^{2^{N-2}} \sum_{m=1}^{4} [I_1 + I_2 + I_3 + I_4] \tag{II.2}$$

where I_A is defined as

$$I_1 = \frac{1}{2} \int_0^{\infty} \int_0^{\frac{\pi}{2} - \arctan(\frac{A_2}{A_1})} \exp\left(\frac{-\tau^2 A_1^2}{2\sin^2\theta} \right) \frac{2\tau}{C_{\tau_1\tau_1}} \exp\left(-K - \frac{\tau^2}{C_{\tau_1\tau_1}} \right)$$
$$\sum_{i=1}^{L} \frac{C_i}{2} \exp(\beta_i K) I_0 \left(\frac{\tau}{\sqrt{C_{\tau_1\tau_1}/2}} \sqrt{2K(1-\beta_i)} \right) d\theta d\tau \tag{II.3}$$

The outer integral in (II.3) can be solved by using [13] can be written as

$$I_1 = \frac{1}{2} \sum_{i=1}^{L} C_i \left[\exp(-K + \beta_i K) \sum_{l_1=0}^{\infty} \frac{1}{l_1!} \left(K(1-\beta_i) \right)^{l_1} \right]$$
$$\int_0^{\frac{\pi}{2} - \arctan(\frac{A_2}{A_1})} \left(\frac{\sin^2\theta}{\frac{A_1^2 C_{\tau_1\tau_1}}{2} + \sin^2\theta} \right)^{l_1+1} d\theta. \tag{II.4}$$

Finally, using the identity from [13, App. A.5],the integral in (II.4) can be solved. Similarly, the integrals I_2, I_3 and I_4 can be solved and combined to form the complete SER expression is given in (8).