Genetic algorithm and polynomial chaos modelling for performance optimization of photonic circuits under manufacturing variability

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Abstract: We propose an efficient technique based on polynomial chaos expansion and genetic algorithms to enable constrained optimization of photonic integrated circuits subject to fabrication tolerances. Simulations on a realistic SOI design confirm its effectiveness.

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1. Introduction

Photonic integration technologies are rapidly evolving towards mature and stable processes, making possible the integration of complex circuits and functionalities on chip and attracting an increasing interest from many market players. This evolution has urgently posed the necessity to deal with manufacturing variability which often limits the sustainable complexity and poses major problems in achieving high production yield [1]. This is particularly true for high-index-contrast technologies where small fabrication deviations in the waveguide geometry and circuit topology have significant impacts on light propagation. For these reasons large efforts have been recently done in the exploitation of advanced stochastic methods to predict the effects of fabrication uncertainties on the final behavior of the devices and circuits [1–3]. On the other hand, the availability of information on manufacturing variability during device and circuit design allows to go beyond the sole prediction of its effects, opening the possibility to include it in the design process and obtain optimized layouts that show for example an increased fabrication yield.

Previously [4], we proposed a technique to efficiently solve unconstrained optimization of circuit designs subject to fabrication tolerances exploiting generalized polynomial chaos (gPC) models. In this work, we demonstrate how the combination of gPC modeling and genetic algorithms can be used to improve optimization results by introducing constraints in the optimization problems. The use of a gPC models to evaluate the average of both the quantity to be optimized and the constrain function at each step of the genetic algorithm ensures a computational time thousands of times faster than the direct use of Monte Carlo approach. Although this technique is general and widely applicable, we focus here on the realistic example shown in Fig. 1(a). The goal is to minimize the average return loss of the silicon-on-insulator coupled ring resonator filter when exposed to fabrication tolerances by optimizing the value of the coupling coefficients K_i. As a constraint, during the optimization it is required that the 3-dB bandwidth of the filter does not deviate from the initial designed value more than 2.5 GHz in order to preserve the filter functionality. The obtained solution shows an increment of the fabrication yield (calculated as the number of devices with a return loss smaller than -15 dB) of about 23% compared to the initial nominal design when exposed to the same fabrication tolerances. The entire optimization requires only 90 seconds on an i5 CPU at 3.5 GHz whereas the use of Monte Carlo simulations would require more than 8 days.

2. Constrained optimization exploiting genetic algorithms and generalized polynomial chaos modeling

In order to set up the optimization problem, we consider the circuit layout sketched in Fig. 1(a). The initial nominal design of the five-ring silicon-on-insulator filter was obtained with a standard Chebyshev synthesis technique for a bandwidth of 25.5 GHz and considering for all the five rings the same effective index $n_{\mathrm{i}} = 2.23$. The obtained nominal values of the six coupling coefficients are K1 = K6 = 0.34, K2 = K5 = 0.04, K3 = K4 = 0.01. The power transfer functions at the drop and through ports in the ideal case (i.e. without fabrication tolerances) are shown in Fig. 1(b)

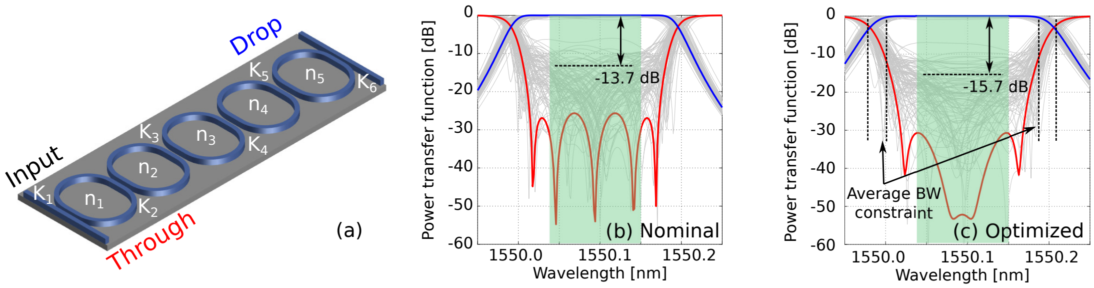

Fig. 1. a) Schematic of the ring-resonator filter structure considered in this work. Power transfer functions at the drop and through ports without manufacturing variability (blue and red bold lines) and considering a Gaussian uncertainties on the waveguide effective indices (thin gray lines) in the case of (b) the nominal design and (c) the constrained optimized design.

with blue and red bold lines, respectively. The ideal response has a return loss of -26.5 dB. To simulate the effect of manufacturing variability, random fluctuations were hence added to the effective indices of the rings with five fabrication tolerance variables $\vec{\xi} = [\Delta n_1,...,\Delta n_5]$, considered as zero-mean Gaussian distributed variables with standard deviation 10^{-5}. As an example, Fig. 1(b) shows few Monte Carlo simulations obtained with this uncertainty (gray thin lines): in this case the average return loss is only around -13.7 dB, calculated in the green highlighted spectral range. The return loss RL($\vec{x}$, $\vec{\xi}$) is hence the quantity of interest for the optimization problem while a constraint is set on the filter bandwidth BW($\vec{x}, \vec{\xi}$), where $\vec{x}$ = [K$_1$,...,K$_6$] is the vector of the six design parameters. The goal is to optimize $\vec{x}$ in order to minimize the average of the return loss $\mathbb{E}_\xi[RL(\vec{x},\vec{\xi})]$ under the defined fabrication tolerances ensuring the average bandwidth deviation from the designed value (25.5 GHz) being smaller than $\pm$2.5 GHz. The constrained optimization problem to be solved by the genetic algorithm can hence be defined as

$$\underset{\vec{x}}{\text{minimize}} \quad \mathbb{E}_\xi\left[RL(\vec{x},\vec{\xi})\right] \quad \text{subject to} \quad \left|\mathbb{E}_\xi\left[BW(\vec{x},\vec{\xi})\right]-25.5\right| \leq 2.5;\ \ a_i \leq x_i \leq b_i,\ i=1,\ldots,m. \tag{1}$$

Constrains are defined also on the acceptable values of the design parameters. An efficient way to calculate the average of both RL and BW is provided by combined generalized polynomial chaos expansion (gPC) models [4]. Since the constraints on x_i are boxed constraints, it is equal to express x_i as uniformly distributed variables in the interval [a$_i$,b$_i$]. In the proposed method, RL and BW are hence expanded on both $\vec{x}$ and $\vec{\xi}$ as

$$RL(\vec{x},\vec{\xi}) \approx \sum_{\vec{\beta},\vec{\gamma}} U_{\vec{\beta},\vec{\gamma}} \mathbb{L}_{\vec{\beta}}(\vec{x}) \Phi_{\vec{\gamma}}(\vec{\xi}); \quad BW(\vec{x},\vec{\xi}) \approx \sum_{\vec{\beta},\vec{\gamma}} G_{\vec{\beta},\vec{\gamma}} \mathbb{L}_{\vec{\beta}}(\vec{x}) \Phi_{\vec{\gamma}}(\vec{\xi}) \tag{2}$$

where $\mathbb{L}_{\vec{\beta}}$ is a multi-variate Legendre polynomial with multi-index β, Φ is a multi-variate polynomial of ξ with multi-index γ and $U_{\vec{\beta},\vec{\gamma}}$ and $G_{\vec{\beta},\vec{\gamma}}$ are the corresponding coefficients that can be computed from an initial pool of Monte Carlo simulations exploiting for example an ℓ_1 minimization solver [4]. Once the models are available, the expectations $\mathbb{E}_\xi[RL(\vec{x},\vec{\xi})]$ and $\mathbb{E}_\xi[BW(\vec{x},\vec{\xi})]$ can be directly calculated from the coefficients $U_{\vec{\beta},\vec{\gamma}}$ and $G_{\vec{\beta},\vec{\gamma}}$. These coefficients are hence passed to an off-the-shelf genetic algorithm to solve problem (1).

3. Numerical results

An initial pool of 5000 Monte Carlo simulations of the filter transfer function was obtained with a transfer-matrix-based circuit simulator and used to calculate both the two combined gPC models for the return loss and the bandwidth. As described in Sec. 2, the optimized solution obtained by the genetic algorithm exploiting these models was hence $\vec{x_{opt}}$ = [0.437, 0.036, 0.015, 0.015, 0.037, 0.453]. The filter power transfer functions are reported in Fig. 1(c) for both the ideal case without any uncertainty (bold lines) and considering manufacturing variability (thin gray lines). As can be seen, the optimized filter has an ideal response (no variability) significantly distorted compared to the nominal design, which is on the contrary outperformed when fabrication tolerances are taken into account: on average the return loss is 2 dB smaller and the average deviation of the 3-dB bandwidth is within the required limit of 2.5 GHz (dashed black lines). Figure 2 (a) and (c) shows the probability density functions of the return loss and bandwidth for the nominal (gray) and optimized (red) filters. Apart from the shift of about 2.5 GHz, the bandwidth does not show significant variations of its statistical behavior between the two cases. As expected, the return loss (shown in linear

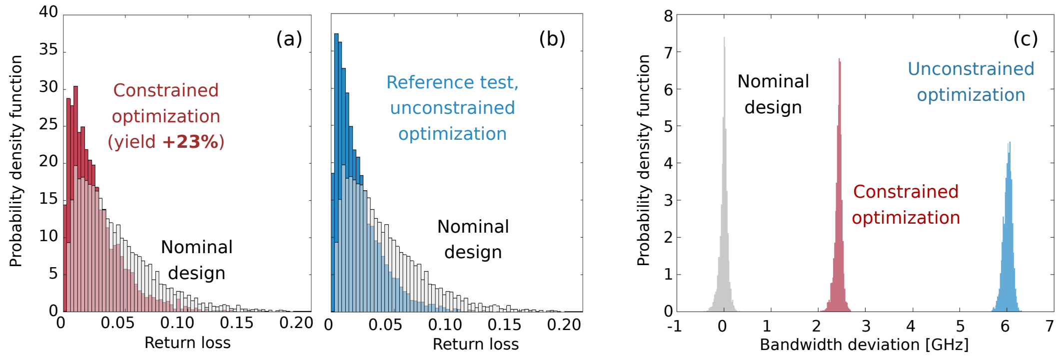

Fig. 2. Probability density function of the return loss of the filter for (a) the constrained optimized design (red) and (b) a reference optimized design (blue) obtained without any constrain on the filter bandwidth. Probability density function for the initial nominal design is shown in gray. (c) Probability density function of the deviation of 3-dB bandwidth from the designed value.

scale for clarity) is less dispersed for the optimized design. If only the filters with a return loss smaller than -15 dB are considered as acceptable devices, than the optimized design shows a fabrication yield of about 73% while the nominal design has a yield of only 50%. To evaluate these results, we performed as a reference test the same optimization described above but removing the constrain on the filter bandwidth, obtaining $\vec{x_{opt}}$ = [0.410, 0.038, 0.018, 0.018, 0.038, 0.417]. Simulations results for this design are shown in Fig. 2 (b) and (c). In this case, the average return loss is slightly smaller compared to the constrained optimization (-16.5 dB) and the yield is about 80%. On the other hand, this marginal improvement comes at the expense of an highly distorted transfer function, with a bandwidth deviation from the designed value that is 6 GHz on average and more sensitive to the fluctuations of the effective indices, as can be seen by the more dispersed probability density function. Lastly, on a workstation with an i5 CPU at 3.5 GHz, the entire optimization, including the computation of the initial MC pool and the two gPC models, and the optimizer time, required in total about 90 seconds. As a comparison, using directly Monte Carlo on the circuit simulator to compute the average of the return loss and bandwidth at each step of the optimizer would require more than 8 days, making the optimization largely impractical for real-use scenario.

4. Conclusions

In this work, we have proposed an efficient method to enable the constrained optimization of integrated photonic devices and circuits by combining generalized polynomial chaos expansion modeling and genetic algorithms. We focused in particular on the optimization of the coupling coefficient of a coupled-ring-resonator filter in order to minimize the average return loss when exposed to fabrication tolerances, allowing only small deviation of the average filter bandwidth from the initial design. The obtained solution showed an increment of the fabrication yield of about 23% compared to the initial nominal design and the optimization was demonstrated to be thousands of times faster than the direct use of Monte Carlo approach.

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