Static output-feedback controller design for vehicle suspensions: an effective two-step computational approach

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1 Introduction

Nowadays, vehicle suspension systems have become a field of increasing relevance in control theory and applications. Taking advantage of new computing tools and efficient numerical algorithms, a significant number of advanced control strategies for active and semi-active suspension systems have been proposed to deal with control problems of growing complexity. Some interesting recent works can be found in [1–8].

Information constraints and, in particular, restricted access to the state information are factors of indubitable importance [9, 10]. When the information available for feedback purposes consists in a reduced number of linear combinations of the states, static output-feedback control strategies constitute an excellent option to facilitate a simple implementation in practice [11–15]. The design of this kind of controllers, however, leads to challenging theoretical problems and serious computational difficulties [11, 16]. To provide a practical solution to these problems, a variety of multi-step numerical algorithms have been proposed, which allow finding suboptimal solutions with a reasonable computational cost [17–25]. These heuristic approaches typically involve a number of free parameters and, for a practical application of the method, a suitable set of parameter values has to be determined. In most cases, however, there is no satisfactory solution for this important issue, which can critically compromise the effectiveness of the method.

Recently, a new computational strategy for static output-feedback controller design was presented in [26]. This approach considers a linear matrix inequality (LMI) formulation of the state-feedback control problem, and uses a suitable transformation of the LMI variables to obtain an LMI formulation for the static output-feedback controller. The definition of the LMI variables transformation involves a matrix $\mathbf{L}$, which can take arbitrary values. For the choice $\mathbf{L} = \mathbf{0}$, this design methodology has produced positive results in the fields of vibration control of large structures [27–30], control of offshore wind turbines [31, 32] and control of active vehicle suspensions [33]. The choice $\mathbf{L} = \mathbf{0}$ has the obvious advantage of its mathematical simplicity, but it presents the drawback of ignoring the specific properties of the considered control problem.

After a detailed study of the $\mathbf{L}$ matrix properties, an improved two-step design methodology has been proposed in [34]. In the initial step, a first LMI optimisation problem is solved to compute an optimal state-feedback controller. As a side product, the LMI solver provides a matrix $\mathbf{X}$ that facilitates a suitable definition of the $\mathbf{L}$ matrix. Next, the output-feedback controller is obtained by solving a second LMI optimisation problem. Overall, the new approach requires solving two LMI optimisation problems. Moreover, the optimal state-feedback controller computed in the first step can be used as a natural reference in the performance assessment of the static output-feedback controller.

The objective of this work is to explore the potential applicability of the new two-step design methodology in the field of vehicle suspensions. In addition, we are also interested in providing a clear and practical presentation of the main theoretical elements of the new approach, which we believe can be of general interest for control engineers in different fields. To this end, two static output-feedback H_∞ controllers are designed for a simplified quarter-car suspension system. The first one uses the suspension deflection and the sprung mass velocity as feedback information, whereas the second one only requires the sprung mass velocity to compute the control actions. The state-feedback H_∞ controller obtained in the first step of the design procedure is used as a reference in the performance assessment. The main contribution of the paper is to provide an effective computational strategy to design static output-feedback controllers for active vehicle suspension systems. This strategy is conceptually simple, allows taking advantage of the existing state-feedback LMI formulations, and can be implemented without determining additional parameter values.

The paper is organised as follows: Section 2 provides a minimal summary of the fundamental theoretical elements involved in the proposed two-step design methodology, and its application to the particular case of H_∞ controller design. In Section 3, a suitable mathematical model for a quarter-car suspension system is provided, and the static output-feedback H_∞ controllers are designed. In Section 4, a suitable set of frequency and time responses are computed to assess the effectiveness of the proposed controllers. Finally, in Section 5, some conclusions and future lines of research are briefly presented.

2 Theoretical background

In this section, we provide a minimal background on the design methodology for static output-feedback controllers proposed in [26, 34]. Next, in Section 2.2, we detail how these general ideas can be applied to the particular case of H_∞ controller design.

2.1 Static output-feedback controller design

Let us consider a state-feedback controller

$$\mathbf{u}(t) = \mathbf{G}_s\,\mathbf{x}(t) \tag{1}$$

where $\mathbf{u}(t) \in \mathbb{R}^m$ and $\mathbf{x}(t) \in \mathbb{R}^n$ denote the control and state vectors, respectively, and $\mathbf{G}_s \in \mathbb{R}^{m\times n}$ is the state gain matrix. Let us also assume that this state-feedback controller can be designed by solving an LMI optimisation problem of the form

$$\begin{cases} \text{minimise } h(\mathbf{X},\mathbf{Y},\boldsymbol{\zeta}) \\ \text{subject to } F(\mathbf{X},\mathbf{Y},\boldsymbol{\zeta}) < \mathbf{0},\ \mathbf{X} > \mathbf{0} \end{cases} \tag{2}$$

where h and F are given affine maps, $\mathbf{X} \in \mathbb{R}^{n\times n}$ and $\mathbf{Y} \in \mathbb{R}^{m\times n}$ are variable matrices, and $\boldsymbol{\zeta} \in \mathbb{R}^{p\times 1}$ is a vector that collects other LMI variables not contained in $\mathbf{X}$ and $\mathbf{Y}$. More precisely, if an optimal solution to the optimisation problem (2) is attained for the triplet$(\tilde{\mathbf{X}}_s, \tilde{\mathbf{Y}}_s, \tilde{\boldsymbol{\zeta}}_s)$, then the state gain matrix can be computed as

$$\mathbf{G}_s = \tilde{\mathbf{Y}}_s\,\tilde{\mathbf{X}}_s^{-1} \tag{3}$$

Now, let us suppose that the available information for feedback purposes consists in a vector of observed outputs $\mathbf{y}(t) \in \mathbb{R}^q$, which can be written in the form

$$\mathbf{y}(t) = \mathbf{C}_y\,\mathbf{x}(t) \tag{4}$$

where $\mathbf{C}_y$ is a $q \times n$ matrix with full row-rank $q < n$. An interesting option in this second scenario consists in considering a static output-feedback controller

$$\mathbf{u}(t) = \mathbf{K}\mathbf{y}(t) \tag{5}$$

which computes the control actions from the observed-output information by means of an output gain matrix $\mathbf{K} \in \mathbb{R}^{m\times q}$.

The problem of obtaining a static output-feedback controller (5) can be seen as a constrained state-feedback control problem, where the state gain matrix $\mathbf{G}$ must admit the factorisation

$$\mathbf{G} = \mathbf{K}\mathbf{C}_y \tag{6}$$

When an LMI formulation of the form (2) is available for the state-feedback controller design, the static output-feedback controller (5) can be computed by solving the following optimisation problem

$$\begin{cases} \text{minimise } h(\mathbf{X},\mathbf{Y},\boldsymbol{\zeta}) \\ \text{subject to } F(\mathbf{X},\mathbf{Y},\boldsymbol{\zeta}) < \mathbf{0},\ \mathbf{X} > \mathbf{0},\ (\mathbf{X},\mathbf{Y}) \in \mathcal{M} \end{cases} \tag{7}$$

where $\mathcal{M}$ is the set of all pairs of matrices $(\mathbf{X},\mathbf{Y})$ for which there exists an $m \times q$ matrix $\mathbf{K}$ satisfying the matrix equation

$$\mathbf{Y}\mathbf{X}^{-1} = \mathbf{K}\mathbf{C}_y \tag{8}$$

Using the results presented in [26], an effective computational strategy to deal with the non-convex optimisation problem (7) and computing the output gain matrix $\mathbf{K}$ can be defined as follows:

(S1) Choose a suitable $(n-q) \times q$ matrix $\mathbf{L}$ and compute

$$\mathbf{R} = \mathbf{C}_y^{\dagger} + \mathbf{Q}\mathbf{L} \tag{9}$$

where $\mathbf{Q}$ is an $n \times (n-q)$ matrix whose columns are a basis of the nullspace of $\mathbf{C}_y$ and $\mathbf{C}_y^{\dagger} = \mathbf{C}_y^{\mathrm{T}}(\mathbf{C}_y\mathbf{C}_y^{\mathrm{T}})^{-1}$ is the Moore–Penrose pseudoinverse of $\mathbf{C}_y$.

(S2) Solve the following LMI optimisation problem with variables $\mathbf{X}_Q, \mathbf{X}_R, \mathbf{Y}_R$ and $\boldsymbol{\zeta}$

$$\begin{cases} \text{minimise } \hat{h}(\mathbf{X}_Q,\mathbf{X}_R,\mathbf{Y}_R,\boldsymbol{\zeta}) \\ \text{subject to } \hat{F}(\mathbf{X}_Q,\mathbf{X}_R,\mathbf{Y}_R,\boldsymbol{\zeta} < \mathbf{0},\ \mathbf{X}_Q) > \mathbf{0},\ \mathbf{X}_R > \mathbf{0} \end{cases} \tag{10}$$

where $\mathbf{X}_Q \in \mathbb{R}^{(n-q)\times(n-q)}$ and $\mathbf{X}_R \in \mathbb{R}^{q\times q}$ are symmetric positive-definite matrices, $\mathbf{Y}_R \in \mathbb{R}^{m\times q}$ and $\boldsymbol{\zeta} \in \mathbb{R}^{p\times 1}$ are arbitrary matrices and the functions $\hat{h}$ and $\hat{F}$ are defined as

$$\begin{aligned} \hat{h}(\mathbf{X}_Q,\mathbf{X}_R,\mathbf{Y}_R,\boldsymbol{\zeta}) &= h(\mathbf{Q}\mathbf{X}_Q\mathbf{Q}^{\mathrm{T}} + \mathbf{R}\mathbf{X}_R\mathbf{R}^{\mathrm{T}}, \mathbf{Y}_R\mathbf{R}^{\mathrm{T}}, \boldsymbol{\zeta}) \\ \hat{F}(\mathbf{X}_Q,\mathbf{X}_R,\mathbf{Y}_R,\boldsymbol{\zeta}) &= F(\mathbf{Q}\mathbf{X}_Q\mathbf{Q}^{\mathrm{T}} + \mathbf{R}\mathbf{X}_R\mathbf{R}^{\mathrm{T}}, \mathbf{Y}_R\mathbf{R}^{\mathrm{T}}, \boldsymbol{\zeta}) \end{aligned} \tag{11}$$

If an optimal solution to the LMI optimisation problem (10) is obtained for the quartet $(\tilde{\mathbf{X}}_Q, \tilde{\mathbf{X}}_R, \tilde{\mathbf{Y}}_R, \tilde{\boldsymbol{\zeta}})$, then the triplet $(\tilde{\mathbf{X}}, \tilde{\mathbf{Y}}, \tilde{\boldsymbol{\zeta}})$ with

$$\tilde{\mathbf{X}} = \mathbf{Q}\tilde{\mathbf{X}}_Q\mathbf{Q}^{\mathrm{T}} + \mathbf{R}\tilde{\mathbf{X}}_R\mathbf{R}^{\mathrm{T}},\ \tilde{\mathbf{Y}} = \tilde{\mathbf{Y}}_R\mathbf{R}^{\mathrm{T}} \tag{12}$$

defines a feasible solution of the optimisation problem (7), and the matrix equation (8) is satisfied by $\tilde{\mathbf{X}}, \tilde{\mathbf{Y}}$ and the output gain matrix

$$\mathbf{K} = \tilde{\mathbf{Y}}_R \tilde{\mathbf{X}}_R^{-1} \tag{13}$$

To date, this computational procedure has been successfully applied to design static output-feedback controllers in the fields of vibration control of large structures [27–30], control of offshore wind turbines [31, 32] and control of active vehicle suspensions [33]. In all these works, a zero matrix $\mathbf{L}$ was selected in the step (S1). The choice $\mathbf{L} = \mathbf{0}$ leads to $\mathbf{R} = \mathbf{C}_y^{\dagger}$ in (9), and has the obvious advantage of its mathematical simplicity. This option, however, presents the drawback of ignoring the specific properties of the considered control problem.

While developing the aforementioned applications, it became apparent that a suitable choice of the matrix $\mathbf{L}$ can exert a critical influence on both, the feasibility of the optimisation problem (10), and the optimality level of the feasible triplet $(\tilde{\mathbf{X}}, \tilde{\mathbf{Y}}, \tilde{\boldsymbol{\zeta}})$. A detailed study of some relevant properties of the matrix $\mathbf{L}$ has been recently presented in [34]. The results obtained in that work led the authors to propose the following $\mathbf{L}$-matrix for the step (S1)

$$\mathbf{L} = \mathbf{Q}^{\dagger} \tilde{\mathbf{X}}_s \mathbf{C}_y^{\mathrm{T}} (\mathbf{C}_y \tilde{\mathbf{X}}_s \mathbf{C}_y^{\mathrm{T}})^{-1} \tag{14}$$

where $\mathbf{Q}^{\dagger} = (\mathbf{Q}^{\mathrm{T}}\mathbf{Q})^{-1}\mathbf{Q}^{\mathrm{T}}$ denotes the Moore–Penrose pseudoinverse of $\mathbf{Q}$ and $\tilde{\mathbf{X}}_s$ is the $\mathbf{X}$-matrix corresponding to an optimal solution $(\tilde{\mathbf{X}}_s, \tilde{\mathbf{Y}}_s, \tilde{\boldsymbol{\zeta}}_s)$ of the LMI optimisation problem (2) associated to the state-feedback controller design. This choice of the matrix $\mathbf{L}$ has proved to be particularly effective in the field of seismic protection of large structures, and it will be used in the present paper to obtain static output-feedback controllers for a quarter-car suspension system with satisfactory results.

2.2 H_∞ controllers

Let us consider a system of the form

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{B}_w \mathbf{w}(t) \\ \mathbf{z}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t) \end{cases} \tag{15}$$

where $\mathbf{x}(t) \in \mathbb{R}^n$ is the state vector, $\mathbf{u}(t) \in \mathbb{R}^m$ is the control input, $\mathbf{w}(t) \in \mathbb{R}^r$ is the disturbance input, $\mathbf{z}(t) \in \mathbb{R}^d$ is the controlled output and $\mathbf{A}, \mathbf{B}, \mathbf{B}_w, \mathbf{C}$ and $\mathbf{D}$ are real constant matrices with appropriate dimensions. For a given state-feedback controller $\mathbf{u}(t) = \mathbf{G}\mathbf{x}(t)$, the following closed-loop system results

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}_G \mathbf{x}(t) + \mathbf{B}_w \mathbf{w}(t) \\ \mathbf{z}(t) = \mathbf{C}_G \mathbf{x}(t) \end{cases} \tag{16}$$

where

$$\mathbf{A}_G = \mathbf{A} + \mathbf{B}\mathbf{G}, \quad \mathbf{C}_G = \mathbf{C} + \mathbf{D}\mathbf{G} \tag{17}$$

In the H_∞ approach, the objective is to obtain a state gain matrix $\mathbf{G}$ that produces an asymptotically stable matrix $\mathbf{A}_G$ and, simultaneously, minimises the H_∞ norm

$$\gamma_G = \|T_G\|_\infty = \sup_{\omega \in \mathbb{R}} \sigma_{\max}[T_G(\mathrm{j}\omega)] \tag{18}$$

where $\sigma_{\max}[\cdot]$ denotes the maximum singular value and

$$T_G(s) = \mathbf{C}_G(s\mathbf{I} - \mathbf{A}_G)^{-1}\mathbf{B}_w \tag{19}$$

is the transfer function from the disturbance input to the controlled output.

According to the 'Bounded Real Lemma' [35], for a given $\gamma > 0$, the closed-loop state matrix $\mathbf{A}_G$ is asymptotically stable and $\gamma_G < \gamma$ if and only if there exists a symmetric positive-definite matrix $\mathbf{X} \in \mathbb{R}^{n \times n}$ that satisfies the matrix inequality

$$\begin{bmatrix} \mathbf{A}_G\mathbf{X} + \mathbf{X}\mathbf{A}_G^{\mathrm{T}} + \gamma^{-2}\mathbf{B}_w\mathbf{B}_w^{\mathrm{T}} & * \\ \mathbf{C}_G\mathbf{X} & -\mathbf{I} \end{bmatrix} < \mathbf{0} \tag{20}$$

where $(*)$ denotes the transpose of the symmetric entry. Using the values of the closed-loop matrices in (17), and by introducing the new variables $\eta = \gamma^{-2}$ and $\mathbf{Y} = \mathbf{G}\mathbf{X}$, we obtain the following LMI

$$\begin{bmatrix} \mathbf{A}\mathbf{X} + \mathbf{X}\mathbf{A}^{\mathrm{T}} + \mathbf{B}\mathbf{Y} + \mathbf{Y}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}} + \eta\mathbf{B}_w\mathbf{B}_w^{\mathrm{T}} & * \\ \mathbf{C}\mathbf{X} + \mathbf{D}\mathbf{Y} & -\mathbf{I} \end{bmatrix} < \mathbf{0} \tag{21}$$

Hence, an optimal state-feedback H_∞ controller can be computed by solving the LMI optimisation problem

$$\begin{cases} \text{maximise } \eta \\ \text{subject to LMI (21)}, \ \eta > 0, \ \mathbf{X} > \mathbf{0} \end{cases} \tag{22}$$

If the triplet $(\tilde{\mathbf{X}}_s, \tilde{\mathbf{Y}}_s, \tilde{\eta}_s)$ provides an optimal solution to (22), then the state gain matrix $\mathbf{G}_s = \tilde{\mathbf{Y}}_s \tilde{\mathbf{X}}_s^{-1}$ defines a state-feedback controller with optimal H_∞-norm

$$\gamma_{G_s} = \tilde{\eta}_s^{-\frac{1}{2}} \tag{23}$$

By setting $\boldsymbol{\zeta} = \eta$, $h(\mathbf{X}, \mathbf{Y}, \eta) = -\eta$, and taking the affine map $F(\mathbf{X}, \mathbf{Y}, \eta)$ as

$$\begin{bmatrix} \mathbf{A}\mathbf{X} + \mathbf{X}\mathbf{A}^{\mathrm{T}} + \mathbf{B}\mathbf{Y} + \mathbf{Y}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}} + \eta\mathbf{B}_w\mathbf{B}_w^{\mathrm{T}} & * & * \\ \mathbf{C}\mathbf{X} + \mathbf{D}\mathbf{Y} & -\mathbf{I} & * \\ \mathbf{0} & \mathbf{0} & -\eta \end{bmatrix} \tag{24}$$

the LMI optimisation problem (22) can be written in the standard form presented in (2). Consequently, for a given observed output $\mathbf{y}(t) = \mathbf{C}_y\mathbf{x}(t)$, the general design methodology presented in Section 2.1 can be applied to obtain a static output-feedback H_∞ controller $\mathbf{u}(t) = \mathbf{K}\mathbf{y}(t)$ for the system (15). In this case, the LMI optimisation problem (10) can be formulated as follows

$$\begin{cases} \text{maximise } \eta \\ \text{subject to } \hat{F}(\mathbf{X}_Q, \mathbf{X}_R, \mathbf{Y}_R, \eta) < \mathbf{0}, \ \mathbf{X}_Q > \mathbf{0}, \ \mathbf{X}_R > \mathbf{0} \end{cases} \tag{25}$$

where $\hat{F}(\mathbf{X}_Q, \mathbf{X}_R, \mathbf{Y}_R, \eta)$ has the form (see (26))

$$\begin{bmatrix} \mathbf{A}\mathbf{Q}\mathbf{X}_Q\mathbf{Q}^{\mathrm{T}} + \mathbf{Q}\mathbf{X}_Q\mathbf{Q}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}} + \mathbf{A}\mathbf{R}\mathbf{X}_R\mathbf{R}^{\mathrm{T}} + \mathbf{R}\mathbf{X}_R\mathbf{R}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}} + \mathbf{B}\mathbf{Y}_R\mathbf{R}^{\mathrm{T}} + \mathbf{R}\mathbf{Y}_R^{\mathrm{T}}\mathbf{B}^{\mathrm{T}} + \eta\mathbf{B}_w\mathbf{B}_w^{\mathrm{T}} & * & * \\ \mathbf{C}\mathbf{Q}\mathbf{X}_Q\mathbf{Q}^{\mathrm{T}} + \mathbf{C}\mathbf{R}\mathbf{X}_R\mathbf{R}^{\mathrm{T}} + \mathbf{D}\mathbf{Y}_R\mathbf{R}^{\mathrm{T}} & -\mathbf{I} & * \\ \mathbf{0} & \mathbf{0} & -\eta \end{bmatrix} \tag{26}$$

If an optimal solution to the optimisation problem (25) is attained for the quartet $(\tilde{\mathbf{X}}_Q, \tilde{\mathbf{X}}_R, \tilde{\mathbf{Y}}_R, \tilde{\eta})$, then the output gain matrix $\mathbf{K} = \tilde{\mathbf{Y}}_R \tilde{\mathbf{X}}_R^{-1}$ defines an output-feedback controller $\mathbf{u}(t) = \mathbf{K}\mathbf{y}(t)$ with an asymptotically stable closed-loop matrix $\mathbf{A}_{G_K}$ and an H_∞-norm γ_{G_K} that satisfies

$$\gamma_{G_K} \leq \tilde{\eta}^{-\frac{1}{2}} \tag{27}$$

where $\mathbf{G}_K = \mathbf{K}\mathbf{C}_y$ is the state gain matrix associated to the output gain matrix $\mathbf{K}$.

Remark 1: It should be noted that the LMI optimisation problem (25) only provides an upper bound for γ_{G_K}. The actual value of the H_∞-norm corresponding to the output-feedback controller $\mathbf{u}(t) = \mathbf{K}\mathbf{y}(t)$ can be obtained by setting $\mathbf{G} = \mathbf{G}_K$ in (20), and solving the LMI optimisation problem

$$\begin{cases} \text{maximise } \eta \\ \text{subject to LMI (20)}, \ \ \eta > 0, \ \ \mathbf{X} > \mathbf{0} \end{cases} \tag{28}$$

If an optimal value $\hat{\eta}$ is obtained in (28), then we have $\gamma_{G_K} = \hat{\eta}^{-1/2}$. Alternatively, γ_{G_K} can also be computed by maximising the maximum singular value of the transfer function T_{G_K}, as indicated in (18).

3 Application to vehicle suspensions

In this section, the design methodology presented in Section 2 is applied to compute two different static output-feedback H_∞ controllers for a quarter-car suspension system. The first controller uses the suspension deflection and the sprung mass velocity as feedback information. The second controller only uses the sprung mass velocity to compute the control actions. A state-feedback H_∞ controller is also designed, which is taken as a natural reference in the performance assessment of the proposed output-feedback controllers, and provides the matrix $\tilde{\mathbf{X}}_s$ to compute the $\mathbf{L}$-matrix defined in (14). The LMI optimisation problems corresponding to the different controller designs have been solved with the 'MATLAB Robust Control Toolbox' [36].

3.1 Quarter-car suspension model

Let us consider a lumped-mass model of a quarter-car suspension system shown in Fig. 1, where m_s and m_u represent the sprung and unsprung masses, respectively; c_s is the damping of the suspension system; k_s and k_u are, respectively, the suspension stiffness and the tyre stiffness; $z_r(t)$ is the vertical road displacement; $z_s(t)$ and $z_u(t)$ represent the vertical displacements of the sprung and unsprung masses, respectively; and $u(t)$ is the active input of the suspension system. The quarter-car motion is governed by the system of second-order differential equations

$$\begin{aligned} m_s\ddot{z}_s(t) &= -c_s[\dot{z}_s(t) - \dot{z}_u(t)] - k_s[z_s(t) - z_u(t)] + u(t) \\ m_u\ddot{z}_u(t) &= c_s[\dot{z}_s(t) - \dot{z}_u(t)] + k_s[z_s(t) - z_u(t)] \\ &\quad - k_u[z_u(t) - z_r(t)] - u(t) \end{aligned} \tag{29}$$

which can be converted into the state-space model

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t) + \mathbf{B}_w w(t) \tag{30}$$

where

$$\mathbf{x}(t) = [z_s(t) - z_u(t), z_u(t) - z_r(t), \dot{z}_s(t), \dot{z}_u(t)]^{\mathrm{T}} \tag{31}$$

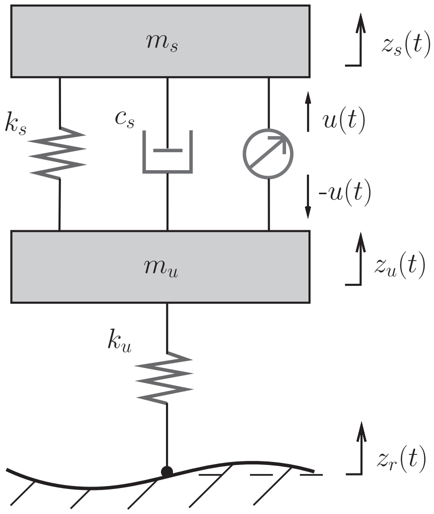

Fig. 1 *Quarter-car suspension model with active suspension*

is the state vector, $w(t) = \dot{z}_r(t)$ is the road displacement velocity, $u(t)$ is the control input and the matrices $\mathbf{A}, \mathbf{B}$ and $\mathbf{B}_w$ have the following form

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \\ -k_s/m_s & 0 & -c_s/m_s & c_s/m_s \\ k_s/m_u & -k_u/m_u & c_s/m_u & -c_s/m_u \end{bmatrix} \tag{32}$$

$$\mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 1/m_s \\ -1/m_u \end{bmatrix}, \quad \mathbf{B}_w = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \end{bmatrix} \tag{33}$$

The following particular values of the parameters [19, 37]

$$\begin{aligned} &m_s = 504.5, \quad m_u = 62\,\text{kg}, \quad k_s = 13100, \\ &k_u = 252000\,\text{N/m}, \quad c_s = 400\,\text{N s/m} \end{aligned} \tag{34}$$

are used in the controllers design of Section 3.2 and in the numerical simulations conducted in Section 4.

The vertical body acceleration is widely used to quantify the ride comfort. Hence, a natural point of interest in the controllers design consists in minimising the sprung mass acceleration $\ddot{z}_s(t)$, especially in the sensitive frequency range of 0–65 rad/s [38]. Additionally, in order to respect the suspension stroke limits and to improve the road holding ability, we are also interested in reducing the suspension deflection $z_s(t) - z_u(t)$ and the tire deflection $z_u(t) - z_r(t)$. Obviously, avoiding high levels of control effort is also desirable. Accordingly, the following vector of controlled outputs is selected

$$\mathbf{z}(t) = [\ddot{z}_s(t), \ \alpha(z_s(t) - z_u(t)), \ \beta(z_u(t) - z_r(t)), \ \rho u(t)]^{\mathrm{T}} \tag{35}$$

where α, β and ρ are weighting coefficients that manage the tradeoff between the conflicting design requirements. Considering the first equation in (29), this vector of controlled outputs can be written in the form

$$\mathbf{z}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}u(t) \tag{36}$$

with

$$\mathbf{C} = \begin{bmatrix} -k_s/m_s & 0 & -c_s/m_s & c_s/m_s \\ \alpha & 0 & 0 & 0 \\ 0 & \beta & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{37}$$

and

$$\mathbf{D} = \begin{bmatrix} 1/m_s \\ 0 \\ 0 \\ \rho \end{bmatrix} \tag{38}$$

The following particular values of the weighting coefficients

$$\alpha = 0.1, \quad \beta = 0.2, \quad \rho = 0.1 \times 10^{-3} \tag{39}$$

are used to compute the controllers presented in the next section.

3.2 Controllers design

Following the discussion in Section 2, we begin by designing an optimal state-feedback H_∞ controller

$$\mathbf{u}(t) = \mathbf{G}_s \, \mathbf{x}(t) \tag{40}$$

which uses the full state $\mathbf{x}(t)$ defined in (31) as feedback information. By solving the LMI optimisation problem (22) with the matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{B}_w$, $\mathbf{C}$ and $\mathbf{D}$ given by (32), (33), (37) and (38), the particular parameter values in (34) and the weighting coefficients in (39), we obtain the state gain matrix

$$\mathbf{G}_s = 10^4 \times [1.1810 \; 0.2333 \; -0.1096 \; 0.0109] \tag{41}$$

and the optimal H_∞-norm

$$\gamma_{G_s} = 7.8365 \tag{42}$$

We also obtain the $\mathbf{X}$-matrix corresponding to this optimal solution

$$\tilde{\mathbf{X}}_s = \begin{bmatrix} 0.5965 & -0.0020 & -0.5075 & -0.0085 \\ -0.0020 & 0.0020 & 0.0020 & -0.0081 \\ -0.5075 & 0.0020 & 0.7659 & -0.0015 \\ -0.0085 & -0.0081 & -0.0015 & 8.0713 \end{bmatrix} \tag{43}$$

Next, we compute a first static output-feedback controller

$$\mathbf{u}(t) = \mathbf{K}_\mathrm{I} \, \mathbf{y}_\mathrm{I}(t) \tag{44}$$

which uses the suspension deflection and the sprung mass velocity as feedback information [19]. In this case, the observed output is given by

$$\mathbf{y}_\mathrm{I}(t) = [z_s(t) - z_u(t), \; \dot{z}_s(t)]^\mathrm{T} \tag{45}$$

which can be written as

$$\mathbf{y}_\mathrm{I}(t) = (\mathbf{C}_y)_\mathrm{I} \, \mathbf{x}(t) \tag{46}$$

with

$$(\mathbf{C}_y)_\mathrm{I} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{47}$$

By computing the nullspace of $(\mathbf{C}_y)_\mathrm{I}$, we obtain the matrix

$$\mathbf{Q}_\mathrm{I} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \tag{48}$$

Using the matrices (43), (47) and (48) in (14), we obtain

$$\mathbf{L}_\mathrm{I} = \begin{bmatrix} -0.0025 & 0.0009 \\ -0.0364 & -0.0261 \end{bmatrix} \tag{49}$$

and, by substituting the matrices (47)–(49) in (9), we finally obtain

$$\mathbf{R}_\mathrm{I} = \begin{bmatrix} 1 & 0 \\ -0.0025 & 0.0009 \\ 0 & 1 \\ -0.0364 & -0.0261 \end{bmatrix} \tag{50}$$

Now, we solve the LMI optimisation problem (25) with the same matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{B}_w$, $\mathbf{C}$ and $\mathbf{D}$ used in the state-feedback controller design, and the matrices $\mathbf{Q}_\mathrm{I}$ and $\mathbf{R}_\mathrm{I}$ given in (48) and (50). As a result, we obtain the output gain matrix

$$\mathbf{K}_\mathrm{I} = 10^4 \times [1.0824 \; -0.2071] \tag{51}$$

The state gain matrix $\mathbf{G}_\mathrm{I} = \mathbf{K}_\mathrm{I}(\mathbf{C}_y)_\mathrm{I}$ associated to the output gain matrix $\mathbf{K}_\mathrm{I}$ is given by

$$\mathbf{G}_\mathrm{I} = 10^4 \times [1.0824 \; 0 \; -0.2071 \; 0] \tag{52}$$

Setting $\mathbf{G} = \mathbf{G}_\mathrm{I}$ in (20), and solving the optimisation problem (28), we obtain an H_∞-norm of

$$\gamma_{G_\mathrm{I}} = 7.8432 \tag{53}$$

To illustrate the flexibility of the proposed design methodology, we compute a second static output-feedback controller

$$\mathbf{u}(t) = \mathbf{K}_\mathrm{II} \, \mathbf{y}_\mathrm{II}(t) \tag{54}$$

which only uses the sprung mass velocity $\dot{z}_s(t)$ as feedback information. In this second case, the observed output can be written as

$$\mathbf{y}_\mathrm{II}(t) = (\mathbf{C}_y)_\mathrm{II} \, \mathbf{x}(t) \tag{55}$$

with

$$(\mathbf{C}_y)_\mathrm{II} = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \tag{56}$$

and the matrices $\mathbf{L}$, $\mathbf{Q}$ and $\mathbf{R}$ take the following values

$$\mathbf{L}_\mathrm{II} = [0.0026 \; -0.6627 \; -0.0019]^\mathrm{T} \tag{57}$$

$$\mathbf{Q}_\mathrm{II} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{R}_\mathrm{II} = \begin{bmatrix} -0.6627 \\ 0.0026 \\ 1.0000 \\ -0.0019 \end{bmatrix} \tag{58}$$

Solving the LMI optimisation problem (25) with the new values of the matrices $\mathbf{Q}$ and $\mathbf{R}$, we obtain the output gain

$$\mathbf{K}_\mathrm{II} = -8.9703 \times 10^3 \tag{59}$$

which has an associated state gain matrix

$$\mathbf{G}_\mathrm{II} = \mathbf{K}_\mathrm{II}(\mathbf{C}_y)_\mathrm{II} = 10^3 \times [0 \; 0 \; -8.9703 \; 0] \tag{60}$$

and H_∞-norm

$$\gamma_{G_\mathrm{II}} = 7.9068 \tag{61}$$

Remark 2: In the first step of the proposed design procedure, the objective is to find a suitable state-feedback controller. This ideal controller has full access to the state information and must satisfy the performance requirements of the problem under consideration. Clearly, if no satisfactory solution can be found for this exploratory step, the possibility of obtaining a suitable static output-feedback controller should be reconsidered. The output-feedback controller presented in [19] has been taken as a reference to compute the control gain matrix $\mathbf{G}_s$ given in (41). In what follows, we will assume that $\mathbf{G}_s$ defines a suitable state-feedback controller for the active suspension system introduced in Section 3.1.

Remark 3: Note that all the controllers presented in this section have been computed using the same controlled output $\mathbf{z}(t)$ defined in (36)–(39). For this choice of $\mathbf{z}(t)$, the state-feedback controller defined by $\mathbf{G}_s$ attains the optimal H_∞-norm $\gamma_{G_s} = 7.8365$. A suboptimal γ-value will be produced by any static output-feedback controller designed using the same $\mathbf{z}(t)$. Looking at the γ-value in (53), it can be seen that the output-feedback controller (44) is practically optimal. Comparing the γ-values in (42) and (61), it can also be appreciated that the H_∞-norm achieved by the second output-feedback controller (54) exceeds the optimal γ-value (42) in less than a 1%. From a practical point of view, the behaviour of these almost-optimal controllers is often very similar to the behaviour exhibited by the optimal state-feedback controller. The numerical simulations carried out in Section 4 illustrate this fact.

Remark 4: An output-feedback controller design for a quarter-car suspension model using a single-step procedure can be found in [33]. In this preliminary work, the state variables are $z_s(t)$, $z_u(t)$, $\dot{z}_s(t)$, $\dot{z}_u(t)$ and a static output-feedback controller of the form given in (44) is computed by using $\mathbf{R} = \mathbf{C}_y^\dagger$, which can be considered as a particular case of (9) with $\mathbf{L} = \mathbf{0}$. However, in this case, the corresponding LMI is unfeasible for an output-feedback controller of the form (54). For the state variables (31), proposed in [19] and used in the present paper, the attempt of computing the output-feedback controllers (44) and (54) by using a null matrix $\mathbf{L}$ in (9) also fails, and the corresponding LMI optimisation problems are reported to be infeasible by the MATLAB LMI solver. Similar feasibility problems associated to the choice $\mathbf{L} = \mathbf{0}$ were encountered in previous works on vibration control of large structures [27–30], and they were circumvented by using a slightly perturbed state matrix of the form $\hat{\mathbf{A}} = \mathbf{A} - \epsilon\mathbf{I}$ with a small $\epsilon > 0$. By means of this computational trick, it was possible to overcome the initial feasibility difficulties and to obtain suitable static output-feedback controllers. This approach, however, has proved to be inappropriate for the output-feedback controllers considered in the present paper and, after extensive numerical testing, no satisfactory results have been obtained by using a perturbed state matrix $\hat{\mathbf{A}}$. These facts come to highlight the singular relevance of the matrix $\mathbf{L}$ defined in (14), showing its ability to avoid unfeasibility and to capture the specific properties of the considered control problem.

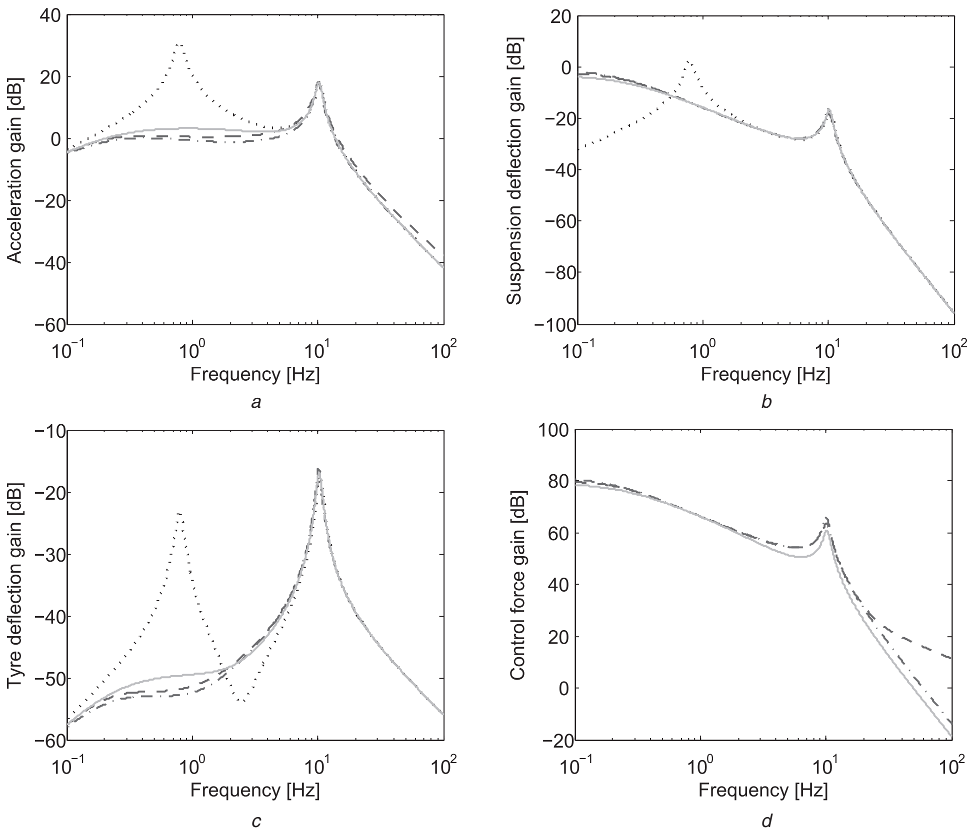

Fig. 2 *Frequency transfer functions from road displacement velocity to*

a Sprung mass acceleration
b Suspension deflection
c Tyre deflection
d Control effort, corresponding to the output-feedback controller I (red dash-dotted line), velocity-feedback controller II (green solid line), state-feedback (blue dashed line) and uncontrolled (black dotted line) configurations

4 Numerical results

In this section, we consider the following control configurations for the quarter-car suspension model:

(i) Controlled system using the active output-feedback controller (44), defined by the two-measurement observed output $\mathbf{y}_{\mathrm{I}}(t) = [z_{\mathrm{s}}(t) - z_{\mathrm{u}}(t), \dot{z}_{\mathrm{s}}(t)]^{\mathrm{T}}$ and the output gain matrix $\mathbf{K}_{\mathrm{I}}$ in (51). This controller will be called output-feedback controller I in the sequel.
(ii) Controlled system using the active output-feedback controller (54), defined by the single-measurement observed output $\mathbf{y}_{\mathrm{II}}(t) = \dot{z}_{\mathrm{s}}(t)$ and the output gain $\mathbf{K}_{\mathrm{II}}$ given in (59). For clarity, this controller will be referred to as velocity-feedback controller II in what follows.
(iii) Controlled system using the active state-feedback controller (40), defined by the full state

$$\mathbf{x}(t) = [z_{\mathrm{s}}(t) - z_{\mathrm{u}}(t),\ z_{\mathrm{u}}(t) - z_{\mathrm{r}}(t),\ \dot{z}_{\mathrm{s}}(t),\ \dot{z}_{\mathrm{u}}(t)]^{\mathrm{T}}$$

and the state gain matrix $\mathbf{G}_{\mathrm{s}}$ in (41).
(iv) Uncontrolled system with no active control implementation.

For these four control configurations, the frequency transfer functions from the road displacement velocity $\dot{z}_{\mathrm{r}}(t)$ to the sprung mass acceleration $\ddot{z}_{\mathrm{s}}(t)$, to the suspension deflection $z_{\mathrm{s}}(t) - z_{\mathrm{u}}(t)$, to the tyre deflection $z_{\mathrm{u}}(t) - z_{\mathrm{r}}(t)$ and to the control force $u(t)$ are presented in Figs. 2a–d, respectively.

In the graphics, the black dotted line corresponds to the uncontrolled system, the red dash-dotted line corresponds to the output-feedback controller I, the green solid line pertains to the velocity-feedback controller II and the blue dashed line represents the state-feedback controlled system, which is taken as a reference to evaluate the performance of the proposed static output-feedback controllers. Looking at the graphics in Fig. 2, it can be clearly appreciated that the frequency response of the output-feedback controller I (which has attained a practically optimal γ-value) is very similar to the response corresponding to the state-feedback controller. For the velocity-feedback controller II, a small loss of performance with respect to the state-feedback controller can be observed. However, we must recall the severe feedback information constraints imposed on this controller.

To provide a better insight into the behaviour exhibited by the proposed output-feedback controllers, we consider the time responses to an isolated bump of the form

$$z_{\mathrm{r}}(t) = \begin{cases} \dfrac{A}{2}\left[1 - \cos\left(\dfrac{2\pi V}{L} t\right)\right] & \text{if } 0 \le t \le \dfrac{L}{V} \\ 0 & \text{otherwise} \end{cases} \tag{62}$$

where A and L are the bump height and bump length, respectively and V is the vehicle forward velocity. The following particular values [39]

$$A = 0.1\,\mathrm{m}, \quad L = 5\,\mathrm{m}, \quad V = 12.5\,\mathrm{m/s} \tag{63}$$

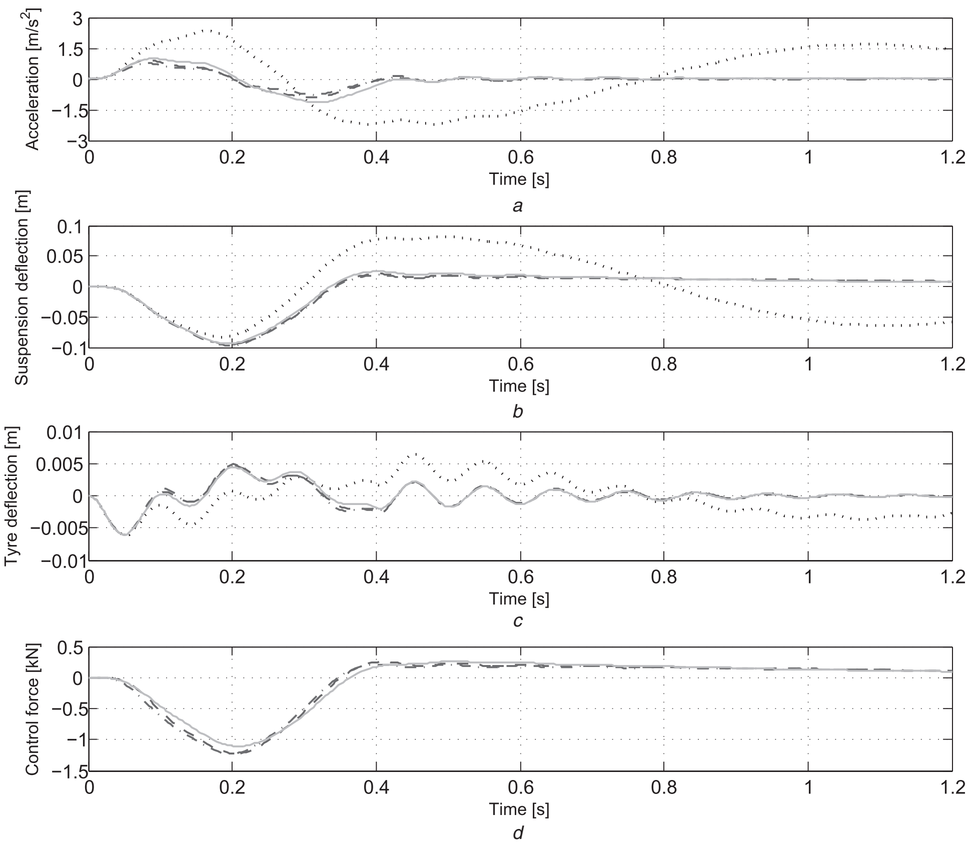

Fig. 3 *Time response to an isolated bump disturbance*
a Sprung mass acceleration
b Suspension deflection
c Tyre deflection
d Control effort, corresponding to the output-feedback controller I (red dash-dotted line), velocity-feedback controller II (green solid line), state-feedback (blue dashed line) and uncontrolled (black dotted line) configurations

have been taken to conduct the numerical simulations. For the control configurations (i)–(iv), the graphics corresponding to the sprung mass acceleration $\ddot{z}_s(t)$ and the control efforts $u(t)$ are respectively presented in Figs. 3a and d, with the same colours and line styles used in the frequency plots. A quick inspection of these figures makes apparent the ability of the active controllers to mitigate the sprung mass acceleration response. It can also be clearly appreciated that the proposed output-feedback controllers achieve similar levels of response mitigation as the state-feedback controller, with similar levels of control effort. The graphics in Figs. 3b and c demonstrate the effectiveness of the active controllers in mitigating the suspension deflection response and the tyre deflection response. In this case, slightly larger peak values are produced by the active controllers during the initial 0.3 s when compared with the uncontrolled configuration. These large initial amplitudes, however, are effectively reduced by the active controllers.

Remark 5: As indicated in Remark 2, the state-feedback controller (40) has been intendedly designed to match the behaviour of the static output-feedback controller presented in [19], which uses the observed outputs $z_s(t) - z_u(t)$, $\dot{z}_s(t)$ as feedback information and has been computed using a genetic algorithm approach. Consequently, the performances of the static output-feedback controller presented in [19] and the proposed output-feedback controller I are very similar. However, it should be highlighted that the design methodology proposed in Section 2 has made it possible to compute the output-feedback controller I by solving two LMI optimisation problems. Moreover, a second output-feedback controller that only uses $\dot{z}_s(t)$ as feedback information has also been obtained by solving a single LMI optimisation problem. In both cases, after obtaining a satisfactory state-feedback controller, no additional parameter values need to be set to implement the design procedure.

5 Conclusions and future directions

In this work, a novel strategy to design static output-feedback controllers for vehicle suspension systems has been presented. To illustrate the main elements of the new approach, two kinds of static output-feedback H_∞ controllers have been designed for a simplified quarter-car suspension system. Numerical simulations show that the proposed static output-feedback H_∞ controllers exhibit a good behaviour in terms of both frequency and time responses, when compared with the corresponding optimal state-feedback H_∞ controller. In fact, from the point of view of H_∞ controller design, the values of the H_∞-norms show that the proposed static output-feedback controllers are practically optimal. The positive results obtained for this simplified problem clearly indicate that further research effort should be invested in applying the new methodology to more complex scenarios, involving more complete vehicle models such as half-car or full-car models, or more sophisticated control strategies such as non-fragile control or limited frequency designs. A detailed study of discrete-time controllers would also be convenient.

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