Risk seeking, non convex remuneration and regime switching

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We investigate asset management in a regime switching framework when the fund manager aims to beat a certain target for the assets under management over an infinite horizon or over a finite horizon. We consider both a full information and a partial information setting. In a full information setting, the asset manager tends to take more risk in the good state and less risk in the bad state with respect to the constant parameter environment. Confidence risk induces the agent to increase his risk exposure.

Keywords: regime switching; bonus; remuneration; investment.

1. Introduction

The recent financial crisis has shed some light on risk incentives and remuneration schemes. While the academic literature has investigated the issue mostly in a constant investment opportunity environment, the financial crisis has suggested the possibility that a non convex remuneration scheme may induce excess risk seeking when financial markets are bubbling. In other words, bonus compensation and irrational exuberance may create a dangerous mix. Charles Prince (the former CEO of Citi) said it explicitly: "As long as the music is playing, you have got to get up and dance".[a]

In this paper we investigate this claim analyzing asset management in a regime switching framework when the fund manager aims to beat a given target for the assets under management (wealth) that coincide with the fund performance. We consider several different remuneration schemes: i) fixed bonus when a target is reached, ii) fixed bonus if the target is reached over a finite horizon, iii) fixed bonus when the manager outperforms a benchmark by a given threshold, iv) high water marks contract. The first two schemes are motivated by absolute return performance fees, i.e., the manager is remunerated according to the absolute performance of the fund. The third one mimics a remuneration based on the performance of the fund relative to a benchmark. Finally, high water marks contracts are widely used in

[a]Financial Times, "Citigroup chief stays bullish on buy-outs", July 9, 2007.

the hedge fund industry. The analysis of these four reward fee schemes allows us to conduct a comprehensive analysis on the effects of non convex remuneration schemes in a regime switching environment.

Regime switching is modeled assuming that the drift and the volatility of the geometric Brownian motion of the asset price evolve as a continuous time Markov chain with two states. We consider both a full information and a partial information setting. In the first case the manager observes the state of the Markov chain, in the second one he does not observe the state of the Markov chain but forms his beliefs on it through the observation of the asset price. The regime switching setting is interesting because it allows us to model in a simple way a mean reverting dynamics addressing some regularities observed in the asset pricing-management literature, see Ang & Timmermann (2012), Cecchetti *et al.* (1990), Guidolin & Timmermann (2007), and in the option pricing literature, see Buffington & Elliot (2002), Eliot *et al.* (2005), Guo & Zhang (2004).

The main goal of the paper is to analyze how a target driven asset allocation is affected by the risk of a switch in the state of the economy and by the estimation risk about the state. The first issue is addressed considering the full information setting, the second one considering the partial information setting.

There are no transaction costs, therefore in a full information setting the manager can redefine the strategy promptly as the state of the economy changes. However, this possibility does not imply that the optimal portfolio coincides with the one obtained in a constant parameter environment. As a matter of fact, in a good (bad) state the manager may decide to overweight or underweight the risky asset position exploiting the momentum and fearing (waiting) a switch to the bad (good) state. This consideration does not affect the optimal portfolio of an agent maximizing the expected utility from terminal wealth. In fact, in Sotomayor & Cadenillas (2009) authors showed that in case of a utility function with constant relative risk aversion, the optimal portfolio is the constant weight obtained in the constant parameter environment with state dependent parameters (drift and variance matrix). Therefore, the regime switching environment does not affect the optimal policy obtained in case of constant parameters, simply the agent switches as the state changes always adopting the investment policy obtained in the state as if the parameters were constant.

Considering a target driven remuneration scheme we show that the risk of a regime switch affects the optimal investment policy obtained in the case of constant parameters. Differently from what is observed in the constant parameter setting, see Browne (1995, 1997), when the agent is rewarded with a constant bonus as the assets under management touch the target, the solution is no more a constant weight. When the assets under management are low and the target is far away, the agent overweights the risky asset in a good state (high Sharpe ratio) compared to the constant parameter solution, and he takes less risk in a bad state underweighting the risky asset. The effect is reversed when the value of the assets approaches the target. Instead, when the target has to be reached over a finite horizon, the asset manager always takes more risk in the good state and less risk in the bad state with respect to the constant parameter environment. The rationale for this behavior is that in the good state the manager exploits the momentum and takes more risk fearing a switch to the bad state. On the other hand, in the bad state, the manager expects a switch to the good state and therefore he takes less risk.

In a constant parameter environment, the investment strategy is not monotone in the Sharpe ratio and therefore the investment in the risky asset in the bad state can be higher than the investment in the good state. If this happens, we demonstrate that the ranking is reversed when the probability of having a switch is large enough. This is due to the above attitude of the manager to exploit the momentum in a good state fearing a switch to the bad state or to wait for a switch in case of a bad state. When the switching probability is high, the above effect becomes significant and the rank obtained in a constant parameter environment is reversed. Therefore, with a high enough switching probability, the agent always takes more risk in the state with the higher Sharpe ratio.

In a partial information setting, the agent faces a confidence risk, i.e., the change of his beliefs about the state of the economy. It is worthwhile to observe that in a two regime switching model beliefs update and asset returns are positively correlated. In other words confidence risk is positively correlated with market risk. As a consequence, an agent more risk averse than a logarithmic utility would attempt to hedge the confidence risk buying less of the risky asset with respect to the constant parameter optimal investment strategy, i.e., the hedging demand is negative, see David (1997), Honda (2003). In our setting we show that the agent's attitude towards confidence risk is similar to what we observe in a full information setting: the agent tends to overweight (underweight) the risky asset when he believes that the good state is more (less) likely. Over an infinite horizon this phenomenon is observed for a low wealth and the reverse occurs when the wealth approaches the target. When the horizon is fixed the effect is observed for every level of wealth. Moreover, we observe that the agent takes a long position in the risky asset also when he assigns a small probability to the favorable state (the one with a positive risk premium).

When we consider the case of a manager who is remunerated as the benchmark is beaten over an infinite or a finite horizon, the above results under full and partial information are confirmed. The high water mark contract case renders an analysis similar to what is observed in the infinite horizon setting. In both cases, the manager takes more (less) risk in the good (bad) state.

This paper adds to the literature on incentive fees and asset management showing that a non convex remuneration scheme leads to excess risk seeking, see Carpenter (2000), Goetzmann *et al.* (2003), Grinblatt & Titman (1989), Panageas & Westerfield (2009), Ross (2004). We can conclude that the regime switching environment induces excess risk taking in the favorable state when the horizon is fixed or is infinite and the target is far away. This confirms that a bubble may be reinforced by a non convex remuneration.

The paper is organized as follows. In Section 2 we introduce the regime switching model in a full and in a partial information environment. In Section 3 we analyze the asset management problem when the manager goal is provided by a fixed bonus when the target for the assets under management is reached. In Section 4 we analyze the case in which the manager goal is provided by maximizing the probability of reaching a given target over a finite horizon. In Section 5 we extend the analysis to the case of a manager who wants to beat a benchmark. In Section 6 we analyze portfolio choices of a manager remunerated through a high water marks contract.

2. The Model

Let $(\Omega, \mathcal{F}, P)$ be a probability space on which a standard Brownian motion Z and an independent two state continuous-time Markov chain Y are defined. The process Y is right-continuous with values in $\{0, 1\}$ and represents the regime of the economy.

In $t = 0$, $Y(0)$ has outcome 1 with probability π and 0 with probability $1 - \pi$. The process $Y(t)$ starting in state i remains in the same state for an exponentially distributed length of time and jumps to state $j \neq i$ with intensity λ_{ij}. In what follows we consider the symmetric case and we set $\lambda_{01} = \lambda_{10} = \lambda$.

The jump times are independent and independent of Z. The regime switching and the Brownian motion generate the information filtration $\mathcal{F} = \{\mathcal{F}_t^{Z,Y}\}$ where $\mathcal{F}_t^{Z,Y} = \sigma(Z(s), Y(s), s \leq t)$, i.e., $\mathcal{F}_t^{Z,Y}$ is the augmented σ-algebra on Ω generated by the observation of Z and Y up to t.

The agent can trade a riskless bond and a risky asset paying no dividend. The riskless bond price $B(t)$ satisfies

$$dB(t) = rB(t)dt, \quad B(0) = 1$$

with a positive constant r, the risky asset price evolves as

$$dS(t) = S(t)\mu(Y(t))dt + S(t)\sigma dZ(t), \quad S(0) = S_0.$$

As far as the information set is concerned, we consider two different information environments: the full information, and the partial information one.

In the partial information setting, the volatility of the asset price is constant σ and the drift is a function of the state $Y(t)$. More precisely, we assume $\mu(0) = \mu_0$ and $\mu(1) = \mu_1$. Instead, under full information, we will also consider the case of switching volatility, i.e., $\sigma = \sigma(Y(t))$. In the following, we denote by $w(t)$ the wealth fraction invested in the risky asset.

In the full information setting, the investor observes $Y(t), Z(t)$, and $S(t)$. In this case, $w(t)$ is adapted to $\mathcal{F}_t^{Z,Y}$. In the partial information setting, the agent only observes the asset price $S(t)$, he does not observe $Y(t)$ and $\mu(Y(t))$. In this case, the investor's information is defined by the filtration $\mathcal{F}^S = \{\mathcal{F}_t^S\}$ where $\mathcal{F}_t^S = \sigma(S(s), s \leq t)$. The investment policy $w(t)$ is adapted to $\mathcal{F}_t^S$. In both cases, the parameters $\sigma,\ \pi,\ \lambda,\ \mu_0,\ \mu_1$ are known constants.

Let $X(t)$ be the assets under management (wealth) of the manager. In the full information setting, $X(t)$ evolves as follows:

$$dX(t) = X(t)(w(t)(\mu(Y(t)) - r) + r)dt + w(t)\sigma X(t)dZ(t), \quad X(0) = x. \tag{2.1}$$

In the partial information case, we can identify a σ-algebra equivalent economy with filtered probability

$$\pi(t) = P(Y(t) = 1|\mathcal{F}_t^S), \quad \pi(0) = \pi.$$

$\pi(t)$ is the probability that the current regime is state 1 given the observation $S(s), s \leq t$. As shown in Honda (2003), filtering techniques yield that $\pi(t)$ satisfies the stochastic differential equation

$$d\pi(t) = \lambda(1 - 2\pi(t))dt + \pi(t)(1 - \pi(t))\frac{\mu_1 - \mu_0}{\sigma}d\overline{Z}(t) \tag{2.2}$$

where $\overline{Z}(t)$ is the standard Brownian motion defined as

$$\overline{Z}(t) := \int_0^t \frac{1}{S(s)\sigma}dS(s) - \int_0^t \frac{\hat{\mu}(\pi(s))}{\sigma}ds$$

with $\hat{\mu}(\pi(s)) = \pi(s)\mu_1 + (1 - \pi(s))\mu_0$.

A σ-algebra equivalent is described by the risk-free asset, the filtered probability space and the risky price process $S(t)$ satisfying

$$dS(t) = S(t)\hat{\mu}(\pi(t))dt + S(t)\sigma d\overline{Z}(t)$$

and the filtration $\mathcal{F}^S$ generated by S.

A trading strategy $w(t)$ is an adapted process and $X(t)$ evolves as follows

$$dX(t) = X(t)(w(t)(\hat{\mu}(\pi(t)) - r) + r)dt + w(t)\sigma X(t)d\overline{Z}(t), \quad X(0) = x. \tag{2.3}$$

Note the following features of the stochastic differential equation (2.2) governing agent's beliefs: a) the larger the difference between the two states (μ_1 and μ_0), the larger the volatility of beliefs, b) the larger the volatility of asset returns, the smaller the volatility of beliefs, c) the process $\{\pi(t)\}_{t\geq 0}$ is mean reverting, the mean reversion speed is high when the switching probability λ is large. These features imply that the degree of confidence on a state is mean reverting and the convergence rate is proportional to the switching probability. Confidence on the state of the economy is extremely volatile when the mean returns in the two states are different and the return volatility is low.

3. Fixed bonus from reaching a target

Let us analyze the asset allocation problem for a manager who is rewarded with a fixed amount of money (normalized to one) when the assets under management $X(t)$ reach a certain target b provided that bankruptcy does not occur before ($X(t) = 0$). Let us denote by

$$\tau_b = \inf\{t > 0 : X(t) = b\}$$

the first hitting time of the target b of the assets under management, and by δ the discount factor of the manager, then the asset allocation problem can be formulated defining the optimal value function as

$$V(x) := \sup_w E\left[e^{-\delta\tau_b}|X(0) = x\right] \tag{3.1}$$

subject to (2.1) under full information and (2.3) under partial information. In the first case $w(t)$ is adapted to $\mathcal{F}_t^{Z,Y}$, in the second case to $\mathcal{F}_t^S$. Notice that the manager goes bankrupt when the assets under management reach the zero level, i.e., the manager cannot leverage his position.

In Browne (1995, 1997) the problem has been analyzed in a no switching setting (constant parameters). As a benchmark for our analysis, we report the main results. Let $\lambda = 0$ and denote by μ the constant drift of the asset price. In this environment the Hamilton Jacobi Bellman (HJB) equation becomes

$$\sup_w \quad -\delta V + (w(\mu - r) + r)xV_x + \frac{\sigma^2 w^2}{2} x^2 V_{xx} = 0, \quad 0 \le x \le b, \tag{3.2}$$

where we denote with V_x and V_{xx} the first and second order derivative of V with respect to the variable x. The HJB equation is coupled with the boundary conditions

$$V(0) = 0, \quad V(b) = 1.$$

The above problem can be solved analytically, providing a solution which satisfies $V_{xx} < 0$, and therefore the optimal investment strategy is given by

$$w = \frac{r - \mu}{\sigma^2} \frac{V_x(x)}{xV_{xx}(x)},$$

and the HJB equation can be rewritten as

$$-\delta V - \frac{1}{2}\frac{(r-\mu)^2}{\sigma^2}\frac{V_x(x)^2}{V_{xx}(x)} + rxV_x(x) = 0, \quad 0 < x < b. \tag{3.3}$$

More precisely, simple computations lead to the value function given by

$$V(x) = \frac{x^C}{b^C} \tag{3.4}$$

with

$$C = \frac{\delta + \frac{1}{2}\frac{(r-\mu)^2}{\sigma^2} + r - \sqrt{(\delta + \frac{1}{2}\frac{(r-\mu)^2}{\sigma^2} + r)^2 - 4\delta r}}{2r}. \tag{3.5}$$

Since $0 < C < 1$, it holds $V_{xx} < 0$, and thus (3.4) is the solution of equations (3.2) and (3.3), and the optimal strategy is given by

$$w = \frac{1}{C - 1}\frac{r - \mu}{\sigma^2}. \tag{3.6}$$

Note that the strategy is a constant weight as the one observed maximizing a logarithmic or a power utility function from terminal wealth. So, the portfolio is the golden rule (the strategy maximizing the expected logarithmic growth of rate) multiplied by the constant $1/(C-1)$ that depends on the parameters of the model. Note that the parameter C depends on the Sharpe ratio of the risky asset. It is easy to show that the relationship between the Sharpe ratio and the fraction of wealth invested in the risky asset is not monotonic.

3.1. *Regime Switching with full information*

We denote by V^0, V^1 and w^0, w^1 the value function and the optimal strategy in state 0 and 1, respectively. The HJB equation for problem (3.1) becomes

$$\sup_{w^0} \; -\delta V^0 + (w^0(\mu_0 - r) + r)xV_x^0 + \frac{\sigma^2(w^0)^2}{2}x^2 V_{xx}^0 - \lambda V^0 + \lambda V^1 = 0, \quad (3.7)$$

$$\sup_{w^1} \; -\delta V^1 + (w^1(\mu_1 - r) + r)xV_x^1 + \frac{\sigma^2(w^1)^2}{2}x^2 V_{xx}^1 - \lambda V^1 + \lambda V^0 = 0, \quad (3.8)$$

$x \in [0, b]$, and the boundary conditions are

$$V^i(0) = 0, \quad V^i(b) = 1.$$

An explicit solution for the value function is not available. Existence of the solution and uniqueness can be addressed via viscosity solution methods, see for example Ceci & Bassan (2004), Chevalier *et al.* (2013), Pham (2009).
In order to compare the optimal investment strategy with the one obtained in the no switching case, we solve the above problem numerically considering a finite difference technique to discretize the partial differential equation, coupled with the Picard iterative scheme. The Picard iteration is an easy way of handling non-linear ordinary differential equations (ODEs): it belongs to the class of fixed point algorithms, and it is based on the idea of considering a known, previously computed solution in the non-linear term so that this term becomes linear in the unknown. The strategy is also known as the method of successive substitutions.
In the Appendix we describe in details our numerical technique. We would like to stress that, considering as guess function the solution obtained in the no switching setting, our numerical scheme provides a sequence of functions $\{V_m^i\}_{m\geq 0}$ which satisfy $(V_m^i)_{xx} < 0$ in all grid points and which converge to the solution V^i. This allows us to establish that the optimal strategies are given by the formula

$$w^i = \frac{r - \mu_i}{\sigma^2} \frac{V_x^i}{xV_{xx}^i}, \quad i = 0, 1.$$

In what follows, we set $b = 5$, $\mu_0 = 0.04$, $\mu_1 = 0.08$, $r = 0.05$, $\delta = 0.04$ and $\sigma = 0.3$. Note that there are no transaction costs, short selling is allowed, and volatility is constant in the two states, then what is relevant in the asset allocation problem is the absolute value of the expected excess return in the two states: $|\mu_0 - r| = 0.01$ and $|\mu_1 - r| = 0.03$. For this set of parameters, state 1 is the good one not because of a higher expected return but because the absolute value of the expected excess return is higher than the one observed in state 0. In Figure 1 we represent the fraction of the wealth invested in the risky asset also assuming $\mu_0 = 0.06$. Note

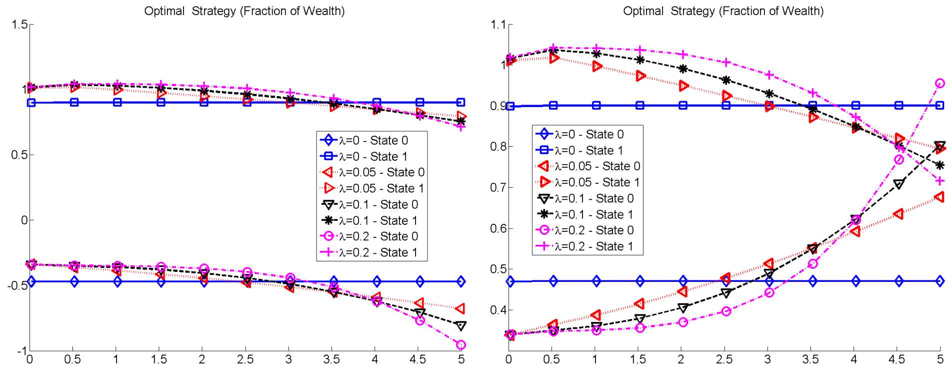

Figure 1. Fixed bonus when a target is reached. No Switching and Switching with Full Information: $b = 5$, $\delta = 0.04$, $r = 0.05$, $\mu_0 = 0.04$ (left) and $\mu_0 = 0.06$ (right), $\mu_1 = 0.08$, $\sigma = 0.3$.

that in both cases the risky asset is more appealing in state 1 than in state 0, the difference is that the manager short sells the risky asset in state 0 when $\mu_0 = 0.04$ and, instead, he invests a positive amount of wealth in both states when $\mu_0 = 0.06$.

Figure 1 shows that the optimal investment strategy is not of a constant weight type: the exposure to the risky asset ($|w|$) decreases (increases) in the good (bad) state as wealth increases. If we compare the optimal portfolio obtained in the regime switching setting with full information with the one obtained in a setting with constant weights (no regime switching, $\lambda = 0$ in Figure 1), we notice that when the wealth is low the manager invests more (less or sells short less) in the risky asset in state 1 (state 0) with respect to the no regime switching case. This attitude is reversed when the reward target is approaching. The departure from the constant weight strategy increases as the switching probability increases (λ goes up). So the bias in the investment policy due to the switching probability is asymmetric with more risk in the good state and less risk in the bad state when the wealth is low. The opposite effect is observed when the wealth is next to the target barrier.

It seems that a reward through a fixed bonus as the target is reached induces the agent to excess (less) risk taking in the good (bad) state with respect to his investment strategy in the no switching setting when the barrier is far away. The rationale is that in the good state the manager exploits the momentum and takes more risk fearing a switch to the bad state. On the other hand, in case of the bad state, when the target is far away the manager expects a switch to the good state and therefore he takes less risk. Instead, when the wealth is close to the target, the probability that the target is reached before a switch increases and therefore the agent's investment attitude is reversed in both states. Notice that when $|\mu_1 - r| = |\mu_0 - r|$ the optimal investment does not depend on λ, the investment weights are constant and coincide with those obtained in case of constant parameters. This interpretation is confirmed by the fact that the above phenomenon is magnified by an increase in the switching probability λ. A similar result is obtained in Panageas

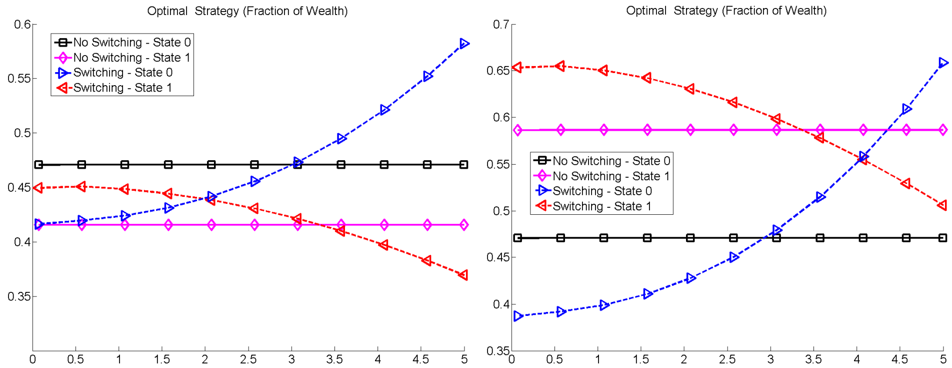

Figure 2. Fixed bonus when a target is reached. No Switching and Switching with Full Information ($\lambda = 0.1$): $b = 5$, $\delta = 0.04$, $r = 0.05$, $\mu_0 = 0.06$, $\mu_1 = 0.08$, $\sigma_0 = 0.3$, $sr_0 = 0.0333$, $\sigma_1 = 0.5$, $sr_1 = 0.06$ (left) and $\sigma_1 = 0.4$, $sr_1 = 0.075$ (right).

& Westerfield (2009) in case of a high water mark fee, where it is shown that the portfolio weight is increasing in the density of the jump determining the termination of the fund.

In the full information setting, we can easily extend the analysis to the case with both the drift and the volatility of the risky asset switching as the state changes ($\mu_i,\ \sigma_i,\ i = 0, 1$). As both the drift and the volatility change in the two states, the risk-return profile can be evaluated according to the Sharpe ratio $sr_i = \frac{\mu_i - r}{\sigma_i},\quad i = 0, 1$. In Figure 2 we consider two different sets of parameters, in both cases the Sharpe ratio in state 1 is higher than the one in state 0. First of all, it is worthwhile to notice that in a constant parameter environment the relationship between the Sharpe ratio and the fraction of wealth invested in the risky asset is not monotone: while in Figure 2-(right) the agent invests more in the risky asset in the higher Sharpe ratio state (state 1) as it happens in Figure 1, the reverse behavior is shown in Figure 2-(left). The figure also shows that when the wealth is low the manager overinvests (underinvests) in the risky asset in the state with the higher (lower) Sharpe ratio (state 1 and state 0, respectively) with respect to the strategy obtained in the constant parameter setting. The reverse occurs when the wealth approaches the target. This result confirms that when the target is far away the asset manager invests more (with respect to the constant parameter setting) in the risky asset in the favorable state fearing the switch to the bad state. This attitude is reversed when the target is approaching.

Comparing the investment strategies in the two states in a switching environment for a low level of wealth, we observe that when a larger investment is obtained in the good state with respect to the bad state for a constant parameter environment (Figure 2-(right)) the order is confirmed in a switching environment. When a smaller investment is obtained in the good state for a constant parameter environment (Figure 2-(left)), the rank between the strategy in the good state and

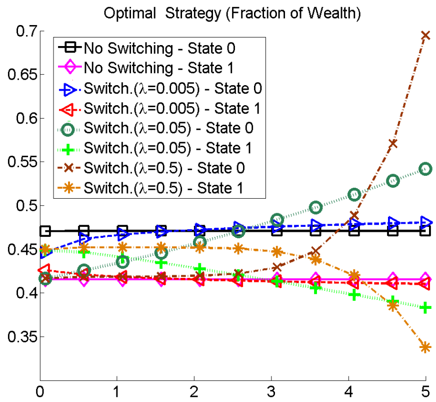

Figure 3. Fixed bonus when a target is reached. No Switching and Switching with Full Information and different values of λ: $b = 5$, $\delta = 0.04$, $r = 0.05$, $\mu_0 = 0.06$, $\sigma_0 = 0.3$, $sr_0 = 0.0333$, $\mu_1 = 0.08$, $\sigma_1 = 0.5$, $sr_1 = 0.06$.

the one in the bad state is not univocally defined. To better understand this point, in Figure 3 we deal with the parameter set analyzed in Figure 2-(left) considering different values of the switching probability λ: the order obtained in the constant parameter environment (lower investment in the state with the higher Sharpe ratio) is confirmed in a switching parameter setting for a small wealth and a small switching probability (e.g. $\lambda = 0.005$); instead if λ is large enough and the target is far away, then the asset manager invests more in the risky asset in the state with the higher Sharpe ratio with respect to the state with the lower Sharpe ratio. This result confirms the interpretation provided above. In a good state, the fear of a switch to the bad state induces always the agent to take more risk. The opposite holds true in case of the bad state. When the switching probability is significant, this attitude may even reverse the rank obtained in a constant parameter setting yielding always an investment in the risky asset in the good state higher than in the bad state.

3.2. *Regime Switching with partial information*

Considering the stochastic differential equation for beliefs (2.2), for a control processes w the generator of a generic process $g = g(t, x, \pi)$ is

$$\mathcal{A}^w g(t, x, \pi) = g_t + (w(\hat{\mu}(\pi) - r) + r)xg_x + \lambda(1 - 2\pi)g_\pi \\ + \frac{1}{2}\sigma^2 w^2 x^2 g_{xx} + \frac{1}{2}\pi^2(1-\pi)^2\frac{(\mu_1 - \mu_0)^2}{\sigma^2} g_{\pi\pi} + w\pi(1-\pi)(\mu_1 - \mu_0)xg_{x\pi} \quad (3.9)$$

where g_t, g_x, $\cdots$, denote the derivatives of the function g.

Defining the optimal value function as

$$V(x, \pi) := \sup_w E\left[e^{-\delta\tau_b} | X(0) = x, \pi(0) = \pi\right],$$

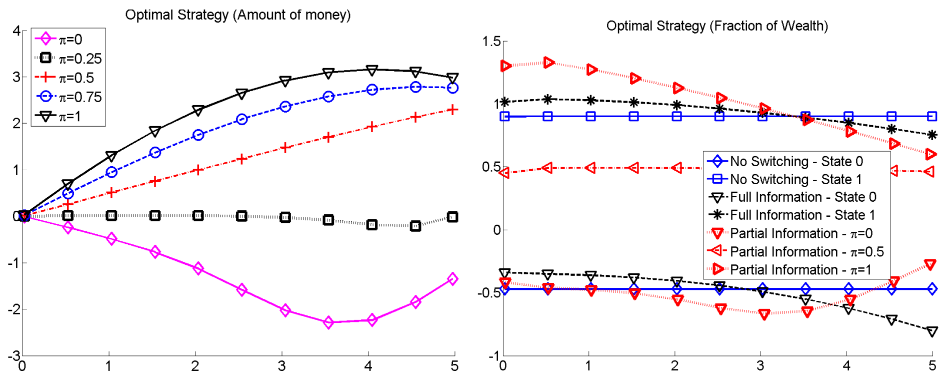

Figure 4. Fixed bonus when a target is reached. Switching with Partial Information, no Switching, Switching with Full Information: $b = 5$, $\delta = 0.04$, $\lambda = 0.1$, $r = 0.05$, $\mu_0 = 0.04$, $\mu_1 = 0.08$, $\sigma = 0.3$.

the HJB equation becomes

$$\sup_w \; -\delta V + (w(\hat\mu(\pi) - r) + r)xV_x + \lambda(1-2\pi)V_\pi + \frac{\sigma^2 w^2}{2}x^2 V_{xx} \tag{3.10}$$
$$+ \frac{\pi^2(1-\pi)^2}{2}\frac{(\mu_1-\mu_0)^2}{\sigma^2}V_{\pi\pi} + w\pi(1-\pi)(\mu_1-\mu_0)xV_{x\pi} = 0, \quad 0 \le x \le b, \; 0 \le \pi \le 1$$

where $\widehat{\mu}(\pi) = \mu_0(1-\pi) + \mu_1\pi$, with boundary conditions

$$V(0,\pi) = 0, \quad V(b,\pi) = 1,$$

for $0 \le \pi \le 1$.

The partial differential equation (3.10) is solved again with our numerical procedure.[b] More precisely, considering as guess function $V_0(x,\pi) = (1-\pi)V^0(x) + \pi V^1(x)$, where V^i, $i = 0, 1$, is the solution of the corresponding problem with full information described in Section 3.1, our numerical scheme provides a sequence of solutions which satisfy $V_{xx} < 0$, and therefore the optimal strategy is given by

$$w = \frac{-(\hat\mu(\pi) - r)V_x - \pi(1-\pi)(\mu_1-\mu_0)V_{x\pi}}{\sigma^2 x V_{xx}}.$$

In Figure 4-(left) we plot the optimal investment strategy for different values of π assuming $b = 5$, $\delta = 0.04$, $\lambda = 0.1$, $r = 0.05$, $\mu_0 = 0.04$, $\mu_1 = 0.08$, $\sigma = 0.3$. First of all we notice that the optimal strategy is not of a constant weight type. For a given level of wealth x, the investment weight is increasing in the initial probability π that the agent assigns to the state with the higher expected return. Note that also for a low level of confidence in the favorable state (e.g. $\pi = 0.25$) the agent is long in the risky asset and not short as we would expect being $\mu_0 < r$. It seems that the agent aiming to reach the target takes a high risk investing in the risky asset

[b]In all our numerical experiments related to the partial information case we discretize the domain $(x,\pi) \in [0,b] \times [0,1]$ with a Cartesian grid of 200×100 points.

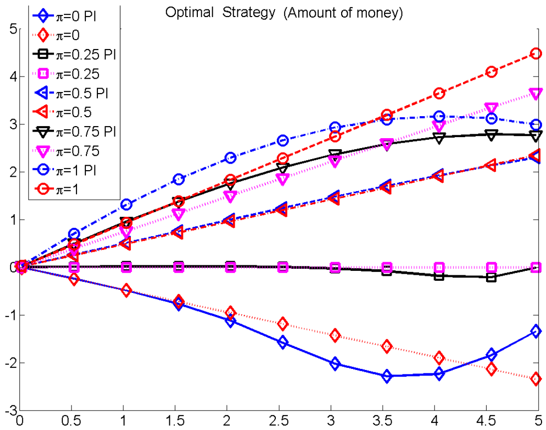

Figure 5. Fixed bonus when a target is reached. Switching with Partial Information (PI), comparison with a certainty equivalent drift strategy: $b = 5$, $\delta = 0.04$, $r = 0.05$, $\mu_0 = 0.04$, $\mu_1 = 0.08$, $\sigma = 0.3$, $\lambda = 0.01$.

also when the probability guess would suggest to sell it short. Again, the rationale is that $|\mu_1 - r| = 0.03 > |\mu_0 - r| = 0.01$ and therefore the agent aiming to reach the target as soon as possible "bets" on the most favorable state (1) also when the probability of being in that state is small. We can conclude that confidence risk induces the agent to take a long position hoping to be in the good state.

For $\pi = 0$ and $\pi = 1$ we can compare the optimal investment strategies obtained under partial information with those obtained under full information (see the previous subsection) and with those obtained with a constant investment opportunities set. The comparison is provided in Figure 4-(right). When the wealth is low, in the good (bad) state, confidence risk induces the agent to invest in the risky asset more (less) than in the full information setting with regime switching and therefore than in the constant parameter setting, when the wealth approaches the target the phenomenon is reversed.

According to these results, confidence risk leads to a higher (lower) risk exposure when the wealth is low (high) with respect to the full information setting. Remember that confidence and market risk are positively correlated, therefore an agent maximizing the expected utility in a partial information setting should take less risk, see David (1997), Honda (2003), instead a pure target bonus induces the agent to take excess risk when the target is far away. We can conclude that a reward through a fixed bonus induces the agent to act as a risk lover with respect to confidence risk. When the target approaches, the attitude changes for arguments similar to those introduced in a full information world.
The interpretation is confirmed looking at Figure 5 where, for different values of π, we compare the optimal investment strategy with partial information with the no switching strategy with a drift equal to the expected drift according to the agent's

initial beliefs, i.e., $\widehat{\mu}(\pi) = \mu_0(1-\pi) + \mu_1\pi$ (certainty equivalent drift strategy). The analysis confirms that the strategy obtained in the partial information setting is riskier than the no switching one if the agent's wealth is low, the reverse occurs when the wealth approaches the target.

4. Reaching a target by a deadline

In this section we analyze the optimal investment strategy for an agent maximizing the probability of reaching a certain target for the assets under management by the end of the horizon $[0,T]$. Set b the target, the optimal value function becomes

$$V(t,x) = E\left[P(X(T))|X(t)=x\right].$$

where $P(x) := \mathbf{1}_{x\geq b}$. We assume that reaching a zero wealth corresponds to bankruptcy. Notice that for $0 \leq t < T$ and for any wealth level $x \geq x_{\max} = x_{\max}(t) := be^{-r(T-t)}$, the value function $V(t,x) = 1$, and the optimal policy consists in investing all the wealth in the risk-free asset to reach the target level at the terminal time with probability 1.

This problem in a constant parameter setting has been addressed in Browne (1999a). The author shows that the problem corresponds to solve the HJB equation

$$\sup_{w} V_t + (w(\mu_0 - r) + r)xV_x + \frac{\sigma^2 w^2}{2}x^2 V_{xx} = 0, \tag{4.1}$$

on $[0,T)\times[0,x_{\max}]$, with boundary condition $V(t,0)=0$ and $V(t,x_{\max}(t))=1$ for any $t\in[0,T)$ and terminal condition $V(T,x)=P(x)$. The author proves that the optimal solution is

$$V(t,x) = \Phi\left(\Phi^{-1}\left(\frac{x}{x_{\max}}\right) + (T-t)\left(\frac{\mu-r}{\sigma}\right)^2\right),$$

$$w = w(t,x) = \frac{\mu - r}{\sigma|\mu-r|\sqrt{T-t}}\frac{x_{\max}}{x}\phi\left(\Phi^{-1}\left(\frac{x}{x_{\max}}\right)\right),$$

where we denote by ϕ (Φ) the density (cumulative) distribution function of a standard normal random variable, respectively. Notice that the terminal condition causes a discontinuity, since $\lim_{t\to T} V(t,x) = \frac{x}{x_{\max}} \neq P(x)$. The solution is not a constant weight, it is time dependent and is affected through the normal density by the distance of x from x_{max}.

4.1. *Regime Switching with full information*

In a regime switching environment with full information, the HJB equation becomes

$$\sup_{w^0} V_t^0 + (w^0(\mu_0 - r) + r)xV_x^0 + \frac{\sigma^2(w^0)^2}{2}x^2V_{xx}^0 - \lambda V^0 + \lambda V^1 = 0,$$

$$\sup_{w^1} V_t^1 + (w^1(\mu_1 - r) + r)xV_x^1 + \frac{\sigma^2(w^1)^2}{2}x^2V_{xx}^1 - \lambda V^1 + \lambda V^0 = 0,$$

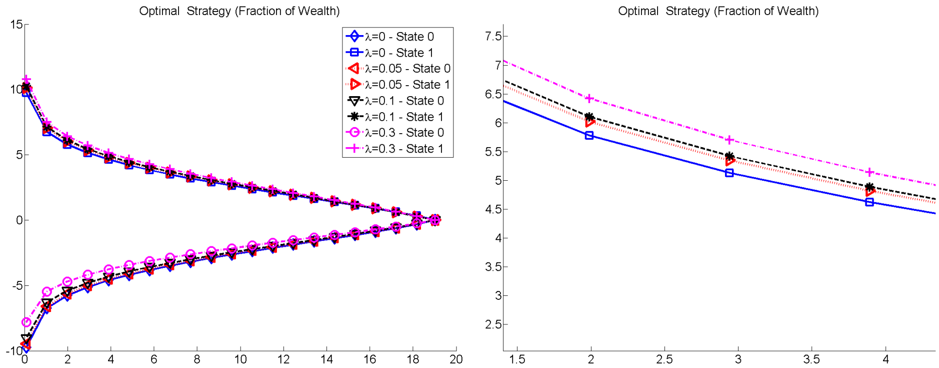

Figure 6. Fixed bonus if the target is reached over a finite horizon. No Switching, Switching with Full Information: $t = 0$, $b = 20$, $T = 1$, $\mu_0 = 0.04$, $\mu_1 = 0.08$, $r = 0.05$, $\sigma = 0.3$. Right: zoom of the state 1 case.

for $0 \le t < T$, $0 \le x \le x_{\max} = x_{\max}(t)$ with terminal condition $V^i(T, x) = P(x)$, $i = 0, 1$. Due to the bankruptcy condition, it must also hold $V^i(t, 0) = 0$ for $0 \le t \le T$, $i = 0, 1$. Moreover, $V^i(t, x_{\max}) = 1$ for $0 \le t < T$, $i = 0, 1$. We also assume that $\lim_{t\to T} V^i(t, x) = \frac{x}{x_{\max}}$ for $i = 0, 1$, to avoid the numerical problems related to the discontinuity of the terminal function.

We solve this problem with our numerical procedure, coupling the finite difference scheme with the Picard fixed-point algorithm and using the solution of the no switching case as guess solution at each time step $t_l = T - l\delta$, $l = 1, \cdots M$, with $\delta = T/M$,[c] obtaining a sequence of concave solutions, i.e., $V^i_{xx} < 0$, $i = 0, 1$; therefore the optimal policy is given by

$$w^i = \frac{r - \mu_i}{\sigma^2} \frac{V^i_x}{x V^i_{xx}}.$$

In Figure 6 we plot the optimal investment strategy in a full information environment in $t = 0$ for different values of λ together with the optimal investment strategy obtained in case of no regime switching ($\lambda = 0$). First of all, we notice that the agent takes less risk (absolute value of the portfolio weight invested in the risky asset) as the wealth increases. As a general result, we have that the agent takes more risk in state 1 (the most favorable one) for all level of wealth and less risk in state 0 (the less favorable one) with respect to the no switching setting. Increasing λ, the departure is amplified.

The interpretation of these results is similar to the one provided in case of a fixed reward as the target is reached. Reaching a goal over a finite horizon induces the agent to take excess (less) risk in the good (bad) state. The manager exploits the fact that the state is good and takes more risk fearing a switch to the bad state. On the other hand, in the bad state, the manager expects a switch to the good

[c]In all our numerical experiments we set $M = 50$.

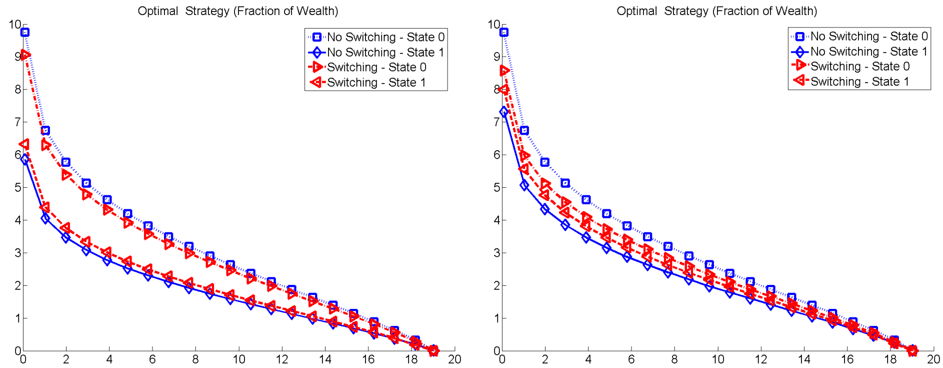

Figure 7. Fixed bonus if the target is reached over a finite horizon. No Switching ($\lambda = 0$) and Switching with Full Information ($\lambda = 0.3$): $t = 0$, $b = 20$, $T = 1$, $r = 0.05$, $\mu_0 = 0.06$, $\mu_1 = 0.08$, $\sigma_0 = 0.3$, $sr_0 = 0.0333$, $\sigma_1 = 0.5$, $sr_1 = 0.06$ (left) and $\sigma_1 = 0.4$, $sr_1 = 0.075$ (right).

state and therefore he takes less risk. This interpretation is confirmed by the fact that this phenomenon is magnified by an increase in the switching probability λ. Differently from what has been observed in case of a reward for reaching a given target over an infinite horizon, a reward over a finite horizon induces excess (less) risk in the good (bad) state for all levels of wealth.

The analysis is confirmed considering the more general case where both the drift and the volatility of the risky asset switch in the two states. In Figure 7 we consider the case where the Sharpe ratio (volatility) in state 1 is higher (lower) than the one in state 0. From this figure it is evident that in a regime switching environment the agent performs a riskier strategy in the state characterized by the higher Sharpe ratio with respect to the strategy obtained in a constant parameter setting, the reverse holds true in the state with the lower Sharpe ratio. Note that in a constant parameter environment the relationship between the Sharpe ratio and the fraction of wealth invested in the risky asset is not monotone as the first depends (inversely) only on the volatility, actually for our parameter sets the agent invests more in the risky asset in the state with the lower Sharpe ratio (state 0). Numerical experiments here not reported show that the rank is reversed as λ is increased: for the parameter set considered in Figure 7-(right), the agent invests more in the risky asset in state 1 (the good state) than in state 0 (the bad one) if $\lambda > 0.5$. Again the explanation of this phenomenon is that in the good state the agent is fearing a switch to the bad state.

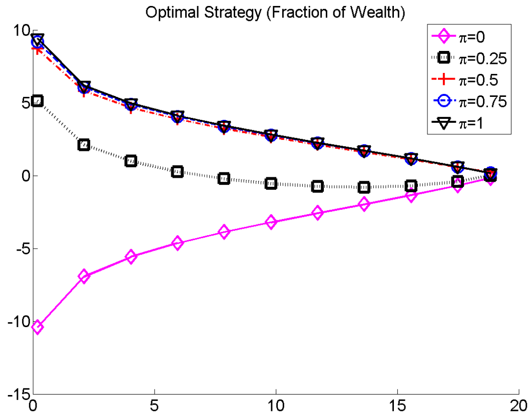

Figure 8. Fixed bonus if the target is reached over a finite horizon. Switching with Partial Information: $t = 0$, $b = 20$, $T = 1$, $\mu_0 = 0.04$, $\mu_1 = 0.08$, $r = 0.05$, $\sigma = 0.3$ and $\lambda = 0.1$

4.2. *Regime Switching with partial information*

In this case the HJB equation becomes

$$\sup_w \; V_t + (w(\hat{\mu}(\pi) - r) + r)xV_x + \lambda(1 - 2\pi)V_\pi \tag{4.2}$$
$$+ \frac{\sigma^2 w^2}{2} x^2 V_{xx} + \frac{\pi^2(1-\pi)^2}{2} \frac{(\mu_1 - \mu_0)^2}{\sigma^2} V_{\pi\pi} + w\pi(1-\pi)(\mu_1 - \mu_0)xV_{x\pi} = 0,$$

for $0 \le t < T$, $0 \le x \le x_{\max}$ and $0 \le \pi \le 1$, with boundary condition $V(t, 0, \pi) = 0$ and $V(t, x_{\max}, \pi) = 1$ for $0 \le \pi \le 1$.

We use our numerical procedure to solve the above problem, obtaining a sequence of solutions which satisfy $V_{xx} < 0$. Therefore, differentiating equation (4.2) with respect to w, the optimal investment strategy is given by

$$w = \frac{-(\hat{\mu}(\pi) - r)V_x - \pi(1-\pi)(\mu_1 - \mu_0)V_{x\pi}}{\sigma^2 x V_{xx}}.$$

In Figure 8 we plot the optimal investment strategy in $t = 0$ for different beliefs π assuming $b = 20$, $T = 1$, $\lambda = 0.1$, $r = 0.05$, $\mu_0 = 0.04$, $\mu_1 = 0.08$, $\sigma = 0.3$. First of all, we notice that the agent takes less risk (absolute value of the portfolio weight invested in the risky asset) as the wealth increases. The optimal strategies for $\pi = 1$, $\pi = 0.75$ and $\pi = 0.5$ are very close one another with a positive investment in the risky asset. Considering the case $\pi = 0.25$ (a small probability guess of being in the favorable state 1), the agent invests a positive fraction of wealth in the risky asset when the wealth is low. This behavior contrasts with the likelihood that he assigns to a favorable regime and therefore the agent assumes a very risky position betting against his beliefs. On the other hand, when the wealth is large enough, the agent invests a small negative amount of wealth in the risky asset.

To conclude, in Figure 9 we compare the no switching setting, the regime switching setting with full information and with partial information. Considering $\pi = 0$ or

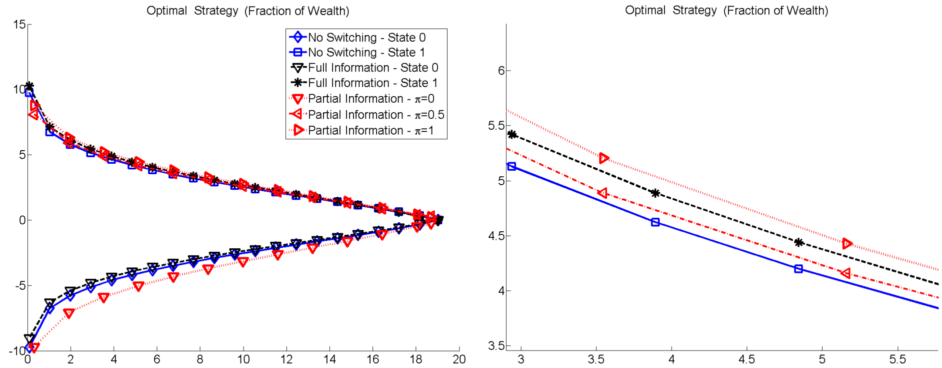

Figure 9. Fixed bonus if the target is reached over a finite horizon. No Switching, Switching with Full and Partial Information: $t = 0$, $b = 20$, $T = 1$, $\mu_0 = 0.04$, $\mu_1 = 0.08$, $r = 0.05$, $\sigma = 0.3$ and $\lambda = 0.1$. Right: zoom of the state 1 case.

$\pi = 1$, we observe that the optimal investment strategies in the partial information setting are riskier than those obtained in the full information and in the constant parameter setting in the bad and in the good state. Note that also the strategy for $\pi = 0.5$ is riskier than the one obtained with constant parameters in the good state.

We can conclude that confidence risk affects the strategy of the agent inducing him to take excess risk overinvesting in the favorable state and underinvesting in the bad state. The phenomenon is observed for all levels of wealth. Moreover, the agent may decide to invest a positive amount of wealth in the risky asset also when the likelihood that he assigns to a favorable state (positive risk premium) is low.

Summing up, the goal to reach a target over a fixed horizon induces a risky strategy in a regime switching environment with full information (in the good state) and partial information reinforces the excessive risk taking attitude. This effect contrasts with what is obtained when the agent maximizes expected utility.

5. Relative performance bonus

The analysis can be extended to a remuneration scheme based on beating a benchmark over an infinite or a finite horizon. The main results on regime switching and risk seeking are confirmed. Let us assume that the benchmark is driven by a geometric Brownian motion with a drift switching at the same time as the asset price does. The Brownian motion of the benchmark is correlated with the one of the asset price. Note that we are in an incomplete market setting, i.e., the manager cannot use the stock to replicate the benchmark.

The benchmark dynamics is provided by

$$dP(t) = P(t)\alpha(Y(t))dt + P(t)\beta dZ(t) + P(t)\gamma d\widehat{Z}(t) \quad P(0) = P_0.$$

with zero correlation between the Brownian motions Z and $\widehat{Z}$. The drift is a function of the state $Y(t)$. More precisely, we assume $\alpha(0) = \alpha_0$ and $\alpha(1) = \alpha_1$. We follow

Browne (1999b) assuming that the asset manager receives a fixed bonus (normalized to one) the first time that the assets under management outperform the benchmark by a multiplicative constant η.

The analysis can be developed considering the setting of Section 3: we look for the optimal investment strategy to beat the benchmark by a fraction η before bankruptcy that occurs for $X(t) = 0$. Let

$$\tau_\eta = \inf\{t > 0 : X(t) \geq (1+\eta)P(t)\} = \inf\{t > 0 : R(t) \geq 1+\eta\},$$

and $R := \frac{X}{P}$ the process of the assets under management normalized by the benchmark, then the asset allocation problem can be formulated as follows

$$\sup_w E\left[e^{-\delta\tau_\eta}|R(0) = X(0)/P(0)\right].$$

Assuming no switch and a drift for the risky asset S equal to μ_0 and for the benchmark P equal to α_0, the problem has been solved in Browne (1999b). In this setting, the evolution of the process $R(t)$ becomes

$$dR = R\left(r - \alpha_0 + \beta^2 + \gamma^2 + w(t)\left(\mu_0 - r - \sigma\beta\right)\right)dt + R(w\sigma - \beta)dZ(t) - R\gamma d\widehat{Z}(t).$$

The optimal investment strategy is a constant weight

$$w = -\frac{\mu_0 - r - \sigma\beta}{C\sigma^2} + \frac{\beta}{\sigma},$$

where C is the unique root which belongs to the interval $(-1, 0)$ of the cubic equation

$$\frac{\gamma^2}{2}C^3 + \left(\widehat{A} + \frac{\gamma^2}{2}\right)C^2 + \left(-\delta + \widehat{A} - \widehat{B}\right)C - \widehat{B} = 0$$

with

$$\widehat{A} = r - \alpha_0 + \gamma^2 + \frac{\mu_0 - r}{\sigma}\beta, \qquad \widehat{B} = \frac{1}{2}\left(\frac{\mu_0 - r - \sigma\beta}{\sigma}\right)^2;$$

see (Browne 1999b Section 6) for details.

Assuming a regime switching model with full information, and defining the value function as

$$V^i(z) = \sup_w E\left[e^{-\delta\tau_\eta}|R(0) = z, Y(0) = i\right], \qquad i = 0, 1,$$

the HJB equation becomes

$$\begin{aligned}
\sup_{w^0}\ & -(\delta+\lambda)V^0 + \left(r - \alpha_0 + \beta^2 + \gamma^2 + w^0\left(\mu_0 - r - \sigma\beta\right)\right)zV^0_z \\
& \qquad + \frac{(w^0\sigma - \beta)^2 + \gamma^2}{2}z^2V^0_{zz} + \lambda V^1 = 0, \\
\sup_{w^1}\ & -(\delta+\lambda)V^1 + \left(r - \alpha_1 + \beta^2 + \gamma^2 + w^1\left(\mu_1 - r - \sigma\beta\right)\right)zV^1_z \\
& \qquad + \frac{(w^1\sigma - \beta)^2 + \gamma^2}{2}z^2V^1_{zz} + \lambda V^0 = 0,
\end{aligned}$$

for $z \in \mathbb{R}^+$.

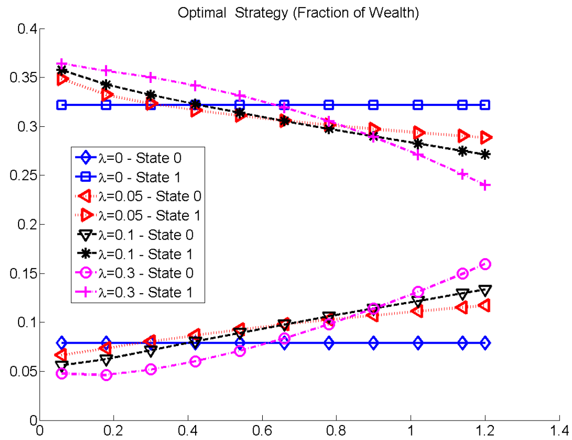

Figure 10. Fixed bonus when that manager outperforms a benchmark. Switching with Full Information: $\eta = 0.2$, $\delta = 0.04$, $r = 0.03$, $\mu_0 = 0.06$, $\mu_1 = 0.08$, $\sigma = 0.3$, $\alpha_0 = 0.045$, $\alpha_1 = 0.085$, $\beta = 0.3$, $\gamma = 0.4$. Zoom on $z \in [0, 1 + \eta]$.

These equations can be solved concurrently with our fixed-point iterative method. In Figure 10 we show the optimal strategies considering different values of λ and $\eta = 0.2$, $\delta = 0.04$, $r = 0.03$, $\mu_0 = 0.06$, $\mu_1 = 0.08$, $\sigma = 0.3$, $\alpha_0 = 0.045$, $\alpha_1 = 0.085$, $\beta = 0.3$, $\gamma = 0.4$. It can be observed that the optimal investment strategy is similar to the one obtained in Section 3: it is not a constant weight, when the wealth is low the agent invests more (less) in the risky asset in state 1 (0) with respect to the case without regime switching. This attitude is reversed when the reward target is approaching.

Under partial information, we obtain results similar to those shown in Section 3 (and therefore not reported for the sake of brevity), i.e., confidence risk induces excess risk seeking with respect to the full information investment strategy. Instead, considering the case of a fixed bonus if the manager beats the benchmark by a fraction η by a terminal date T, we obtain results similar to those obtained in Section 4.

6. High water marks remuneration scheme

In Panageas & Westerfield (2009) authors address the manager's optimal investment problem when he is remunerated by a high water marks contract: the manager receives a fraction of the increase in fund value in excess of the last recorded maximum, the so-called high water mark, if such an increase took place. Mathematically, assuming that the fund manager can invest in a risk-free and in a risky asset, and that k is the fraction of the maximum increase that the manager receives

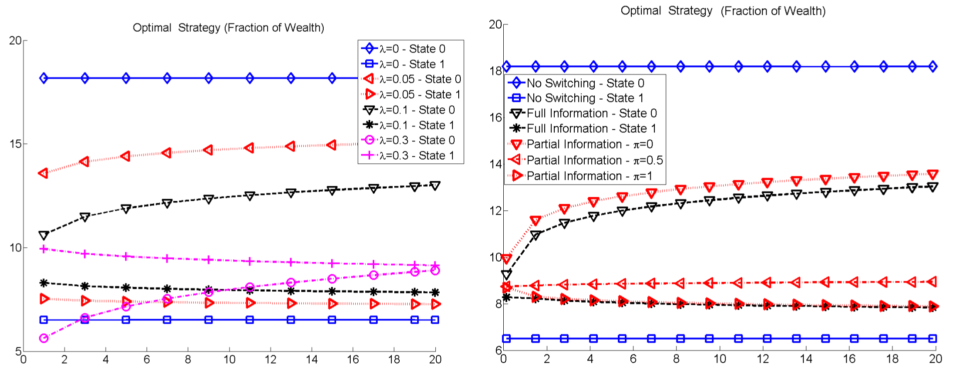

Figure 11. High water marks contract. No Switching and Switching with Full Information and different values of λ (left) and Switching with both Full and Partial Information ($\lambda = 0.1$) (right): $\delta = 0.04$, $\eta = 0.1$, $r = 0.05$, $\mu_0 = 0.06$, $\mu_1 = 0.08$, $\sigma = 0.3$, $k = 0.2$, $h = 20$.

as compensation, the asset management problem becomes

$$\sup_w E\left[\int_0^\tau e^{-(\delta+\eta)t} k dH(t)\right],$$

where τ is the (random) termination time of the fund, η is the constant intensity of the Poisson process that models the termination time of the fund, and $H(t)$ is the running maximum of the wealth $X(t)$, which evolves as follows

$$dX(t) = X(t)(w(t)(\mu(Y(t)) - r) + r)dt + w(t)\sigma X(t)dZ(t) - kdH(t), \quad X(0) = x.$$

In Panageas & Westerfield (2009), the HJB equation is given by

$$\sup_w \quad -(\delta+\eta)V + (w(\mu - r) + r)xV_x + \frac{\sigma^2 w^2}{2} x^2 V_{xx} = 0,$$

with $V = V(x,h)$ for any $0 \le x \le h$ and $h \ge 0$. The HJB equation is coupled with boundary conditions $V(0,h) = 0$ for any $h \ge 0$, i.e., for any value of the maximum process, due to the bankruptcy condition, and $V_x(h,h) = 1 + \frac{1}{k} V_h(h,h)$. In Panageas & Westerfield (2009) authors provide a closed form solution for this problem. We can provide a formulation of the problem in the full and in the partial information case proceeding as above, solving it numerically. In Figure 11-(left) we compare the no switching case with the full information one assuming $\delta = 0.04$, $\eta = 0.1$, $r = 0.05$, $\mu_0 = 0.06$, $\mu_1 = 0.08$, $\sigma = 0.3$, $k = 0.2$, $h = 20$.

As obtained in Panageas & Westerfield (2009), we observe that in a constant parameter environment the fraction of wealth invested in the risky asset in the good state (state 1) is smaller than the one in the bad state (state 0). As expected, in the good state, the agent invests in the risky asset more than in the no switching case as λ increases, the reverse happens in the bad state. Moreover, if λ is large enough ($\lambda \ge 0.3$), the agent invests more in the risky asset in the good state than in the bad one reverting the order obtained in a constant parameter environment.

The results are similar to those obtained in case of a fixed bonus when the target is reached. A reward related to the maximum dynamics induces the agent to excess (less) risk taking in the good (bad) state for any level of wealth. In this case, in the good state the manager exploits the momentum and takes more risk fearing a switch to the bad state or the end of the fund. On the other hand, in case of the bad state, the manager expects a switch to the good state and therefore he takes less risk. As far as confidence risk concerns, in Figure 11-(right) we show that it induces the agent to increase its exposure to the risky asset.

7. Conclusions

There are some anecdotes on how a non convex remuneration may affect management decisions in a non constant environment. The claim is that a manager remunerated through a bonus when a target is reached will take risk in excess in a bull market.

In this paper, considering several different target driven non convex remuneration schemes, we have demonstrated this claim showing that in a two state regime switching environment the manager's risk exposure is high in a good state and is low in a bad state. More precisely, in a full information setting, we have shown two results.

In the good state (higher Sharpe ratio), the investment in the risky asset is higher than the one obtained in the corresponding constant parameter environment. On the other hand, in the bad state (lower Sharpe ratio) the investment in the risky asset is lower than the one obtained in the corresponding constant parameter environment. The effect is increasing in the switching probability. The rationale of this behavior is that in the good state the manager exploits the momentum and takes more risk fearing a switch to the bad state. On the other hand, in the bad state, the manager expects a switch to the good state and therefore he takes less risk. The effect is observed for all levels of wealth in case of a bonus over a finite horizon or a high water marks remuneration, when the horizon is infinite this effect is observed only when the target is far away.

In a constant parameter environment, the investment strategy is not monotone in the Sharpe ratio and therefore the investment in the risky asset/risk exposure in the bad state can be higher than those obtained for the good state. We have shown that if the risk exposure in the good state without switching is higher than the risk exposure in the bad state without switching, then the order is preserved when the states switch. Otherwise, if the risk exposure in the bad state is higher than the one obtained in the good state when no switching occurs, we have demonstrated that this ranking is reversed in a regime switching environment if the probability of having a switch is large enough. This is due to the above attitude of the manager to exploit the momentum in a good state fearing a switch to the bad state or to wait for a switch in case of a bad state. When the switching probability is high, this effect can be so relevant that the rank obtained in a constant parameter environment is reversed when a regime switching setting is considered.

We have also shown that, contrary to what is observed in case of the maximization of the expected utility, confidence risk induces the agent to take more risk in a partial information environment.

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Appendix A. Numerical scheme

Let us consider the full information case and the 'fixed bonus from reaching target level' problem (Section 3.1). In this case the Picard iteration scheme consists in computing a sequence of solutions $\{V^i_m\}_{m\geq 0}$, and therefore a sequence of optimal strategies $\{w^i_m\}_{m\geq 0}$, $i = 0, 1$, such that, given (w^0_m, w^1_m), V^i_{m+1} is the solution of

$$-\delta V^0_{m+1}+(w^0_m(\mu_0-r)+r)x(V^0_{m+1})_x+\frac{\sigma^2(w^0_m)^2}{2}x^2(V^0_{m+1})_{xx}-\lambda V^0_{m+1}+\lambda V^1_{m+1}=0,$$
$$-\delta V^1_{m+1}+(w^1_m(\mu_1-r)+r)x(V^1_{m+1})_x+\frac{\sigma^2(w^1_m)^2}{2}x^2(V^1_{m+1})_{xx}-\lambda V^1_{m+1}+\lambda V^0_{m+1}=0, \tag{A.1}$$

and the iteration scheme stops when the difference between two consecutive solutions, $\max\{||V^0_{m+1}-V^0_m||, ||V^1_{m+1}-V^1_m||\}$, computed in a suitable norm, falls below a given tolerance level. Notice that, if the solution V^i_m is concave, i.e., $(V^i_m)_{xx} < 0$, then the related optimal portfolio strategy in state i can be computed as[d]

$$w^i_m = \frac{r-\mu_i}{\sigma^2}\frac{(V^i_m)_x}{x(V^i_m)_{xx}}, \quad i = 0, 1. \tag{A.2}$$

As already stressed above, given w^i_m, equation (A.1) consists of two coupled linear ODEs with respect to V^i_{m+1}, and thus can be easily solved numerically considering a finite difference scheme. More precisely, we introduce a set of nodes, i.e., a mesh, $\{x_j\}_{j=0}^N$, with $x_j = j\Delta x$, $\Delta x = b/N$; given the guess vector $\mathbf{w}^i_0$ defined as $\{\mathbf{w}^i_0\}_j = w^i_0(x_j)$, $i = 0, 1$, $j = 1, \cdots, N-1$, for $m = 0, \cdots$, we solve the linear

[d]The couple (w^0_m, w^1_m) in equation (A.1), given (V^0_m, V^1_m), is the solution of

$$w^0_m = \arg\sup_w -\delta V^0_m + (w(\mu_0-r)+r)x(V^0_m)_x + \frac{\sigma^2 w^2}{2}x^2(V^0_m)_{xx} - \lambda V^0_m + \lambda V^1_m,$$
$$w^1_m = \arg\sup_w -\delta V^1_m + (w(\mu_1-r)+r)x(V^1_m)_x + \frac{\sigma^2 w^2}{2}x^2(V^1_m)_{xx} - \lambda V^1_m + \lambda V^0_m.$$

system

$$-\delta\mathbf{V}^0_{m+1}+(\mathbf{w}^0_m(\mu_0-r)+r)\cdot\mathbf{x}\cdot\mathcal{T}\mathbf{V}^0_{m+1}+\frac{\sigma^2(\mathbf{w}^0_m\cdot\mathbf{x})^2}{2}\cdot\mathcal{D}\mathbf{V}^0_{m+1}-\lambda\mathbf{V}^0_{m+1}+\lambda\mathbf{V}^1_{m+1}=0$$
$$-\delta\mathbf{V}^1_{m+1}+(\mathbf{w}^1_m(\mu_1-r)+r)\cdot\mathbf{x}\cdot\mathcal{T}\mathbf{V}^1_{m+1}+\frac{\sigma^2(\mathbf{w}^1_m\cdot\mathbf{x})^2}{2}\cdot\mathcal{D}\mathbf{V}^1_{m+1}-\lambda\mathbf{V}^1_{m+1}+\lambda\mathbf{V}^0_{m+1}=0,$$

where the finite difference operator $\mathcal{D}$ is given by

$$\{\mathcal{D}\mathbf{V}\}_j := \frac{V(x_{j-1})-2V(x_j)+V(x_{j+1})}{\Delta x^2},$$

with $V(x_0)=0$ and $V(x_N)=1$. Similarly $\mathcal{T}$ is the upwind finite difference operator for the first order derivative, see Quarteroni *et al.* (2007) for further details. Notice that in the above formulation $\mathbf{f}\cdot\mathbf{g}$ and $(\mathbf{f})^2$ represent the element-wise product and square operators, respectively. The iterative procedure is repeated till the distances $\mathbf{V}^0_m-\mathbf{V}^0_{m-1}$ and $\mathbf{V}^1_m-\mathbf{V}^1_{m-1}$, computed according to the l^2 norm, fall below a 10^{-6} tolerance threshold. Finally, in all the numerical experiments related to the full information case we set the number of grid points $N+1$ equal to 2000.

It is well known that the Picard iterative scheme converges if the guess function (V^0_0, V^1_0) (and thus (w^0_0, w^1_0)) is close enough to the solution of equations (3.7)-(3.8) (and therefore to the optimal investment strategy). We refer to (McDonough 2008 Section 4.1.3) and Quarteroni *et al.* (2007) for details. In our numerical experiments, we consider as guess function the solution obtained in the no switching setting, and therefore w^i_0, $i=0,1$, is computed according to (3.6). Our numerical scheme provides a sequence of solutions $\{V^i_m\}_{m\geq 0}$ which satisfy $(V^i_m)_{xx}<0$ in all grid points: therefore the values $\{w^i_m\}_{m\geq 0}$ (and thus the vectors $\mathbf{w}^i_m$, with $(\mathbf{w}^i_m)_j = w^i_m(x_j)$) are computed according to (A.2).