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Sampled-Data Exponential Synchronization of Chaotic Lur'e Systems

YUEYING WANG¹, (Member, IEEE), YINGXIN ZHU², HAMID REZA KARIMI³, (Senior Member, IEEE), AND XIAOHANG LI¹

¹School of Electronic and Electrical Engineering, Shanghai University of Engineering Science, Shanghai 201620, China
²School of Astronautics, Harbin Institute of Technology, Harbin 150001, China

³Department of Mechanical Engineering, Politecnico di Milano, 20156 Milan, Italy

Corresponding author: Yueying Wang (wyy676@126.com)

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ABSTRACT This paper is concerned with the problem of sampled-data exponential synchronization for chaotic Lur'e systems (CLSs) in the form of master–slave framework. An improved time-dependent Lyapunov functional (TDLF) is put forward to fully exploit the accessible information about sampling characteristics and nonlinearities of the CLS. By resorting to the improved TDLF, a new synchronization criterion is established, which ensures the synchronization error system is globally exponentially stable. An illustrative example is offered to demonstrate the validity and virtue of the proposed design methodology.

INDEX TERMS Master–slave synchronization, sampled-data, chaotic Lur'e systems.

I. INTRODUCTION

Chaotic Lur'e systems (CLSs) are a class of typical nonlinear systems with nonlinearities satisfying a sector condition, and can exactly describe many dynamical systems. For instance, the well-known neural networks [1]–[9] and complex networks [10]–[12] belong to this class. As a fundamental issue in CLSs, the master-slave synchronization (MSS) problem has persistently received great research attention during the past years. Up to now, a variety of MSS methodologies have been developed, including delay feedback control approach [13], sliding mode control technique [14], and so on.

In practical control systems, the traditional analogcircuit-based controllers are rapidly being replaced by microprocessor-based digital controllers, where the latter ones have a better reliability and flexibility than former ones. In this situation, the aforementioned control strategies based on continuous measurements may not be applicable. To reflect the reality, it is necessary to study the MSS problem for CLSs by resorting to sampled-data control technique [15]–[20]. During the past years, considerable research efforts have focused on sampled-data MSS of timedelay or delay-free CLSs in the framework of input delay approach [21]–[28]. It is worth mentioning that a common objective of these results is to achieve MSS of CLSs under a larger allowable sampling interval, which is motivated by the fact that less consumption of transmission channel and more reliability will be ensured with the increase of sampling interval. To this end, a novel TDLF was proposed in [21] to reduce conservativeness and enlarge the allowable sampling. Hua et al. [23] further took the characteristics of nonlinear part into account, which can help to enlarge the sampling interval. In [26], a helpful triple term was introduced into the proposed TDLF, and a larger sampling period than previous results was obtained by partly resorting to free-matrix-based integral inequality. Two methodologies called sampling-instant-to-present-time fragmentation and free-matrix -based TDLF were adopted in [28] to increase the sampling interval. Although the sampled-data MSS problem for CLSs has attracted considerable attention and some methods have been obtained, there still exists the room for improvement. One important reason is that the available information about nonlinear part of CLS is not fully utilized in the constructed TDLF, which can lead to considerable conservativeness.

In this paper, we further investigate the sampled-data MSS problem for CLSs. An MSS criterion is provided to guarantee the exponential stability of synchronization error system in presence of sampled data. An illustrative example is given to demonstrate the effectiveness and advantage of proposed methodology over some recent results. The contributions of the note can be highlighted as: 1) To fully use the available

information about the nonlinear part of the system, an improved TDLF is constructed by introducing a novel term; 2) A more relaxed condition that ensures the positive definiteness of the TDLF on sampling intervals is presented.

II. PRELIMINARIES

Consider the master-slave type chaotic Lur'e systems, which are described by

$$\mathcal{M} : \begin{cases} \dot{y}(t) = \mathcal{D}y(t) + \mathcal{E}\sigma \left(\mathcal{F}y(t)\right) \\ z(t) = \mathcal{G}y(t) \end{cases}$$
$$\mathcal{S} : \begin{cases} \dot{\bar{y}}(t) = \mathcal{D}\bar{\bar{y}}(t) + \mathcal{E}\sigma \left(\mathcal{F}\bar{\bar{y}}(t)\right) + u(t) \\ \bar{z}(t) = \mathcal{G}\bar{\bar{y}}(t) \end{cases}$$
$$\mathcal{C} : u(t) = \mathcal{K} \left(z(t_k) - \bar{z}(t_k)\right) \tag{1}$$

where \mathcal{M} , \mathcal{S} , and \mathcal{C} denote the master system, slave system and controller, respectively. $y(t) \in \Re^n$ and $\overline{y}(t) \in \Re^n$ represent the state vectors. $u(t) \in \Re^n$ is the control input with gain matrix $\mathcal{K} \in \Re^{l \times n}$. $z(t) \in \Re^l$ and $\overline{z}(t) \in \Re^l$ represent output vectors. $\mathcal{D} \in \Re^{n \times n}$, $\mathcal{G} \in \Re^{l \times n}$, $\mathcal{F} \in \Re^{m \times n}$, and $\mathcal{E} \in \Re^{n \times m}$ are system matrices. $\sigma(\cdot) : \Re^m \to \Re^m$ is a nonlinear vector with elements $\sigma_i(\cdot)$ belonging to sector $[0, w_i]$. For the sampled-data MSS, it is assumed that measurement signals $z(t_k)$ and $\overline{z}(t_k)$ are only available at the sampling instant t_k with $0 = t_0 \cdots < t_k < \cdots + \infty$. The sampling interval is bounded by

$$t_{k+1} - t_k \le h, \quad \forall k \ge 0 \tag{2}$$

where h > 0 represents the maximum upper bound of sampling intervals.





The basic framework of the sampled-data MSS is provided in Fig. 1. Denote the synchronization error as $\delta(t) = y(t) - \bar{y}(t)$. From the MSS scheme (1), the following synchronization error system can be obtained

$$\dot{\delta}(t) = \mathcal{D}\delta(t) + \mathcal{E}\mathcal{J}\left(\mathcal{F}\delta(t)\right) - \mathcal{K}\mathcal{G}\delta(t_k), \quad t_k \le t < t_{k+1}$$
(3)

with $\mathcal{J}(\mathcal{F}\delta(t)) = \sigma (\mathcal{F}\delta(t) + \mathcal{F}\bar{y}(t)) - \sigma (\mathcal{F}\bar{y}(t))$. For any δ, \bar{y} ,

$$0 \leq \frac{\mathcal{J}_i(f_i^T \delta, \bar{y})}{f_i^T \delta} \\ = \frac{\sigma_i(f_i^T \delta, \bar{y}) - \sigma_i(f_i^T \bar{y})}{f_i^T \delta} \leq w_i, \quad i = 1, 2, \dots, m$$

where f_i^T represents the *i*th row for matrix \mathcal{F} .

III. MAIN RESULTS

During this subsection, a new sampled-data MSS criterion for CLSs will be provided by resorting to the constructed TDLF. Before proceeding, we denote the following notations:

$$\tau(t) = t - t_k, \quad h_{\tau}(t) = h - \tau(t),$$

$$\varsigma(t) = \begin{bmatrix} \delta^T(t) & \dot{\delta}^T(t) & \delta^T(t_k) & \int_{t_k}^t \delta^T(\alpha) d\alpha \\ & \mathcal{J}^T(\mathcal{F}\delta(t)) & \int_{t_k}^t \mathcal{J}^T(\mathcal{F}\delta(\alpha)) d\alpha \end{bmatrix}^T$$

$$\bar{\varsigma}(t) = \begin{bmatrix} \delta^T(t) & \delta^T(t_k) & \int_{t_k}^t \delta^T(\alpha) d\alpha & \int_{t_k}^t \mathcal{J}^T(\mathcal{F}\delta(\alpha)) d\alpha \end{bmatrix}^T$$
(4)

Theorem 1: For given scalars h > 0, $\lambda \ge 0$, ε_1 , ε_2 ,

$$\kappa = \begin{cases} 1, & \text{if } \lambda = 0\\ 0, & \text{if } \lambda \neq 0, \end{cases}$$

the exponential MSS of system (1) can be ensured, if there exist positive diagonal matrices S_i , T_i , i = 1, 2, $\mathcal{L} = diag(l_1, l_2, ..., l_m)$, $\mathcal{W} = diag(w_1, w_2, ..., w_m)$, positive matrices $\mathcal{P}, \mathcal{Q}, \mathcal{Z}$,

$$\mathcal{R} = \begin{bmatrix} \mathcal{R}_{11} & \mathcal{R}_{12} \\ * & \mathcal{R}_{22} \end{bmatrix}, \quad \mathcal{U} = \begin{bmatrix} \mathcal{U}_{11} & \mathcal{U}_{12} \\ * & \mathcal{U}_{22} \end{bmatrix},$$
$$\mathcal{V} = \begin{bmatrix} \mathcal{V}_{11} & \mathcal{V}_{12} \\ * & \mathcal{V}_{22} \end{bmatrix},$$

and appropriately dimensioned matrices $\mathcal{H}, \bar{\mathcal{K}}, \Pi, \hat{\Pi}$ and \mathcal{X} such that the following conditions are feasible

$$\Xi + \Gamma < 0 \tag{5}$$
$$\bar{\Pi} \qquad h\hat{\Pi}$$

$$\begin{bmatrix} * & -hQ - he^{-2\lambda h}\mathcal{R}_{11} & 0\\ * & * & -2e^{2\lambda h}\mathcal{Z} \end{bmatrix} < 0 \quad (6)$$
$$\Xi_2 = \begin{bmatrix} \mathcal{P} + e^{-2\lambda h}Q & -e^{-2\lambda h}Q & 0 & 0\\ * & e^{-2\lambda h}Q & 0 & 0\\ * & * & 0 & 0\\ * & * & * & 0 \end{bmatrix} + h\mathcal{X} > 0 \quad (7)$$

where

 $\Gamma \Xi + \overline{\Gamma}$

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & -\varepsilon_1 \mathcal{H}\mathcal{E} + \Pi_5^T & \Xi_{16} \\ * & \Xi_{22} & \Xi_{23} & -e^{-2\lambda h} \hat{\Pi}_2 & -\kappa \mathcal{F}^T \mathcal{L}^T - \mathcal{H}\mathcal{E} & 0 \\ * & * & \Xi_{3,3} & \Xi_{34} & -\varepsilon_2 \mathcal{H}\mathcal{E} - \Pi_5^T & \Xi_{36} \\ * & * & * & \Xi_{44} & -e^{-2\lambda h} \hat{\Pi}_5^T & \Xi_{46} \\ * & * & * & * & 0 & 0 \\ * & * & * & * & & \varepsilon_{66} \end{bmatrix}$$
$$\Gamma = \begin{bmatrix} \Theta_{11} & \Theta_{12} & \Theta_{13} & \Theta_{14} & h \mathcal{X}_{14} + \mathcal{F}^T \mathcal{W}S_2 & \Theta_{16} \\ * & \Theta_{22} & \Theta_{23} & h \mathcal{X}_{13} & 0 & h \mathcal{X}_{14} \\ * & * & \Theta_{33} & 2\lambda h \mathcal{X}_{23} & h \mathcal{X}_{24} + h \mathcal{V}_{12} & 2\lambda h \mathcal{X}_{24} \\ * & * & * & * & & -2S_2 - \mathcal{T}_2 + h \mathcal{V}_{22} & h \mathcal{X}_{44} \\ * & * & * & * & & & & 2\lambda h \mathcal{X}_{44} \end{bmatrix}$$

Then, gain matrix \mathcal{K} of sampled-data controller can be computed as follows:

$$\mathcal{K} = \mathcal{H}^{-1}\bar{\mathcal{K}}.$$
 (8)

Proof: The following improved TDLF is constructed for synchronization error system (3):

$$V(t) = \sum_{i=1}^{4} V_i(t), \quad t \in [t_k, t_{k+1})$$
(9)

where

$$\begin{split} V_{1}(t) &= 2\kappa e^{2\lambda t} \sum_{i=1}^{m} \int_{0}^{f_{i}^{T}\varepsilon} \left[l_{i}(w_{i}\alpha - \mathfrak{h}_{i}(\alpha)) \right] d\alpha \\ V_{2}(t) &= e^{2\lambda t} \delta^{T}(t) \mathcal{P}\delta(t) + h_{\tau}(t) e^{2\lambda t} \bar{\varsigma}^{T}(t) X \bar{\varsigma}(t) \\ &+ h_{\tau}(t) \int_{t_{k}}^{t} e^{2\lambda \alpha} \dot{\delta}^{T}(\alpha) \mathcal{Q}\dot{\delta}(\alpha) d\alpha \\ V_{3}(t) &= h_{\tau}(t) \int_{t_{k}}^{t} e^{2\lambda \alpha} \left[\dot{\varepsilon}^{T}(\alpha) \quad \delta^{T}(t_{k}) \right] \\ &\times \mathcal{R} \left[\dot{\delta}^{T}(\alpha) \quad \delta^{T}(t_{k}) \right]^{T} d\alpha \\ &+ h_{\tau}(t) \int_{t_{k}}^{t} e^{2\lambda \alpha} \left[\delta^{T}(t_{k}) \quad \delta^{T}(\alpha) \right] \\ &\times \mathcal{U} \left[\delta^{T}(t_{k}) \quad \delta^{T}(\alpha) \right]^{T} d\alpha \\ &+ h_{\tau}(t) \int_{t_{k}}^{t} e^{2\lambda \alpha} \left[\delta^{T}(t_{k}) \quad \mathcal{J}^{T}(\mathcal{F}\delta(\alpha)) \right] \mathcal{V} \\ &\times \left[\delta^{T}(t_{k}) \quad \mathcal{J}^{T}(\mathcal{F}\delta(\alpha)) \right]^{T} d\alpha \\ &+ h_{\tau}(t) \int_{-\tau(t)}^{0} \int_{t+\beta}^{t} e^{2\lambda \alpha} \dot{\delta}^{T}(\alpha) \mathcal{Z}\dot{\delta}(\alpha) d\alpha d\beta \end{split}$$

From (4) and the assumption of theorem, we know that $V_1(t)$, $V_3(t)$ and $V_4(t)$ are positive definite. It can be verified that

$$V_{2}(t) \geq h^{-1}\tau(t)e^{2\lambda t}\delta^{T}(t)\mathcal{P}\delta(t) + h^{-1}h_{\tau}(t)e^{2\lambda t}\delta^{T}(t)\mathcal{P}\delta(t) + h_{\tau}(t)e^{2\lambda t}\bar{\varsigma}^{T}(t)X\bar{\varsigma}(t) + h^{-1}h_{\tau}(t)e^{2\lambda(t-h)} \times [\delta(t) - \delta(t_{k})]^{T} \times \mathcal{Q}[\delta(t) - \delta(t_{k})] = h^{-1}\tau(t)e^{2\lambda t}\delta^{T}(t)\mathcal{P}\delta(t) + h^{-1}h_{\tau}(t)e^{2\lambda t}\bar{\varsigma}^{T}(t)\Xi_{2}\bar{\varsigma}(t)$$
(10)

From condition (7), we can conclude that $V_2(t) \ge 0$, $t \in [t_k, t_{k+1})$, then the whole TDLF (9) is positive definite.

It is noted that V(t) is a continuous function on $[0, \infty)$ except the jumps t_k (k = 0, 1, 2, ...). When $t = t_k$, the second and third terms in $V_2(t)$ and the terms in $V_3(t)$ are not less than zero when $t \to t_k^-$ and equal to zero when $t = t_k^+$. This implies V(t) does not increase along the sampling instants. By taking the derivatives of V(t) along system (3), we have

$$\begin{split} \dot{V}_{1}(t) &= 2\kappa e^{2\lambda t} \delta^{T}(t) \mathcal{F}^{T} \mathcal{WLF} \dot{\delta}(t) \\ &- 2\kappa e^{2\lambda t} \mathcal{J}^{T}(\mathcal{F}\delta(t)) \mathcal{LF} \dot{\delta}(t) \qquad (11) \\ \dot{V}_{2}(t) &\leq 2e^{2\lambda t} \delta^{T}(t) \mathcal{P} \dot{\delta}(t) + 2\lambda \delta^{T}(t) \mathcal{P}\delta(t) \\ &+ 2h_{\tau}(t) e^{2\lambda t} \bar{\varsigma}^{T}(t) \mathcal{X} \dot{\varsigma}(t) - e^{2\lambda t} \bar{\varsigma}^{T}(t) \mathcal{X} \bar{\varsigma}(t) \\ &+ 2\lambda h_{\tau}(t) e^{2\lambda t} \delta^{T}(t) \mathcal{Q} \dot{\delta}(t) \\ &- e^{2\lambda t} e^{-2\lambda h} \int_{t_{k}}^{t} \delta^{T}(\alpha) \mathcal{Q} \dot{\delta}(\alpha) d\alpha \qquad (12) \\ \dot{V}_{3}(t) &\leq h_{\tau}(t) e^{2\lambda t} \left[\dot{\delta}^{T}(t) - \delta^{T}(t_{k}) \right] \mathcal{R} \left[\dot{\delta}^{T}(t) - \delta^{T}(t_{k}) \right]^{T} \\ &- e^{2\lambda(t-h)} \int_{t_{k}}^{t} \left[\dot{\delta}^{T}(\alpha) - \delta^{T}(t_{k}) \right] \\ &\times \mathcal{R} \left[\dot{\delta}^{T}(\alpha) - \delta^{T}(t_{k}) \right]^{T} d\alpha \end{split}$$

$$+ h_{\tau}(t)e^{2\lambda t} \left[\delta^{T}(t_{k}) \quad \delta^{T}(t) \right] \mathcal{U} \left[\delta^{T}(t_{k}) \quad \delta^{T}(t) \right]^{T}$$

$$- \tau(t)e^{2\lambda(t-h)}\delta^{T}(t_{k})\mathcal{U}_{11}\delta(t_{k})$$

$$- h^{-1}e^{2\lambda(t-h)}\int_{t_{k}}^{t}\delta^{T}(\alpha)d\alpha$$

$$\times U_{22}\int_{t_{k}}^{t}\delta(\alpha)d\alpha - 2e^{2\lambda(t-h)}\delta^{T}(t_{k})\mathcal{U}_{12}\int_{t_{k}}^{t}\delta(\alpha)d\alpha$$

$$- 2e^{2\lambda(t-h)}\delta^{T}(t_{k})\mathcal{V}_{12}\int_{t_{k}}^{t}\mathcal{J}\left(\mathcal{F}\delta(\alpha)\right)d\alpha + h_{\tau}(t)e^{2\lambda t}$$

$$\times \left[\delta^{T}(t_{k}) \quad \mathcal{J}^{T}\left(\mathcal{F}\delta(t)\right) \right]$$

$$\times \mathcal{V} \left[\delta^{T}(t_{k}) \quad \mathcal{J}^{T}\left(\mathcal{F}\delta(t)\right) \right]^{T}$$

$$- h^{-1}e^{2\lambda(t-h)}\int_{t_{k}}^{t}\mathcal{J}^{T}\left(\mathcal{F}\delta(\alpha)\right)d\alpha \mathcal{V}_{22}$$

$$\times \int_{t_{k}}^{t}\mathcal{J}\left(\mathcal{F}\delta(\alpha)\right)d\alpha$$

$$- \tau(t)e^{2\lambda(t-h)}\delta^{T}(t_{k})\mathcal{V}_{11}\delta(t_{k})$$

$$(13)$$

$$\dot{V}_{4}(t) \leq 0.25\left(h_{\tau}(t) + \tau(t)\right)^{2}e^{2\lambda t}\dot{\delta}^{T}(t)\mathcal{Z}\dot{\delta}(t)$$

$$- \int_{-\tau(t)}^{0}\int_{t+\beta}^{t}e^{2\lambda\alpha}\dot{\delta}^{T}(\alpha)\mathcal{Z}\dot{\delta}(\alpha)d\alpha d\beta$$

$$\leq 0.25h\left(h_{\tau}(t) + \tau(t)\right)e^{2\lambda t}\dot{\delta}^{T}(t)\mathcal{Z}\dot{\delta}(t)$$

$$- e^{2\lambda(t-h)}\int_{-\tau(t)}^{0}\int_{t+\beta}^{t}\dot{\delta}^{T}(\alpha)\mathcal{Z}\dot{\delta}(\alpha)d\alpha d\beta$$

$$\leq 0.25he^{2\lambda t}h_{\tau}(t)\dot{\delta}^{T}(t)\mathcal{Z}\dot{\delta}(t) + 0.25he^{2\lambda t}\tau(t)\dot{\delta}^{T}(t)$$

$$\times \mathcal{Z}\dot{\delta}(t) - e^{2\lambda(t-h)}\int_{-\tau(t)}^{0}\int_{t+\beta}^{t}\dot{\delta}^{T}(\alpha)\mathcal{Z}\dot{\delta}(\alpha)d\alpha d\beta$$

$$(14)$$

On the other hand, the following inequality holds

$$0 = 2\varsigma^{T}(t)\hat{\Pi}\bigg[\tau(t)\delta(t) - \int_{t_{k}}^{t}\delta(\alpha)d\alpha - \int_{-\tau(t)}^{0}\int_{t+\beta}^{t}\dot{\delta}^{T}(\alpha)d\alpha d\beta\bigg]$$

$$\leq 2\tau(t)\varsigma^{T}(t)\hat{\Pi}\delta(t) - 2\varsigma^{T}(t)\hat{\Pi}\int_{t_{k}}^{t}\delta(\alpha)d\alpha + 0.5\tau^{2}(t)\varsigma^{T}(t)\hat{\Pi}$$

$$\times \mathcal{Z}^{-1}\hat{\Pi}^{T}\xi(t) + 2\tau^{-2}(t)\bigg[\int_{-\tau(t)}^{0}\int_{t+\beta}^{t}\dot{\delta}^{T}(\alpha)d\alpha d\beta\bigg]\mathcal{Z}$$

$$\times\bigg[\int_{-\tau(t)}^{0}\int_{t+\beta}^{t}\dot{\delta}(\alpha)d\alpha d\beta\bigg]$$

$$\leq 2\tau(t)\varsigma^{T}(t)\hat{\Pi}\delta(t) - 2\varsigma^{T}(t)\hat{\Pi}\int_{t_{k}}^{t}\delta(\alpha)d\alpha + 0.5\tau^{2}(t)\varsigma^{T}(t)\hat{\Pi}$$

$$\times \mathcal{Z}^{-1}\hat{\Pi}^{T}\xi(t) + \int_{-\tau(t)}^{0}\int_{t+\beta}^{t}\dot{\delta}^{T}(\alpha)\mathcal{Z}\dot{\delta}(\alpha)d\alpha d\beta \qquad (15)$$

which implies

$$-e^{2\lambda(t-h)} \int_{-\tau(t)}^{0} \int_{t+\beta}^{t} \dot{\delta}^{T}(\alpha) Z \dot{\delta}(\alpha) d\alpha d\beta$$

$$\leq 2\tau(t) e^{2\lambda(t-h)} \varsigma^{T}(t) \hat{\Pi} \delta(t) - 2e^{2\lambda(t-h)} \varsigma^{T}(t) \hat{\Pi} \int_{t_{k}}^{t} \delta(\alpha) d\alpha$$

$$+ 0.5h\tau(t) e^{2\lambda(t-h)} \varsigma^{T}(t) \hat{\Pi} Z^{-1} \hat{\Pi}^{T} \varsigma(t)$$
(16)

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For any matrix Π with appropriate dimension, the following equation holds:

$$0 = 2e^{2\lambda t}\varsigma^{T}(t)\Pi\left[\delta(t) - \delta(t_{k}) - \int_{t_{k}}^{t} \dot{\delta}(\alpha)d\alpha\right] \quad (17)$$

Similarly, for any matrix $\mathcal{H} \in \Re^{n \times n}$, and scalars ε_1 and ε_2 , we obtain from (3) that

$$0 = 2e^{2\lambda t} \left[\varepsilon_1 \delta^T(t) \mathcal{H} + \dot{\mathcal{E}}^T(t) \mathcal{H} + \varepsilon_2 \delta^T(t_k) \mathcal{H} \right] \\ \times \left[\dot{\mathcal{E}}(t) - \mathcal{D}\delta(t) - \mathcal{E}\mathcal{J} \left(\mathcal{F}\delta(t) \right) + \mathcal{K}\mathcal{G}\delta(t_k) \right]$$
(18)

Moreover, for any $S_i = diag(s_{i1}, s_{i2}, \dots, s_{im}) \ge 0$, and $\mathcal{T}_i = diag(t_{i1}, t_{i2}, \dots, t_{im}) \ge 0$, i = 1, 2, it follows from (4) that

$$2h^{-1}\tau(t)e^{2\lambda t} \left[\delta^{T}(t)\mathcal{F}^{T}\mathcal{W} - \mathcal{J}^{T}(\mathcal{F}\delta(t))\right] \mathcal{S}_{1}\mathcal{J}(\mathcal{F}\delta(t)) + 2h^{-1}\tau(t)e^{2\lambda t} \left[\delta^{T}(t)\mathcal{F}^{T}\mathcal{W} - \mathcal{J}^{T}(\mathcal{F}\delta(t))\right] \mathcal{S}_{2}\mathcal{J}(\mathcal{F}\delta(t)) \geq 0 \qquad (19) h^{-1}\tau(t)e^{2\lambda t}\delta^{T}(t)\mathcal{F}^{T}\mathcal{W}\mathcal{T}_{1}\mathcal{W}\mathcal{F}\delta(t) - h^{-1}\tau(t)e^{2\lambda t}\mathcal{J}^{T}(\mathcal{F}\delta(t)) \times \mathcal{T}_{1}\mathcal{J}(\mathcal{F}\delta(t)) + h^{-1}h_{\tau}(t)e^{2\lambda t}\delta^{T}(t)\mathcal{F}^{T}\mathcal{W}\mathcal{T}_{2}\mathcal{W}\mathcal{F}\delta(t) - h^{-1}h_{\tau}(t)e^{2\lambda t}\mathcal{J}^{T}(\mathcal{F}\delta(t))\mathcal{T}_{2}\mathcal{J}(\mathcal{F}\delta(t)) \geq 0 \qquad (20)$$

Then, by adding the right sides of (17), (18) and the left sides of (19) and (20) into $\dot{V}(t)$, and setting $\bar{\mathcal{K}} = \mathcal{HK}$, we can have from (11)-(14) and (16) that for $t \in [t_k, t_{k+1})$,

$$\begin{split} \dot{V}(t) &\leq \frac{h_{\tau}(t)}{h} e^{2\lambda t} \varsigma^{T}(t) (\Xi + \Gamma) \varsigma(t) + h^{-1} e^{2\lambda t} \int_{t_{k}}^{t} \begin{bmatrix} \varsigma(t) \\ \dot{\delta}(\alpha) \end{bmatrix}^{T} \\ &\times \left(\begin{bmatrix} \Xi + \bar{\Gamma} & \bar{\Pi} \\ * & -hQ - he^{-2\lambda h} \mathcal{R}_{11} \end{bmatrix} \\ &+ 0.5h^{2} e^{-2\lambda h} \begin{bmatrix} \hat{\Pi} \\ 0 \end{bmatrix} \mathcal{Z}^{-1} \begin{bmatrix} \hat{\Pi} \\ 0 \end{bmatrix}^{T} \right) \begin{bmatrix} \varsigma(t) \\ \dot{\delta}(\alpha) \end{bmatrix} d\alpha \end{split}$$
(21)

Following the similar analysis in [21], we can obtain from (21) that there exists a scalar $\kappa > 0$ such that $||e(t)|| \leq \kappa e^{-\lambda t} ||e(0)||$, if conditions (5) and (6) hold. This implies that the chaotic Lur'e systems (1) are exponentially synchronous. The proof is completed.

Remark 1: Compared with previous TDLFs in [21]–[28], the main difference in the constructed TDLF (9) is that the useful term $\int_{t_k}^t \mathcal{J}(\mathcal{F}\delta(\alpha)) d\alpha$ is introduced into the second term of $V_2(t)$ to fully use the available information about nonlinear part, which will contribute to the enlargement of sampling interval. Moreover, motivated by [29], the double integral term in $V_4(t)$ is adopted for the first time to reduce design conservativeness.

Remark 2: It is worth mentioning the constraint condition (7) is used to ensure the positive definiteness of the whole TDLF on sampling intervals, which is more relaxed than the ones in [21]–[28] by imposing P + hX > 0. The relaxation comes from the introduction of an additional free matrix Q, which will reduce conservativeness to some extent. It should be pointed out that there are some other ways that

can further improve the obtained result. For instance, an effective way is to use sampling-instant-to-present-time fragmentation to fully exploit the inner sampling behavior and reduce conservativeness.

Remark 3: The number of decision variables involved in our MSS criterion obtained in Theorem 1 is $42.5n^2 + 9m^2 + 36mn + 4.5n + 7m + nl$, which is bigger than previous results [21]–[28]. This means that the solution of the criterion can be searched in a larger set. As a consequence, more computational burden will be consumed.

IV. ILLUSTRATIVE EXAMPLE

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During this subsection, an illustrative example about Chua's circuit will be utilized to verify the effectiveness and advantage of the proposed sampled-data MSS scheme.

The nonlinear model of a Chua's circuit can be presented as follows:

$$\dot{y}_1(t) = \bar{\alpha} (y_2(t) - \pi_1 y_1(t) + \sigma (y_1(t)))$$

$$\dot{y}_2(t) = y_1(t) - y_2(t) + y_3(t)$$

$$\dot{y}_3(t) = -\beta y_2(t)$$
(22)

where parameters $\pi_0 = -1/7$, $\pi_1 = 2/7$, $\bar{\alpha} = 9$, $\bar{\beta} = 14.28$, and nonlinear part

$$\sigma(y_1(t)) = 0.5(|y_1(t) + 1| - |y_1(t) - 1|),$$

belongs to sector [0, 1].

Then, the related system matrices are provided as

$$\mathcal{D} = \begin{bmatrix} -\bar{\alpha}\pi_1 & \bar{\alpha} & 0\\ 1 & -1 & 1\\ 0 & -\bar{\beta} & 0 \end{bmatrix}, \quad \mathcal{E} = \begin{bmatrix} -\bar{\alpha}(\pi_0 - \pi_1) \\ 0\\ 0 \end{bmatrix},$$
$$\mathcal{G} = \mathcal{F} = \begin{bmatrix} 1\\ 0\\ 0 \end{bmatrix}^T.$$

We first verify the virtue of the obtained MSS criterion. By choosing $\varepsilon_1 = 0$, $\varepsilon_2 = 1$ and $\lambda = 0$, the exponential MSS criterion obtained in Theorem 1 reduces to the asymptotic one. In this situation, the obtained maximum allowable sampling interval *h* is 0.6728, while using the methodologies given in [25]–[28], the corresponding ones are 0.5218, 0.5319, 0.5342 and 0.5368, respectively. So, a larger sampling interval has been obtained by using our MSS criterion. Denote h = 0.32, the maximum allowable decay rate λ in [21] is 0.07, while adopting Theorem 1, the maximum decay rate is 0.22. This implies that a shorter convergence time can be achieved by our approach. Based on (8), the corresponding gain matrix of sampled-data controller can be computed as

$$\mathcal{K} = \begin{bmatrix} 3.1789 & 0.8559 & -1.6012 \end{bmatrix}^T$$

Let $y(0) = \begin{bmatrix} 0.1 & -0.3 & 0.4 \end{bmatrix}^T$, $\bar{y}(0) = \begin{bmatrix} -0.4 & 0.2 & -0.1 \end{bmatrix}^T$ be initial conditions of CLS. Under the sampling interval h = 0.32, decay rate $\lambda = 0.22$ and the above gain matrix for the sampled-data controller, the responses about states y(t), $\bar{y}(t)$, and synchronization error $\delta(t)$ are depicted in Fig. 2, which verifies that under the designed controller,



FIGURE 2. Responses of states y(t), $\bar{y}(t)$ and synchronization error $\delta(t)$.

the synchronization error $\delta(t)$ can exponentially converge to zero despite the divergence of state for master system.

V. CONCLUSION

In this paper, the sampled-data master-slave synchronization (MSS) problem has been studied for chaotic Lur'e systems (CLSs) subject to the sampled-data. To fully use the information of nonlinear part, an improved TDLF has been established. A relaxed constraint condition is also presented to keep the positive definiteness of the whole TDLF on sampling intervals. By resorting to the TDLF and constraint condition, a less conservative MSS criterion has been achieved. The validity and virtue of the developed methodology has been demonstrated by an illustrative example. In future, we shall study sampled-data fuzzy systems [30]–[40] and sampled-data -based state estimation/consensus problems [41]–[49].

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YUEYING WANG (M'16) received the B.Eng. degree in mechanical engineering and automation from the Beijing Institute of Technology, Beijing, China, in 2006, the M.Eng. degree in navigation, guidance, and control and the Ph.D. degree in control science and engineering from Shanghai Jiao Tong University, Shanghai, China, in 2010 and 2015, respectively.

From 2010 to 2017, he was a Research Associate and Post-Doctoral Fellow with the School of

Aeronautics and Astronautics, Shanghai Jiao Tong University, Shanghai. He is currently an Associate Professor with the School of Electronic and Electrical Engineering, Shanghai University of Engineering Science, Shanghai. His current research interests include sliding mode control/observation, sampleddata control/filtering, reset control/filtering, and flight control.



in 1976. He received the B.Sc. (Hons.) degree in power systems from the Sharif University of Technology, Tehran, Iran, in 1998, and the M.Sc. and Ph.D. (Hons.) degrees in control systems engineering from the University of Tehran, Tehran, in 2001 and 2005, respectively. He is currently a Professor of applied mechanics with the Department of Mechanical Engineering, Politecnico di Milano, Milan, Italy. His current research interests include

HAMID REZA KARIMI (M'06-SM'09) was born

control systems and mechatronics.

Dr. Karimi is a member of the IEEE Technical Committee on Systems with Uncertainty, the Committee on Industrial Cyber-Physical Systems, the IFAC Technical Committee on Mechatronic Systems, the Committee on Robust Control, and the Committee on Automotive Control. He received the 2016 Web of Science Highly Cited Researcher in Engineering recognition. He is currently the Editor-in-Chief of Designs with MDPI, Switzerland, and an Editorial Board Member for some international journals, such as the IEEE TRANSACTIONS ON INDUSTRIAL ELECTRONICS, the IEEE TRANSACTIONS ON CIRCUIT AND SYSTEMS-I, the IEEE/ASME TRANSACTIONS ON MECHATRONICS, the IEEE TRANSACTIONS ON SYSTEMS, MAN, and CYBERNETICS: SYSTEMS, INFORMATION SCIENCES, the IEEE ACCESS, Mechatronics (IFAC), Neurocomputing, the Asian Journal of Control, Automation, and Systems, the International Journal of Control, Automation, and Systems, the International Journal of Fuzzy Systems, the International Journal of e-Navigation and Maritime Economy, and the Journal of Systems and Control Engineering.



YINGXIN ZHU received the B.Eng. degree from the Hebei University of Science and Technology, Shijiazhuang, China. She is currently pursuing the M.Eng. degree with the Harbin Institute of Technology, Harbin, China.

Her research interests include sampled-data control and flight control.



XIAOHANG LI received the Ph.D. degree in control theory and control engineering from Tongji University, Shanghai, China, in 2016. She is currently a Lecturer of the Shanghai University of Engineering Science.

Her research interests include observer design, model-based fault detection, fault estimation, and fault tolerant control.