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Attitude tracking with an adaptive sliding mode response to reaction wheel failure

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Abstract

This paper proposes an attitude tracking control for a rigid spacecraft that adapts to two types of faults that commonly occur in reaction wheels: a gain fault and a deviation fault. In its normal operating mode the tracking controller replicates that of a continuous quaternion feedback controller. When a fault occurs in the system the attitude of the spacecraft will deviate from the reference trajectory and will consequently trigger a sliding mode response of the control which introduces robustness. For the proposed control law, we construct a suitable Lyapunov function to prove the closed-loop system is asymptotically stable in the presence of such faults. However, the proposed control is not practically suitable over long periods as the gain on the sliding mode component will always increase unless the sliding surface is exactly zero (in practise this is never the case because of sensor noise). To address this problem a simple adaptive parameter is defined such that it converges to an appropriate upper-limit. Simulations of the attitude dynamics of a spacecraft are undertaken which compares the tracking performance in the presence of a fault with and without the adaptive sliding mode component.

Keywords: Unknown external disturbances, Actuator limited output torques, Actuator failures, Robust gain control, Sliding surface, Adaptive control

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1. Introduction

Fault-tolerant attitude control has been an active area of research in recent years. Conventional spacecraft are designed with very high reliability due to their high cost and it is necessary to design fault-tolerant controls to further reduce the risk of mission failure given the failure of an actuator. Furthermore, nano-spacecraft have shifted the emphasis of reliability dominated design to efficiency design and thus have a much higher risk of failure than conventional spacecraft. Thus, for nano-spacecraft where the risk of failure of an actuator, such as a reaction wheel, is much higher it is essential to design controllers that ensure a mission can still be undertaken in the event of failure. There have been a number of fault-tolerant-controls [1]-[3] (FTC) developed based on adaptive control and sliding mode control methods.

Adaptive attitude control methods [4]-[7] can guarantee the performance of a system when there exists actuator uncertainty by autonomously updating control parameters. An adaptive quaternion feedback fault-tolerant control was presented in [8] to deal with a gain fault and a deviation fault that commonly occur in reaction wheels. However, this type of control is not useful over long time periods as the adaptive parameter increases continuously and can lead to excessive control demands beyond the limits of the reaction wheels. The attitude stabilization problem has been investigated using adaptive control in the presence of external disturbances, unknown inertia parameters, and actuator uncertainties including faults in [9]. In [10] both passive and active designs based on variable structure reliable control (VSR-C) are presented which uses an observer to actively identify faults. A robust adaptive control strategy using a fuzzy compensator for MEMS triaxial gyroscope is proposed in [11]. Adaptive controllers have also been combined with extended state observers that estimate the unknown disturbance torques and inertia and compensate for them at each sampling period to provide accurate pointing and tracking [12, 13]. The use of an extended state observer (ESO) improves tracking performance when the disturbances are compensated for at each sampling period, but the ESO can add significant additional on-board computational expense.

Recently, sliding mode control [14]-[17] has been used for robust tracking in the presence of uncertainties, however, these have not been applied to address actuator faults. An indirect (non-regressor-based) approach based on adaptive sliding mode control (SMC) has been used for attitude tracking control in the presence of modeling uncertainties, unknown disturbances, actuator failures and limited resources in [18]. In [19] high precision attitude tracking of spacecraft in the presence of actuator failures and saturations with finite-time convergence was achieved by using a non-singular terminal sliding-mode control law. The FTC based on an integral-type sliding mode strategy to compensate for actuator faults is proposed in [20]. However, although the sliding mode controllers can provide robustness to faults they can induce chattering or non-smooth feedback that is detrimental to the reaction wheels. Moreover, a continuous proportional tracking control can provide asymptotic tracking without actuator faults and provides a smooth feedback that can be efficiently produced by the reaction wheel. Thus, it is intuitive to consider that proportional type tracking controllers should be used when there are no faults or significant uncertainties and only use sliding mode control if the fault or disturbances significantly effect performance. In this paper a control is proposed where a quaternion feedback control is used to track a reference trajectory and when a fault occurs a sliding mode component is adaptively introduced where the gain increases to ensure robustness to faults.

In this paper, we propose an adaptive control law based on a conventional continuous tracking control and an adaptive sliding mode component that can be implemented using reaction wheels. When there are no disturbances, actuator constraints or actuator faults the proposed control operates in the same way as a conventional proportional control and will accurately track a prescribed, smooth, reference attitude. However, when there is an actuator failure the adaptive control triggers a sliding mode component to induce robustness into the tracking. The adaptive gain parameter will continue to increase until it is greater than the magnitude of the disturbance and induce the closed loop system to asymptotically stable. The actuator failures considered in this paper are typical of those for reaction wheels and include a gain fault and a deviation fault [21],[22]. However, one of the weaknesses of the proposed adaptive parameter is that it increases continuously for all values of attitude errors except for when they are exactly zero i.e. only when the sliding surface is zero. This could pose problems practically as there will be noise and errors from the sensors and therefore the sliding surface will never be exactly zero. Thus, we augment the adaptive parameter such that it is bounded from above to ensure that it does not saturate the reaction wheels. The adaptive control os proved to asymptotically stabilize the closed-loop system and track the reference trajectory in the presence of disturbances and specific actuator faults.

The attitude kinematics and dynamics of the spacecraft in the presence of disturbances, actuator limited output torque and actuator failures are formulated including a real control torque model in Section 2. Section 3 then addresses the problem of developing a adaptive sliding mode control and proving the closed-loop system stability. Firstly, adaptive sliding mode control using unbounded adaptive parameter has been designed in 3.1. For the proposed control law, we construct a suitable Lyapunov function to prove the closed-loop system is asymptotically stable. However, the adaptive parameter will continue to increase and eventually lead to a control law that is not feasible given the actuator constraints. In 3.2 then improves a novel adaptive parameter to address this problem based on adaptive update law and uses an appropriate Lyapunov function to prove the closed-loop system is asymptotically stable. In Section 4, simulations are undertaken using the model of a spacecraft which illustrates the robustness of the proposed control method to tracking desired quaternion trajectory and angular velocity with unknown disturbances and actuator failures.

2. Attitude dynamics and kinematics of a rigid-spacecraft

In this section the attitude dynamics and kinematics considered nonlinear saturated characteristic and failures of the actuator are formulated in the appropriate form for the application of the control law design.

The spacecraft is assumed to be a rigid body described by the dynamics and kinematics:

$$J\dot{\boldsymbol{\omega}} = -\boldsymbol{\omega}^{\times} J\boldsymbol{\omega} + \boldsymbol{\tau} + \boldsymbol{d} \tag{1}$$

$$\begin{aligned} \dot{\boldsymbol{q}} &= \tfrac{1}{2}\left(q_4\boldsymbol{\omega} - \boldsymbol{\omega}^{\times}\boldsymbol{q}\right) \\ \dot{q}_4 &= -\tfrac{1}{2}\boldsymbol{\omega}^T\boldsymbol{q} \end{aligned} \tag{2}$$

where, $J \in \mathbb{R}^{3\times 3}$ is the positive definite and symmetric inertia tensor, the angular velocity vector of nano-spacecraft is $\boldsymbol{\omega}=[\omega_1, \omega_2, \omega_3]^T$ with respect to the inertial frame and expressed in the body coordinates, $\boldsymbol{\tau} = [\tau_1, \tau_2, \tau_3]^T$ denotes the real control torque and $\boldsymbol{d}$ is the unknown external disturbance.

Assumption 1. *Considering two types actuator failures of practical spacecraft,* $\boldsymbol{F}_g = \mathrm{diag}(\mathrm{g}_1, \mathrm{g}_2, \mathrm{g}_3)$ *is multiplication fault which denotes efficiency of actuator and* $0 \le g_i \le 1 (i = 1, 2, 3)$*,* $\boldsymbol{F}_d = [F_{d1}, F_{d2}, F_{d3}]^T$ *is plus fault and satisfied* $\|\boldsymbol{F}_d\| \le l_d$*. Since the output of controller is bounded in practice, the actuator nonlinear saturated limit is* $\boldsymbol{u}_{max}$ *and the beyond part denotes* $\bar{\boldsymbol{u}}$*. Hence, the real control torque is*

$$\boldsymbol{\tau} = \boldsymbol{F}_g(\boldsymbol{u} + \bar{\boldsymbol{u}}) + \boldsymbol{F}_d \tag{3}$$

where $\bar{\boldsymbol{u}} = [\bar{u}_1, \bar{u}_2, \bar{u}_3]^T$ *is bounded* $\|\bar{\boldsymbol{u}}\| \leq l_\theta$ *and satisfies*

$$\bar{u}_i = \begin{cases} \operatorname{sgn}(u_i)\, u_{max} - u_i, & |u_i| \geq u_{max} \\ 0, & otherwise \end{cases}$$

The unit quaternion is $\bar{\boldsymbol{q}} = [q_1, q_2, q_3, q_4]^T$, which can be expressed equivalently as $\bar{\boldsymbol{q}} = \left[\boldsymbol{q}^T, q_4\right]^T$ with $\boldsymbol{q} = [q_1, q_2, q_3]^T$ and such that $\boldsymbol{q}^T\boldsymbol{q} + q_4^2 = 1$, the $^\times$ denotes an operator, such that

$$\boldsymbol{\omega}^\times = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}$$

The attitude tracking control problem is to track a desired quaternion $\bar{\boldsymbol{q}}_d$, where $\bar{\boldsymbol{q}}_d = \left[\boldsymbol{q}_d^T, q_{d4}\right]^T$ with $\boldsymbol{q}_d = [q_{d1}, q_{d2}, q_{d3}]^T$ satisfying $\boldsymbol{q}_d^T\boldsymbol{q}_d + q_{d4}^2 = 1$ denotes the desired attitude quaternion and $\boldsymbol{\omega}_d$ is the target angular velocity.

Considering the error quaternion $\bar{\boldsymbol{q}}_e = \left[\boldsymbol{q}_e^T, q_{4e}\right]^T$ where $\boldsymbol{q}_e = [q_{1e}, q_{2e}, q_{3e}]^T$ and the error angular velocity $\boldsymbol{\omega}_e$ are defined as

$$\begin{aligned} &\boldsymbol{q}_e = q_{d4}\boldsymbol{q} - \boldsymbol{q}_d^\times\boldsymbol{q} - q_4\boldsymbol{q}_d \\ &q_{e4} = \boldsymbol{q}_d^T\boldsymbol{q} + q_4 q_{d4} \\ &\boldsymbol{\omega}_e = \boldsymbol{\omega} - \boldsymbol{C}\boldsymbol{\omega}_d \end{aligned} \tag{4}$$

where $\boldsymbol{C} = \left(q_{e4}^2 - \boldsymbol{q}_e^T\boldsymbol{q}_e\right) I_3 + 2\boldsymbol{q}_e\boldsymbol{q}_e^T - 2q_{e4}\boldsymbol{q}_e^\times$ is the direction cosine matrix from inertial frame A to body coordinate B, where I_3 an identity matrix.

The kinematics can be expressed as error quaternion form [23]

$$\begin{aligned} &\dot{\boldsymbol{q}}_e = \tfrac{1}{2}\left(q_{e4}\boldsymbol{\omega}_e - \boldsymbol{\omega}_e^\times\boldsymbol{q}_e\right) \\ &\dot{q}_{e4} = -\tfrac{1}{2}\boldsymbol{\omega}_e^T\boldsymbol{q}_e \end{aligned} \tag{5}$$

where $\bar{\boldsymbol{q}}_e$ satisfies $\boldsymbol{q}_e^T\boldsymbol{q}_e + q_{e4}^2 = 1$.

Substitution the $\boldsymbol{\omega}_e = \boldsymbol{\omega} - \boldsymbol{C}\boldsymbol{\omega}_d$ into the Eq.(1), considered $\dot{\boldsymbol{C}} = -\boldsymbol{\omega}_e^\times\boldsymbol{C}$ [24], the error dynamic equations is

$$J\dot{\boldsymbol{\omega}}_e = -(\boldsymbol{\omega}_e + \boldsymbol{C}\boldsymbol{\omega}_d)^\times J\left(\boldsymbol{\omega}_e + \boldsymbol{C}\boldsymbol{\omega}_d\right) + J\boldsymbol{\omega}_e^\times\boldsymbol{C}\boldsymbol{\omega}_d - J\boldsymbol{C}\dot{\boldsymbol{\omega}}_d + \boldsymbol{\tau} + \boldsymbol{d} \tag{6}$$

3. Adaptive Sliding Mode Control Law and Stability Proof

In this section, we propose a an adaptive control law that consists of a (conventional) proportional component and an adaptive sliding mode component. The robust sliding mode control law is designed and proved to track a feasible attitude with asymptotic convergence. The sliding manifold will be used as

$$\boldsymbol{S} = \boldsymbol{\omega}_e + \boldsymbol{\kappa}\boldsymbol{q}_e \tag{7}$$

where $\boldsymbol{\kappa} = diag(\kappa_1, \kappa_2, \kappa_3)$, $\kappa_i (i = 1, 2, 3)$ are positive scalars.

Then the Eq.(6) has became

$$\begin{aligned} J\dot{\boldsymbol{S}} &= -(\boldsymbol{\omega}_e + \boldsymbol{C}\boldsymbol{\omega}_d)^{\times} J(\boldsymbol{\omega}_e + \boldsymbol{C}\boldsymbol{\omega}_d) + J\boldsymbol{\omega}_e^{\times}\boldsymbol{C}\boldsymbol{\omega}_d - J\boldsymbol{C}\dot{\boldsymbol{\omega}}_d \\ &+ \boldsymbol{\tau} + \boldsymbol{d} + \frac{1}{2}J\kappa\big(q_{e4}\boldsymbol{\omega}_e - \boldsymbol{\omega}_e^{\times}\boldsymbol{q}_e\big) \\ &= \boldsymbol{G} + \boldsymbol{u} + \boldsymbol{E}_d \end{aligned} \tag{8}$$

where $\boldsymbol{E}_d = (\boldsymbol{F}_g - I_3)\boldsymbol{u} + \boldsymbol{F}_g\bar{\boldsymbol{u}} + \boldsymbol{F}_d + \boldsymbol{d}$, $\boldsymbol{G} = -(\boldsymbol{\omega}_e + \boldsymbol{C}\boldsymbol{\omega}_d)^{\times} J(\boldsymbol{\omega}_e + \boldsymbol{C}\boldsymbol{\omega}_d) + J\boldsymbol{\omega}_e^{\times}\boldsymbol{C}\boldsymbol{\omega}_d - J\boldsymbol{C}\dot{\boldsymbol{\omega}}_d + \frac{1}{2}J\kappa(q_{e4}\boldsymbol{\omega}_e - \boldsymbol{\omega}_e^{\times}\boldsymbol{q}_e)$. It is assumed that $\boldsymbol{E}_d$ is bounded.

3.1. Robust adaptive control with unbounded adaptive parameter

We present the design of a sliding mode controller with unbounded adaptive parameter for the spacecraft with external disturbance and actuator failures.

Theorem 1. *Considering the spacecraft system model by Eq.(5) and Eq.(8), for any initial $\boldsymbol{S}(0)$, the adaptive sliding mode feedback control law:*

$$\boldsymbol{u} = -\boldsymbol{G} - \sigma\boldsymbol{S} - \Xi(S)\boldsymbol{\rho} \tag{9}$$

with the function $\Xi(S)$ is given by

$$\Xi(S) = diag[\mathrm{sgn}(\mathrm{S}_1), \mathrm{sgn}(\mathrm{S}_2), \mathrm{sgn}(\mathrm{S}_3)] \tag{10}$$

and adaptive parameter $\boldsymbol{\rho} = [\rho_1, \rho_2, \rho_3]^T$ satisfies

$$\dot{\boldsymbol{\rho}} = \boldsymbol{\lambda}|\boldsymbol{S}| \tag{11}$$

with $\boldsymbol{\rho}(0) = \boldsymbol{0}$ and $\boldsymbol{\lambda} = diag(\lambda_1, \lambda_2, \lambda_3)$, $|\boldsymbol{S}| = [|S_1|, |S_2|, |S_3|]^T$. $\lambda_i \geq 0$ are scalar constants. with the parameter $\sigma > 0$. Given the control law (9) the sliding surface (7) will converge to the zero vector as $t \to \infty$, such as $\lim_{t\to\infty} \boldsymbol{S} = 0$.

Proof: Defining the Lyapunov function as:

$$V = \frac{1}{2}\boldsymbol{S}^T J \boldsymbol{S} \tag{12}$$

The time derivative of the Lyapunov function is

$$\dot{V} = \boldsymbol{S}^T J \dot{\boldsymbol{S}} \tag{13}$$

Recalling Eq.(8), the time derivative of this Lyapunov function becomes

$$\dot{V} = \boldsymbol{S}^T \left(\boldsymbol{G} + \boldsymbol{u} + \boldsymbol{E}_d\right) \tag{14}$$

Substituting the control law Eq.(9) into Eq.(14) gives

$$\begin{aligned}
\dot{V} &= \boldsymbol{S}^T \left[-\sigma\boldsymbol{S} - \Xi(S)\boldsymbol{\rho} + \boldsymbol{E}_d\right] \\
&= -\sigma\boldsymbol{S}^T\boldsymbol{S} - \sum_{i=1}^{3}|S_i|\rho_i + \boldsymbol{S}^T\boldsymbol{E}_d \\
&\leq -\sigma\sum_{i=1}^{3}S_i^2 - \sum_{i=1}^{3}|S_i|\rho_i + \sum_{i=1}^{3}|S_i||E_{di}| \\
&= -\sigma\sum_{i=1}^{3}S_i^2 + \sum_{i=1}^{3}(|E_{di}| - \rho_i)|S_i|
\end{aligned} \tag{15}$$

then from (15) it follows that there is a finite time t_1 when all $\rho_i > |E_{di}|$ and therefore $\dot{V} < 0$ when $t \in [t_1, \infty)$. Therefore, $|S| \to 0$ as $t \to \infty$. □

Note that if ρ_i is set to zero for all time then (9) reduces to a proportional tracking controller. This adaptive sliding mode control is effective in practise as it can provide convergence in the presences of faults without knowledge of the upper bound on unknown disturbances. However, care must be taken in implementation as the adaptive gain is always increasing unless $|S| = 0$ which in practise is never the case due to sensor noise. This means that the gain on the sliding component will continue to increase beyond the required value and can lead to aggressive chattering. However, the gain parameter in the adaptive control law will continue to increase and eventually lead to a control law that is not feasible given the actuator constraints. In the following section the adaptive parameter update law is designed to avoid saturation of the actuators. Thus, the following adaptive parameter is proposed to bound the adaptive parameter.

3.2. Robust adaptive control with bounded adaptive parameter

$$\dot{\boldsymbol{\rho}} = \boldsymbol{\lambda}\,|\boldsymbol{S}| - [\hat{S}]\boldsymbol{\rho} \tag{16}$$

where $[\hat{S}] = diag[|S_1|\,,|S_2|\,,|S_3|]$. This adaptive parameter rate of increase is proportional to $|\boldsymbol{S}|$. Furthermore, there are two equilibrium points in the adaptive gain $|\boldsymbol{S}| = 0$ which corresponds to the desired state and $\boldsymbol{\lambda} = \boldsymbol{\rho}$. It is clear to see that if $\boldsymbol{\lambda} > \boldsymbol{\rho}$ then $\dot{\boldsymbol{\rho}} > 0$ and if $\boldsymbol{\lambda} < \boldsymbol{\rho}$ then $\dot{\boldsymbol{\rho}} < 0$ so $\boldsymbol{\rho} \to \boldsymbol{\lambda}$. In this case the proof of asymptotic stability of the closed-loop system is given by the following Lyapunov function as:

$$V = \frac{1}{2}\boldsymbol{S}^T J \boldsymbol{S} + \frac{1}{2}\boldsymbol{\rho}^T \boldsymbol{\lambda}^{-1}\boldsymbol{\rho} \tag{17}$$

The time derivative of the Lyapunov function is

$$\dot{V} = \boldsymbol{S}^T J \dot{\boldsymbol{S}} + \boldsymbol{\rho}^T \boldsymbol{\lambda}^{-1}\dot{\boldsymbol{\rho}} \tag{18}$$

Recalling Eq.(8) and Eq.(16), the time derivative of this Lyapunov function becomes

$$\dot{V} = \boldsymbol{S}^T\left(\boldsymbol{G} + \boldsymbol{u} + \boldsymbol{E}_d\right) + \boldsymbol{\rho}^T\boldsymbol{\lambda}^{-1}(\boldsymbol{\lambda}\,|\boldsymbol{S}| - [\hat{S}]\boldsymbol{\rho}) \tag{19}$$

Substituting the control law Eq.(9), Eq.(10) and Eq.(16) into Eq.(19) gives

$$\dot{V} = \boldsymbol{S}^T\left[-\sigma\boldsymbol{S} - \Xi(S)\boldsymbol{\rho} + \boldsymbol{E}_d\right] + \boldsymbol{\rho}^T|\boldsymbol{S}| - \boldsymbol{\rho}^T\boldsymbol{\lambda}^{-1}[\hat{S}]\boldsymbol{\rho} \tag{20}$$

$$\dot{V} = -\sigma\boldsymbol{S}^T\boldsymbol{S} + \boldsymbol{S}^T\boldsymbol{E}_d - \boldsymbol{\rho}^T\boldsymbol{\lambda}^{-1}[\hat{S}]\boldsymbol{\rho} \tag{21}$$

then

$$\dot{V} \le -\sigma\sum_{i=1}^{3} S_i^2 + \sum_{i=1}^{3}|S_i||E_{di}| - \sum_{i=1}^{3}\frac{|S_i|}{\lambda_i}\rho_i^2 \tag{22}$$

in this case $\dot{V} < 0$ if $|E_{di}| < \frac{\rho_i^2}{\lambda_i}$. Note that as $t \to \infty$ $\rho_i \to \lambda_i$ so the condition for asymptotic stability is $|E_{di}| < \lambda_i$.

3.3. Practical considerations

There are several practical aspects to implementing this control in practise. The first is related to a well-known problem of sliding mode control related to chattering. To avoid this problem we replace the sign function in (10) with $\mathrm{Sn}(S_i)(i = 1, 2, 3)$ defined by

$$\mathrm{Sn}(S_i) = \begin{cases} S_i/\varepsilon, & |S_i| \le \varepsilon \\ \mathrm{sgn}(S_i), & otherwise \end{cases} \tag{23}$$

where ε is a small constant scalar. In addition the adaptive parameter must be adjusted such that when $|S_i| \leq \varepsilon$ then $\boldsymbol{\rho}$ must also be held constant such that:

$$\dot{\boldsymbol{\rho}} = \begin{cases} \mathbf{0}, & |S_i| \leq \varepsilon \\ \boldsymbol{\lambda}\,|\boldsymbol{S}| - [\hat{S}]\boldsymbol{\rho}, & otherwise \end{cases} \tag{24}$$

The second practicality of implementing the control is to consider the tuning. The initial stage of the tuning process for a known reference attitude is to consider the perfect case with no constraints, disturbances or actuator failures. In this case $\sigma, \boldsymbol{\kappa}$ are tuned experimentally to achieve a good tracking performance. The maximum torque required to track the reference in an ideal case is computed as $|\boldsymbol{u}_{ideal}|$. Then given the maximimum torque that can be provided by the actuator $|\boldsymbol{u}_{max}|$ we can define

$$|\boldsymbol{u}_{max}| \geq |\boldsymbol{u}_{ideal}| + |\boldsymbol{\rho}| \tag{25}$$

From the adaptive law (16) if we select $\boldsymbol{\rho}(0) = 0$ then $\boldsymbol{\rho}$ will increase until it reaches $\boldsymbol{\lambda}$ therefore we can choose an upper limit for $\rho_i = \lambda_i$ such that

$$\lambda_i = |u_{i,max}| - |u_{i,ideal}| \tag{26}$$

Using this strategy for selecting λ_i gives the stability condition

$$|u_{i,max}| - |u_{i,ideal}| > E_{di} \tag{27}$$

4. Numerical Example

In this section, the proposed control law (9) with (16) will be tested effectively compared with a sliding mode control such as

$$\boldsymbol{\tau} = -\boldsymbol{G} - \sigma\boldsymbol{S} \tag{28}$$

The inertial matrix of a representative rigid spacecraft in simulation has been given as

$$J = \begin{bmatrix} 20 & 1.2 & 0.9 \\ 1.2 & 17 & 1.4 \\ 0.9 & 1.4 & 15 \end{bmatrix} \quad kg \cdot m^2 \tag{29}$$

The multiplication fault model is given

$$g_i = \begin{cases} 1, & t < 80s \\ 0.6 + 0.1\sin(0.5t + \pi/3), & t \geq 80s \end{cases} \tag{30}$$

the plus fault model is

$$F_{di} = \begin{cases} 0, & t < 80s \\ 0.01 + 0.005\sin(0.5\pi t), & t \geq 80s \end{cases} \tag{31}$$

and the external disturbance torque $\boldsymbol{d}$ is assumed:

$$\boldsymbol{d} = \begin{bmatrix} \sin(0.1t) \\ 2\sin(0.2t) \\ 3\sin(0.3t) \end{bmatrix} \times 10^{-3} N \cdot m \tag{32}$$

In order to test the robust tracking effectiveness of the proposed control law (9), it will be compared with the feedback control Eq.(28) in presence of multiplication fault $\boldsymbol{F}_g$, plus fault $\boldsymbol{F}_d$ and external disturbance $\boldsymbol{d}$. The reference quaternion trajectory will be generated by the use of Eq.(28) and spacecraft model Eq.(1) without $\boldsymbol{d}$ and Eq.(2).

Let the desired quaternion be as

$$\bar{\boldsymbol{q}}(t_f) = \begin{cases} [0.5, 0.5, 0.5, 0.5]^T, & t < 150s \\ [0.1, 0.7, 0.1, 0.7]^T, & t \geq 150s \end{cases} \tag{33}$$

The initial angular velocity $\boldsymbol{\omega}(0) = [0.01, 0.01, 0.01]^T\, rad/s$, initial quaternion $\bar{\boldsymbol{q}}\,(0) = [0, 0, 0, 1]^T$. The gain parameters of adaptive control are $\lambda_i = 0.1$, $\hat{\rho}_i\,(0) = 0$, $\kappa_i = 0.5$, $\sigma = 10$, $\varepsilon = 0.1$ and maximum of control torque $u_{max} \leq 0.1 N \cdot m$.

The quaternion $\bar{\boldsymbol{q}}$ tracking with the proposed control law (9) with (16) are respectively shown in Fig.1 and Fig.3 and the corresponding error $\boldsymbol{q}_e$ denotes in Fig.5. The comparison results without or with actuator failures using the control law by Eq.(28) indicate in Fig.2, Fig.4 and Fig.6. Since the reference attitude designed by the control law by Eq.(28), there is no error in the first 10 seconds under a perfect actuator. It is clear shown in Fig.2, Fig.4 and Fig.6. In contrast of the quaternion feedback control, the proposed control has a small tracking error because of needing a dynamic response process to track the reference trajectory. However, the tracking performance of the simply quaternion feedback control is much worse than the proposed control with the actuator failures happened. It is obviously illustrated by comparing Fig.5 and Fig.6. The proposed control strategy has such high tracking precision because the adaptive parameter can accurately estimate the total uncertainties including actuator failures, external disturbance and inertia uncertainty.

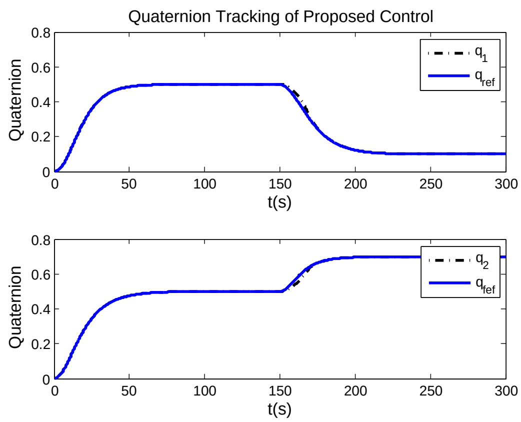


Figure 1: Quaternion q_1 and q_2 of proposed control

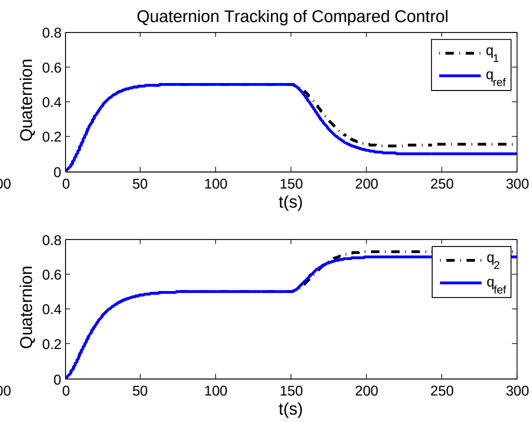
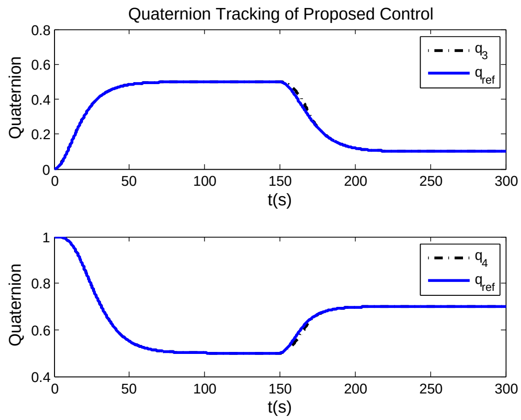


Figure 2: Quaternion q_1 and q_2 of compared control

It has shown that the tracking error of proposed control is much smaller than conventional control with actuator fault and external disturbance, very closed to zero. It can be seen that the quaternion tracking error of the proposed control has significantly stable to reject uncertainty. This is extremely significant for high tracking accuracy attitude control of nano-spacecraft. The Fig.7 and Fig.9 illustrate the angular velocity and tracking error of the proposed control in contrast of feedback control shown in Fig.8 and Fig.10. We can know that the angular velocity of spacecraft can great track the desired angular velocity with extremely small error. The corresponding control torque of two control methods are shown in Fig.11 and Fig.12. In Fig.14 the initial value of $\hat{\rho}_{ei} = 0$, then when there is a fault the sliding mode is triggered. The stable values have changed old ones of no failures to new values after happening fault. In this example, it can be seen from the figures that the proposed controller is effective at dealing with the uncertainties(actuator fault and external disturbance) and shows a significant improvement in tracking capability in the presence of uncertainties.

Next, the simulations at more operating conditions should be performed and discussed to verify the performance of the proposed control strategy under an another external disturbance and inertia uncertainty. Therefore, a desired attitude trajectory is designed by Eq.(34) and Eq.(2). The comparison simulation results of tracking the reference attitude quaternion trajectory with the external disturbance and inertia uncertainty respectively calculated



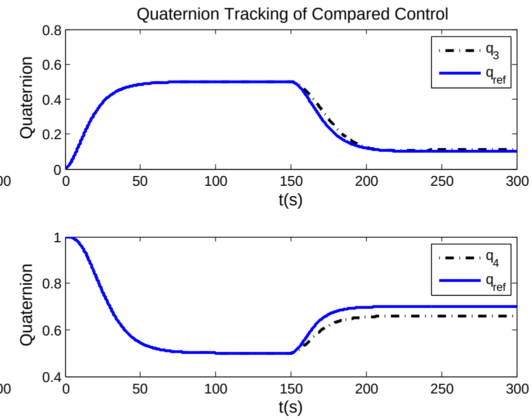
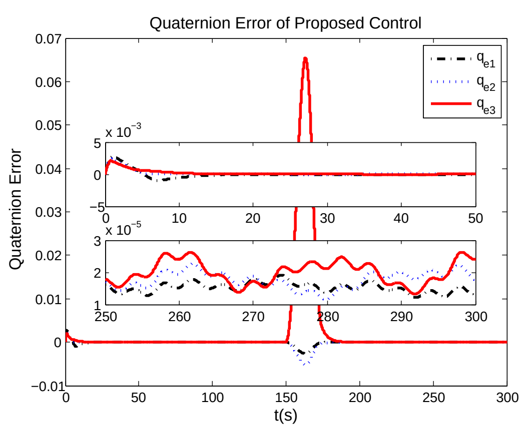


Figure 3: Quaternion q_3 and q_4 of proposed control

Figure 4: Quaternion q_3 and q_4 of compared control



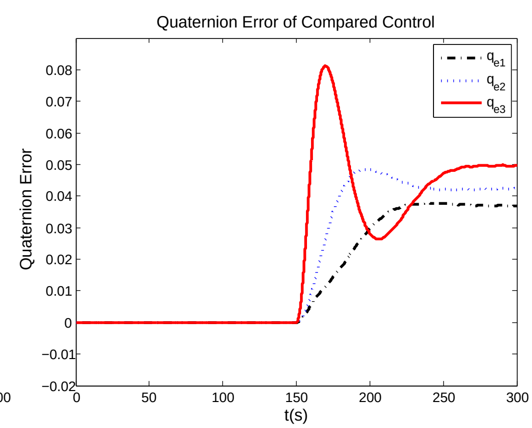
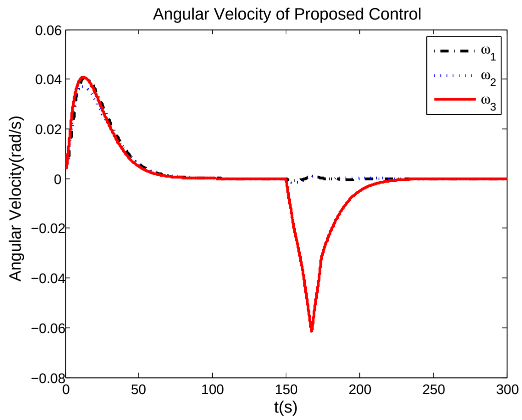


Figure 5: Quaternion error of proposed control

Figure 6: Quaternion error of compared control



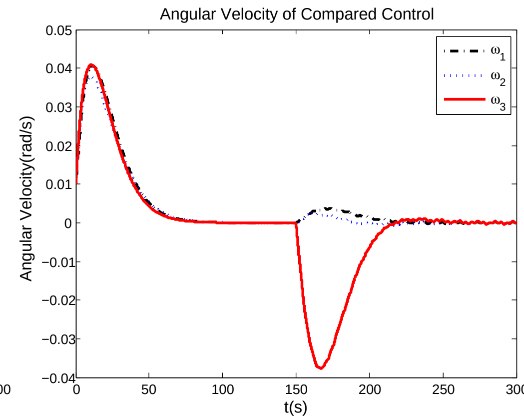
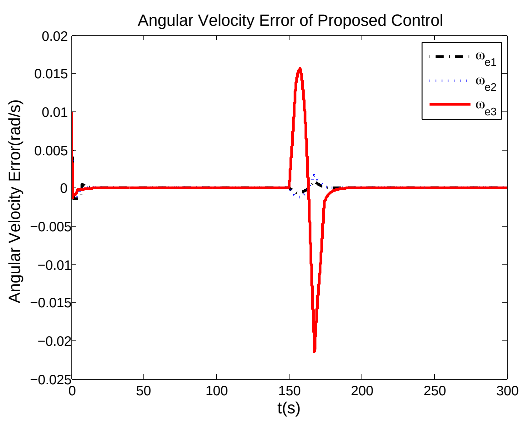


Figure 7: Angular velocity of proposed control

Figure 8: Angular velocity of compared control



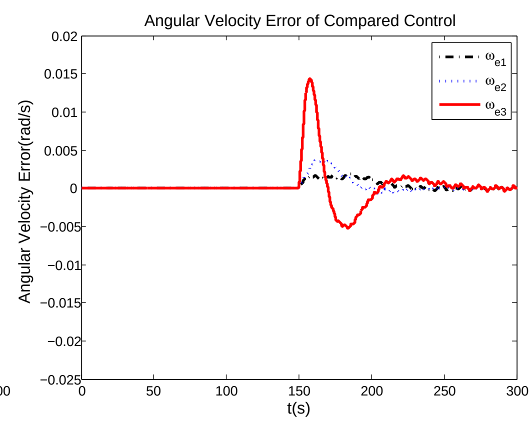


Figure 9: Angular velocity error of proposed control

Figure 10: Angular velocity error of compared control

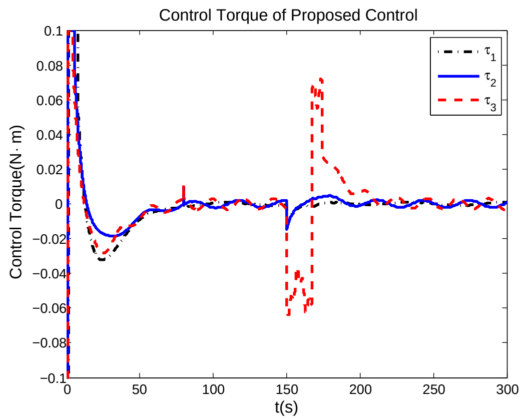


Figure 11: Control torque of proposed control

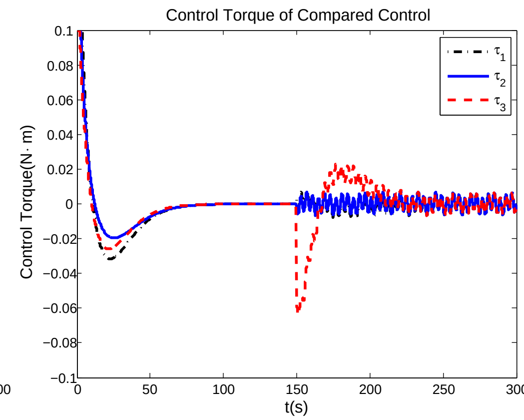


Figure 12: Control torque of compared control

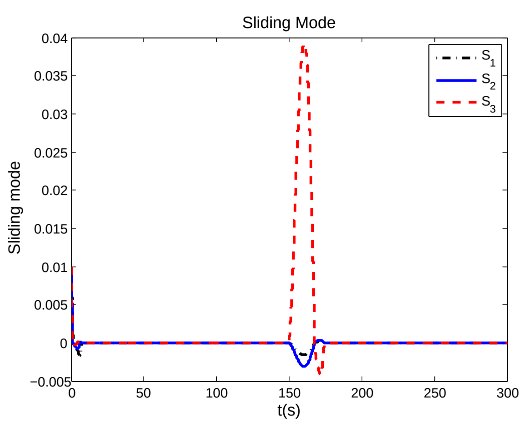


Figure 13: Sliding surface of proposed control

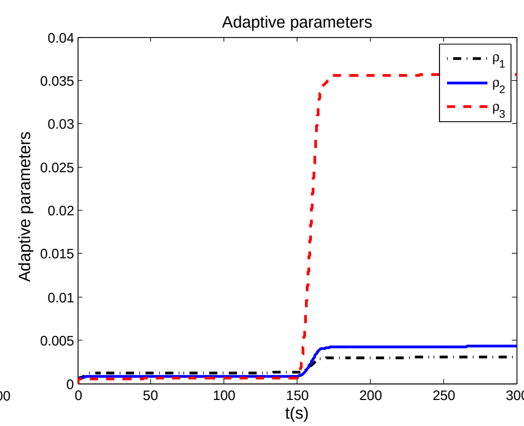


Figure 14: Adaptive parameters of proposed control

by Eq.(35) and Eq.(36) are shown in Fig.15 to Fig.26.

$$\boldsymbol{\omega} = \begin{bmatrix} 0.03\sin(2\pi t/200) \\ 0.02\sin(2\pi t/200) \\ 0.01\sin(2\pi t/200) \end{bmatrix} rad/s \tag{34}$$

the external disturbance torque $\boldsymbol{d}$ is assumed:

$$\boldsymbol{d} = \begin{bmatrix} 3 + 2\sin(0.1t) \\ 1 + 4\sin(0.2t) \\ 2 + 3\sin(0.3t) \end{bmatrix} \times 10^{-3} N \cdot m \tag{35}$$

and the inertia uncertainty is as follow

$$\Delta J = 0.1J \tag{36}$$

The quaternion $\bar{\boldsymbol{q}}$ tracking with the proposed control law (9) with (16) are respectively shown in Fig.15 and Fig.17 and the corresponding error $\boldsymbol{q}_e$ denotes in Fig.19. The comparison results without or with actuator failures using the control law by Eq.(28) indicate in Fig.16, Fig.18 and Fig.20. Since the reference attitude designed by the control law by Eq.(28), there is no error in the first 10 seconds under a perfect actuator. It is clear shown in Fig.16, Fig.18 and Fig.20. In contrast of the quaternion feedback control, the proposed control has a small tracking error because of needing a dynamic response process to track the reference trajectory. However, the tracking performance of the simply quaternion feedback control is much worse than the proposed control with the actuator failures happened. It is obviously illustrated by comparing Fig.19 and Fig.20. The proposed control strategy has such high tracking precision because the adaptive parameter can accurately estimate the total uncertainties including actuator failures, external disturbance and inertia uncertainty.

It has shown that the tracking error of proposed control is much smaller than conventional control with actuator fault and external disturbance, very closed to zero. It can be seen that the quaternion tracking error of the proposed control has significantly stable to reject uncertainty. This is extremely significant for high tracking accuracy attitude control of nano-spacecraft. The Fig.21 and Fig.23 illustrate the angular velocity and tracking error of the proposed control in contrast of feedback control shown in Fig.22 and Fig.24. We can know that the angular velocity of spacecraft can great track

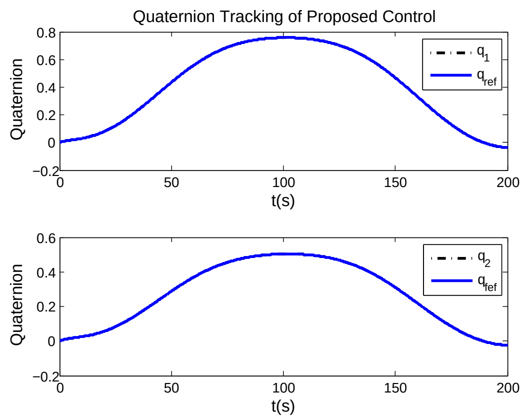


Figure 15: Quaternion q_1 and q_2 of proposed control

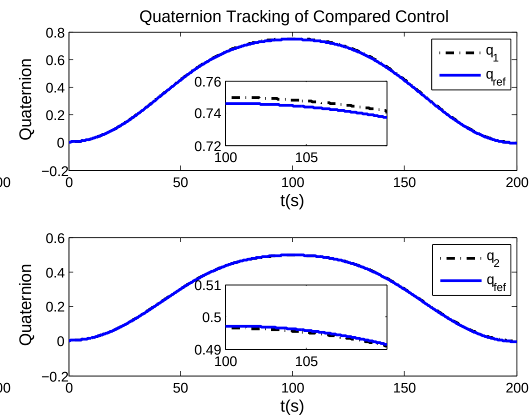


Figure 16: Quaternion q_1 and q_2 of compared control

the desired angular velocity with extremely small error. The corresponding control torque of two control methods are shown in Fig.25 and Fig.26. In this example, it can be seen from the figures that the proposed controller is effective at dealing with the uncertainties(actuator fault and external disturbance) and shows a significant improvement in tracking capability in the presence of uncertainties.

5. Conclusion

The control law proposed in this note is not only robust to unknown disturbance, but also adapts to limited output torque and actuator failures which consist of a gain fault and a deviation fault. It is proved that the closed-loop system is asymptotically stable by Lyapunov's direct method. Contrary to traditional quaternion feedback control which have fixed gains, the adaptive parameter can be automatically tuned as functions of the quaternion and angular velocity of spacecraft. Furthermore, a significant advantage of this control is that it is tolerant to faults and unexpected disturbances.

6. Acknowledgments

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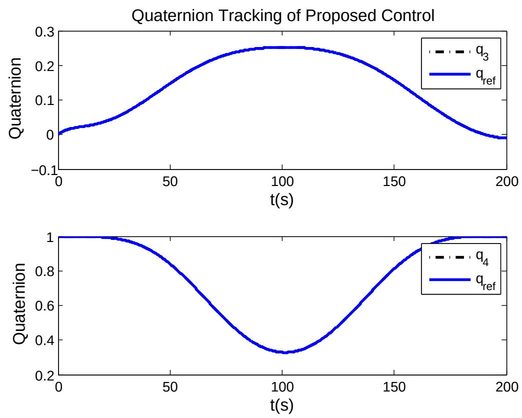

Figure 17: Quaternion q_3 and q_4 of proposed control

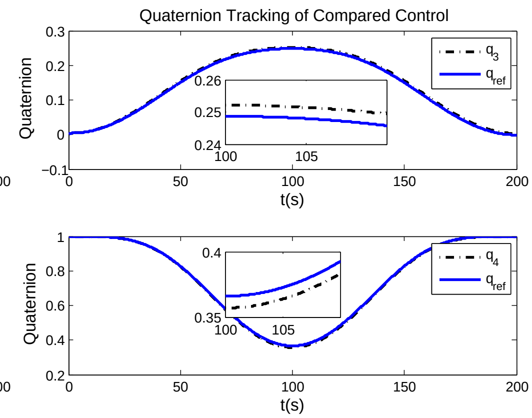

Figure 18: Quaternion q_3 and q_4 of compared control

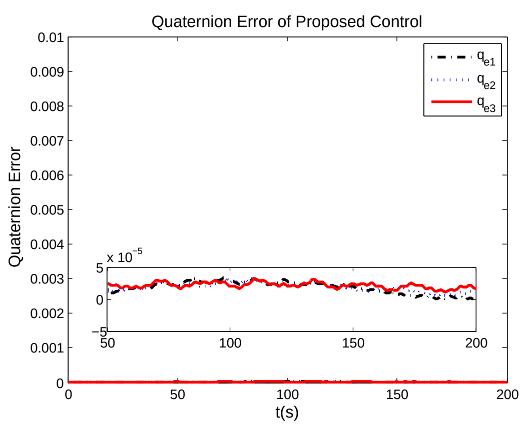

Figure 19: Quaternion error of proposed control

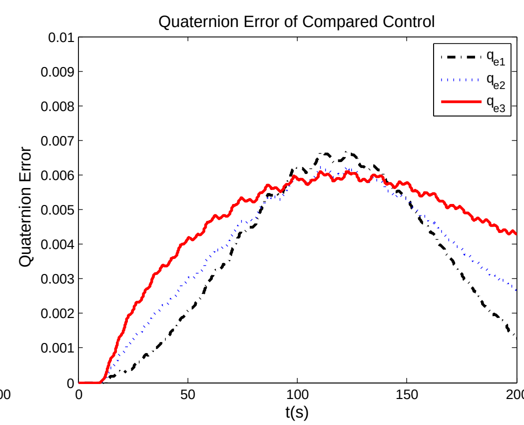

Figure 20: Quaternion error of compared control

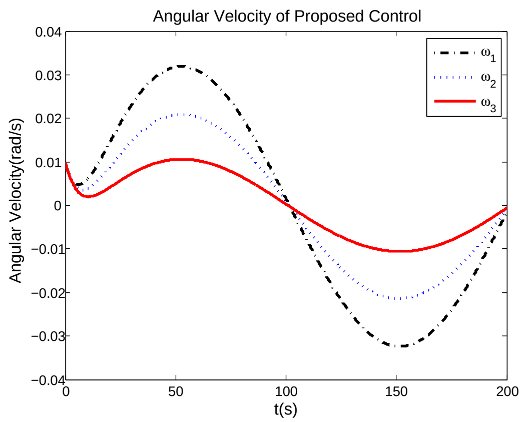

Figure 21: Angular velocity of proposed control

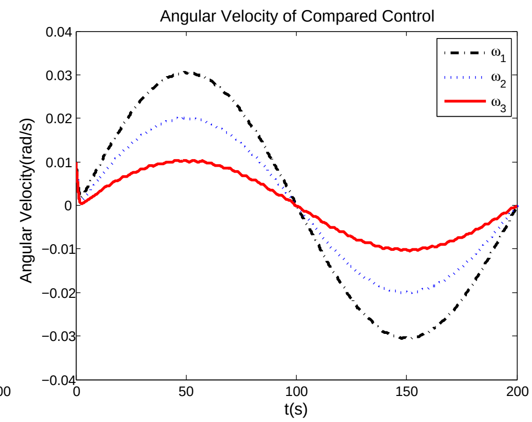

Figure 22: Angular velocity of compared control

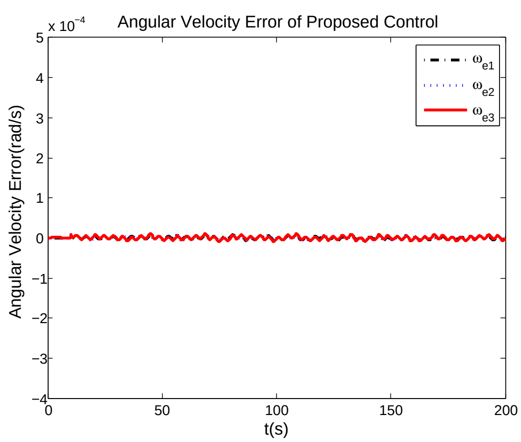

Figure 23: Angular velocity error of proposed control

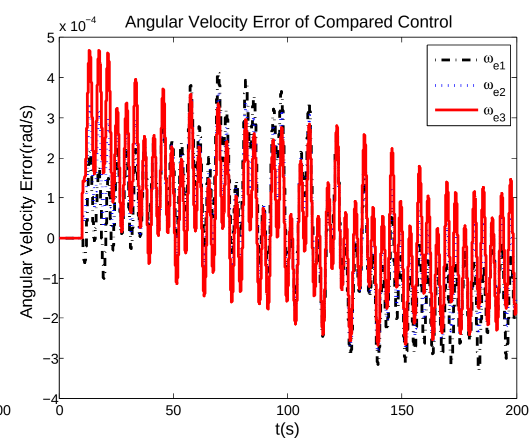

Figure 24: Angular velocity error of compared control

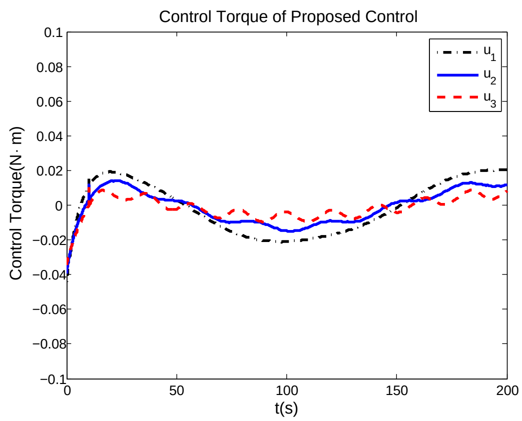

Figure 25: Control torque of proposed control

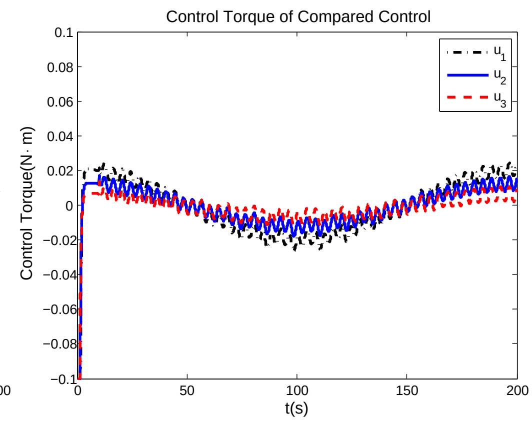

Figure 26: Control torque of compared control

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