A global approach for scaling the mode shapes estimated by operational modal analysis

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ABSTRACT
This paper describes a new method for estimating the modal mass associated to mode shapes found through operational modal analysis. The knowledge of the modal mass is a key information for estimating the frequency response function of the structure/system under analysis. The newly proposed method is based on the use of mono-harmonic excitation tests at frequency values at (or close to) resonances. The method allows for simpler and cheaper experimental set-ups, compared to the other methods already available in the literature. Furthermore, it does not require any assumption on the structure/system considered. The paper presents the theory behind the method and then validates it by means of experimental tests.

1. INTRODUCTION

The Operational Modal Analysis (OMA) is a well-known technique for estimating the modal properties of systems and structures without the need of measuring the forces exciting the structure/system (thus, often exploiting the natural environmental excitation). This is a key tool in all those cases where measuring/providing excitations is complicated, such as the case of huge civil structures. OMA provides estimates of the eigenfrequency and non-dimensional damping ratio values. Mode shapes can be also estimated, but they are non-scaled, which in turn means that they are associated to an unknown modal mass. This implies that it is not possible to reconstruct the dynamic response of the considered system/structure to a given excitation [1].

There are different methods in the literature to evaluate the scaling of the mode shapes estimated by means of OMA. They can rely on different approaches; among them, it is possible to use additional masses to be placed on the structure (e.g. [2–5]), to employ measured broad-band exogenous forces (e.g. [6]), to couple the system under analysis with known dynamic systems [7] and to use accurate finite element models [8]. These approaches have some drawbacks: they are based on rather elaborate experimental procedures/set-ups, or they rely on the accuracy of a computational model, which latter must be based on assumptions, making any uncertainty larger, as the measurement uncertainties combine with the model approximation. The aim of this paper is to present a new approach for estimating modal masses, and thus scaling the OMA mode shapes, employing a readily applicable and fast experimental procedure based on the employment of relatively inexpensive and general-purpose actuators as well as simple signal processing, thus without the need of any structure modifications and/or models. The next two sections describe the basic procedure to be followed for applying the proposed method. Then, Section 4 explains in detail the advantages of this technique and Section 5 shows an experimental campaign allowing for the validation of the method.

2. THE NEW METHOD FOR SCALING THE OMA MODES

The new method here proposed relies on the use of small actuators which are employed to excite the structure with mono-harmonic force profiles at frequency values equal to (or close to) the eigenfrequency values of the modes that have to be scaled. The whole procedure for applying the newly proposed method can be summarized as follows:

1) Carry out an OMA. With the data coming from the OMA, a modal extraction can be performed. This allows the estimation of the poles s_r (where r indicated the order of the pole) and the associated non-scaled eigenvector ψ_r. The poles assume the following expression:

$$s_r = - \omega_r \xi_r + j \omega_r (1- \xi_r^2)^{1/2} \quad (1)$$

Where ω_r is the r-th eigenfrequency and ξ_r is the associated non-dimensional damping ratio. The symbol j represents the imaginary unit.

2) Perform the tests with the mono-modal force profiles at frequency values equal to (or close to) the eigenfrequency values of the modes which have to be scaled. In these tests, it is essential that the points where the structural response is measured and where the force is applied are points where the ψ_r were computed with the OMA.

3) The frequency response functions (FRFs) at the selected frequency values can be then estimated by using simple digital signal processing tools (see Section 3).

4) The knowledge of these FRF samples is the base of the whole method presented herein. The theoretical expression of these FRF values, in case of proportional damping, is [9]:

$$H_{p,q} (j\omega_{ex}) = \Sigma[\psi_r^p \psi_r^q /(m_r (j\omega_{ex} - s_r) (j\omega_{ex} - s_r^*))] + D_{pq} + (C_{pq} / \omega_{ex}^2) \quad (2)$$

The summation must be carried out from $r=h$ and $r=h+g-1$, where h is the index of the first mode to be scaled, and g is the number of modes to be scaled. ω_{ex} is the frequency at which the excitation is provided, p and q are the points where the response of the structure is collected and input force is applied, respectively. Furthermore, D_{pq} and C_{pq} are constants which allow to take into account the influence of out-of-band modes (D_{pq} for higher modes and C_{pq} for lower modes); m_r is the r-th unknown modal mass and * represents complex conjugation.
The knowledge of many FRF samples allows to build a system of equations where the unknowns are the modal masses and residual terms accounting for the influence of out-of-band modes. The other parameters in Eq. (2) are set by the user (i.e. the excitation frequency ω_{ex} that must be set equal to, or close to, the eigenfrequency values), or estimated by OMA tests (i.e. ψ_r and s_r), or known from the harmonic tests (i.e. $H_{p,q}$ (jω_{ex})). With few of the mentioned mono-harmonic tests, it is possible to write a system of equations to be solved, characterized by many equations. Thus, it is easy to make the system overdetermined. Therefore, the estimation of the unknowns requires the use of a least square approach or of the pseudo-inverse matrix. The use of many equations leads to an increased accuracy of the results (modal mass values) because it allows to filter out all the random effects affecting the estimated non-scaled mode components ψ_r coming from OMA.
As an example, if there are five modal masses to be estimated (g=5) and two forcing points must be used because there are no points where all the five modes under analysis show a significant eigenmode component, one has to build the mentioned system of equations by choosing, at first, the number of points where the structural response must/can be measured during the harmonic tests. The higher the number of these points is, the more effective the filtering of the random effects affecting the estimated values of ψ_r is. On the other hand, this makes the experimental set-up more complicated and expensive. Let us suppose to set this quantity equal to six. Once the number of response points has been set, the number of excitation frequencies ω_{ex} must be set. This depends on the number of unknowns that must be estimated. In this example, then unknowns are 29 (5 modal masses and the terms D_{pq} and C_{pq} for 12 FRFs). Therefore, more than (or at least) 29 equations must be written. Supposing to use different ω_{ex} values in the two different excitation points, at least 5 ω_{ex} values must be employed (5 different ω_{ex} values allow to write 30 equations because, at each ω_{ex} value, six FRFs can be written).

Before discussing the advantages of the newly proposed method, it is worth highlighting two points. First, the mathematical approach, that implies a least square minimization or the use of the pseudo-inverse, must guarantee the avoidance of ill-conditioned solutions. This aspect has already been treated in [10], where it is shown that the mathematical solution is guaranteed, provided to have, for each mode considered, at least one FRF sample characterized by a high eigenmode component in both the excitation and response points, which is readily accomplished. The second point to be faced regards the possible random and bias effects in the results of the proposed method, that could be introduced by the method itself. To investigate this point, different simulations were carried out considering a practical case which is of high complexity: a system with axis-symmetry where twin (repeated) modes appear, nominally at the same frequency, but with mode shapes rotated with respect to each other, so that the maximum of one of the twin modes lies on the nodal line of the other mode. As an example, if we consider a disc, twin modes like those in Figure 1 will be encountered. In theory, the two modes at a particular frequency for the circular disc are undetermined in space, i.e. any linear combination of the two orthogonal modes are modes of the plate. This will mean that applying only one shaker will excite one of the modes, whereas the other mode will not be excited at all. In cases where a non-perfect axis-symmetry exists, which is a common case in practical applications, the two modes will be fixed in space and their eigenfrequencies will be slightly different. This case is taken into consideration here, because it is of high complexity due to the extreme modal superimposition between the two modes. Therefore, this case is fine for testing the proposed method.

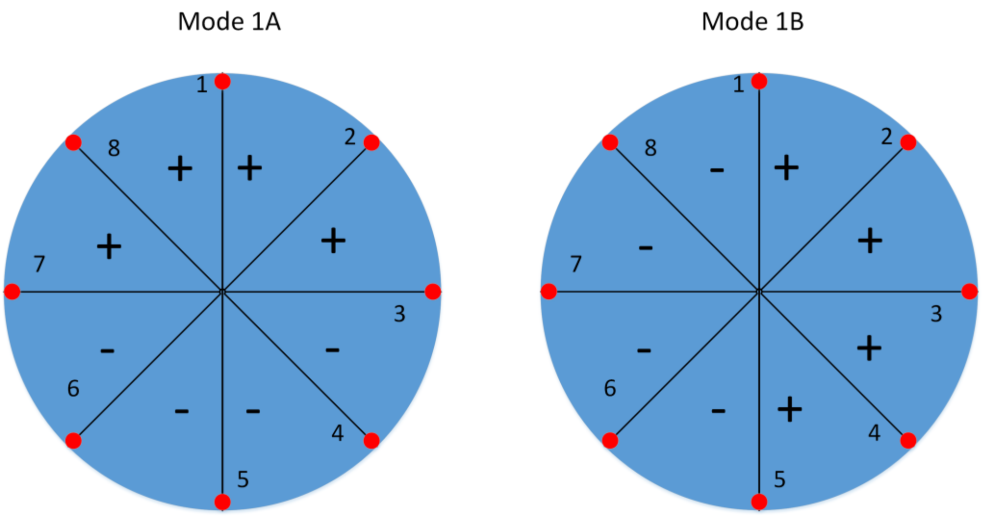

Figure 1: Twin modes of an axis-symmetric structure

The usual practice in experimental modal analysis (EMA) with modes like those in Figure 1 is to use a multiple–reference approach. The optimal choice of the reference DOFs in EMA would be to use DOF 1 as reference for identifying mode 1A (it is noticed that DOF 1 is a node for mode 1B) and DOF 3 for mode 1B (it is noticed that DOF 3 is a node for mode 1A), allowing to have response measurements with perfectly decoupled modes. However, any other choice of the reference DOFs is valid. Something similar can be done with the method presented here for scaling the modal model found with OMA. The reference modal parameters chosen here for modes 1A and 1B are: ω_1 =5.00 Hz, ω_2 =5.02 Hz, ξ_1= ξ_2=0.50%. The eigenvector components are described by cosine functions with unitary amplitude.
Two different types of simulations were carried out: exciting the structure in DOFs 1 and 3 (mode responses completely decoupled, best case) and then in DOFs 2 and 4 (mode responses completely coupled, worst case with very high modal coupling). For both the cases (thus, even in case of

excitation in DOFs 2 and 4 and high modal coupling in the responses), the first numerical test was performed by considering the modal parameters (eigenfrequencies, non-dimensional damping ratios and non-scaled eigenmodes) coming from OMA as unaffected by any error. The estimated modal mass values for the two modes were always so close to the actual values that the differences could be considered as negligible. This means that the numerical procedure of the method does not introduce any bias or random effects in the results and it is numerically well-conditioned, even in case of very high modal superimposition.
Then, errors between the actual modal parameters of the structure and the values supposed to be estimated by means of OMA and then used for finding the modal masses were imposed. Different error configurations were considered. Here, just one is presented for the sake of conciseness. The errors on the modal parameters were imposed by extracting from given probability density functions the value of the errors for the different modal parameters. The error configuration taken into consideration here was characterized by:

- errors on the two eigenfrequency values characterized by Gaussian distribution with null mean value and a standard deviation of 0.05% of the eigenfrequency values;
- for the non-dimensional damping ratios, the mean value of the error was still null and the standard deviation was 10% of their values (Gaussian distribution);
- finally, for the non-scaled eigenvector components, the mean value of the error was still null and the standard deviation was 5% of their values (Gaussian distribution).

The distributions described in the previous list are more than reasonable in practical cases. 2000 simulations exciting in DOFs 1 and and 3 and exciting in DOFs 2 and 4 were carried out, and in each of them the errors imposed on the values of the modal parameters estimated with OMA were randomly extracted according to the distributions described previously. Figure 2 shows the results in terms of μ (mean value of the resulting modal masses of the 2000 simulations) and $\mu \pm 2\sigma$ (where σ is the standard deviation of the resulting modal masses of the 2000 simulations). The resulting distribution were almost Gaussian. The results are similar in both the cases: exciting the system in DOFs 1 and 3 and in DOFs 2 and 4. This evidences that possible random and bias effects on the estimations of the method are directly due to random and bias effects on the OMA estimates and they are not due to the method itself.

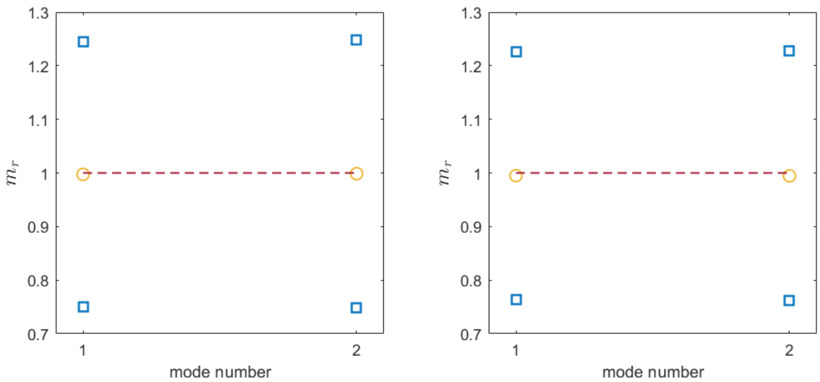

Figure 2: μ (circles) and $\mu \pm 2\sigma$ (squares) for the 2000 simulations with excitations in DOFs 1 and 3 (left) and in DOFs 2 and 4 (right). The dashed lines show the reference values of the modal masses (i.e. 1)

3. THE ESTIMATION OF THE FRF AT SELECTED FREQUANCY VALUES

In point 3 of the numbered list in Section 2, it was mentioned that it is necessary to estimate one or more FRFs at the excitation frequencies when carrying out the mono-harmonic test. To estimate the FRF, it is necessary to estimate the complex amplitude of the of the exciting force and of the structural response. These complex amplitudes have been estimated in two different ways in this work. One approach is the sine fit in the time domain, while the other is the use of the Fourier transform (thus, in the frequency domain).

The former technique has been widely studied in the past. More specifically, closed formulations show that the complex amplitude estimation improves by increasing the number of samples used for the estimation itself [11,12].
Something similar occurs when working in the frequency domain. Indeed, the harmonic signals do not change their amplitude increasing the acquisition time, while the random disturbance (e.g. electrical noise, response to traffic and wind) show a decrease of their amplitudes increasing the acquisition time [9] and thus decreasing the frequency resolution. This implies that the connect related to the harmonic components becomes more and more evident and easy to estimate when increasing the acquisition time.
As mentioned, both the methods were applied, and they provided similar results.

4. ADVANTAGES OF THE METHOD

Once described the procedure behind the method proposed, it is possible to evidence its strengths and advantages. First, the signal processing approaches to be applied for deriving the FRFs when exciting the structure with mono-harmonic forces are simple (see Section 3). They can be both in the frequency and in the time domain, and it can be demonstrated that they work properly even with low signal-to-noise ratio (SNR), provided that the acquisition time of the input and output data is increased enough. Furthermore, since the method requires to excite the structure just at (or close to) resonances, small (and thus not expensive) actuators and easy set-ups can be used to obtain responses with enough SNR. According to the previous points, the forces that must be applied to the structure under analysis are low, compared to the techniques based on exogenous excitation usually adopted to scale OMA modes. In general, the procedure for performing the proposed method is much easier and cheaper compared to the other methods available in the literature. Furthermore, no models and assumptions on the structure/system are needed. In addition, the method is general and can be applied also in case that more than one excitation point is needed (e.g. because there are no points where all the modes to be scaled show a significant response).

5. EXPERIMENTAL VALIDATION

Different test campaigns were conducted in order to validate the proposed method. Here, an example is provided to evidence the reliability of the new approach. In this case, the dynamic tests were carried out on a staircase located at the Bovisa campus of Politecnico di Milano (see Figure 3). At first, an OMA was performed, evidencing that the first two modes of the structure are characterised by the following modal parameters: ω_1 =7.84 Hz, ω_2 =8.88 Hz, ξ_1=0.22%, ξ_2=0.39%. The first mode is a pure bending, while the second shows bending coupled to torsion. Then, the mono-harmonic tests were conducted according to the procedure described in Section 2. This allowed to estimate the modal masses. The OMA tests together with the mono-harmonic excitation tests thus allowed to estimate all the modal parameters for the first modes of the structure, making it possible to reconstruct the FRFs of the structure in the frequency range of these modes. These reconstructed FRFs were then compared to FRFs found experimentally by exciting the structure with a random force profile. This comparison, shown in Figure 4, highlights a good agreement, especially at the resonances, and confirms (as well as many other tests not shown here for the sake of conciseness) the reliability of the proposed method.

Figure 3: The staircase used for the experimental tests

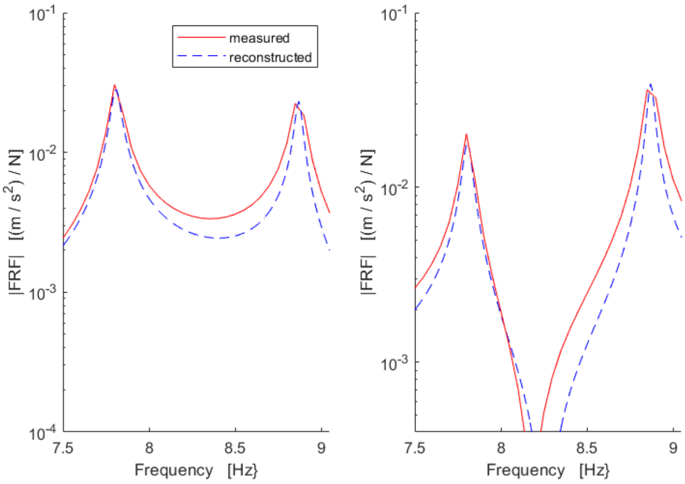

Figure 4: FRF amplitudes for two different response locations (left and right)

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