Integrating Random Shocks into Multi-State Physics Models of Degradation Processes for Component Reliability Assessment

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Index Terms – Component Degradation, Random shocks, Multi-state physics model, Semi-Markov process, Monte Carlo simulation.

Abstract - We extend a multi-state physics model (MSPM) framework for component reliability assessment by including semi-Markov and random shock processes. Two mutually exclusive types of random shocks are considered: extreme and cumulative. The former leads the component to immediate failure, whereas the latter influences the component degradation rates. General dependences between the degradation and the two types of random shocks are considered. A Monte Carlo simulation algorithm is implemented to compute component state probabilities. An illustrative example is presented and a sensitivity analysis is conducted on the model parameters. The results show that our extended model is able to characterize the influences of different types of random shocks onto the component state probabilities and the reliability estimates.

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Acronyms

MSPM Multi-state physics model

Notations

$\boldsymbol{S}$	The states set of component degradation processes
τ_i	The residence time of component being in the state i since the last transition
$\boldsymbol{\theta}$	The external influencing factors
$\lambda_{i,j}(\tau_i, \boldsymbol{\theta})$	The transition rate between state *i* and state *j*
t	Time
$(t, t+\Delta t)$	Infinitesimal time interval
X_k	The state of the component after *k* transitions
T_k	The time of arrival at X_k of component
$P(t)$	The state probability vector
$p_i(t)$	The probability of component being in state *i* at time *t*
$R(t)$	The component reliability
$N(t)$	The number of random shocks occurred until time t
μ	The constant Arrival rate of random shocks
$\tau'_{i,m}$	The residence time of the component in the current degradation state *i* after *m* cumulative shocks
$p_{i,m}(\tau'_{i,m})$	The probability that one shock results in extreme damage
$\lambda_{i,j}^{(m)}(\tau'_{i,m}, \boldsymbol{\theta})$	The transition rates after *m* cumulative random shocks
$\boldsymbol{S}'$	The state space of the integrated model
$\lambda_{(i,m),(j,n)}(\tau'_{i,m}, \boldsymbol{\theta})$	The transition rate between state (i,m) and state (j,n)
$f_{(i,m),(j,n)}(\tau'_{i,m} \mid t, \boldsymbol{\theta})$	The transition probability density function
$P_{(i,m)}(\tau'_{i,m} \mid t, \boldsymbol{\theta})$	The probability that, given that the component arrives at the state (i,m) at *t* and $\boldsymbol{\theta}$, no transition will occur in $(t, t+\tau'_{i,m})$
$\lambda_{(i,m)}(\tau'_{i,m}, \boldsymbol{\theta})$	The conditional probability that, given that the component is in the state (i,m) at time *t*, having arrived there at time $t-\tau'_{i,m}$, and $\boldsymbol{\theta}$, it will depart from (i,m) during $(t, t+d\tau'_{i,m})$
$\psi_{(i,m)}(\tau'_{i,m} \mid \theta)$	The probability density function for $\tau'_{\mathrm{i,m}}$ in the state $(\mathrm{i,m})$, given $\boldsymbol{\theta}$
$\pi_{(i,m),(j,n)}(\tau'_{i,m} \mid \boldsymbol{\theta})$	The conditional probability that, for the transition out of state (i,m) after holding time $\tau'_{i,m}$ and $\boldsymbol{\theta}$, the transition arrival state will be (j,n)
N_{max}	The maximum number of replications
$\widehat{\boldsymbol{P}}(t) = \{\widehat{p_M}(t), \widehat{p_{M-1}}(t), \dots, \widehat{p_0}(t)\}$	The estimation of the state probability vector
$var_{\widehat{p_\iota}(t)}$	The sample variance of estimated state probability $\widehat{p_\iota}(t)$
δ	The predetermined constant which controls the influence of the degradation onto the probability $p_{i,m}(\tau'_{i,m})$

ε The relative increment of transition rates after one cumulative shock happens

1. INTRODUCTION

Failures of components generally occur in two modes: degradation failures due to physical deterioration in the form of wear, erosion, fatigue, etc, and catastrophic failures due to damages caused by sudden shocks in the form of jolts, blows, etc [1]-[2].

In the past decades, a number of degradation models have been proposed in the field of reliability engineering [3]-[9]. They can be grouped into the following categories [9]: statistical distributions (e.g. Bernstein distribution [3]), stochastic processes (e.g. Gamma process and Wiener process) [4]-[5], and multi-state models [6]-[8].

Most of the existing models are typically built on degradation data from historical collection [3], [5]-[7] or degradation tests [4], which however are suited for components of relatively low cost or/and high failure rates (e.g. electronic devices and vehicle components) [10]-[12]. In industrial systems, there are a number of critical components (e.g. valves and pumps in nuclear power plants or aircraft [13]-[14], engines of airplanes, etc.) designed to be highly reliable to ensure system operation and safety, but for which degradation experiments are costly. In practice, it is then often difficult to collect sufficient degradation/failure samples to calibrate the degradation models mentioned above.

An alternative is to resort to failure physics and structural reliability, to incorporate knowledge on the physics of failure of the particular component (passive and active) [13-17]. Recently, Unwin *et al.* [16] have proposed a multi-state physics model (MSPM) for modeling nuclear component degradation, also accounting for the effects of environmental factors (e.g. temperature and stress) within certain predetermined ranges [17]. In a previous work by the authors [9], the model has been formulated under the framework of inhomogeneous continuous time Markov chain and solved by Monte Carlo simulation.

Random shocks need to be accounted for on top of the underlying degradation processes, because they can bring variations to influencing environmental factors, even outside their predetermined boundaries [18], that can accelerate the degradation processes. For example, thermal and mechanical shocks (e.g. internal thermal shocks and water hammers) [17], [19]-[20] onto power plant components can lead to intense increases in temperatures and stresses, respectively; under these extreme conditions, the original physics functions in MSPM might be insufficient to characterize the influences of random shocks onto the degradation processes and must, therefore, be modified. In the literature, random shocks are typically modeled by Poisson processes [1], [18], [21]-[23], distinguishing two main types, extreme shock and cumulative shock processes [21], according to the severity of the damage. The former could directly lead the component to immediate failure [24]-[25], whereas the latter increases the degree of damage in a cumulative way [26]-[27].

Random shocks have been intensively studied [1]-[2], [22]-[23], [28]-[33]. Esary *et al.* [23] have considered extreme shocks in a component reliability model, whereas Wang *et al.* [2], Klutke and Yang [30], and Wortman *et al.* [31] have modeled the influences of cumulative shocks onto a degradation process. Both extreme and cumulative random shocks have been considered by Li and Pham [1], Wang and Pham [22]. Additionally, Ye *et al.* [28] and Fan *et al.* [29] have considered that high severity of degradation can lead to high probability that a random shock causes extreme damage. However, the fact that the effects of cumulative shocks can vary according to the severity of degradation has also to be considered.

Among the models mixing the multi-state degradation models and random shocks, Li and Pham [1] divided the underlining continuous and monotonically increasing degradation processes into a finite number of states and combined them with independent random shocks. Wang and Pham [22], further considered the dependences among the continuous and monotone (increasing or decreasing) degradation processes and between degradation processes and random shocks. Yang et al. [33] integrated random shocks into a Markov degradation model. Becker et al. [32] combined semi-Markov degradation model, which is more general than Markov model, with random shocks in a dynamic reliability formulation, where the influence of random shocks is characterized by the change of continuous degradation variables (e.g. structure strength). To the best knowledge of the authors, this is the first work of semi-Markov degradation modeling that represents the influence of random shocks by changing the transition rates, which might also be physics functions.

The contribution of the paper is that it generalizes the MSPM framework to handle both degradation and random shocks, which have not been previously considered by the existing MSPMs. More specifically: first, we extend our previous MSPM framework [9] to semi-Markov modeling, which more generally describes the fact that the time of transition to a state can depend on the residence time in the current state, and hence is more suitable for including maintenance [34]; then, we propose a general random shock model, where the probability of a random shock resulting in extreme or cumulative damage, and the cumulative damages, are both dependent on the current component degradation condition (the component degradation state and residence time in the state); finally, we integrate the random shock model into the MSPM framework to describe the influence of random shocks on the degradation processes. The rest of this paper is organized as follows. Section 2 introduces the semi-Markov scheme into the MSPM framework. Section 3 presents the random shock model; in Section 4, its integration into MSPM is presented. Monte Carlo simulation procedures to solve the integrated model are presented in Section 5. Section 6 uses a numerical example regarding a case study of literature, to illustrate the proposed model. Section 7 concludes the work.

2. MSPM OF COMPONENT DEGRADATION PROCESSES

A continuous-time stochastic process is called a semi-Markov process if the embedded jump chain is a Markov Chain and the times between transitions may be random variables with any distribution [35]. The following assumptions are made for the extended MSPM framework [9] based on semi-Markov processes:

- The degradation process has a finite number of states $\boldsymbol{S} = \{0,1,\dots,M\}$ where states '*0*' and '*M*' represent the complete failure state and perfect functioning state, respectively; The generic intermediate degradation states *i* (0<*i*<*M*) are established according to the degradation development and condition, wherein the component is functioning or partially functioning.
- The degradation follows a continuous-time semi-Markov process; the transition rate between state *i* and state *j*, denoted by $\lambda_{i,j}(\tau_i, \boldsymbol{\theta})$, is a function of τ_i, which is the residence time of the component being in the current state *i* since the last transition, and $\boldsymbol{\theta}$, which represents the external influencing factors (including physical factors).
- The initial state (at time *t* = *0*) of the component is *M*.
- Maintenance can be carried out from any degradation state, except the complete failure state (in other words, there is no repair from failure).

Fig. 1 presents the diagram of the semi-Markov component degradation process.

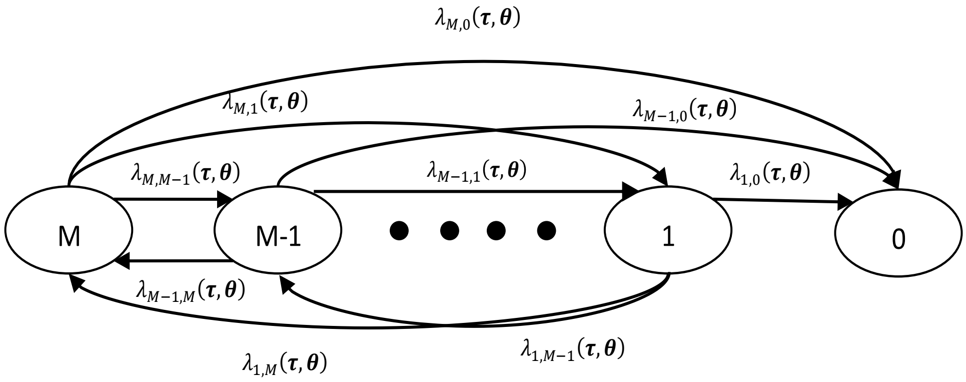

Fig 1. The diagram of the semi-Markov process

The probability that the continuous time semi-Markov process will step to state *j* in the next infinitesimal time interval $(t, t + \Delta t)$, given that it has arrived at state *i* at time T_n after *n* transitions and remained stable in *i* from T_n until time *t* , is defined as follows,

$$P[X_{n+1} = j, T_{n+1} \in [t, t + \Delta t] \mid \{X_{k,} T_k\}_{k=0}^{n-1}, (X_n = i, T_n), T_n \leq t \leq T_{n+1}, \boldsymbol{\theta}]$$
$$= P[X_{n+1} = j, T_{n+1} \in [t, t + \Delta t] \mid (X_n = i, T_n)\ , T_n \leq t \leq T_{n+1}, \boldsymbol{\theta}]$$
$$= \lambda_{i,j}(\tau_i = t - T_n, \boldsymbol{\theta})\Delta t,\ \forall\, i, j\ \in\ \boldsymbol{S}, i \neq j \quad (1)$$

where X_k denotes the state of the component after k transitions and T_k denotes the time of arrival at X_k. The degradation transition rates can be obtained from the structural reliability analysis of the degradation processes (e.g. the crack propagation process ([15], [17]), whereas the transition rates related to maintenance tasks can be estimated from the frequencies of maintenance activities). For example, the authors of [17] divided the degradation process of the alloy metal weld into six states dependent on the underlying physics phenomenon, and some degradation transition rates are represented by corresponding physics equations.

The solution to the semi-Markov process model is the state probability vector $P(t) = \{p_M(t), p_{M-1}(t), \ldots, p_0(t)\}$, where $p_i(t)$ is the probability of the component being in state *i* at time *t*. Since no maintenance is carried out from the component failure state and the component is regarded as functioning in all other intermediate alternative states, its reliability can be expressed as

$$R(t) = 1 - p_0(t) \quad (2)$$

where $p_0(t)$ is the probability of the complete failure state at time t. Analytically solving the continuous time semi-Markov model with state residence time-dependent transition rates is a difficult or sometimes impossible task, and the Monte Carlo simulation method is usually applied to obtain $P(t)$ [36]-[37].

3. RANDOM SHOCKS

The following assumptions are made on the random shock process:

- The arrivals of random shocks follow a homogeneous Poisson process $\{N(t), t \geq 0\}$ [21] with constant arrival rate μ, where the random variable $N(t)$ denotes the number of random shocks occurred until time *t*. The random shocks are independent of the degradation process, but they can influence the degradation process (see Fig. 2).

- The damages of random shocks are divided into two types: extreme and cumulative.
- Extreme shock and cumulative shock are mutually exclusive.
- The component fails immediately upon occurrence of extreme shocks.
- The probability of a random shock resulting in extreme or cumulative damage is dependent on the current component degradation.
- The damage of cumulative shocks can only influence the degradation transition departing from the current state and its impact on the degradation process is dependent on the current component degradation.

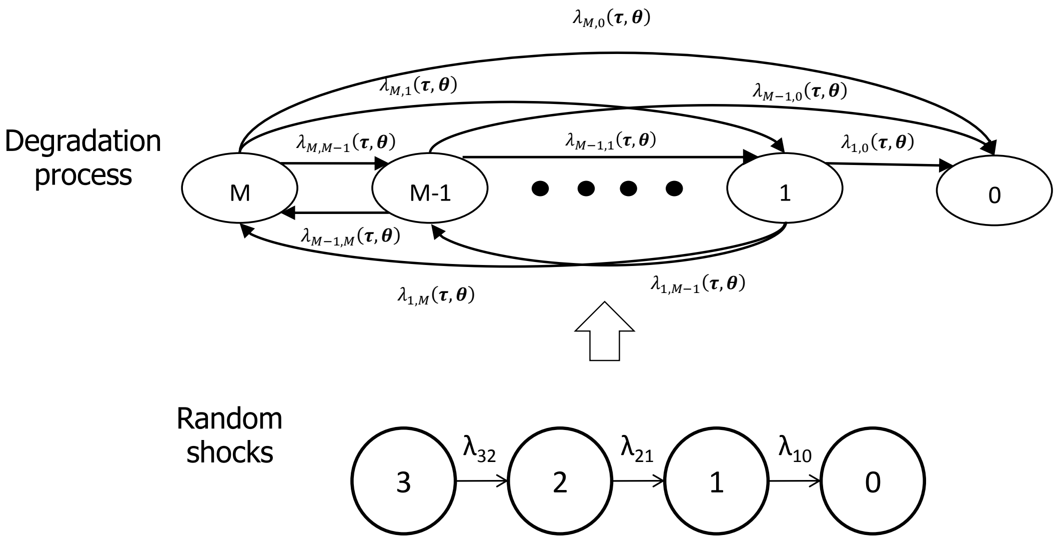

Fig 2. Degradation and random shock processes

The first five assumptions are taken from [22]. The sixth assumption reflects the aging effects addressed in Fan *et al.*'s shock model [29], where the random shocks are more fatal to the component (i.e. more likely lead to extreme damages) when the component is in severe degradation states. However, the influences of cumulative shocks under aging effects have not been considered in Fan *et al.*'s model, as in the last assumption. In addition, the random shock damage is assumed to depend on the current degradation, characterized by three parameters: 1) the current degradation state *i*, 2) the number of cumulative shocks *m* occurred while in the current degradation state since the last degradation state transition, 3) the residence time $\tau'_{i,m}$ of the component in the current degradation state *i* after *m* cumulative shocks $\tau'_{i,m} \geqslant 0$.

Let $p_{i,m}(\tau'_{i,m})$ denote the probability that one shock results in extreme damage (the cumulative damage probability is then $1 - p_{i,m}(\tau'_{i,m})$). In case of cumulative shock, the degradation transition rates for the current state change at the moment of occurrence of the shock, whereas the other transition rates are not affected. Let $\lambda_{i,j}^{(m)}(\tau'_{i,m}, \boldsymbol{\theta})$ denote the transition rates after *m* cumulative random shocks, where $\lambda_{i,j}^{(0)}(\tau'_{i,0}, \boldsymbol{\theta})$ holds the same expression as the transition rate $\lambda_{i,j}(\tau'_{i,0}, \boldsymbol{\theta})$ in the pure degradation model, and the other transition rates (i.e. *m*>0) depend on the degradation and the external influencing factors. Because the influences of random shocks can render invalid the original physics functions, we propose a general model which allows the formulation of 'physics' functions dependent on the effects of shocks. The modified transition rates can be obtained by material science knowledge and/or data from shock tests [38]. These quantities will be used as the key linking elements in the integration work of next section.

4. INTEGRATION OF RANDOM SHOCKS IN THE MSPM

Based on the first and second assumptions on random shocks, the new model that integrates random shocks into MSPM is shown in Fig 3. In the model, the states of the component are represented by pair (*i*,*m*), where *i* is the degradation state and *m* is the number of cumulative shocks occurred during the residence time in the current state. For all the degradation states of component except for the state '*0*', the number of cumulative shocks could range from *0* to positive infinity. If the transition to a new degradation state occurs, the number of cumulative shocks is set to *0*, coherently with the last assumption on random shocks. The state space of the new integrated model is denoted by $\boldsymbol{S'} = \{(M,0),(M,1),(M,2),\dots,(M-1,0),(M-1,1),\dots,(0,0)\}$. The component is failed whenever it reaches (*0,0*). The transition rate denoted by $\lambda_{(i,m),(j,n)}(\tau'_{i,m}, \boldsymbol{\theta})$ is residence time-dependent, thus rendering the process a continuous time semi-Markov process.

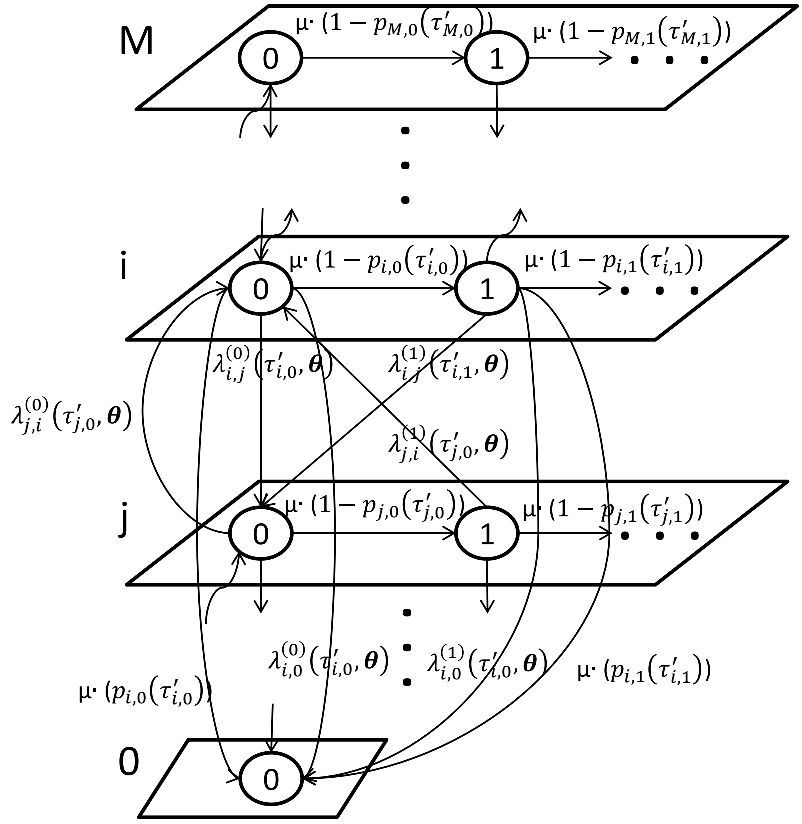

Fig 3. Degradation and random shock processes

Suppose that the component is in a non-failure state (*i*,*m*); then, we have three types of outgoing transition rates:

$$\lambda_{(i,m),(0,0)}(\tau'_{i,m}, \boldsymbol{\theta}) = \mu \cdot (p_{i,m}(\tau'_{i,m})) \tag{3}$$

the rate of occurrence of an extreme shock which will cause the component to go to state (*0*,*0*),

$$\lambda_{(i,m),(i,m+1)}(\tau'_{i,m}, \boldsymbol{\theta}) = \mu \cdot (1 - p_{i,m}(\tau'_{i,m})) \tag{4}$$

the rate of occurrence of a cumulative shock which will cause the component to go to state (*i*,*m+1*) and

$$\lambda_{(i,m),(j,0)}(\tau'_{i,m}, \boldsymbol{\theta}) = \lambda_{i,j}^{(m)}(\tau'_{i,j}, \boldsymbol{\theta}) \tag{5}$$

the rate of transition (i.e. degradation or maintenance) which will cause the component to make the transition to state (j,0).

The effect of random shocks on the degradation processes is shown in equation (5) by using the superscript (m) where m is the number of cumulative shocks occurred during the residence time in the current state. It means that the transition rate functions depend on the number of cumulative shocks. This is a general formulation.

The first two types (equation (3) and equation (4)) depend on the probability of a random shock resulting in extreme damage and in cumulative damage, respectively; the last type of transition rates (equation (5)) depends on the cumulative damage of random shocks. In this model, we do not directly associate a failure threshold to the cumulative shocks, since the damage of cumulative shocks can only influence the degradation transition departing from the current state and its impact on the degradation process is dependent on the current component degradation. The cumulative shocks can only aggravate the degradation condition of the component instead of leading it suddenly to failure (which is the role of extreme shocks). The effect of the cumulative shocks is reflected in the change of transition rates. The probability of a shock becoming an extreme one depends on the degradation condition of the component. The extreme shocks immediately lead the component to failure, whereas the damage of cumulative shocks aggravates the degradation processes of the component.

The proposed model is based on semi-Markov process and random shocks. Under this general structure, as explained in the paragraph above, the physics lies in the transition rates of the semi-Markov process. We name it a ‘physics’ model because the stressors (e.g. the crack in the case study) that cause the component degradation are explicitly modeled, differently from the conventional way of estimating the transition rates from historical failure/degradation data, which are relatively rare for the critical components. More information about MSPM can be found in [9]. In addition, the random shocks are integrated into the MSPM in a way that they may change the ‘physics’ functions of the transition rates, within a general formulation.

Similarly to what was said for the semi-Markov process presented in Section 2, the state probabilities of the new integrated model can be obtained by Monte Carlo simulation and the expression of component reliability is:

$$R(t) = 1 - p_{(0,0)}(t) \tag{6}$$

5. RELIABILITY ESTIMATION

5.1 Basics of Monte Carlo simulation

The key theoretical construct upon which Monte Carlo simulation is based is the transition probability density function $f_{(i,m),(j,n)}(\tau'_{i,m} \mid t, \boldsymbol{\theta})$, defined as follows

$f_{(i,m),(j,n)}(\tau'_{i,m} \mid t, \boldsymbol{\theta}) d\tau'_{i,m} \equiv$ probability that, given that the system arrives at the state (i, m) at time *t* and physical factors $\boldsymbol{\theta}$, the next transition will occur in the infinitesimal time interval $(t + \tau'_{i,m}, t + \tau'_{i,m} + d\tau'_{i,m})$ and will be to the state (j, n) [36]. (7)

By using the previously introduced transition rates, equation (7) can be expressed as

$$f_{(i,m),(j,n)}(\tau'_{i,m} \mid t, \boldsymbol{\theta}) d\tau'_{i,m} = P_{(i,m)}(\tau'_{i,m} \mid t, \boldsymbol{\theta}) \lambda_{(i,m),(j,n)}\left(\tau'_{i,m}, \boldsymbol{\theta}\right) d\tau'_{i,m} \tag{8}$$

where $P_{(i,m)}(\tau'_{i,m} \mid t, \boldsymbol{\theta})$ is the probability that, given that the component arrives at the state (i, m) at time *t* and physical factors $\boldsymbol{\theta}$, no transition will occur in the time interval $(t, t + \tau'_{i,m})$ and it satisfies:

$$\frac{dP_{(i,m)}(\tau'_{i,m} \mid t, \boldsymbol{\theta})}{P_{(i,m)}(\tau'_{i,m} \mid t, \boldsymbol{\theta})} = -\lambda_{(i,m)}\left(\tau'_{i,m}, \boldsymbol{\theta}\right) d\tau'_{i,m} \tag{9}$$

where

$$\lambda_{(i,m)}\left(\tau'_{i,m}, \boldsymbol{\theta}\right) = \sum_{(i\prime,m\prime)} \lambda_{(i,m),(i\prime,m\prime)}\left(\tau'_{i,m}, \boldsymbol{\theta}\right) \tag{10}$$

and $\lambda_{(i,m)}\left(\tau'_{i,m}, \boldsymbol{\theta}\right) d\tau'_{i,m}$ is the conditional probability that, given that the component is in the state (i, m) at time *t*, having arrived there at time $t - \tau'_{i,m}$, and physical factors $\boldsymbol{\theta}$, it will depart from (i, m) during $(t, t + d\tau'_{i,m})$.

Taking the integral at both sides of equation (9) with the initial condition $P_{(i,m)}(0 \mid t, \boldsymbol{\theta}) = 1$, we obtain

$$P_{(i,m)}(\tau'_{i,m} \mid t, \boldsymbol{\theta}) = exp[-\int_0^{\tau'_{i,m}} \lambda_{(i,m)}(s, \boldsymbol{\theta}) ds] \tag{11}$$

Substituting equation (11) into equation (8), we obtain

$$f_{(i,m),(j,n)}(\tau'_{i,m} \mid t, \boldsymbol{\theta}) = \lambda_{(i,m),(j,n)}(\tau'_{i,m}, \boldsymbol{\theta}) exp[-\int_0^{\tau'_{i,m}} \lambda_{(i,m)}(s, \boldsymbol{\theta}) ds] \quad (12)$$

To derive a Monte Carlo simulation procedure, equation (12) is rewritten as

$$f_{(i,m),(j,n)}(\tau'_{i,m} \mid t, \boldsymbol{\theta}) = \frac{\lambda_{(i,m),(j,n)}(\tau'_{i,m}, \boldsymbol{\theta})}{\lambda_{(i,m)}(\tau'_{i,m}, \boldsymbol{\theta})} \cdot \lambda_{(i,m)}(\tau'_{i,m}, \boldsymbol{\theta}) exp[-\int_0^{\tau'_{i,m}} \lambda_{(i,m)}(s, \boldsymbol{\theta}) ds]$$

$$= \pi_{(i,m),(j,n)}(\tau'_{i,m} \mid \boldsymbol{\theta}) \cdot \psi_{(i,m)}(\tau'_{i,m} \mid \boldsymbol{\theta}) \quad (13)$$

where

$$\psi_{(i,m)}(\tau'_{i,m} \mid \boldsymbol{\theta}) = \lambda_{(i,m)}(\tau'_{i,m}, \boldsymbol{\theta}) exp[-\int_0^{\tau'_{i,m}} \lambda_{(i,m)}(s, \boldsymbol{\theta}) ds] \quad (14)$$

is the probability density function for the holding time $\tau'_{i,m}$ in the state (i,m), given the physical factors $\boldsymbol{\theta}$, and

$$\pi_{(i,m),(j,n)}(\tau'_{i,m} \mid \boldsymbol{\theta}) = \frac{\lambda_{(i,m),(j,n)}(\tau'_{i,m}, \boldsymbol{\theta})}{\lambda_{(i,m)}(\tau'_{i,m}, \boldsymbol{\theta})} \quad (15)$$

is regarded as the conditional probability that, for the transition out of state (i,m) after holding time $\tau'_{i,m}$ and the physical factors $\boldsymbol{\theta}$, the transition arrival state will be (j,n).

In the Monte Carlo simulation, for the component arriving at any non-failure state (i,m) at any time *t*, the process at first samples the holding time at state (i,m) corresponding to equation (14), and then determines the transition arrival state (j,n) from state (i,m) according to equation (15). This procedure is repeated until the accumulated holding time reaches the predefined time horizon or the component reaches the failure state $(0,0)$.

5.2 The simulation procedure

To generate the holding time $\tau'_{i,m}$ and the next state (j,n) for the component arriving in any non-failure state (i,m) at any time *t*, one proceeds as follows: two uniformly distributed random numbers u_1 and u_2 are sampled in the interval [0, 1]; then, $\tau'_{i,m}$ is chosen so that

$$\int_0^{\tau'_{i,m}} \lambda_{(i,m)}(s, \boldsymbol{\theta})\, ds = \ln(1/u_1) \quad (16)$$

and $(j,n) = a^*$ that satisfies

$$\sum_{k=0}^{a^*-1} \lambda_{(i,m),k}(\tau'_{i,m}, \boldsymbol{\theta}) < u_2 \lambda_{(i,m)}(\tau'_{i,m}, \boldsymbol{\theta}) \leq \sum_{k=0}^{a^*} \lambda_{(i,m),k}(\tau'_{i,m}, \boldsymbol{\theta}) (17)$$

where a^* represents one state in the ordered sequence of all possible outgoing states of state (i,m).The state a^* is determined by going through the ordered sequence of all possible outgoing states of state (i,m) until the equation (17) is satisfied. The algorithm of Monte Carlo simulation for solving the integrated MSPM on a time horizon $[0,t_{max}]$ is presented as follows:

Set N_{max} (the maximum number of replications) and $k=0$

While $k<N_{max}$

  Initialize the system by setting $s=(M,0)$ (initial state of perfect performance), setting the time $t=0$ (initial time)

  Set $t'=0$ (state holding time)

  While $t<t_{max}$

    Calculate the equation (10)

    Sample a t' by using equation (16)

    Sample an arrival state (j,n) by using equation (17)

    Set $t=t+t'$

    Set $s=(j,n)$

    If $s=(0{,}0)$

    then break

    End if

  End While

  Set $k=k+1$

End While □

The estimation of the state probability vector $\widehat{\boldsymbol{P}}(t)=\{\widehat{p_M}(t),\widehat{p_{M-1}}(t),\dots,\widehat{p_0}(t)\}$ at time t is done as,

$$\widehat{\boldsymbol{P}}(t)=\frac{1}{N_{max}}\{n_M(t),n_{M-1}(t),\dots,n_0(t)\} \tag{18}$$

where $\{n_i(t)|i=M,\dots,0,t\le t_{max}\}$ is the total number of visits to state *i* at time *t*, with sample variance [39] defined as follows

$$var_{\widehat{p_\iota}(t)} = \widehat{p_\iota}(t)(1 - \widehat{p_\iota}(t))/(N_{max} - 1) \quad (19)$$

6. CASE STUDY AND RESULTS

6.1 Case study

We illustrate the proposed modeling framework on a case study slightly modified from an Alloy 82/182 dissimilar metal weld in a primary coolant system of a nuclear power plant in [17]. The MSPM of the original crack growth is shown in Fig. 4.

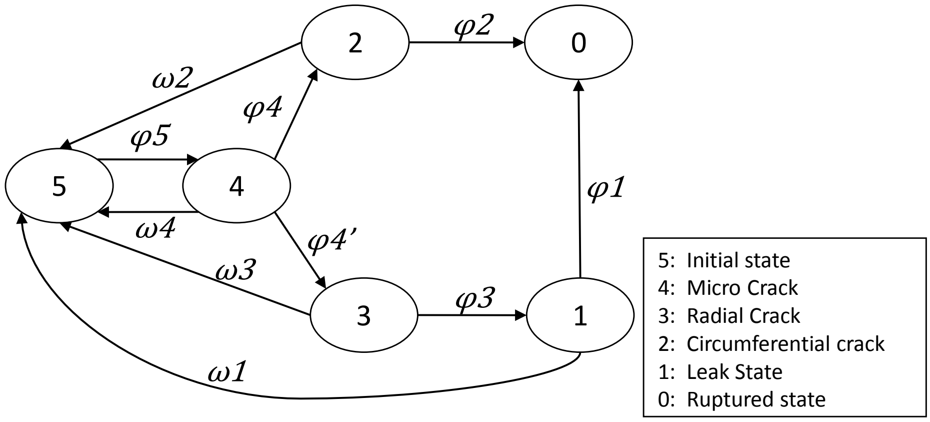

Fig 4. MSPM of crack development in Alloy 82/182 dissimilar metal welds

where φ_i and ω_i represent the degradation transition rate and maintenance transition rate, respectively. Except for φ_5,φ_4, $\varphi_{4'}$ and φ_3, all the other transition rates are assumed to be constant. The expressions of the variable transition rates are as follows:

$$\varphi_5 = \left(\frac{b}{\tau}\right) \cdot \left(\frac{\tau_5}{\tau}\right)^{b-1} \quad (20)$$

$$\varphi_4 = \begin{cases} \frac{a_C P_C}{\dot{a}_M \tau_4{}^2 (1 - P_C(1 - a_C/(u\dot{a}_M)))}, & if\ \tau_4 > a_C/\dot{a}_M \\ 0, & else \end{cases} \quad (21)$$

$$\varphi_4{}' = \begin{cases} \frac{a_D P_D}{\dot{a}_M \tau_4{}^2 (1 - P_D(1 - a_D/(u\dot{a}_M)))}, & if\ \tau_4 > a_D/\dot{a}_M \\ 0, & else \end{cases} \quad (22)$$

$$\varphi_3 = \begin{cases} \frac{1}{\tau_3}, & if\ \tau_3 > (a_L - a_D)/\dot{a}_M \\ 0, & else. \end{cases} \quad (23)$$

The other transition rates and the parameters values are presented in Table I below.

Table I Parameters and constant transition rates [17]

b –Weibull shape parameter for crack initiation model	2.0
τ – Weibull scale parameter for crack initiation model	4 years
a_D – Crack length threshold for radial macro-crack	10 mm
P_D – Probability that micro-crack evolves as radial crack	0.009
$\dot{a}_M$ – Maximum credible crack growth rate	9.46 mm/yr
a_C – Crack length threshold for circumferential macro-crack	10 mm
P_C – Probability that micro-crack evolves as circumferential crack	0.001
a_L – Crack length threshold for leak	20 mm
ω_4 – Repair transition rate from micro-crack	1 x10-3 /yr
ω_3 – Repair transition rate from radial macro-crack	2 x10-2 /yr
ω_2 – Repair transition rate from circumferential macro-crack	2 x10-2 /yr
ω_1 – Repair transition rate from leak	8 x10-1 /yr
φ_1 – Leak to rupture transition rate	2x10-2 /yr
φ_2 – Macro-crack to rupture transition rate	1x10-5 /yr

The random shocks correspond to the thermal and mechanical shocks (e.g. internal thermal shocks and water hammers) [17], [19]-[20] to the dissimilar metal welds. The damage of random shocks can accelerate the degradation processes, and hence, increase the rate of component degradation. Note that Yang *et al* [33] have related random shocks to the degradation rates in their work. To assess the degree of impact of shocks, we may use 1) physics functions for the influence of random shocks through material science knowledge; 2) transition times, speed of cracking development and other related information obtained from shock tests [38]. We set the occurrence rate $\mu = 1/15\, y^{-1}$ and the probability of a random shock becoming extreme shock $p_{i,m}(\tau'_{i,m}) = 1 - exp\left[-\delta m(6-i)\left(2 - e^{-\tau'_{i,m}}\right)\right]$, taking the exponential formulation from Fan *et al.*'s work [29]. In this formula, we use $m(6-i)(2-e^{-\tau'_{i,m}})$ to quantify the component degradation. It is noted that the quantity $2 - e^{-\tau'_{i,m}}$ ranges from *1* to *2*, representing the relatively small effect of $\tau'_{i,m}$ onto the degradation situation in comparison with the other two parameters m and i, and δ is a predetermined constant which controls the influence of the degradation onto the probability $p_{i,m}(\tau'_{i,m})$. In this study, we set $\delta = 0.0001$. The value of δ was set considering the balance between showing the impact of extreme shocks and reflecting the high reliability of the critical component. In addition, we assume the corresponding degradation transition rates after *m* cumulative shocks to be $\lambda_{i,j}^{(m)}(\tau'_{i,m}, \boldsymbol{\theta}) = (1+\varepsilon)^m \lambda_{i,j}(\tau'_{i,m}, \boldsymbol{\theta})$, where $\varepsilon = 0.3$ is the relative increment of transition rates after one cumulative shock happens, and the formulation $(1+\varepsilon)^m$ is used to characterize the accumulated effect of such shocks. In order to characterize the increase of the transition rates, in the case study we have used the parameter ε to represent the relative increment of degradation transition rate after one cumulative shock occurs. For the sake of simplicity, but without loss of generality in the framework for integration, we assume that the values of ε for each cumulative shock are equal. But the model can handle different εs for different stages of the crack process.

6.2 Results and analysis

The Monte Carlo simulation over a time horizon of $t_{max} = 80$ years is run $N_{max} = 10^6$ times. The results are collected and analyzed in the following sections.

6.2.1 Results of state probabilities

The estimated state probabilities without and with random shocks throughout the time horizon are shown in Figs. 5 and 6, respectively.

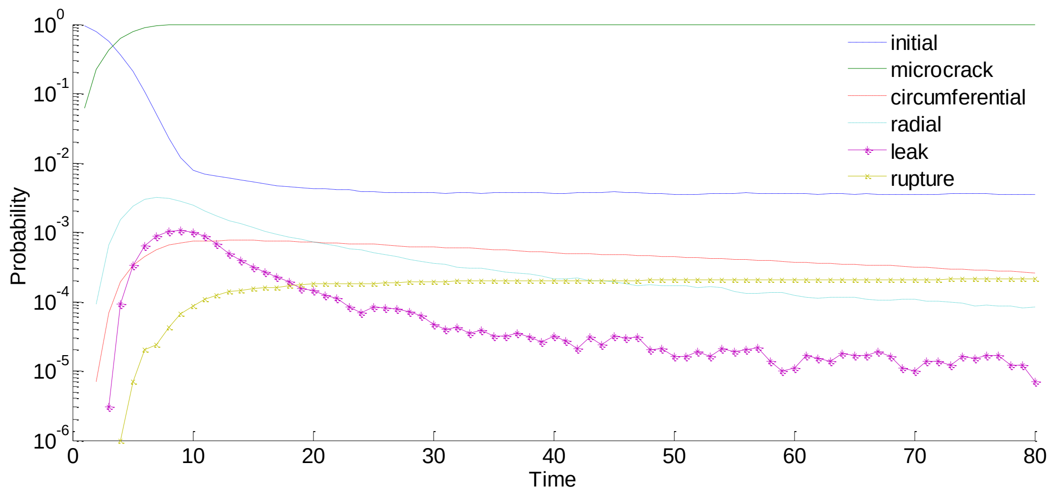

Fig 5. State probabilities obtained without random shocks

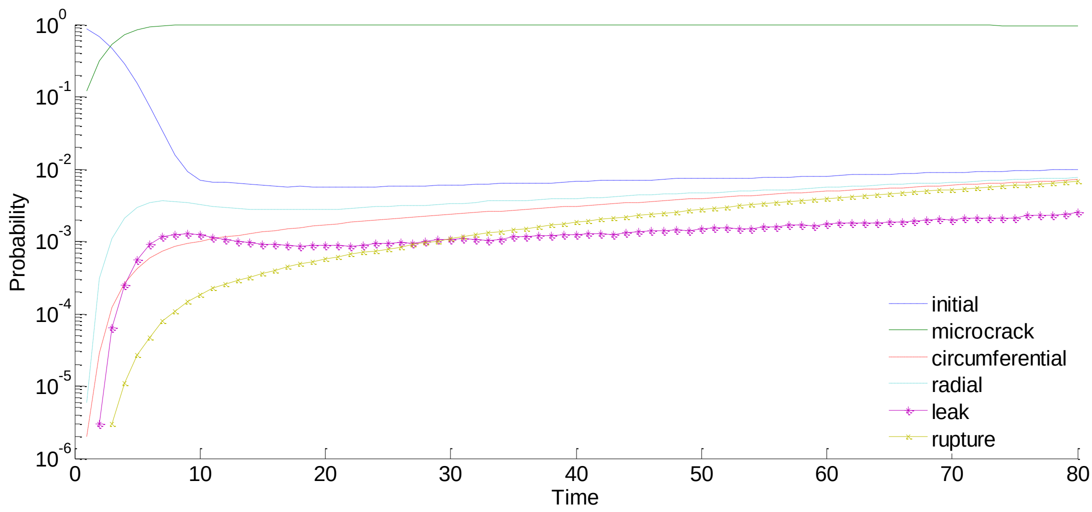

Fig 6. State probabilities obtained with random shocks

Comparing the above two Figures, it can be observed that as expected the random shocks drive the component to higher degradation states than the micro-crack state. The numerical comparisons on the state probabilities w/o random shocks at year 80 are reported in Table II. It is seen that except for the micro-crack state probability, all the other state probabilities at year 80 have increased due to the random shocks, with the increase in leak probability being the most significant.

Table II Comparison of state probabilities w/o random shocks (at year 80)

State	Probability without random shocks	Probability with random shocks	Relative difference
Initial	3.52e-3	9.82e-3	180.00%
Micro-crack	0.9959	0.9661	-2.99%
Circumferential crack	3.05e-4	7.28e-3	2286.89%
Radial crack	1.00e-4	7.75e-3	7650.00%
Leak	1.30e-5	2.59e-3	19823.08%
Rupture state	2.06e-4	7.00e-3	3298.06%

The fact that the probability of the initial state (compared with no random shocks) at 80 years has increased is attributed to the maintenance tasks. All the maintenance tasks lead the component to the initial state and the repair rates from radial macro-crack state, circumferential macro-crack state and leak state are higher than that from micro-crack state. The shocks generally increase the speed of the component to step back to further degradation states from where it steps to the initial state more quickly. In summary, this phenomenon is due to the combined effects of shocks.

6.2.2 Results of component reliability

The estimated component reliabilities with and without random shocks throughout the time horizon are shown in Fig. 7, respectively. At year 80, the estimated component reliability with random shocks is 0.9930, with sample variance equal to 6.95e-9. Compared with the case without random shocks (reliability equals to 0.9998, with sample variance 2.00e-10), the component reliability has decreased by 0.68%.

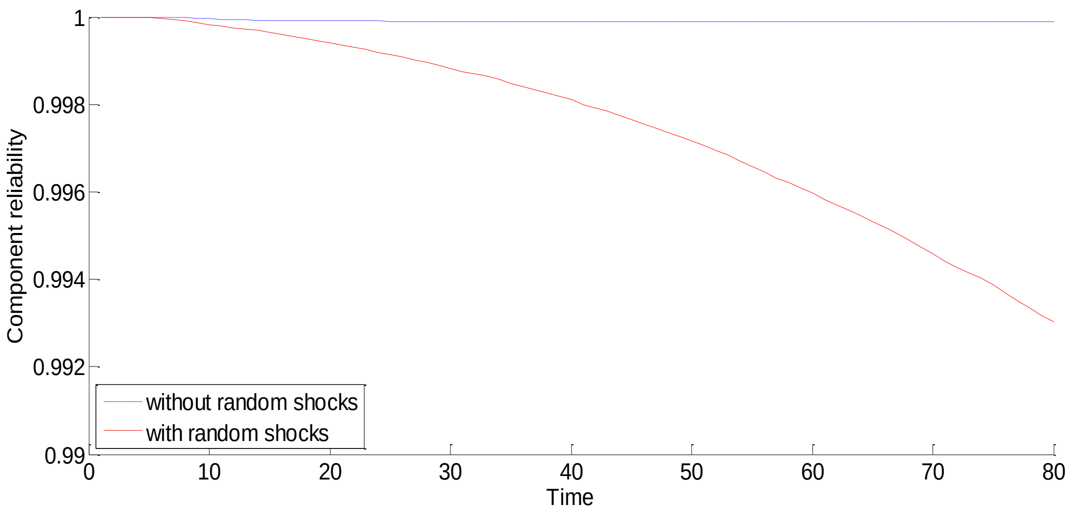

Fig 7. Component reliability estimation w/o random shocks.

6.2.3 Analysis of the extreme shocks

Table III presents the frequencies of different numbers of random shocks occurred per simulation trial. The most likely number is around 5, which is consistent with our assumption on the value of the occurrence rate ($\mu = 1/15y^{-1}$) of random shocks.

Table III Frequency of the number of random shocks occurred per trial (mission time *t* = 80 years)

Nb of random shocks/trial	0	1	2	3	4	5	6	7	8	9	>9
Percentage (%)	0.63	3.14	8.00	13.55	17.15	17.56	14.91	10.83	6.87	3.90	3.45

In total, 6973 trials ended in failure, among which 4531 trials (64.98%) are caused by extreme shocks. Table IV reports the number of trials ending with extreme shocks, for different numbers of cumulative shocks occurred per trial.

Table IV Number of trials ended with extreme shocks for different numbers of cumulative shocks (mission time *t* = 80 years)

Nb of cumulative shocks per trial	Nb of trials	Nb of trials ending with extreme shock

0	6345	0
1	31739	367
2	80292	633
3	135676	812
4	171526	809
5	175569	743
6	148844	500
7	108101	332
8	68579	172
9	38964	90
10	19569	43
11	8998	19
>11	5798	11

The influence of the number of cumulative shocks occurred per trial on the probability of the next random shock being extreme is shown in Fig. 8: as expected, the larger the number of cumulative shocks the higher the probability of extreme shock.

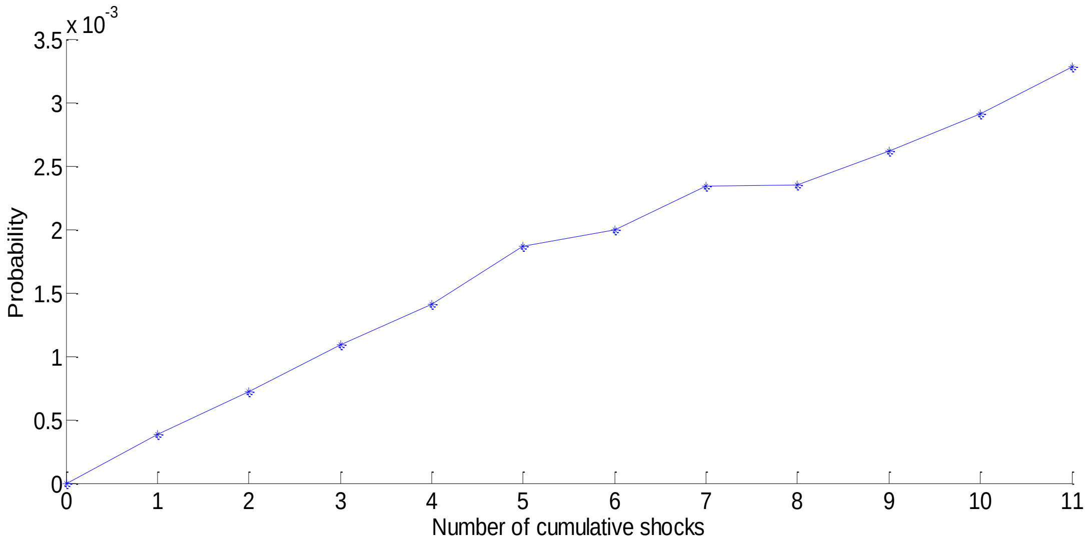

Fig 8. Probability of the next random shock being extreme as a function of the number of cumulative shocks occurred per trial.

The influence of the degradation state on the probability of the next random shock being extreme is shown in Fig. 9: as expected, the likelihood of extreme shocks is higher when the component degradation state is closer to the failure state.

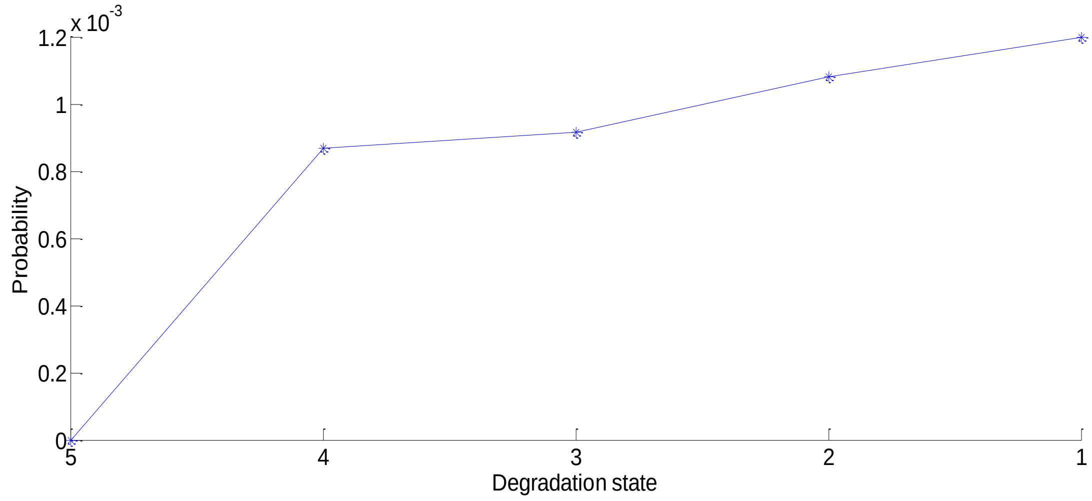

Fig 9. Probability of the next random shock being extreme as a function of the degradation state of the component.

6.2.4 Influence of cumulative shocks on degradation

In order to characterize the influence of cumulative shocks on the degradation processes, we set to *0* the probability of a random shock being extreme, so that all random shocks will be cumulative. The estimated state probabilities are shown in Fig. 10.

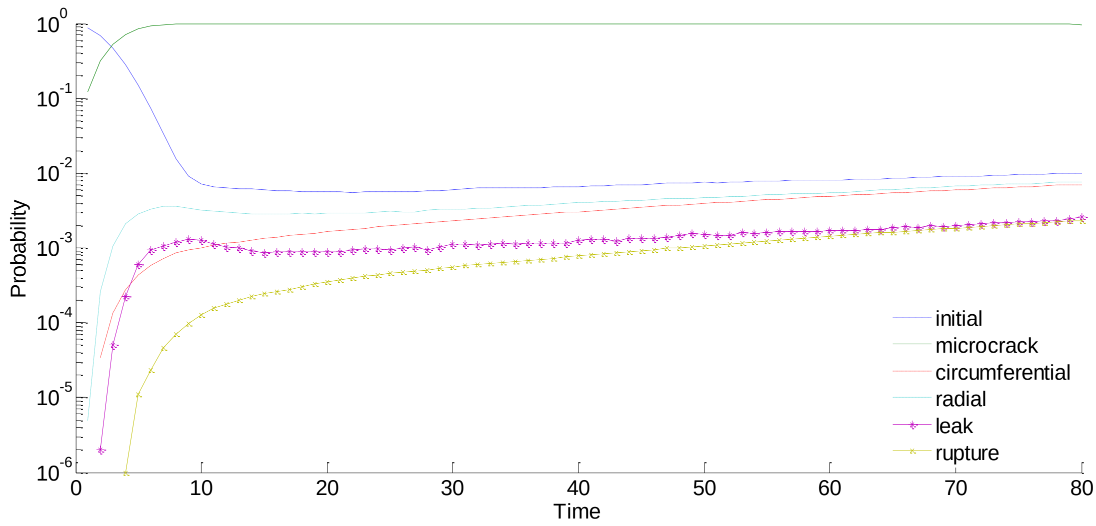

Fig 10. State probabilities obtained with cumulative shocks only.

The state probabilities with cumulative shocks exhibit similar patterns as those in Fig. 6; only the rupture state probability has decreased due to the lack of extreme shocks. The numerical comparisons on the state probabilities without random shocks and with cumulative shocks at year 80 are reported in Table V.

Table V Comparison of state probabilities without random shocks and with cumulative shocks
(at year 80)

State	Probability without random shocks	Probability with cumulative shocks	Relative difference
Initial	3.52e-3	9.94e-3	184.11%
Micro-crack	0.9959	0.9704	-2.56%
Circumferential crack	3.05e-4	7.05e-3	2210.16%
Radial crack	1.00e-4	7.52e-3	7419.00%
Leak	1.30e-5	2.76e-3	21161.54%
Rupture	2.06e-4	2.70e-3	1212.62%

As for the case with random shocks, cumulative shocks have a similar influence on the state probabilities. In Fig. 11, we compare the estimated component reliability with cumulative shocks with the other two estimated probabilities of Fig. 7. At year 80, the estimated component reliability with cumulative shocks is 0.9973 and the sample variance equals to 2.69e-9. Considering cumulative shocks only, the component reliability has decreased by 0.26%.

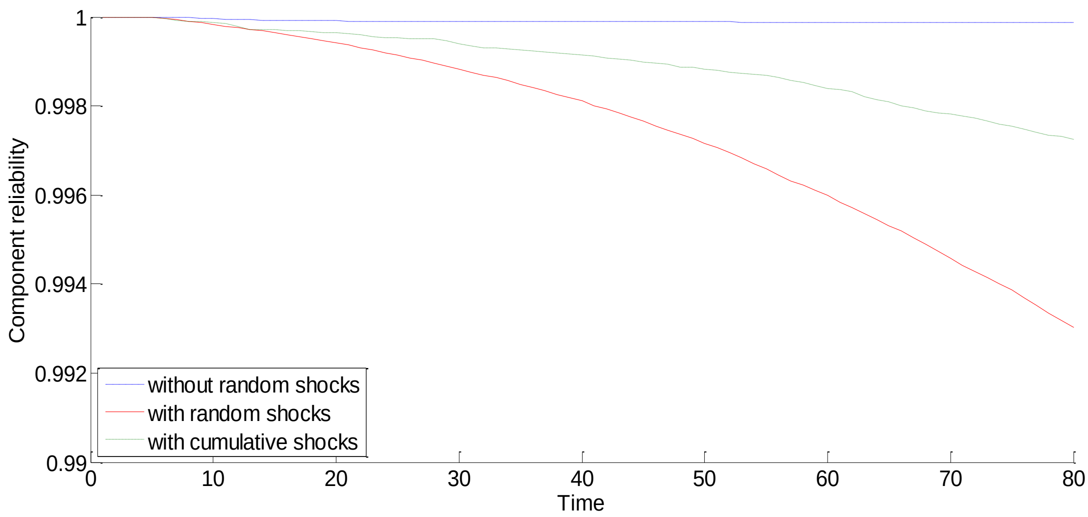

Fig 11. Component reliability w/o random shocks and with only cumulative shocks.

6.3 Sensitivity analysis

With the model specifications of Section 6.1, two important parameters are: the constant δ in $p_{\mathrm{i,m}}(\tau'_{\mathrm{i,m}})$ and the relative increment ε in $\lambda_{i,j}^{(m)}(\tau'_{i,m},\boldsymbol{\theta})$. To analyze the sensitivity of the component reliability estimates to these two parameters, we take values of δ within the range [0.0001, 0.0002] and ε within the range [0.2, 0.4].

Fig. 12 shows the estimated component reliabilities with different combinations of the two parameters. In general, the component reliability decreases when any of the parameters increases. In fact, higher δ in $p_{\mathrm{i,m}}(\tau'_{\mathrm{i,m}})$ leads to higher probability of the random shock being extreme, which is more critical to the component, and higher relative increment ε in $\lambda_{i,j}^{(m)}(\tau'_{i,m},\boldsymbol{\theta})$ results in larger degradation transition rates. We can also see from the Figure that in this situation, when the same percentage of variation applies to the two parameters, ε is more influential than δ on the component reliability. The corresponding variances of the estimated component reliability computed using equation (19) are shown in Fig. 13, where it is seen that the high reliability estimates have low variance levels.

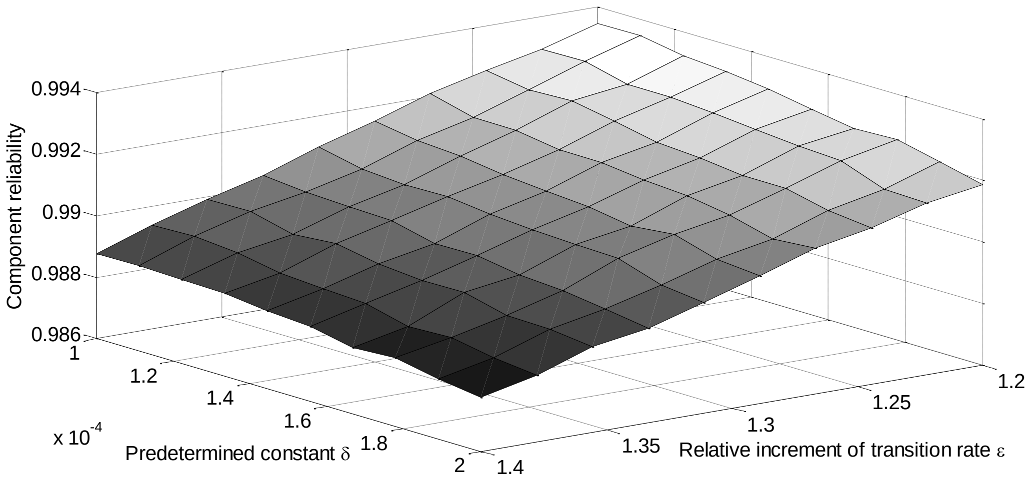

Fig 12. Component reliability estimate as a function of ε and δ (at year 80).

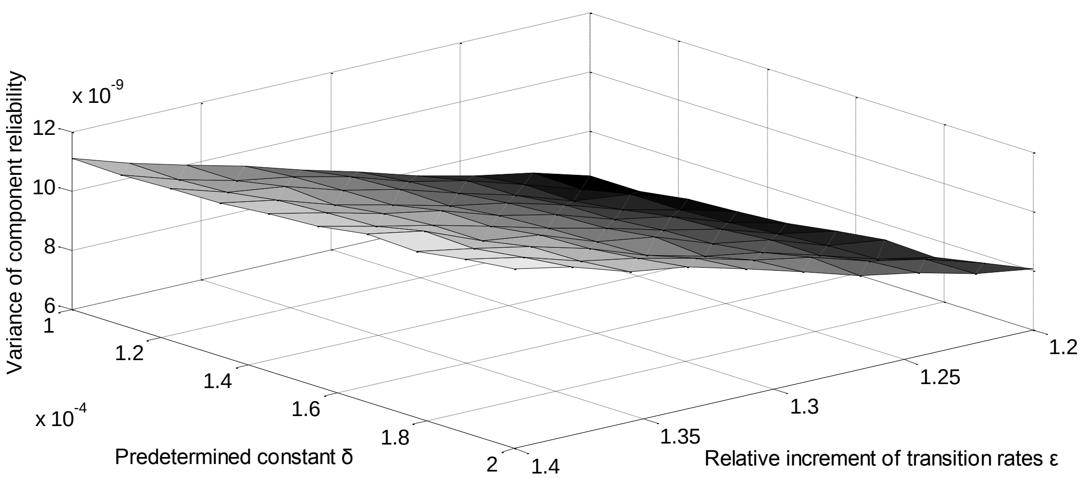

Fig 13.Variance of component reliability estimate as a function of ε and δ (at year 80).

7. CONCLUSIONS

An original, general model of a degradation process dependent on random shocks has been proposed and integrated into a MSPM framework with semi-Markov processes, which also considers two types of random shocks: extreme and cumulative. General dependences between the degradation and the effects of shocks can be considered.

A literature case study has been illustrated to show the effectiveness and modeling capabilities of the proposal, and a crude sensitivity analysis has been applied to a pair of characteristic parameters newly introduced. The significance of the findings in the case study considered is that our extended model is able to characterize the influences of different types of random shocks onto the component state probabilities and the reliability estimates.

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