Evaluating the Load-Bearing Capacity of Wood Elements in Early 19th-Century France

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1. INTRODUCTION

Whenever the issue of preservation has to be considered in relation to monumental buildings, the history of the structure has to be deeply studied, starting from the analysis of the original conceptual design and considering modifications and interventions occurred in its lifetime. This kind of study should cover as well rules and specific criteria employed for the design; this holds in particular in the case of timber structures dating back to the 18th and 19th centuries, at the time when design criteria were progressively changing from the traditional heuristic approach to a new scientifically based knowledge. Within this renovation process, both experimental and theoretical research were promoted, thus opening the way to new design criteria; the process extended over a period of time that is well defined, from Galileo's formulation of the scientific method (1638) to Navier's expression of the theory of bending (1826).

As it is well known, Galileo (1638) clearly understood the need of assigning proper dimensions to the elements constituting the structural scheme; in other words, he introduced the quantitative approach to design. Moreover, Galileo stated the double role of the experimental research in both suggesting and verifying physical laws. A classical application of his method refers to the problem of beam bending; the solution he proposed for the bearing capacity, referred to as *Galileo's rule*, established the reference point for the long debate that took place among researchers for almost two centuries. In this rule, the parameters governing the problem are correctly considered; what is wrong is the multiplication factor, which depends on the stress distribution over the transversal section.

The debate that followed involved theoretical and experimental contributions as well; it had a major reference point at the *Ecole des Ponts et Chaussées* in France and is widely reflected in a large number of engineering and architecture treatises of the time. All such studies had to face a double difficulty, as the knowledge of the equilibrium conditions was still limited, both at the element level in terms of load typology, support conditions and span length, and at the level of the transversal section as well, in terms of stress distribution.

In parallel to the theoretical studies, a wide experimental activity had started. Two distinct trends can be easily recognized in this kind of works: in some cases experimental tests were performed in order to support theoretical knowledge, according to Galileo's method; in other cases the aim simply consisted in the compilation of load capacity tables for timber beams, differing in length and transversal section, subject to specific load patterns. These tables were intended to directly support the design activity of builders.

All of these studies were carried on for more than 100 years through the entire 18th century, and reached a conclusion with Navier's work (1826), when the correct solution to the problem of beam bending was formulated. This formulation was expressed in simple terms, suitable for practical use in design.

The research here presented is an attempt to move through the long search for the correct approach to the design of timber beams, highlighting a few meaningful contributions selected from the numerous French treatises written in the 18th century; they provide an overview of the variety of the design criteria in

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use at the time. Finally, Navier's formulation is discussed and the first documented application to the verification of a bridge beam in France is presented.

2. LOAD-BEARING CAPACITY COMPARISON DESIGN RULE

One of the first documented computations of the bearing capacity of a timber beam may be found in the manuscript *Calcul de la Résistance des Jambes de Force Doublées Pour Chaque Coté de la Longueur du Pont*, now at the *Fonds Ancien* of the *Ecole des Ponts et Chaussées* in Paris (École Nationale des Ponts et Chaussées, Fonds Ancien 1793). In this application, the design problem is relative to a beam spanning 15 feet (4.87 m) and with a cross-section 12 inches (0.33 m) wide and 30 inches (0.81 m) deep. The beam is simply supported and loaded at the mid-span.

According to a procedure which was common at the time, the load-bearing capacity is determined by comparison with an experimental reference case, which, in this case, presents different support conditions: a cantilever beam, 7 feet 8 inches (2.49 m) long with square cross-section of 2 inches wide (0.054 m $\times$ 0.054 m), loaded at the free end. The collapse load for the cantilever had been found to be 185.5 lbs (0.91 kN); however, a lower load value was considered allowable in service and utilized in fact for the computation of capacity: 22 lbs (0.11 kN). The ratio between the two loads, which is close to 8, is in line with the values proposed some years later by Jean Baptiste Rondelet in his *Traité Théorique et Pratique de l'Art de Bâtir* (1802) and by Claude Louis Navier (1826).

In the manuscript, the procedure followed to link the bearing capacities of the two structural elements is not explained; a reasonable interpretation, however, has been found and is presented in the following. In the spirit of Galileo's rule, in order to evalu-ate the capacity of the simply supported beam, the author makes use of a proportion between the geometric dimensions of the reference beam and the one under study. Specifically, geometric data and loads are in the proportion:

$$B \cdot H^2 : P = b \cdot h^2 : p \tag{1}$$

where b and h are the width and the depth of the specimen cross-section, respectively, and p is the capacity; B and H are the width and the depth of the cross-section of the element for which the capacity P has to be computed.

From this proportion, a value of 29700 lbs (145.38 kN) is found for the load capacity, P. In the calculation, however, consideration is neither given to the different lengths of the two elements nor to the different support conditions; the result, therefore, is wrong. With respect to the cantilever, indeed, the span of the simply supported beam is about the double, and the bearing capacity, therefore, is also the double.

Another interesting manuscript, *Pont de Bois: Question et Majeure et Neuve* (École Nationale des Ponts et Chaussées, Fonds Ancien 1798) kept in the same archive of the *Ecole des Ponts et Chaussées*, also deals with the bending strength of a wood element. Contrary to the former manuscript author, the author of this manuscript demonstrates to know that in the com-putation it is necessary to consider the length of the elements. This author affirms that the strength is directly proportional to the product of the width times the square of the depth of the cross-section and inversely proportional to the length, thus recalling the concept expressed in Galileo's rule (1638).

3. ANALYTICALLY INTERPRETING EXPERIMENTAL DATA

In the first decades of the 19th century, numerous trea-tises of architecture and engineering were published and widely diffused. They became the collection point for both mathematical studies and experimental results, equally fundamental for the development of new formulations. The works that comprehend both aspects, that is, the theoretical formulation and the experimentation, are particularly interesting. Those works by Jean-Baptiste Rondelet, Jean Henri Hassenfratz, and Claude Louis Navier are presented here in this perspective.

The *Traité de l'Art de Bâtir* by Jean-Baptiste Rondelet (1743–1829), published in 1802, constitutes a significant step forward in the painstaking path toward formalizing a science of building structures; it contains good intuitions, which are not yet translated into final rules. In order to evaluate the effect of the beam span on the load-bearing capacity, Rondelet considers the experimental results obtained by George Leclerc Comte de Buffon (1741) from a series of specimens with a square cross-section of 5 inches wide (0.135 m $\times$ 0.135 m) and with length values varying between 7 and 28 feet (2.27 m and 9.1 m). Such experimental results are represented with a solid line in Figure 1, while those corresponding to the analyti-cal formulation for bending strength (R_b) proposed by Rondelet (Equation 2) are drawn with a dashed line:

$$R_b = \frac{a - \frac{b}{3} \cdot e \cdot e}{b} \tag{2}$$

where a indicates the tension strength, then called *primitive force*, and b indicates the ratio between the element length, l, the depth of the cross-section, e; with the solid line the results of the experimental tests by Buffon are reported. The value of the primitive or absolute force adopted by Rondelet is 57.32 MPa.

The two curves differ minimally: according to Rondelet, this difference is due to not having a constant strength value for all the test specimens and the analytical computations. In a first attempt, Rondelet expressed the bending strength according to the indications of Galileo:

$$R_b = \frac{1}{2} \cdot \sigma_t \cdot \frac{b \cdot h^2}{l} \tag{3}$$

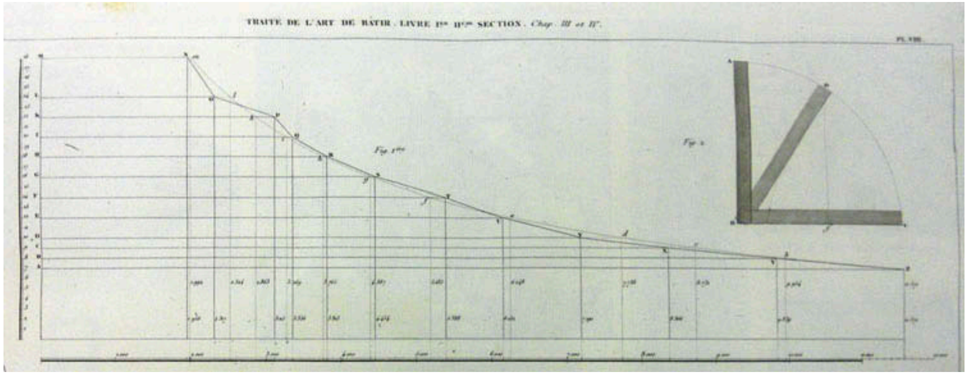


FIG. 1. Bending resistance depending on specimen length.

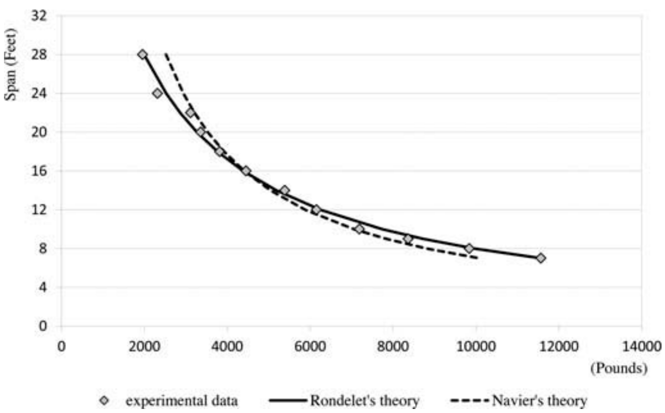


FIG. 2. Experimental and theoretical data in bending.

3.1. Capacity Tables: Jean Henri Hassenfratz

The contribution of the *Traité de l'Art de la Charpenterie*, by Jean Henry Hassenfratz (1804) is of fundamental importance for the design practice. Such treatise is organized in two parts: the first is further subdivided into five chapters. Paragraph VI of the first chapter deals with the mechanical strength of wood. The first two subparagraphs examine the aspect of the *horizontal strength*, that is, *bending*.

Hassenfratz (1804) indicates the strength of a wood element in bending as proportional to the width (b) and to the square of the depth (h) of the cross-section, and inversely proportional to the length (l) in line with Galileo's formulation for the bending capacity (R):

$$R = k \cdot \frac{b \cdot h^2}{l} \tag{4}$$

where b is the width of the cross section, h is the depth l the length and σ_t is the tensile resistance, but he later dis-carded it, for not being, in his opinion, close enough to the real data.

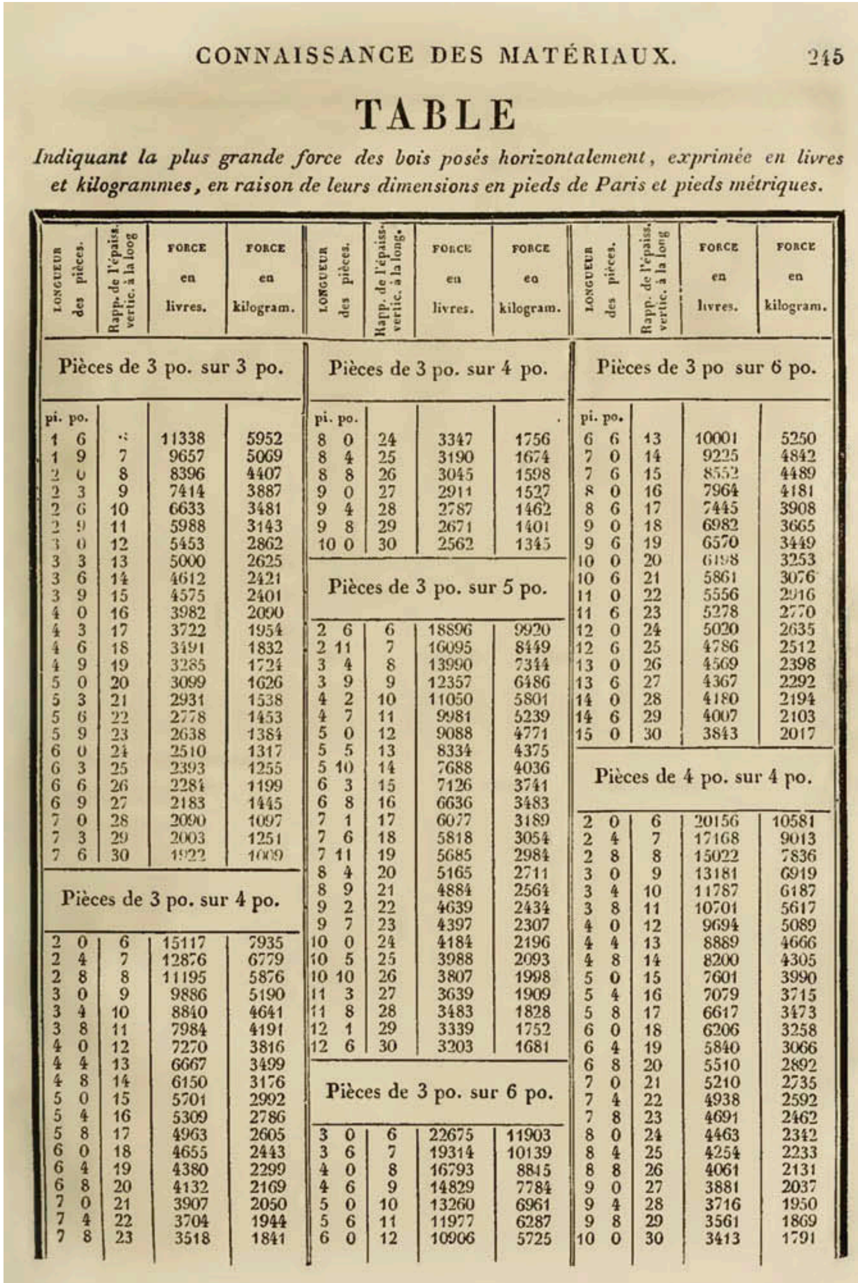
Figure 2 shows the experimental data, the correct theoretical curve based on Navier's theory, drawn on the mean value of experimental strengths, and the theoretical curve computed by Rondelet according to Equation (2), which in spite of its evident limits interpolates surprisingly well the test data.

The experimental results relative to elements with length within 2 and 10 feet (0.65 m and 3.25 m) were organized by Rondelet (1802) in five tables for use in design practice; one of them, referring to oak wood tests is reported in Figure 3. A suggestion, presented in the comments, is particularly worth noting: in order to act in favor of safety, only 1/10 of the value reported in the tables should be adopted for the design strength, by simply eliminating the last digit. This indication, later recon-sidered by Navier (1826), acknowledges the difference between the value of strength at collapse and what may be adopted in service, the two being assumed to differ by an order of magni-tude. This indication is in agreement with what put into practice by the author of the previous manuscript.

The purpose of the work by Hassenfratz (1804) is to find a mean value for the constant k depending on the constraints typology and on wood resistance, that may confirm the validity of the for-mula to express the bearing capacity of wood elements and for organizing the theoretical data into tables suitable for practical use.

According to Hassenfratz (1804), both theory and experience agree in demonstrating that different modalities of constraint on the specimens determine different values of capacity, even if the pertinent coefficients had not yet been correctly expressed in all cases. So, on the one side, Hassenfratz (1804) knows that the load carrying capacity of a simply supported specimen, as indi-cated in drawing 10 of Figure 4, can be doubled if rotations are restrained at the supports, as in drawings 13 and 14. On the other side, about 10 years after the French manuscript and in spite of the many experimental data, Hassenfratz is still convinced that the load provoking failure on the simply supported specimen shown in drawing 10 is the same that causes failure of an ele-ment half its length, built-in at one extreme, loaded at the free end (drawing 18).

A series of 20 tables, one of which is shown in Figure 5, reports the strength values for oak wood elements of different



CONNAISSANCE DES MATÉRIAUX. 245

TABLE

Indiquant la plus grande force des bois posés horizontalement, exprimée en livres et kilogrammes, en raison de leurs dimensions en pieds de Paris et pieds métriques.

Pièces de 3 po. sur 3 po.

LONGUEUR des pièces. (pi. po.)	Rapp. de l'épaiss. vertic. à la long	FORCE en livres.	FORCE en kilogram.
1 6	[illegible]	11338	5952
1 9	7	9657	5069
2 0	8	8396	4407
2 3	9	7414	3887
2 6	10	6633	3481
2 9	11	5988	3143
3 0	12	5453	2862
3 3	13	5000	2625
3 6	14	4612	2421
3 9	15	4575	2401
4 0	16	3982	2090
4 3	17	3722	1954
4 6	18	3491	1832
4 9	19	3285	1724
5 0	20	3099	1626
5 3	21	2931	1538
5 6	22	2778	1453
5 9	23	2638	1384
6 0	24	2510	1317
6 3	25	2393	1255
6 6	26	2284	1199
6 9	27	2183	1445
7 0	28	2090	1097
7 3	29	2003	1251
7 6	30	1922	1009

Pièces de 3 po. sur 4 po.

LONGUEUR des pièces. (pi. po.)	Rapp. de l'épaiss. vertic. à la long	FORCE en livres.	FORCE en kilogram.
2 0	6	15117	7935
2 4	7	12876	6779
2 8	8	11195	5876
3 0	9	9886	5190
3 4	10	8840	4641
3 8	11	7984	4191
4 0	12	7270	3816
4 4	13	6667	3499
4 8	14	6150	3176
5 0	15	5701	2992
5 4	16	5309	2786
5 8	17	4963	2605
6 0	18	4655	2443
6 4	19	4380	2299
6 8	20	4132	2169
7 0	21	3907	2050
7 4	22	3704	1944
7 8	23	3518	1841
8 0	24	3347	1756
8 4	25	3190	1674
8 8	26	3045	1598
9 0	27	2911	1527
9 4	28	2787	1462
9 8	29	2671	1401
10 0	30	2562	1345

Pièces de 3 po. sur 5 po.

LONGUEUR des pièces. (pi. po.)	Rapp. de l'épaiss. vertic. à la long	FORCE en livres.	FORCE en kilogram.
2 6	6	18896	9920
2 11	7	16095	8449
3 4	8	13990	7344
3 9	9	12357	6486
4 2	10	11050	5801
4 7	11	9981	5239
5 0	12	9088	4771
5 5	13	8334	4375
5 10	14	7688	4036
6 3	15	7126	3741
6 8	16	6636	3483
7 1	17	6077	3189
7 6	18	5818	3054
7 11	19	5685	2984
8 4	20	5165	2711
8 9	21	4884	2564
9 2	22	4639	2434
9 7	23	4397	2307
10 0	24	4184	2196
10 5	25	3988	2093
10 10	26	3807	1998
11 3	27	3639	1909
11 8	28	3483	1828
12 1	29	3339	1752
12 6	30	3203	1681

Pièces de 3 po. sur 6 po.

LONGUEUR des pièces. (pi. po.)	Rapp. de l'épaiss. vertic. à la long	FORCE en livres.	FORCE en kilogram.
3 0	6	22675	11903
3 6	7	19314	10139
4 0	8	16793	8845
4 6	9	14829	7784
5 0	10	13260	6961
5 6	11	11977	6287
6 0	12	10906	5725
6 6	13	10001	5250
7 0	14	9225	4842
7 6	15	8552	4489
8 0	16	7964	4181
8 6	17	7445	3908
9 0	18	6982	3665
9 6	19	6570	3449
10 0	20	6198	3253
10 6	21	5861	3076
11 0	22	5556	2916
11 6	23	5278	2770
12 0	24	5020	2635
12 6	25	4786	2512
13 0	26	4569	2398
13 6	27	4367	2292
14 0	28	4180	2194
14 6	29	4007	2103
15 0	30	3843	2017

Pièces de 4 po. sur 4 po.

LONGUEUR des pièces. (pi. po.)	Rapp. de l'épaiss. vertic. à la long	FORCE en livres.	FORCE en kilogram.
2 0	6	20156	10581
2 4	7	17168	9013
2 8	8	15022	7836
3 0	9	13181	6919
3 4	10	11787	6187
3 8	11	10701	5617
4 0	12	9694	5089
4 4	13	8889	4666
4 8	14	8200	4305
5 0	15	7601	3990
5 4	16	7079	3715
5 8	17	6617	3473
6 0	18	6206	3258
6 4	19	5840	3066
6 8	20	5510	2892
7 0	21	5210	2735
7 4	22	4938	2592
7 8	23	4691	2462
8 0	24	4463	2342
8 4	25	4254	2233
8 8	26	4061	2131
9 0	27	3881	2037
9 4	28	3716	1950
9 8	29	3561	1869
10 0	30	3413	1791

FIG. 3. Jean Baptiste Rondelet. Bending tests results. Ultimate load.

length, simply supported, and loaded at midspan. The width of the cross-section is indicated in the first column, while the depth in the first line; the length, ranging from 1 m to 15 m, is in the table heading. Dimensions are indicated in meters.

Analyzing the data in the table, it is evident that Hassenfratz (1804) used Equation (4) adopting the same coefficient, k, for computing the bearing capacity of all the elements. The value adopted for this coefficient is 500. Since the same load and constraint conditions apply to all the specimens, it is possi-ble to determine the value of the normal stress starting from k. Given that:

$$k = \frac{2}{3} \cdot \sigma \tag{5}$$

the reference value of σ is about 75 MPa, a rather high value, but one in line with what indicated by the European Standard (EN) 338 (2002) for oak of good quality.

3.2. An Easily Accessible Bending Theory

The long search for a solution to the problem of bending found accomplishment with Claude Louis Navier (1785–1836),

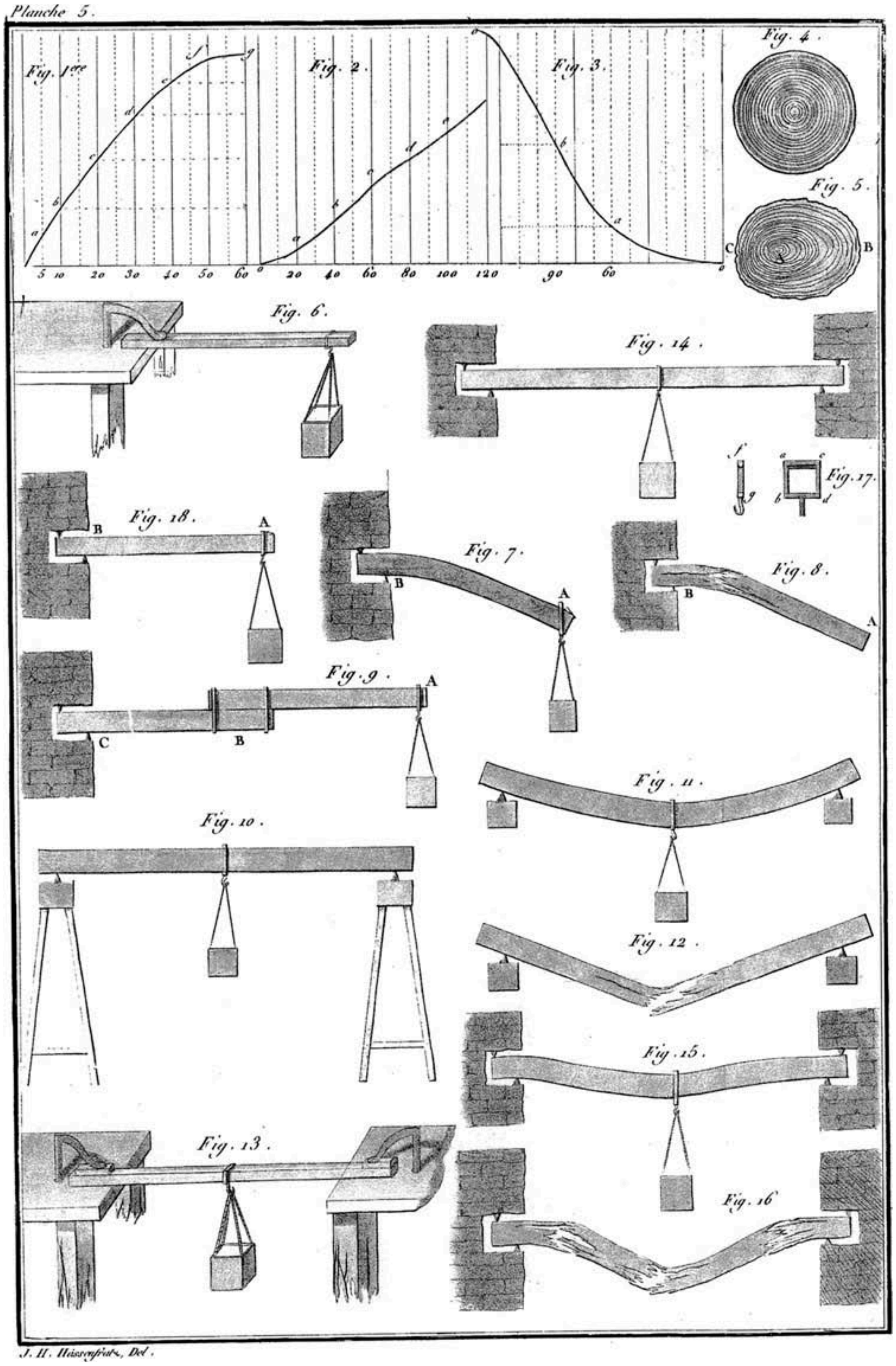


FIG. 4. Bending resistance.

who was able to express it in terms correct and at the same time suitable for current practice; in his teaching activity at the *Ecole des Ponts et Chaussées*, indeed, he was able to perform a synthesis of all the previous research, making the correct analytical formulation of the problem accessible and easily applicable. His approach to bending is discussed in the *Résumé des leçons* (Navier 1826), the class notes of the course of applied mechanics written for his students. The notes became available starting

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Résistance du bois de chêne de 2 mètres de longueur.

HAUTEUR.

Largeur.	002	004	006	008	010	012	014	016	018	020	022	024	026	028	030
0,02	20	80	180	320	500	720	980	1280	1620	2000	2420	2880	3380	3920	4500
0,04	40	160	360	640	1000	1440	1960	2560	3240	4000	4840	5760	6760	7840	9000
0,06	60	240	540	960	1500	2160	2940	3840	4860	6000	7260	8640	10140	11760	13500
0.08	80	320	720	1280	2000	2880	3920	5120	6480	8000	9680	11520	13520	15680	18000
0,10	100	400	900	1600	2500	3600	4900	6400	8100	10000	12100	14400	16900	19600	12500
0,12	120	480	1080	1920	3000	4320	5880	7680	9720	12000	14520	17280	20280	23520	27000
0,14	140	560	1260	2240	3500	5040	6860	8960	11340	14000	16940	20160	23660	27440	31500
0,16	160	640	1440	2560	4000	5760	7840	10240	12960	16000	19360	23040	27040	31360	36000
0,18	180	720	1620	2880	4500	6480	8820	11520	14580	18000	21780	25920	30420	35280	40500
0,20	200	800	1800	3200	5000	7200	9800	12800	16200	20000	24200	28800	33800	39200	45000
0,22	220	880	1980	3520	5500	7920	10780	14080	17820	22000	26620	31680	37180	43120	49500
0 24	240	960	2160	3840	6000	8640	11760	15360	19440	24000	29040	34560	40560	47040	54000
0,26	260	1040	2340	4160	6500	9360	12740	16640	21060	26000	31460	37440	43940	50960	58500
0,28	280	1120	2520	4480	7000	10080	13720	17920	22680	28000	33880	40320	47320	54880	63000
0,30	300	1200	2700	4800	7500	10800	14700	19200	24300	30000	36300	43200	50700	58800	67500

FIG. 5. Resistance of 2-mt long oak beam of different cross sections.

from 1819 as a lithographed text among students and were subsequently printed in 1826.

In the *Résumé* the discussion of the problem of bending follows the study of the element in compression and in tension. Navier clearly expresses the concept that the section, when subjected to bending, rotates around the neutral axis remaining plane. From this assumption, he expresses the deformation at the generic point of the cross-section as "the proportion according to which the fiber elongated" (Navier 1826, Part 1, Article III, paragraph 77), namely:

$$\frac{v}{r} \tag{6}$$

where r is the radius of curvature and v is the quote of the point with respect of the neutral axis. From Equation (6) and by indicating with E the elastic modulus of the material it is possible to express the normal stress R as:

$$R = E \cdot \frac{v}{r} \tag{7}$$

The radius of curvature is, thus, the unknown reference parameter through which all the others are defined.

Navier then deals with the equilibrium conditions. The equilibrium to horizontal translation, expressed by the equality of resultants from compression and tension stresses, leads to the following relation:

$$\int_0^b du \cdot \int_0^{f_1,u} dv \cdot v = \int_0^b du \cdot \int_0^{f_2,u} dv \cdot v \tag{8}$$

where u and v indicate the abscissa and the ordinate, respectively, of a particle of the cross-section, while functions $f_{1,u}$ and $f_{2,u}$ describe the profiles of the two portions of the cross-section, above and below the neutral axis, respectively as in Figure 6. The equation states the position of the neutral axis at the centroid.

Lastly, Navier (1826) applies rotational equilibrium, making reference to the specific case of a cantilever of length a with a concentrated load P at the free end; at distance x from the fixed end, equilibrium requires:

$$\frac{E}{r} \cdot \left(\int_0^b du \cdot \int_0^{f_1,u} dv \cdot v^2 + \int_0^b du \cdot \int_0^{f_2,u} dv \cdot v^2 \right) = P \cdot (a - x) \tag{9}$$

while recognizing the moment of inertia I as:

$$I = \left(\int_0^b du \cdot \int_0^{f_1,u} dv \cdot v^2 + \int_0^b du \cdot \int_0^{f_2,u} dv \cdot v^2 \right) \tag{10}$$

The solution is thus expressed as a function of the external moment ρ in the form:

$$\frac{1}{r} = \frac{\rho}{EI} \tag{11}$$

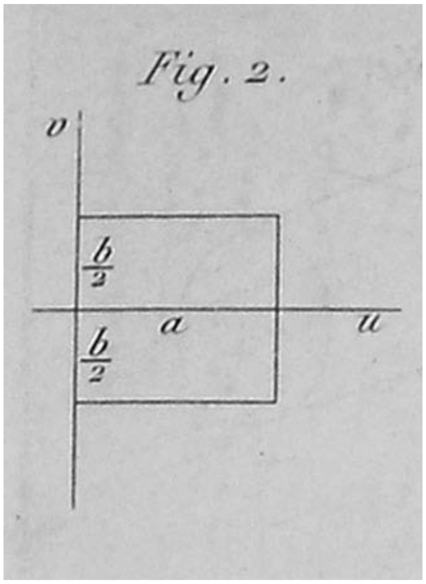

FIG. 6. Cross-section of a cantilever beam.

Navier underlines the importance of the bending stiffness, called *flexure moment* $\varepsilon = \mathrm{E\,I}$, a quantity that "has for each body a value depending from the nature of the body itself and from the shape of the cross-section" (Navier 1826, Part 1, Article III, paragraph 78). For the case where the solid has rectangular cross-section with width b and depth c:

$$\varepsilon = 2E \int_0^b du \cdot \int_0^{\frac{c}{2}} dv \cdot v^2 = E \cdot \frac{b \cdot c^3}{12} \tag{12}$$

In the 1826 *Résumé*, after presenting the theoretical formulation of the problem of bending, Navier discusses some applications: starting from experimental results obtained by other researchers, he derives the values of both bending strength and modulus of elasticity for several materials, first of all for wood.

For the wood element in bending, Navier makes reference to results by Henri Louis Duhamel du Monceau (1768) Charles Aubry (1790), Charles Dupin (1815), Jean-Baptiste Rondelet (1802), Peter Barlow (1826), and Thomas Tredgold (1820). Figure 7 reports, as an example, some of the experimental data published by Tredgold. In this table a and c are two coefficients experimentally determined to be applied in practice in order to evaluate the load-bearing capacity; c depends on the constraints typology and on wood strength and beam deflection:

$$P = \frac{b \cdot h^2}{l} \cdot c \tag{13}$$

and a is a coefficient experimentally determined in order to evaluate beam deflection d that according to Tredglod is expressed as:

$$d = \frac{a \cdot P \cdot l^3}{40 \cdot b \cdot h^2} \tag{14}$$

Using, for instance, maximum deflection measures, f, of a simply supported beam of span $2a$, loaded at midspan with a load $2P$, Navier is in condition to compute the modulus of elasticity of a section having width b and depth c as:

$$E = 2P \cdot \frac{(2a)^3}{4 \cdot b \cdot c^3 \cdot f} \tag{15}$$

after combining the expression of the *flexure moment* in Equation (12) with that of the maximum deflection:

$$f = \frac{2P}{\varepsilon} \cdot \frac{(2a)^3}{48} \tag{16}$$

Analogously, Navier proceeds to evaluate the normal stress, R, at ultimate, assuming linear behavior of the material up to failure. In order to express the moment, ρ (Equation (11)), as a function of the maximum normal stress, R (Equation (7)), he obtains:

$$\rho = \frac{2R}{v'} \cdot \int_0^b du \cdot \int_0^{f,u} dv \cdot v^2 \tag{17}$$

which may be synthetically rewritten as:

$$\rho = R \cdot \frac{b \cdot c^2}{6} \tag{18}$$

Assuming for v' the value of $c/2$. Here, the resisting moment appears as the product of the maximum stress times the section modulus.

For the cantilever of length a, with end load P, Navier writes,

$$R \cdot \frac{b \cdot c^2}{6} = P \cdot a \tag{19}$$

from which the maximum stress becomes:

$$R = P \cdot a \cdot \frac{6}{b \cdot c^2} \tag{20}$$

The value of R is then derived from the experimental results obtained by different authors: George Leclerc Comte de Buffon (1741), an example of which is reported in the table of Figure 8, Bernard Fôret de Bélidor (1729), Jean-Baptiste Rondelet (1802) and George Buchanan (1825). After obtaining the value of numerical results for the ultimate strength, Navier (1826) adopts a reduced value in formulas for checking the design resistance, similarly to what already proposed by Rondelet (1802). This sort of allowable stress is $1/10$ of the limit strength and corresponds approximately to 6 MPa. By the same procedure,

48 RESISTANCE OF TIMBER. [SECT. II.

88 *a.—Table of Experiments on Oak from the Royal Forests.*

Name of Forest.	Specific gravity.	Length in inches.	Breadth in inches.	Depth in inches.	Depression increased with time when loaded to this degree.		At the first fracture.		Values of *a*.	Values of *c*.
					Load in pounds.	Deflexion in inches.	Load in pounds.	Deflexion in inches.		
High meadow Forest	·7926	22	·97	·96	80	0·25	400	2·9	·0175	826
Ditto	·7563	22	·95	·95	60	0·162	390	2·4	·0143	835
Parkhurst Forest (sawed)	·8770	22	·98	·95	90	0·235	370	2·6	·0138	770
Dean Forest	·747	22	·95	·95	70	0·18	340	1·15	·0137	730
Ditto	·799	22	·97	·97	80	0·155	410	1·45	·0112	820
New Forest (cleft) .	·822	22	1·0	1·0	70	0·21	410	4·0*	·0195	751
Ditto (sawed)	·723	22	1·0	1·0	80	0·112	415	1·35	·0091	760
Bere Forest (sawed)	·714	22	1·0	1·0	70	0·155	360	1·15	·0142	660
Ditto	·732	22	1·0	1·0	70	0·1	477	1·5	·0093	875
Ditto	·839	22	1·0	1·0	80	0·14	380	1·1	·0112	698
								Means	·0134	773

FIG. 7. Load-bearing capacity and deflexion of oak beam.

Navier determines also the strength and modulus of elasticity of other materials, such as wrought iron, steel, cast iron, after describing a small series of tests on stone specimens in bending.

An interesting use of the bending theory concerning wood bridges is discussed in the fourth section, *Article X*, paragraph 579. The text is subdivided in three parts, starting from the simplest case of bridges with limited span, usually composed of a deck supported by beams and struts. From equilibrium equations, Navier (1826) determines the maximum forces in the deck beam.

With reference to Figure 9, the segment D-D' is considered; the span is $2a$ and the distributed load intensity is indicated as p, hinges are assumed at D and D', resulting in a statically determined scheme.

After expressing the maximum moment as:

$$\rho = \frac{p \cdot a^2}{2} \tag{21}$$

the cross section is assumed rectangular, $b \times c$. The resulting maximum normal stress at mid-span is obtained combining the contributions of the axial load and bending moment:

$$R' = \frac{p}{b \cdot c} \cdot \left[\left(a + \frac{1}{2} a' \right) \cdot \tan\alpha + \frac{3a^2}{c} \right] \tag{22a}$$

where a and a' are the distances CD and BD, respectively; $a + a'/2$ is the influence length of the element AD; α is the angle BAD.

Equation (22a) may be rewritten as:

$$R' = \frac{p \cdot \tan\alpha \cdot (a + a'/2)}{b \cdot c} + \frac{p \cdot a^2/2}{b \cdot c^2/6} = \frac{N}{A} + \frac{\rho}{W} \tag{22b}$$

so that the first term clearly points out the contribution of axial load, N, which is divided by the cross section, A, and the second the contribution of the moment, ρ, which is divided by the section modulus, W.

4. CHECKING HEURISTIC DESIGN CRITERIA

With the application of the new building science to current design practice, the modern approach to structural design was started, which is based on the sizing of structural elements in relation to both external loads and material strength as well. The method grows by subsequent attempts, at the start still strongly bound to the construction practice of the time, which remains the main reference. Initially, only a strength check or a comparison of active and resisting moments is performed, to which new contributions and steps are progressively added. The first applications of the new tools offered by structural mechanics to design are documented in the *Annales* of the *Ecole des Ponts et Chaussées*. One of these cases, presented in the following section, refers to the Vaudreuil Bridge that had collapsed because of a flood. It was rebuilt in wood, adopting the structural scheme patented by Ithiel Town (USPTO 1820). This is the case where the new formulas were applied for checking the design, still conceived, looking at the past, on the basis of heuristic criteria and experience.

SEPTIÈME TABLE.

Comparaison de la résistance du bois, trouvée par les expériences précédentes, & de la résistance du bois suivant la règle que cette résistance est comme la largeur de la pièce, multipliée par le quarré de la hauteur, en supposant la même longueur.

* Les astérisques marquent que les expériences n'ont pas été faites.

Long.r des Pièces.	GROSSEURS.				
	4 pouces.	5 pouces.	6 pouces.	7 pouces.	8 pouces.
Pieds.	Livres.	Livres.	Livres.	Livres.	Livres.
7.	5312... 5901...	11525..	18950.. 19915 $\frac{2}{3}$..	*32280.. 31624 $\frac{1}{3}$.	48100. 47649 $\frac{1}{3}$. 47198 $\frac{2}{3}$.
8.	4550... 5011 $\frac{1}{3}$..	9787..	15525.. 16912 $\frac{4}{5}$.	26050.. 26856 $\frac{9}{10}$.	*39750. 40089 $\frac{1}{3}$.
9.	4025... 4253 $\frac{13}{17}$.	8308 $\frac{1}{3}$..	13150.. 14356 $\frac{4}{5}$.	22350.. 22798 $\frac{1}{3}$.	*32800. 34031.
10.	3612... 3648...	7125...	11250.. 12312..	19475.. 19551..	27750. 29184.
12.	2987 $\frac{1}{2}$.. 3110 $\frac{2}{3}$..	6075...	9100,. 10497 $\frac{1}{3}$.	16175.. 16669 $\frac{4}{5}$.	23450. 24883 $\frac{1}{3}$.
14.	....	5100...	7475.. 8812 $\frac{4}{5}$.	13225.. 13995 $\frac{1}{3}$.	19775. 20889 $\frac{1}{3}$.
16.	....	4350...	6362 $\frac{1}{4}$. 9516 $\frac{4}{5}$.	11000.. 11936 $\frac{2}{3}$.	16375. 17817 $\frac{1}{3}$.
18.	....	3700...	5562 $\frac{1}{2}$. 6393 $\frac{1}{3}$.	9425.. 10152 $\frac{4}{5}$.	13200. 15155 $\frac{1}{3}$.
20.	....	3225...	4950,, 5572 $\frac{4}{5}$.	8275.. 8849 $\frac{2}{3}$.	*11487 $\frac{1}{2}$. 13209 $\frac{1}{3}$.

FIG. 8. Bending tests results.

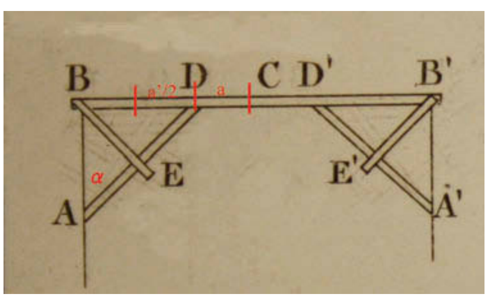

FIG. 9. Wood bridge layout.

4.1. First French Application of Navier's Bending Theory: Vaudreuil Bridge

The first examples of application of the bending theory, as documented in the previously mentioned *Annales des Ponts et Chaussées* (1841), relate to the checking of provisional structures, often ordered to permit the construction of other works, or aimed at a fast rehabilitation of masonry structures that were damaged or demolished.

The extraordinary flood of 1841 in Vaudreuil had caused the collapse of the main pier and of the two adjacent decks. The part to be recovered had a total span of about 17 m. The simplest and cheapest solution appeared to be a wood deck, which permitted to reopen transit in a fast, albeit temporary way.

The structural scheme patented by Ithiel Town in 1820 (Town 1821) in the United States and reported in Figure 10 was considered suitable for the purpose. The structure is composed of two main beams as in Figure 11, which are in turn formed by 12 St. Andrew's crosses each. The elements of this lattice are interconnected without notches, by means of oak pegs. The net width of the bridge is 3.6 m. The longitudinal chords, also in oak wood, are 23.50 m long, with cross-section of 25 cm by 15 cm; the distance between upper and lower chords is 1.45 m.

The report published in the *Annales des Ponts et Chaussées* continues with a paragraph on the resistance of beams, which usually is not present in this type of report. The paragraph follows the description of the structure and of its construction. It appears to be a final checking of the sizes of structural elements rather than part of their design. The resisting moment of the two beams is correctly expressed considering the moment of inertia of a cross-section composed of the upper and lower chord with a rigid link between them coming from the diagonals. The adopted formula makes reference explicitly to Navier's text (1826):

$$\rho = \frac{\mathrm{R} \cdot \mathrm{a} \cdot (\mathrm{b}'^3 - \mathrm{b}''^3)}{6 \cdot \mathrm{b}'} \tag{23}$$

where a is the global width of the 4 chords (4×15 cm) = 0.60 m; b' represents the total depth of the beam = 1.95 m; b'' is the clear distance between upper and lower chord = 1.45 m; R is the wood strength = 6 MPa; and ρ is the resisting moment developed by the two beams = 1343.46 kNm.

The resisting moment must be greater than or equal to the moment generated by external forces, which is correctly expressed by:

$$M = \frac{1}{2} \cdot \mathrm{p} \cdot \mathrm{c}^2 \tag{24}$$

where p is the distributed load and c is the half span.

The self-weight of the structure is 230 kN, distributed over 17 m of length, corresponding to a distributed load of 13.53 kN/m. The bridge width being 3.6 m and assuming a live load of 2 kN/m^2 resulting in 7.2 kN/m, the total load is p = 13.53 + 7.20 = 20.73 kN/m with a maximum moment of:

$$\mathrm{M} = \frac{1}{2} \cdot \mathrm{p} \cdot \mathrm{c}^2 = \frac{20.73 \cdot 8.5^2}{2} = 748.87\,\mathrm{kNm} \tag{25}$$

Comparing results, it is evident that the moment due to loads is about 55% of the resisting moment.

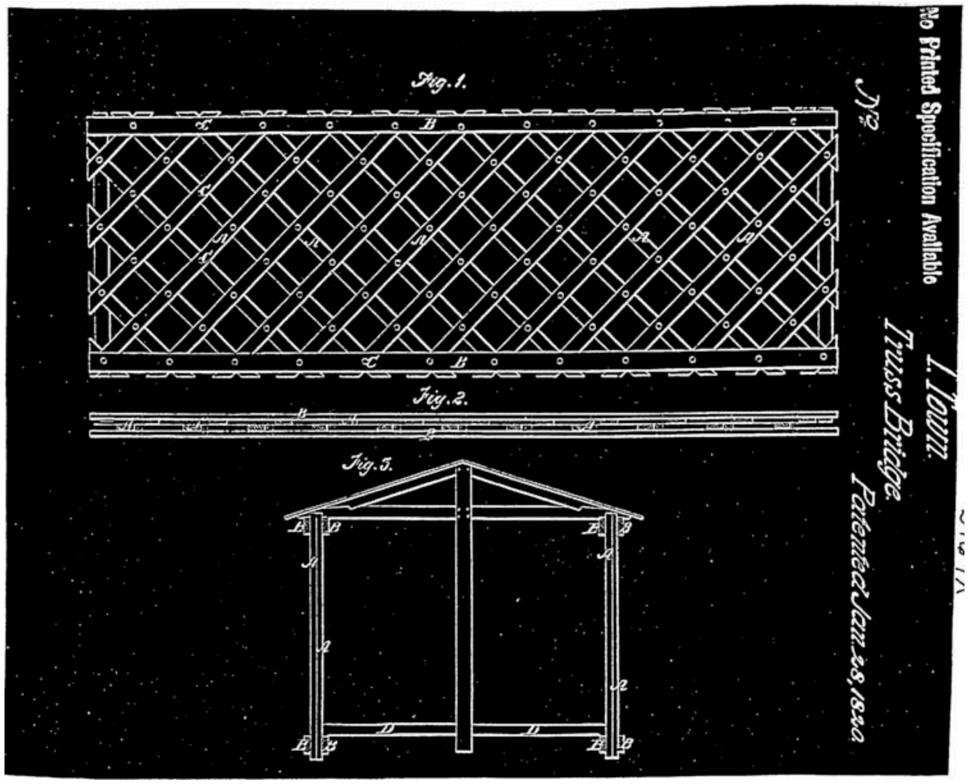

FIG. 10. Ithiel Town's 1820 Patent. United States Patent and Trademark Office. Patent n° 3169X.

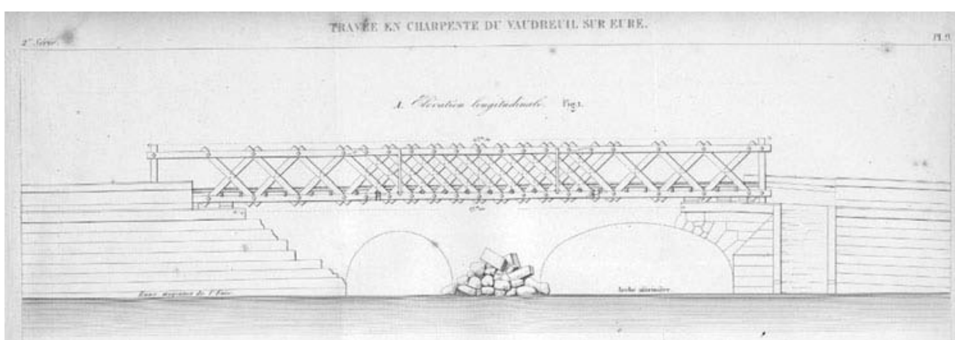

FIG. 11. Vaudreuil bridge.

The check of bending is formally correct, yet neither a shear check, that is a check of diagonal elements, nor a check of deformability were performed. The bridge was opened to traffic in about 3 weeks, but after being in service for 6 days, a mid-span deflection of approximately 8 cm, about 1/200 the deck length, was measured. Monitoring of the deck displacements was continued for some time; 6 weeks later, the deflection had doubled to a 1/100 of the span. In order to increase stiffness and avoid further deflections the decision to add five St. Andrew's crosses in the central area, was taken.

5. CONCLUSIONS

The modern approach to structural design dates back to the beginning of the 17th century, when Galileo (1638) formulated the need for a quantitative approach to the problem, i.e., the necessity to define the size of the cross-section of structural elements in relation to both the applied loads and the material resistance.

In the present work, this problem has been analyzed with reference to the theory of bending, discussing the two centuries long search for a correct solution to the question originally raised by Galileo. In that period of time, the advancement of research went through some innovations and, at the same time, through mistakes. Researchers, indeed, had to face the difficult task of operating a distinction among several new interesting concepts: material strength on the one side and global load carrying capacity on the other side; the role of the cross-section dimensions and the possibility of setting proportions between the capacities of different sections; collapse load rather than design load. All these studies had a constant reference point: the experimental activity which, starting with Galileo, always maintained a fundamental role. The long debate was mainly referred to one specific material, i.e., timber, because of its capability to resist both in tension and compression. In the 19th century, steel will take the place of timber; also, structural schemes developed with reference to timber will be extended to steel structures. The study shows how, as the new approach to design started, the major concern was for checking bending resistance; deformability verifications, although important, will come later. In the same way, also the concern for shear verification belongs to a subsequent phase.

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