Non-fragile H_∞ control for switched stochastic delay systems with application to water quality process

Dong Wang[1,*,†], Peng Shi[2,3], Wei Wang[1] and Hamid R. Karimi[4]

[1]*Faculty of Electronic Information and Electrical Engineering, Dalian University of Technology, Dalian, 116024, China*

[2]*School of Engineering and Science, Victoria University, Melbourne, Vic 8001, Australia*
[3]*School of Electrical and Electronic Engineering, The University of Adelaide, Adelaide, SA 5005, Australia*
[4]*Department of Engineering, Faculty of Engineering and Science, University of Agder, N-4898 Grimstad, Norway*

SUMMARY

In this paper, the problem of non-fragile observer-based H_∞ control for discrete-time switched delay systems is investigated. Both data missing and time delays are taken into account in the links from sensors to observers and from controllers to actuators. Because data missing satisfies the Bernoulli distribution, such problem is transformed into an H_∞ control problem for stochastic switched delay systems. Average dwell time approach is used to obtain sufficient conditions on the solvability of such problems. A numerical exam-ple and a real example for water quality control are provided to illustrate the effectiveness and potential applications of the proposed techniques.

Received 28 February 2012; Revised 29 November 2012; Accepted 8 December 2012

1. INTRODUCTION

Switched systems have attracted considerable attention from various communities of science and engineering during the past few years [1–4]. A switched system is composed of some subsystems with the continuous states and a switching signal which specifies a subsystem being activated at some time instant. In real applications, time delay is also inevitable because of the signal transmission in the long links, mechanical structures, and so on. Therefore, switched systems with state delays are extensively studies in the references [5–12] and the references therein. Switched Lyapunov function (SLF) approach, see [13], is used for analysis and synthesis of switched delay systems in [9–11]. As pointed out in [10], when SLF is employed to deal with the terms in Lyapunov function associated with time delays, these relevant Lyapunov functions are common. As is well-known, common Lyapunov function may not exist or be difficult to find [3]. To circumvent this obstacle, an average dwell time (ADT) technique [14] is used to cope with the Lyapunov function related to time delays [12, 15, 16]. It is indicated that the constructed Lyapunov function is piecewise, that is, a corresponding Lyapunov function is selected for each subsystem. Therefore, ADT approach has become a powerful and effective tool for tackling stability problem of switched delay systems.

As a matter of fact, the signal transmission in the wires or mechanical structures, for instance, do not only give rise to time delays but also bring about the measurement data missing [17–19].

*Correspondence to: Dong Wang, Faculty of Electronic Information and Electrical Engineering, Dalian University of Technology, Dalian, 116024, China.

†E-mail: wangdongandy@gmail.com

This is also a source to destroy the system performance and its effect on the stability is necessarily evaluated. The filter design problem of switched delay systems with missing measurements is studied in [20]. In [21], missing measurements from sensors to filters is investigated for the filter design problem of switched delay systems. Similarly, the measurement missing may happen from sensors to controllers and from controllers to actuators when the problem of controller design is considered. And even when system states are not able to be directly measured but estimated by the observers, it is possible to take place such data missing from sensors to observers. These emphasize the importance of taking the data missing into account. On the other hand, in practice, uncertainties always exist in observers and controllers due to the imprecision inherent in analog systems and the need for additional tuning of parameters in the implementation [22–25]. Meanwhile, the real systems are often affected by the unknown disturbances. An effective method has been pursued, namely, the design of the control law that guarantees the effect of disturbances on the systems to be under a desired level and the closed-loop systems to be asymptotically stable in absence of the disturbances. In time-domain formulation, this is called as H_∞ control problem, which effectively defines how the effect of an unknown but bounded disturbance on the systems can be measured. So far, to the best of our knowledge, no results are reported with regard to observer-based non-fragile H_∞ control problem for switched delay systems in the presence of missing data and such problem still remains challenging and open.

The main contribution of this paper is to solve the problem of non-fragile H_∞ control for switched delay systems. Time delays and data missing are simultaneously taken into account in two links from sensors to observers and from controllers to actuators. Data missing is described by the Bernoulli distribution in the probability space. The uncertainties of the controllers and observers are considered as well. Then, we transform the problem under consideration into a problem of non-fragile H_∞ control for stochastic switched delay systems on the base of the observers. Piecewise Lyapunov–Krasovskii functions are constructed by combining ADT technique such that delay-dependent sufficient conditions are obtained for the existence of non-fragile observers and controllers for discrete-time switched delay systems, which are great improvements compared with the results in [9, 21]. Two examples (a numerical example and a practical example) are provided to show the effectiveness of the proposed approach.

Notation. The notation used in this paper is fairly standard. The superscript T stands for the transposition of vectors or matrices. R^n denotes the n dimensional Euclidean space with the norm $\|x\| = (x^T x)^{1/2}$, and $E(\cdot)$ represents the mathematical expectation operator. $l_2[0,\ \infty)$ is the space of square summable infinite sequences with the norm $\|w_k\|_2 = \left(\sum_{k=0}^{\infty} \|w_k\|^2\right)^{1/2}$. In addition, we use $*$ to denote the symmetry entries of symmetry matrices and $diag\{\ldots\}$ represents a block diagonal matrix. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations. The notation $P > 0$ $(\geqslant 0)$ means P is real symmetric positive (semi-positive) definite. I and 0 represent, respectively, identity matrix and zero matrix. $(*)_n$ denotes n blocks of vectors or matrices with the proper dimensions for avoidance of confusion.

2. PROBLEM FORMULATION

We assume that the controlled plant is modeled as discrete-time switched systems with state delay

$$\begin{aligned}
x_{k+1} &= A_\sigma x_k + A_{d\sigma} x_{k-\tau_0} + B_\sigma u_k + D_{1\sigma} w_k \\
z_k &= G_\sigma x_k + G_{d\sigma} x_{k-\tau_0} + D_{2\sigma} w_k \\
u_k &= \beta_k \hat{u}_{k-\tau_1} \\
y_k &= \alpha_k C_{d\sigma} x_{k-\tau_2} + D_{3\sigma} w_k \\
x_k &= \phi_k,\ k = -\tilde{d},\ -\tilde{d}+1,\ \cdots,\ 0,\ \tilde{d} = max\{\tau_0,\ \tau_1,\ \tau_2\}
\end{aligned} \tag{1}$$

where $x_k \in R^n$ is the state, $u_k \in R^m$ is the input of the actuator, $z_k \in R^q$ is the output to be controlled, and $w_k \in R^p$ is the disturbance input which belongs to $l_2[0, \infty)$. The positive integers τ_0, τ_1, and τ_2 represent time delays in the system, the control input channel and the measured output channel. $\hat{u}_k \in R^m$ is the output of the designed controller which is also transmitted over a non-ideal links with data loss at the probability β_k and time delay τ_1. The stochastic variable β_k is also a Bernoulli distributed white sequence with

$$\begin{aligned} &Prob\{\beta_k = 1\} = E\{\beta_k\} = \bar{\beta} \\ &Prob\{\beta_k = 0\} = 1 - E\{\beta_k\} = 1 - \bar{\beta} \end{aligned} \tag{2}$$

where $\bar{\beta}$ is a constant, denoting the expectation of stochastic variable β. Similarly, $y_k \in R^r$ is the measured output with the probability of data missing α and time delay τ_2 over a non-ideal channel. The stochastic variable α_k is a Bernoulli distributed white sequence with

$$\begin{aligned} &Prob\{\alpha_k = 1\} = E\{\alpha_k\} = \bar{\alpha} \\ &Prob\{\alpha_k = 0\} = 1 - E\{\alpha_k\} = 1 - \bar{\alpha} \end{aligned} \tag{3}$$

where $\bar{\alpha}$ is a constant, denoting the expectation of stochastic variable α. We assume that the stochastic variables α_k and β_k are mutually independent, thus

$$\begin{aligned} &E\left\{\alpha_k - \bar{\alpha}\right\} = 0 \\ &E\left\{\beta_k - \bar{\beta}\right\} = 0 \\ &E\left\{(\alpha_k - \bar{\alpha})\left(\beta_k - \bar{\beta}\right)\right\} = 0 \\ &E\left\{(\alpha_k - \bar{\alpha})^2\right\} = \bar{\alpha}\left(1 - \bar{\alpha}\right) = \hat{\alpha}^2 \\ &E\left\{\left(\beta_k - \bar{\beta}\right)^2\right\} = \bar{\beta}\left(1 - \bar{\beta}\right) = \hat{\beta}^2 \end{aligned} \tag{4}$$

σ_k is denoting σ for simplicity and $\sigma_k : [0, \infty) \rightarrow \mathcal{P} = \{1, \cdots, p\}$ is a switching signal to specify, at time instant k, which subsystem is activated. $\sigma = i$ means that the ith subsystem is activated, which is denoted by the constant matrices A_i, A_{di}, B_i, D_{1i}, G_i, G_{di}, D_{2i}, C_i, C_{di}, and D_{3i} with appropriate dimensions.

Because the states of system (1) are directly unmeasurable, we are interested in designing an observer-based controller with random data loss and time delays described by

$$\begin{aligned} \hat{x}_{k+1} &= A_\sigma \hat{x}_k + A_{d\sigma}\hat{x}_{k-\tau_0} + B_\sigma \bar{\beta}\hat{u}_k + L_\sigma\left(y_k - \bar{\alpha} C_\sigma \hat{x}_k\right) \\ \hat{u}_k &= K_\sigma \hat{x}_k \end{aligned} \tag{5}$$

where $\hat{x}_k \in R^n$ is the estimation of the state of system (1). $\sigma = i$ means that the ith observer and controller are activated. The matrices L_i and K_i, $i \in \mathcal{P}$ are the parameters to be determined. The observer-based controller (5) is assumed to be switched synchronously by the switching signal σ in system (1).

Remark 1

It is noted from system (1) and observer (5) that data missing and time delays are considered simultaneously in both links from sensors to observers and from controllers to actuators. In [9], the control problem of switched delay systems without the consideration of data missing is investigated. In [21], data missing satisfying the Bernoulli distributed white sequences α_k and β_k are taken into consideration in only one link from sensors to filters. Additionally, time delays are not involved. In the present paper, data missing and time delays are simultaneously studied in the system and both links. Therefore, the problem under consideration is more general compared with the existing results in [9] and [21].

For the actual implement, the non-fragile observer and controller gains are considered,

$$\begin{aligned} K_i &= \bar{K}_i + \Delta K_i,\ \Delta K_i = M_{1i}F_{1i}(k)N_{1i} \\ L_i &= \bar{L}_i + \Delta L_i,\ \Delta L_i = M_{2i}F_{2i}(k)N_{2i} \end{aligned} \tag{6}$$

where matrices M_{1i}, N_{1i}, M_{2i}, and N_{2i} are known with appropriate dimensions. The uncertain functions $F_{1i}(k)$ and $F_{2i}(k)$ satisfying

$$
\begin{aligned}
F_{1i}(k)^T F_{1i}(k) &\leqslant I \\
F_{2i}(k)^T F_{2i}(k) &\leqslant I
\end{aligned}
$$

Remark 2

Because the imprecision is an inherent property in analog systems such as the values of the resistances and electric capacities, it is required to take into consideration uncertainties in designed components. The components considering such uncertainties are often called non-fragile components. This paper is concerned with non-fragile observer-based H_∞ control problem, which can be considered as the difference compared with [9] and [21].

By the error $e_k = x_k - \hat{x}_k$, the original systems (1) and (5) are transformed into a stochastic switched delay system

$$
\begin{aligned}
x_{k+1} &= A_\sigma x_k + A_{d\sigma} x_{k-\tau_0} + \bar{\beta} B_\sigma K_\sigma \left(x_{k-\tau_1} - e_{k-\tau_1}\right) + D_{1\sigma} w_k \\
&\quad + \left(\beta - \bar{\beta}\right) B_\sigma K_\sigma \left(x_{k-\tau_1} - e_{k-\tau_1}\right) \\
e_{k+1} &= A_\sigma e_k + A_{d\sigma} e_{k-\tau_0} + \bar{\beta} B_\sigma K_\sigma \left(x_{k-\tau_1} - e_{k-\tau_1}\right) - \bar{\beta} B_\sigma K_\sigma (x_k - e_k) \\
&\quad + \bar{\alpha} L_\sigma C_\sigma (x_k - e_k) - \bar{\alpha} L_\sigma C_\sigma \left(x_{k-\tau_2} - e_{k-\tau_2}\right) + (D_{1\sigma} - L_\sigma D_{3\sigma}) w_k \\
&\quad - (\alpha_k - \bar{\alpha}) L_\sigma C_\sigma \left(x_{k-\tau_2} - e_{k-\tau_2}\right) + \left(\beta - \bar{\beta}\right) B_\sigma K_\sigma \left(x_{k-\tau_1} - e_{k-\tau_1}\right)
\end{aligned}
\tag{7}
$$

By such transformation, the stability of closed-loop system (7) means the solvability of the original H_∞ control problem in the presence of data missing and time delays. In the sequel, by selecting a new piecewise Lyapunov function and using the ADT technique, the conditions for such stability are given. The following assumptions and lemma are quite important for obtaining the main results.

Assumption 1 ([20]*)*

The matrix B_i is of full column rank, that is, $rank(B_i) = m$.

For Assumption 1, we have the singular value decomposition of B_i as follows,

$$
B_i = U_i \begin{bmatrix} \Sigma_i^T & 0 \end{bmatrix}^T V_i \tag{8}
$$

where $U_i \in R^{n \times n}$ and $V_i \in R^{m \times m}$ are unitary matrices, $\Sigma_i = diag\{\sigma_{1i}, \sigma_{2i}, \ldots, \sigma_{mi}\}$ is a diagonal matrix with nonnegative real numbers on the diagonal, $\sigma_{ji}, j = 1, 2, \ldots m$ are nonzero singular values of B_i.

Lemma 1 ([20]*)*

For B_i satisfying Assumptions 1 and (8), $B_i X_i = \tilde{X}_i B_i$ holds for a nonsingular matrix X_i if and only if there exists $\tilde{X}_i$ such that

$$
\tilde{X}_i = U_i \; diag\left\{\tilde{X}_{11i}, \tilde{X}_{22i}\right\} \; U_i^T \tag{9}
$$

where $\tilde{X}_{11i} \in R^{m \times m}$ and $\tilde{X}_{22i} \in R^{(n-m)\times(n-m)}$.

Before proposing the problem addressed in this paper, the following definitions are presented.

Definition 1
The closed-loop system (7) with $w_k \equiv 0$ is said to be exponentially mean-square stable if there exist constants $\phi > 0$ and $\tau \in (0, 1)$, such that

$$E\{\|\varepsilon_k\|^2\} \leqslant \phi\tau^k \sup_{-d\leqslant j\leqslant 0} E\{\|\varepsilon_j\|^2\},\ k \in N^+ \tag{10}$$

where $\varepsilon_k = \begin{bmatrix} x_k^T & e_k^T \end{bmatrix}^T$.

Definition 2 ([14]*)*
For $k_v > k_s > 0$, $\tau^* > 0$, and $N_0 \geqslant 0$, we have

$$N_\sigma(k_s,\ k_v) \leqslant N_0 + \frac{k_v - k_s}{\tau^*} \tag{11}$$

where $N_\sigma(k_s, k_v)$ denotes the switching numbers of σ during $[k_s, k_v]$. τ^* and N_0 are called ADT and the chattering bound, respectively. Here, we assume $N_0 = 0$ as commonly used in literature [12, 16].

Dwell time represents the running time of each subsystem between the consecutive discontinuous switching instants. ADT means that dwell time on the average is no less than a specified positive constant τ^* [14]. In this case, it allows to select one Lyapunov function for each subsystem at the interval of dwell time.

The problem addressed in this paper can be formulated as follows. Given (1) and $\gamma > 0$, design non-fragile observer and controller (5) such that under ADT switching signals, (i) the closed-loop system (7) with $w_k = 0$ is exponentially mean-square stability and (ii) the closed-loop system (7) guarantees, under zero-initial condition, $\|z_k\|_2 \leqslant \gamma\|w_k\|_2$ for all nonzero $w_k \in l_2[0, \infty)$.

The following Lemma plays an important role in the derivation of our main results.

Lemma 2 ([26]*)*
Given symmetric matrix Y and matrices H, $\bar{C}$, then

$$Y + H^T\bar{F}(t)\bar{C} + \bar{C}^T\bar{F}^T(t)H < 0$$

for all $\bar{F}(t)$ satisfying $\bar{F}^T(t)\bar{F}(t) \leqslant I$, if and only if there exists a scalar $\varepsilon > 0$ such that

$$Y + \varepsilon^{-1}H^TH + \varepsilon\bar{C}^T\bar{C} < 0.$$

3. PERFORMANCE ANALYSIS

This section presents the mean-square stability and H_∞ performance analysis for system (7).

Theorem 1
Given scalars $\tau_m > 0$, $0 < \rho < 1$, $\gamma > 0$, $\mu \geqslant 1$, and the switching instants $0 < k_1 < \cdots < k_{s-1} < k_s$, $s = 1, 2, \cdots$, during $[0, k]$, system (7) is exponentially mean-square stable with an H_∞ norm bound γ, if there exist symmetric and positive definite matrices P_i, Q_i, R_{mi}, S_{mi}, $m = \{0, 1, 2\}$, and ADT switching signal σ satisfying

$$\begin{bmatrix} -\Xi_1 & * \\ \Xi_2 & -\Xi_3 \end{bmatrix} < 0 \tag{12}$$

$$\begin{aligned} &P_i \leqslant \mu P_j,\ Q_i \leqslant \mu Q_j,\ R_{0i} \leqslant \mu R_{0j},\ S_{0i} \leqslant \mu S_{0j} \\ &R_{1i} \leqslant \mu R_{1j},\ S_{1i} \leqslant \mu S_{1j},\ R_{2i} \leqslant \mu R_{2j},\ S_{2i} \leqslant \mu S_{2j},\ \forall i, j \in \mathcal{P} \end{aligned} \tag{13}$$

$$\tau_a \geqslant \tau^* = -(ln\rho)^{-1}ln\mu \tag{14}$$

where

$$
\begin{aligned}
\Xi_1 = diag\Big\{ & \rho P_i - \sum_{m=0}^{2} R_{mi}, \quad \rho^{\tau_0} R_{0i}, \quad \rho^{\tau_1} R_{1i}, \quad \rho^{\tau_2} R_{2i}, \\
& \rho Q_i - \sum_{m=0}^{2} S_{mi}, \quad \rho^{\tau_0} S_{0i}, \quad \rho^{\tau_1} S_{1i}, \quad \rho^{\tau_2} S_{2i}, \quad \gamma^2 I \Big\} \\
\Xi_2 = & \begin{bmatrix} \Xi_{21}^T & \Xi_{22}^T & \Xi_{22}^T & \Xi_{23}^T & \Xi_{24}^T & \Xi_{25}^T \end{bmatrix}^T \\
\Xi_3 = diag & \left\{ P_i^{-1}, \quad P_i^{-1}, \quad Q_i^{-1}, \quad Q_i^{-1}, \quad Q_i^{-1}, \quad I \right\} \\
\Xi_{21} = & \begin{bmatrix} A_i & A_{di} & \bar{\beta} B_i K_i & 0_3 & \bar{\beta} B_i K_i & 0 & D_{1i} \end{bmatrix} \\
\Xi_{22} = & \begin{bmatrix} 0_2 & \hat{\beta} B_i K_i & 0_3 & \hat{\beta} B_i K_i & 0_2 \end{bmatrix} \\
\Xi_{23} = & \big[\bar{\alpha} L_i C_i - \bar{\beta} B_i K_i \quad 0 \quad \bar{\beta} B_i K_i \quad -\bar{\alpha} L_i C_i \\
& A_i - \bar{\alpha} L_i C_i + \bar{\beta} B_i K_i \quad A_{di} \quad -\bar{\beta} B_i K_i \quad \bar{\alpha} L_i C_i \quad D_{1i} - L_i D_{3i} \big] \\
\Xi_{24} = & \begin{bmatrix} 0_3 & \hat{\alpha} L_i C_i & 0_3 & \hat{\alpha} L_i C_i & 0 \end{bmatrix} \\
\Xi_{25} = & \begin{bmatrix} G_i & G_{di} & 0_6 & D_{2i} \end{bmatrix}
\end{aligned}
$$

Proof
For the ith subsystem, choose the following Lyapunov–Krasovskii function as

$$
V_i(k) = V_{1i}(k) + V_{2i}(k) + V_{3i}(k) + V_{4i}(k) \tag{15}
$$

where

$$
\begin{aligned}
V_{1i}(k) &= x_k^T P_i x_k \\
V_{2i}(k) &= e_k^T Q_i e_k \\
V_{3i}(k) &= \sum_{m=0}^{2} \sum_{j=k-\tau_m}^{k-1} \rho^{k-j-1} x_j^T R_{mi} x_j \\
V_{4i}(k) &= \sum_{m=0}^{2} \sum_{j=k-\tau_m}^{k-1} \rho^{k-j-1} e_j^T S_{mi} e_j
\end{aligned}
$$

In terms of

$$
E\{\Delta V_i(k)\} = E\{V_i(k+1) - \rho V_i(k)\}
$$

and

$$
\hat{\eta}_k = \begin{bmatrix} x_k^T & x_{k-\tau_0}^T & x_{k-\tau_1}^T & x_{k-\tau_2}^T & e_k^T & e_{k-\tau_0}^T & e_{k-\tau_1}^T & e_{k-\tau_2}^T & w_k \end{bmatrix}^T
$$

we have

$$
\begin{aligned}
E\{\Delta V_{1i}\} &= E\left\{x_{k+1}^T P_i x_{k+1}\right\} - \rho x_k^T P_i x_k \\
&= (\Xi_{21}\hat{\eta}_k)^T P_i \Xi_{21}\hat{\eta}_k + (\Xi_{22}\hat{\eta}_k)^T P_i \Xi_{22}\hat{\eta}_k - \rho x_k^T P_i x_k
\end{aligned} \tag{16}
$$

$$
\begin{aligned}
E\{\Delta V_{2i}\} = E\left\{e_{k+1}^T Q_i e_{k+1}\right\} - \rho e_k^T Q_i e_k = (\Xi_{23}\hat{\eta}_k)^T \\
\times Q_i \Xi_{23}\hat{\eta}_k + (\Xi_{22}\hat{\eta}_k)^T Q_i \Xi_{22}\hat{\eta}_k + (\Xi_{24}\hat{\eta}_k)^T Q_i \Xi_{24}\hat{\eta}_k - \rho e_k^T Q_i e_k
\end{aligned} \tag{17}
$$

$$E\{\Delta V_{3i}\} = \sum_{m=0}^{2}\left(\sum_{j=k-\tau_m+1}^{k}\rho^{k-j}x_j^T R_{mi}x_j - \sum_{j=k-\tau_m}^{k-1}\rho^{k-j}x_j^T R_{mi}x_j\right)$$
$$= \sum_{m=0}^{2}\left(x_k^T R_{mi}x_k - \rho^{\tau_m}x_{k-\tau_m}^T R_{mi}x_{k-\tau_m}\right) \tag{18}$$

$$E\{\Delta V_{4i}\} = \sum_{m=0}^{2}\left(\sum_{j=k-\tau_m+1}^{k}\rho^{k-j}e_j^T S_i e_j - \sum_{j=k-\tau_m}^{k-1}\rho^{k-j}e_j^T S_{mi}e_j\right)$$
$$= \sum_{m=0}^{2}\left(e_k^T S_{mi}e_k - \rho^{\tau_m}e_{k-\tau_m}^T S_{mi}e_{k-\tau_m}\right) \tag{19}$$

Suppose $w_k = 0$ for system (7), from (16)–(19), we can get

$$E\{\Delta V_i(k)\} = \left(\bar{\Xi}_{21}\eta_k\right)^T P_i\bar{\Xi}_{21}\eta_k + \left(\bar{\Xi}_{22}\eta_k\right)^T P_i\bar{\Xi}_{22}\eta_k + \left(\bar{\Xi}_{22}\eta_k\right)^T Q_i\bar{\Xi}_{22}\eta_k$$
$$+ \left(\bar{\Xi}_{23}\eta_k\right)^T Q_i\bar{\Xi}_{23}\eta_k + \left(\bar{\Xi}_{24}\eta_k\right)^T Q_i\bar{\Xi}_{24}\eta_k - \eta_k^T\Psi_{1i}\eta_k = \eta_k^T\Psi_i\eta_k \tag{20}$$

where

$$\eta_k = \left[\begin{array}{cccccccc} x_k^T & x_{k-\tau_0}^T & x_{k-\tau_1}^T & x_{k-\tau_2}^T & e_k^T & e_{k-\tau_0}^T & e_{k-\tau_1}^T & e_{k-\tau_2}^T \end{array}\right]^T$$
$$\hat{\phi} = \left[\begin{array}{cc} I & 0_8^T \end{array}\right]^T$$
$$\Psi_i = -\Psi_{1i} + \bar{\Xi}_{21}^T P_i\bar{\Xi}_{21} + \bar{\Xi}_{22}^T P_i\bar{\Xi}_{22} + \bar{\Xi}_{22}^T Q_i\bar{\Xi}_{22} + \bar{\Xi}_{23}^T Q_i\bar{\Xi}_{23} + \bar{\Xi}_{24}^T Q_i\bar{\Xi}_{24}$$
$$\Psi_{1i} = \Xi_1\hat{\phi},\ \bar{\Xi}_{21} = \Xi_{21}\hat{\phi},\ \bar{\Xi}_{22} = \Xi_{22}\hat{\phi},\ \bar{\Xi}_{23} = \Xi_{23}\hat{\phi},\ \bar{\Xi}_{24} = \Xi_{24}\hat{\phi}$$

It follows from (12) and Schur complement that $E\{\Delta V_i(k)\} < 0$. Suppose the switching instants happen at $0 < k_1 < \cdots < k_{s-1} < k_s$, $s = 1, 2, \cdots$, during $[0, k]$, by making use of (13), we have

$$\begin{aligned} &E\left\{V_{\sigma_{k_s}}(k)\right\} \leqslant E\left\{\rho^{k-k_s}V_{\sigma_{k_s}}(k_s)\right\} \\ &\leqslant E\left\{\rho^{k-k_s}\mu V_{\sigma_{k_{s-1}}}(k_s)\right\} \\ &\leqslant E\left\{\rho^{k-k_{s-1}}\mu V_{\sigma_{k_{s-1}}}(k_{s-1})\right\} \leqslant \cdots \\ &\leqslant E\left\{\rho^k\mu^{N_\sigma(0,k)}V_{\sigma_0}(0)\right\} \end{aligned} \tag{21}$$

Using (14), we can obtain

$$E\left\{V_{\sigma_{k_s}}(k)\right\} \leqslant e^{\lambda k}E\left\{V_{\sigma_0}(0)\right\} \tag{22}$$

where $\lambda = (ln\mu)/\tau_a + ln\rho < 0$.

From (15), we have

$$\begin{aligned} &aE\{\|\varepsilon_k\|^2\} \leqslant E\left\{V_{\sigma_{k_s}}(k)\right\} \\ &E\left\{V_{\sigma_0}(0)\right\} \leqslant b\sup_{-d\leqslant j\leqslant 0}E\{\|\varepsilon_j\|^2\} \end{aligned} \tag{23}$$

where

$$a = min\left\{\lambda_{min}\left(P_{\sigma_{k_s}}\right), \lambda_{min}\left(Q_{\sigma_{k_s}}\right)\right\}$$
$$b = max\left\{\lambda_{max}\left(P_{\sigma_0}\right), \lambda_{max}\left(Q_{\sigma_0}\right)\right.$$
$$\left.\lambda_{max}\left(R_{\sigma_0}\right), \lambda_{max}\left(S_{\sigma_0}\right)\right\}$$

Combining (22) and (23), we obtain

$$E\{\|\varepsilon_k\|^2\} \leqslant (b/a)e^{\lambda k} \sup_{-d\leqslant j\leqslant 0} E\{\|\varepsilon_j\|^2\} \tag{24}$$

Therefore, by Definition 2, the closed-loop switched system (7) is exponentially mean-square stable.
Next, we consider the following performance index

$$\begin{aligned} J &= E\left\{\sum_{k=0}^{\infty} \Upsilon(k)\right\} = E\left\{\sum_{k=0}^{\infty} \left(z_k^T z_k - \gamma^2 w_k^T w_k\right)\right\} \\ &= E\left\{\|z_k\|_2^2 - \gamma^2\|w_k\|_2^2\right\} \end{aligned} \tag{25}$$

For the ith subsystem, we have

$$\begin{aligned} &E\left\{\Delta V_i(k) + z_k^T z_k - \gamma^2 w_k^T w_k\right\} \\ &\quad = (\Xi_{21}\hat{\eta}_k)^T P_i \Xi_{21}\hat{\eta}_k + (\Xi_{22}\hat{\eta}_k)^T P_i \Xi_{22}\hat{\eta}_k + (\Xi_{22}\hat{\eta}_k)^T Q_i \Xi_{22}\hat{\eta}_k + (\Xi_{23}\hat{\eta}_k)^T Q_i \Xi_{23}\hat{\eta}_k \\ &\qquad + (\Xi_{24}\hat{\eta}_k)^T Q_i \Xi_{24}\hat{\eta}_k - \hat{\eta}_k^T \Xi_{1i}\hat{\eta}_k + z_k^T z_k \\ &\quad = \hat{\eta}_k^T \Xi_i \hat{\eta}_k \end{aligned} \tag{26}$$

where

$$\begin{aligned} \Xi_i = {} & \Xi_{21}^T P_i \Xi_{21} + \Xi_{22}^T P_i \Xi_{22} + \Xi_{22}^T Q_i \Xi_{22} + \Xi_{23}^T Q_i \Xi_{23} \\ & + \Xi_{24}^T Q_i \Xi_{24} + \Xi_{25}^T \Xi_{25} - \Xi_1 \end{aligned}$$

It follows from (12) and Schur complement that $E\left\{\Delta V_i(k) + z_k^T z_k - \gamma^2 w_k^T w_k\right\} < 0$.
Furthermore,

$$E\{V_i(k+1)\} = E\left\{\rho V_i(k) + z_k^T z_k - \gamma^2 w_k^T w_k\right\} \tag{27}$$

By combining (13), (15), and (27), at the switching instants $0 < k_1 < \cdots < k_{s-1} < k_s$, $s = 1, 2, \cdots$, during $[0, k]$, one can obtain

$$\begin{aligned} E\left\{V_{\sigma_{k_s}}(k)\right\} &\leqslant E\left\{\rho^{k-k_s} V_{\sigma_{k_s}}(k_s) - \sum_{j=k_s}^{k-1} \rho^{k-j-1}\Upsilon(j)\right\} \\ &\leqslant E\left\{\rho^{k-k_s}\mu V_{\sigma_{k_{s-1}}}(k_s) - \sum_{j=k_s}^{k-1} \rho^{k-j-1}\Upsilon(j)\right\} \\ &\leqslant E\left\{\rho^{k-k_{s-1}}\mu V_{\sigma_{k_{s-1}}}(k_{s-1}) - \mu\sum_{j=k_{s-1}}^{k_s-1} \rho^{k-j-1}\Upsilon(j) - \sum_{j=k_s}^{k-1} \rho^{k-j-1}\Upsilon(j)\right\} \leqslant \cdots \\ &\leqslant E\left\{\rho^k \mu^{N_\sigma(0,k)} V_{\sigma_0}(0) - \sum_{j=0}^{k-1} \mu^{N_\sigma(j,k)}\rho^{k-j-1}\Upsilon(j)\right\} \\ &\leqslant E\left\{e^{\lambda k} V_{\sigma_0}(0) - \sum_{j=0}^{k-1} e^{\lambda(k-j)}\rho^{-1}\Upsilon(j)\right\} \end{aligned} \tag{28}$$

For any nonzero $w_k \in l_2[0, \infty)$ and under zero-initial condition, one has

$$E\left\{\sum_{j=0}^{k-1} e^{\lambda(k-j)}\rho^{-1}\Upsilon(j) \leqslant 0\right\}$$

Note that

$$E\left\{\sum_{j=0}^{k-1} e^{\lambda(k-j)}\rho^{-1}z_j^T z_j\right\}$$

is summable from 0 to ∞ because

$$E\left\{\sum_{j=0}^{k-1} e^{\lambda(k-j)}\rho^{-1}w_j^T w_j\right\}$$

is summable for any $w_j \in l_2[0,\infty)$ when $k=\infty$. Then, we obtain that

$$E\left\{\sum_{k=1}^{\infty}\sum_{j=0}^{k-1} e^{\lambda(k-j)}\rho^{-1}\Upsilon(j)\right\} \leqslant 0.$$

Rearranging the double sum area yields

$$\begin{aligned}&E\left\{\sum_{j=0}^{\infty}\Upsilon(j)\sum_{k=j+1}^{\infty} e^{\lambda(k-j)}\rho^{-1}\right\}\\ &= E\left\{\frac{e^{\lambda}\rho^{-1}}{1-e^{\lambda}}\sum_{j=0}^{\infty}\Upsilon(j)\right\} \leqslant 0\end{aligned}$$

which means $J \leqslant 0$, or $E\left\{\|z_k\|_2^2\right\} \leqslant E\left\{\gamma^2\|w_k\|_2^2\right\}$ for nonzero $w_k \in l_2[0,\infty)$. The proof is completed.

Remark 3

It is mentioned in [9–11] and [21] when SLF method is utilized to address analysis and synthesis problems of switched delay systems, Lyapunov function associated with time delays are common for all subsystems. However, in practice, common Lyapunov function may not exist or be difficult to find [3]. In addition, arbitrary switching signal is obtained by SLF method. However, frequent switchings results in the poor performance and also destroy the hardware of systems. ADT approach can avoid the two drawbacks previously mentioned by constructing piecewise Lyapunov function associated with time delays. It is observed from (15) that Lyapunov matrices R_{mi} and S_{mi} involved with time delay are piecewise and from (22) and (28) that ADT technique is employed. This is the third difference between the present paper and the references [9] and [21].

4. OBSERVER-BASED CONTROLLER DESIGN

In this section, a sufficient condition is presented for the existence of non-fragile observer and controller for system (1) in the presence of data missing and time delays.

Theorem 2

Given $\tau_m > 0$, $0 < \rho < 1$, $\gamma > 0$, and $\mu \geqslant 1$, for system (1) in the presence of data missing and time delays, the observer-based non-fragile H_∞ controller (5) can be designed if there exist symmetric and positive definite matrices P_i, Q_i, R_{mi}, S_{mi}, $m = \{0,1,2\}$, matrices Ω_{1i}, Ω_{2i}, W_i, Y_i, and constants υ_{ci}, $c = 1,2,3,4$, such that (13), (14) and the following inequalities hold

$$\begin{bmatrix} \tilde{\Phi} & * \\ \tilde{M}_i^T & -\Pi_i \end{bmatrix} < 0 \tag{29}$$

where

$$\begin{aligned}
\tilde{\Phi} &= \begin{bmatrix} -\Xi_1 & * \\ \tilde{\Xi}_2 & \hat{\Xi}_3 \end{bmatrix} \\
\tilde{\Xi}_2 &= \begin{bmatrix} \tilde{\Xi}_{21}^T & \tilde{\Xi}_{22}^T & \tilde{\Xi}_{22}^T & \tilde{\Xi}_{23}^T & \tilde{\Xi}_{24}^T & \Xi_{25}^T \end{bmatrix}^T \\
\tilde{\Xi}_{21} &= \begin{bmatrix} \Omega_i A_i & \Omega_i A_{di} & \bar{\beta} B_i W_i & 0_3 & \bar{\beta} B_i W_i & 0 & \Omega_i D_{1i} \end{bmatrix} \\
\tilde{\Xi}_{22} &= \begin{bmatrix} 0_2 & \hat{\beta} B_i W_i & 0_3 & \hat{\beta} B_i W_i & 0_2 \end{bmatrix} \\
\tilde{\Xi}_{23} &= [\ \bar{\alpha} Y_i C_i - \bar{\beta} B_i W_i \quad 0 \quad \bar{\beta} B_i W_i \quad -\bar{\alpha} Y_i C_i \\
&\qquad \Omega_i A_i - \bar{\alpha} Y_i C_i + \bar{\beta} B_i W_i \quad \Omega_i A_{di} \quad -\bar{\beta} B_i W_i \quad \bar{\alpha} Y_i C_i \quad \Omega_i D_{1i} - Y_i D_{3i}\ ] \\
\tilde{\Xi}_{24} &= \begin{bmatrix} 0_3 & \hat{\alpha} Y_i C_i & 0_3 & \hat{\alpha} Y_i C_i & 0 \end{bmatrix} \\
\hat{\Xi}_3 &= diag\left\{ \left(P_i - \Omega_i^T - \Omega_i\right)_2, \quad \left(Q_i - \Omega_i^T - \Omega_i\right)_3, \quad -I \right\} \\
\tilde{M}_i &= \begin{bmatrix} \bar{M}_{1i} & \bar{M}_{2i} & \bar{M}_{3i} & \bar{M}_{4i} & \upsilon_{1i} \bar{N}_{1i}^T & \upsilon_{2i} \bar{N}_{2i}^T & \upsilon_{3i} \bar{N}_{3i}^T & \upsilon_{4i} \bar{N}_{4i}^T \end{bmatrix}^T \\
\bar{M}_{1i} &= \begin{bmatrix} 0_9 & \bar{\beta} M_{1i}^T B_i^T \Omega_i^T & \hat{\beta} M_{1i}^T B_i^T \Omega_i^T & \hat{\beta} M_{1i}^T B_i^T \Omega_i^T & 0_3 \end{bmatrix}^T \\
\bar{N}_{1i} &= \begin{bmatrix} 0_2 & N_{1i} & 0_3 & -N_{1i} & 0_8 \end{bmatrix} \\
\bar{M}_{2i} &= \begin{bmatrix} 0_{12} & -M_{2i}^T \Omega_i^T & 0_2 \end{bmatrix}^T \\
\bar{N}_{2i} &= \begin{bmatrix} \bar{\alpha} N_{2i} C_i & 0_2 & -\bar{\alpha} N_{2i} C_i & -\bar{\alpha} N_{2i} C_i & 0_2 & \bar{\alpha} N_{2i} C_i & N_{2i} D_{3i} & 0_6 \end{bmatrix} \\
\bar{M}_{3i} &= \begin{bmatrix} 0_{13} & \hat{\alpha} M_{2i}^T \Omega_i^T & 0 \end{bmatrix}^T \\
\bar{N}_{3i} &= \begin{bmatrix} 0_2 & N_{2i} C_i & 0_3 & N_{2i} C_i & 0_8 \end{bmatrix} \\
\bar{M}_{4i} &= \begin{bmatrix} 0_{12} & \bar{\beta} M_{1i}^T B_i^T \Omega_i^T & 0_2 \end{bmatrix}^T \\
\bar{N}_{4i} &= \begin{bmatrix} -N_{1i} & 0 & N_{1i} & 0 & N_{1i} & 0 & -N_{1i} & 0_8 \end{bmatrix} \\
B_i &= \begin{bmatrix} U_{1i} & U_{2i} \end{bmatrix} \begin{bmatrix} \Sigma_i & 0 \end{bmatrix}^T V_i \\
\Omega_i &= U_{1i} \Omega_{1i} U_{1i} + U_{2i} \Omega_{2i} U_{2i} \\
\Pi_i &= diag\left\{ \upsilon_{1i}, \quad \upsilon_{2i}, \quad \upsilon_{3i}, \quad \upsilon_{4i}, \quad \upsilon_{1i}, \quad \upsilon_{2i}, \quad \upsilon_{3i}, \quad \upsilon_{4i} \right\}
\end{aligned}$$

and 0_N denotes number N of zero matrix of the appropriate dimensions. Then, under ADT switching signals satisfying (14), there exist non-fragile observer and controller gains constructed by

$$\bar{K}_i = V_i^{-1} \Sigma_i^{-1} \Omega_{1i}^{-1} \Sigma_i V_i W_i, \ \bar{L}_i = \Omega_i^{-1} Y_i. \tag{30}$$

Proof

By Theorem 1, the closed-loop system (7) under switching signals with ADT τ_a is exponentially mean-square stable with an H_∞ norm bound γ if (12), (13), and (14) hold.

Because $P_i > 0$, we select nonsingular matrices Ω_i such that $(P_i - \Omega_i) P_i^{-1} (P_i - \Omega_i)^T \geqslant 0$. Then, $-\Omega_i P_i^{-1} \Omega_i^T \leqslant P_i - \left(\Omega_i + \Omega_i^T\right)$. By following the same line, we have $-\Omega_i Q_i^{-1} \Omega_i^T \leqslant Q_i - \left(\Omega_i + \Omega_i^T\right)$. Therefore, via $diag\left\{I_9, \ \Omega_i^T, \ \Omega_i^T, \ \Omega_i^T, \ \Omega_i^T, \ \Omega_i^T, \ I\right\}$, performing a congruence transformation to (12) yields

$$\begin{bmatrix} -\hat{\Xi}_1 & * \\ \begin{bmatrix} \Xi_{21}^T \Omega_i^T & \Xi_{22}^T \Omega_i^T & \Xi_{22}^T \Omega_i^T & \Xi_{23}^T \Omega_i^T & \Xi_{24}^T \Omega_i^T & \Xi_{25}^T \end{bmatrix}^T & -\hat{\Xi}_3 \end{bmatrix} < 0 \tag{31}$$

In view of (6), (31) is equivalent to

$$\begin{aligned}
&\bar{\Phi} + \bar{M}_{1i} F_{1i}(k) \bar{N}_{1i} + \bar{N}_{1i}^T F_{1i}^T(k) \bar{M}_{1i}^T + \bar{M}_{2i} F_{2i}(k) \bar{N}_{2i} + \bar{N}_{2i}^T F_{2i}^T(k) \bar{M}_{2i}^T \\
&\quad + \bar{M}_{3i} F_{2i}(k) \bar{N}_{3i} + \bar{N}_{3i}^T F_{2i}^T(k) \bar{M}_{3i}^T + \bar{M}_{4i} F_{1i}(k) \bar{N}_{4i} + \bar{N}_{4i}^T F_{1i}^T(k) \bar{M}_{4i}^T < 0
\end{aligned} \tag{32}$$

where

$$\bar{\Phi} = \begin{bmatrix} -\Xi_1 & * \\ \check{\Xi}_2 & -\Xi_3 \end{bmatrix}$$

$$\begin{aligned}
\check{\Xi}_2 &= \begin{bmatrix} \check{\Xi}_{21}^T & \check{\Xi}_{22}^T & \check{\Xi}_{22}^T & \check{\Xi}_{23}^T & \check{\Xi}_{24}^T & \Xi_{25}^T \end{bmatrix}^T \\
\check{\Xi}_{21} &= \begin{bmatrix} \Omega_i A_i & \Omega_i A_{di} & \bar{\beta}\Omega_i B_i \bar{K}_i & 0_3 & \bar{\beta}\Omega_i B_i \bar{K}_i & 0 & \Omega_i D_{1i} \end{bmatrix} \\
\check{\Xi}_{22} &= \begin{bmatrix} 0_2 & \hat{\beta}\Omega_i B_i \bar{K}_i & 0_3 & \hat{\beta}\Omega_i B_i \bar{K}_i & 0_2 \end{bmatrix} \\
\check{\Xi}_{23} &= \begin{bmatrix} \bar{\alpha}\Omega_i \bar{L}_i C_i - \bar{\beta}\Omega_i B_i \bar{K}_i & 0 & \bar{\beta}\Omega_i B_i \bar{K}_i & -\bar{\alpha}\Omega_i \bar{L}_i C_i \end{bmatrix. \\
&\qquad \Omega_i A_i - \bar{\alpha}\Omega_i \bar{L}_i C_i + \bar{\beta}\Omega_i B_i \bar{K}_i \quad \Omega_i A_{di} \quad -\bar{\beta}\Omega_i B_i \bar{K}_i \quad \bar{\alpha}\Omega_i \bar{L}_i C_i \quad \Omega_i D_{1i} - \Omega_i \bar{L}_i D_{3i} \left.\vphantom{\bar{K}}\right] \\
\check{\Xi}_{24} &= \begin{bmatrix} 0_3 & \hat{\alpha}\Omega_i \bar{L}_i C_i & 0_3 & \hat{\alpha}\Omega_i \bar{L}_i C_i & 0 \end{bmatrix}
\end{aligned}$$

By Lemma 2, we can get

$$\begin{aligned}
&\bar{\Phi} + \frac{1}{\upsilon_{1i}}\bar{M}_{1i}\bar{M}_{1i}^T + \upsilon_{1i}\bar{N}_{1i}^T\bar{N}_{1i} + \frac{1}{\upsilon_{2i}}\bar{M}_{2i}\bar{M}_{2i}^T \\
&\quad + \upsilon_{2i}\bar{N}_{2i}^T\bar{N}_{2i} + \frac{1}{\upsilon_{3i}}\bar{M}_{3i}\bar{M}_{3i}^T + \upsilon_{3i}\bar{N}_{3i}^T\bar{N}_{3i} + \frac{1}{\upsilon_{4i}}\bar{M}_{4i}\bar{M}_{4i}^T + \upsilon_{4i}\bar{N}_{4i}^T\bar{N}_{4i} < 0
\end{aligned} \tag{33}$$

By Lemma 1, choosing $\Omega_i = U_{1i}\Omega_{1i}U_{1i} + U_{2i}\Omega_{2i}U_{2i}$, we have $\Omega_i B_i = B_i\hat{\Omega}_i$ with $\hat{\Omega}_i = V_i^{-1}\Sigma_i^{-1}\Omega_{1i}\Sigma_i V_i$.

Substituting

$$W_i = \hat{\Omega}_i\bar{K}_i, \; Y_i = \Omega_i\bar{L}_i \tag{34}$$

into (33) and using Schur complement lead to (29), which means that (12) holds. Thus, if a solution to LMIs (13), (14), and (29) exists, the parameters of non-fragile observer and controller gains are given by (30), which is obtained from (34). The proof is completed.

Remark 4

The conditions (12), (13), and (14) in Theorem 2 are presented in terms of strict LMIs. In the proof of Theorem 2, Assumption 1 is satisfied and further Lemma 1 is employed. It is required to design the observer and the controller gains, two parametric matrices, based on the singular value decomposition of input matrix B. For non-switched systems, the costs of designing two or four matrices are not clearly different. For switched systems, however, the design cost will be obviously increased with the number rise of subsystems. From this point of view, the results of this paper is less conservative. Also, we can assume that the matrices C_i are of full row rank and $X_i C_i = C_i\tilde{X}_i$ similar to Lemma 1 will be used [24]. However, the difference lies in that the former can deal with the output y_k with the disturbance w_k, that is, $y_k = \alpha_k C_\sigma x_k + D_{3\sigma}w_k$, whereas the latter can only cope with $y_k = \alpha_k C_\sigma x_k$. In the latter, if the disturbance is introduced in the output, the condition will not be obtained in terms of LMIs.

5. EXAMPLES

In this section, two examples are presented to demonstrate the validity of the controller design method in the presence of data missing and time delays. The first is a numerical example used to show that the problems of the systems can be solved. The second is derived from the water quality control, an industrial system, to illustrate the potential applicability of the proposed theoretical results.

5.1. Example 1

Consider discrete-time switched delay system (1) with the composite systems of observer and controller (5), which consists of two subsystems as follows.
Subsystem 1

$$A_1 = \begin{bmatrix} 0.2 & -0.4 \\ 0.2 & 0.4 \end{bmatrix},\ A_{d1} = \begin{bmatrix} -0.3 & 0.2 \\ 0.1 & -0.1 \end{bmatrix},\ B_1 = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix},\ D_{11} = \begin{bmatrix} -0.5 \\ 0.25 \end{bmatrix}$$
$$G_1 = \begin{bmatrix} 0.4 & 0.3 \end{bmatrix},\ G_{d1} = \begin{bmatrix} 0 & 0 \end{bmatrix},\ D_{21} = 0.2,\ C_{d1} = \begin{bmatrix} 0.1 & 0.5 \end{bmatrix},\ D_{31} = 0.5$$

Subsystem 2

$$A_2 = \begin{bmatrix} -0.1 & 0.1 \\ 0.2 & -0.4 \end{bmatrix},\ A_{d2} = \begin{bmatrix} 0.5 & 0.1 \\ 0.2 & 0.3 \end{bmatrix},\ B_2 = \begin{bmatrix} 0.3 \\ 0.4 \end{bmatrix},\ D_{12} = \begin{bmatrix} 0.25 \\ 0.2 \end{bmatrix}$$
$$G_2 = \begin{bmatrix} 0.2 & 0.3 \end{bmatrix},\ G_{d2} = \begin{bmatrix} 0 & 0 \end{bmatrix},\ D_{22} = 0.1,\ C_{d2} = \begin{bmatrix} 0.5 & 0.2 \end{bmatrix},\ D_{32} = 0.3$$

The parameters of uncertainties of observers and controllers are

$$M_{11} = M_{12} = N_{21} = N_{22} = 0.05,\ M_{21} = M_{22} = \begin{bmatrix} 0 & 0.05 \end{bmatrix}^T,\ N_{11} = N_{12} = \begin{bmatrix} 0.05 & 0 \end{bmatrix}$$

Let $\tau_0 = 1$, $\tau_1 = 2$, $\tau_2 = 1$, $\rho = 0.95$, $\mu = 1.5$, $\bar{\alpha} = 0.85$, and $\bar{\beta} = 0.9$, by Theorem 2, we obtain the minimal H_∞ normal bound $\gamma = 0.8$ and non-fragile observer and controller gains

$$L_1 = \begin{bmatrix} -0.0007 & 0.0052 \end{bmatrix}^T,\ K_1 = \begin{bmatrix} -0.0043 & -0.0036 \end{bmatrix}$$
$$L_2 = \begin{bmatrix} -0.0004 & -0.0026 \end{bmatrix}^T,\ K_2 = \begin{bmatrix} 0.0009 & 0.0027 \end{bmatrix}$$

and $\tau^* = 7.9048$ is given by (14). Because the real state of the plant is not measured and the estimated state can be set, we can assume that the initial states of systems and observers are $x_0 = \begin{bmatrix} 1 & -2 \end{bmatrix}$ and $\hat{x}_0 = \begin{bmatrix} 0 & 0 \end{bmatrix}$, respectively. The sample time is 1 s. Under $w_k = 2exp(-0.1k)sin(0.5\pi k)$ and $F_{11}(k) = F_{12}(k) = F_{21}(k) = F_{22}(k) = sin(k)$, the following two figures are obtained. It is shown from Figure 1 that the errors of the state estimation of the closed-loop systems converge to zero. Figure 2 shows the switching signal activating the subsystem at time interval. From the aforementioned simulation results, it can be clearly observed that the proposed method is effective.

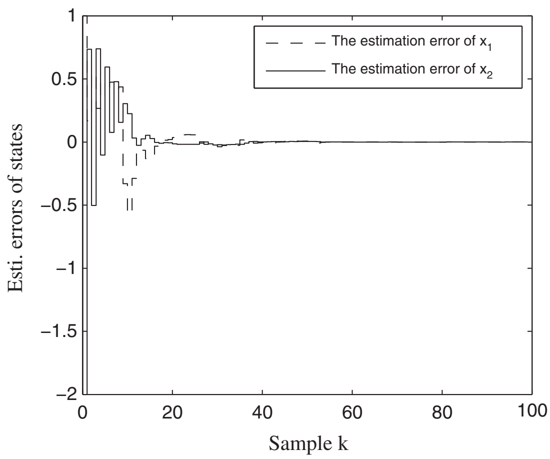

Figure 1. The estimation errors of states of the closed-loop systems.

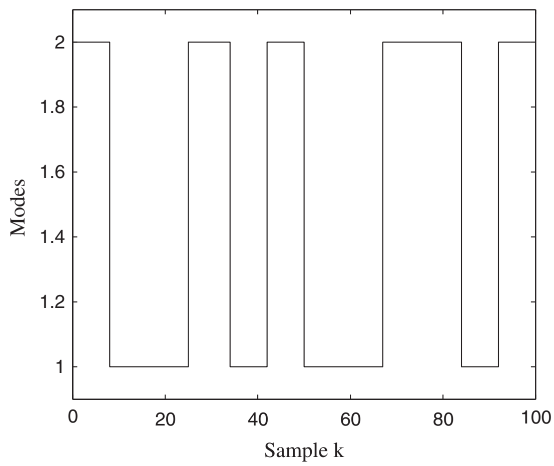

Figure 2. The switching signal of the systems.

5.2. Example 2

The problem of preserving the water quality constituents to standard levels in multi-reach fresh water streams is investigated in [9], where a multi-model representative of water pollution in multi-reach fresh water streams of the River Nile are presented. In order to demonstrate the effectiveness of the proposing methods in the practical example, the parameters of system (1) are borrowed from those in [9] as follows.

Subsystem 1

$$A_1 = \begin{bmatrix} 0.3 & 0.1 \\ -0.4 & 0.2 \end{bmatrix}, \; A_{d1} = \begin{bmatrix} 0.6 & 0 \\ 0.2 & 0.3 \end{bmatrix}, \; B_1 = \begin{bmatrix} 0.2 \\ 0 \end{bmatrix}, \; D_{11} = \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix}$$

$$G_1 = \begin{bmatrix} 0.1 & 0.3 \end{bmatrix}, \; G_{d1} = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix}, \; D_{21} = 0.6, \; C_{d1} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \; D_{31} = 0.01$$

Subsystem 2

$$A_2 = \begin{bmatrix} 0.1 & 0.2 \\ 0.3 & 0.4 \end{bmatrix}, \; A_{d2} = \begin{bmatrix} -0.5 & 0.1 \\ 0 & -0.4 \end{bmatrix}, \; B_2 = \begin{bmatrix} 0 \\ 0.2 \end{bmatrix}, \; D_{12} = \begin{bmatrix} 0.1 \\ 0.5 \end{bmatrix}$$

$$G_2 = \begin{bmatrix} 0.6 & 0.2 \end{bmatrix}, \; G_{d2} = \begin{bmatrix} 0.4 & 0.6 \end{bmatrix}, \; D_{22} = 0.3, \; C_{d2} = \begin{bmatrix} 0.2 & 0 \end{bmatrix}, \; D_{32} = 0.2$$

Subsystem 3

$$A_3 = \begin{bmatrix} 0.2 & 0.1 \\ -0.6 & 0.3 \end{bmatrix}, \; A_{d3} = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.4 \end{bmatrix}, \; B_3 = \begin{bmatrix} 0.2 \\ 0.2 \end{bmatrix}, \; D_{13} = \begin{bmatrix} 0.2 \\ 0.8 \end{bmatrix}$$

$$G_3 = \begin{bmatrix} 0.7 & 0.3 \end{bmatrix}, \; G_{d3} = \begin{bmatrix} 0.01 & 0.02 \end{bmatrix}, \; D_{23} = 0.1, \; C_{d3} = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}, \; D_{33} = 0.2$$

The states x_k denote water quality constituents such as algae and ammonia nitrogen. The disturbance w_k arises from irregular discharge of effluents. Time delay τ_0 represents an average time to clear water streams up, which reflects the mixing effect of biochemical constituents in the reach, assuming $\tau_0 = 4$. By means of the controller design, the water quality constituents are remained in the standard level. Three subsystems $i = \{1, 2, 3\}$ are set up to represent three operating points, each of which depicts the influence of environmental factors. The switching signal σ is not known a priori but its instantaneous value is available online by water pollution management. The switching among these subsystems is dependant of the industrial dumped discharges followed by irregular patterns. In [9], a switching signal is arbitrary, which may lead to the frequent switchings destroying the system performance and even destabilizing the system. In order to circumvent the obstacle, it requires to set up the minimum operation time when certain subsystem is active. So, switching signal with ADT is employed here with the following parameters $\rho = 0.96$ and $\mu = 1.8$, then, we have ADT $\tau_a = 14.4$. Because the disturbance belongs to l_2 space, the H_∞ performance index γ is determined as the objective of the controller design. The control signals are generated by modifying water speed or discharging an amount of effluents. In the practical implementation, it is quite expensive to directly measure the states of systems and even impossible. The observers are designed to overcome the difficulties. As the measurement devices in the river are far from the observers, in general, they are linked by the local area networks or wireless networks. The use of networks brings some advantages to the control systems such as low cost, easy installation and maintenance, and flexible architectures. This results in the drawbacks, data missing and delays, which are the inherent disadvantages of the networks and are not considered in [9]. Therefore, it is necessary to consider the data missing and delays. The cases occur between the controllers and actuators and between sensors and observers. It is desired to design the composite system (5) to attain the control objective under the assumption that the successful transmission probabilities are $\bar{\alpha} = 0.91$ and $\bar{\beta} = 0.90$. Time delays in control input channel and output measurement channel are assumed as $\tau_1 = 2$ and $\tau_2 = 2$, respectively. On the basis of the aforementioned consideration, the problem of water quality control is cast into an H_∞ control problem of switched stochastic delay systems. When $\gamma = 6$, LMIs-based conditions in Theorem 2 have a feasible solution that yields the following observer and controller gains

$$
\begin{aligned}
K_1 &= \begin{bmatrix} -0.0005 & 0.0036 \end{bmatrix}, \; L_1 = \begin{bmatrix} 0.0018 & -0.0055 \end{bmatrix}^T \\
K_2 &= \begin{bmatrix} 0.0056 & -0.0012 \end{bmatrix}, \; L_2 = \begin{bmatrix} 0.0009 & 0.0046 \end{bmatrix}^T \\
K_3 &= \begin{bmatrix} -0.0022 & -0.0019 \end{bmatrix}, \; L_3 = \begin{bmatrix} 0.0285 & 0.0220 \end{bmatrix}^T
\end{aligned}
$$

To further display the validity of the results obtained, the simulation curves will be drawn. Assume that the initial states of systems and observers are $x_0 = \begin{bmatrix} 1 & -2 \end{bmatrix}$ and $\hat{x}_0 = \begin{bmatrix} 0 & 0 \end{bmatrix}$, respectively. The disturbance signal $w_k = 2exp(-0.1k)sin(0.05\pi k)$ comes from that in [9]. The uncertain ranges of the controller parameters are assumed to change in the interval $[-1,\ 1]$. Without loss of generality, define $F_{11}(k) = F_{21}(k) = F_{12}(k) = F_{22}(k) = F_{13}(k) = F_{23}(k) = sin(k)$, which are uncertain parameter structures in the observers and controllers and not considered in [9] and [21]. The running time in the simulation is 100 s. The sample time is 1 s. Figure 3 indicates the data missing between sensors and observers, where 1 and 0 mean that the data are transmitted successfully and aborted, respectively. Figure 4 displays the data missing between controllers and actuators, where 1 and 0 are the same meaning with those in Figure 3. The estimation errors of states of the systems are shown in Figure 5, which illustrate that the controllers are effective in clearing the disturbed water systems.

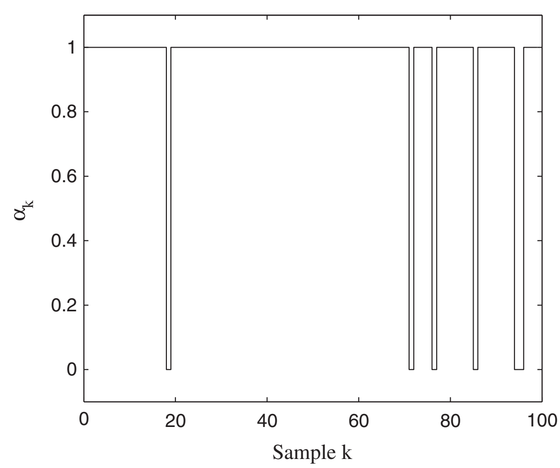

Figure 3. Data missing case between sensors and observers.

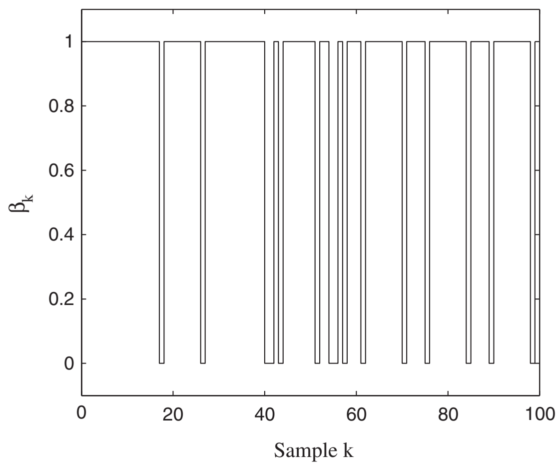

Figure 4. Data missing case between controllers and actuators.

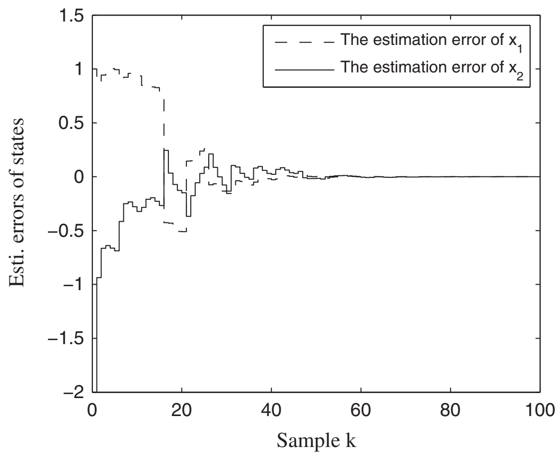

Figure 5. The estimation errors of system states.

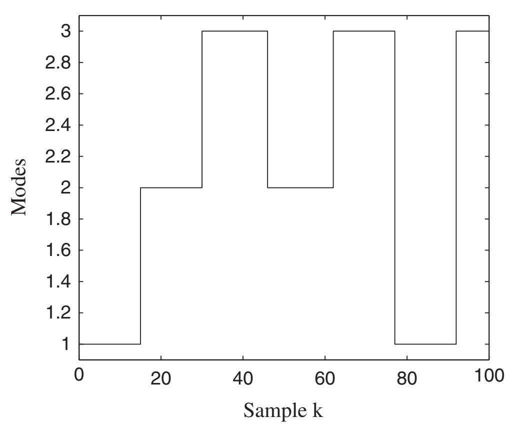

Figure 6. The switching signal of the systems.

The switching signal is provided in Figure 6, displaying which one of the three subsystems is active on the time horizon. From the feasible solutions of LMIs and simulation curves mentioned earlier, it is clearly shown that the techniques proposed in this paper is effective and practically useful.

6. CONCLUSION

We have studied the problem of non-fragile observer-based H_∞ control for discrete-time switched delay systems with data missing and time delays. The links from sensors to observers and from controllers to actuators have been considered. By transforming such problem into an H_∞ control problem for switched stochastic delay systems and using ADT method, an LMI-based condition has been obtained to design the observer and controller gains. The result has been extended to the case where there exists only data missing in two links, which shows that the results obtained are an improvement over the existing results. Two examples have been given to verify the effectiveness and applicability of the proposed techniques.

The deterministic switching signal has been considered, but the result could be extended to Markovian systems by considering Markov chain. In addition, the state-dependent switching signal is worth studying. On the other hand, the proposed method could be applied to control the devices or plant based on the network. In particular, it is potential for applications to control the dangerous devices such as nuclear reactor and chemical reactor.

ACKNOWLEDGEMENTS

The authors are greatly indebted to the editor and the reviewers for their helpful and valuable comments and suggestions for improving this paper. This work is in part supported by National Natural Science Foundation of China (Grants No. 61004040, 61004020, 61104114), the DSO National Laboratories, Singapore (Grant No. DSOCL06184), China Postdoctoral Science Foundation (Grant No. 20110490141, 2012T50254), and Fundamental Research Funds for the Central Universities (Grant No. DUT10RC(3)111).

REFERENCES

1. Sun Z, Ge SS. *Switched Linear Systems: Control and Design*. Springer-Verlag: New York, 2005.
2. Lin H, Antsaklis PJ. Stability and stabilizability of switched linear systems: a survey of recent results. *IEEE Transactions on Automatic Control* 2009; **54**(2):308–322.
3. Zhao J, Hill DJ. On stability, L_2-gain and H_∞ control for switched systems. *Automatica* 2008; **44**(5):1220–1232.
4. Shi P, Xia Y, Liu G, Rees D. On designing of sliding mode control for stochastic jump systems. *IEEE Transactions on Automatic Control* 2006; **51**(1):97–103.
5. Zhang G, Han C, Guan Y, Wu L. Exponential stability analysis and stabilization of discrete-time nonlinear switched systems with time delays. *International Journal of Innovative Computing, Information and Control* 2012; **8**(3(A)):1973–1986.
6. Attia S, Salhi S, Ksouri M. Static switched output feedback stabilization for linear discrete-time switched systems. *International Journal of Innovative Computing, Information and Control* 2012; **8**(5(A)):3203–3213.
7. Liu Q, Wang W, Wang D. New results on model reduction for discrete-time switched systems with time delay. *International Journal of Innovative Computing, Information and Control* 2012; **8**(5(A)):3431–3440.
8. Wang G, Liu Y, Wen C, Chen W. Delay-dependent stability criterion and H_∞ state-feedback control for uncertain discrete-time switched systems with time-varying delays. *International Journal of Innovative Computing, Information and Control* 2011; **7**(5(A)):2473–2484.
9. Mahmoud MS. Switched delay-dependant control policy for water-quality systems. *IET Control Theory and Applications* 2009; **3**(12):1599–1610.
10. Zhang L, Shi P, Boukas EK, Wang C. Robust L_2-L_∞ filtering for switched linear discrete time-delay systems with polytopic uncertainties. *IET Control Theory and Applications* 2007; **1**(3):722–730.
11. Wang D, Wang W, Shi P. Design on H_∞ filtering for discrete-time switched delay systems. *International Journal of Systems Science* 2011; **42**(12):1965–1973.
12. Wang D, Wang W, Shi P. Exponential H_∞ filtering for switched linear systems with interval time-varying delay. *International Journal of Robust and Nonlinear Control* 2009; **19**(5):532–551.
13. Daafouz J, Riedinger P, Iung C. Stability analysis and control synthesis for switched systems: a switched Lyapunov function approach. *IEEE Transactions on Automatic Control* 2002; **47**(11):1883–1887.
14. Hespanha JP, Morse AS. Stability of switched systems with average dwell-time. *Proceedings of the 38th Conference on Decision and Control*, Phoenix, AZ, 1999; 2655–2660.

15. Lian J, Feng Z, Shi P. Observer design for switched recurrent neural networks: an average dwell time approach. *IEEE Transactions on Neural Networks* 2011; **22**(10):1547–1556.
16. Zhang L, Shi P. Stability, l_2-Gain and asynchronous control of discrete-time switched systems with average dwell time. *IEEE Transactions on Automatic Control* 2009; **54**(9):2193–2200.
17. Xia Y, Liu X, Liu G, Rees D. Stabilization analysis and implementation for mimo ncs with time-varying delays. *International Journal of Adaptive Control and Signal Processing* 2011; **25**(7):639–652.
18. Xia Y, Shang J, Chen J, Liu G. Networked data fusion with packet losses and variable delays. *IEEE Transactions on Systems, Man, and Cybernetics: Part B* 2009; **39**(5):1107–1120.
19. Xia Y, Fu M, Yang H, Liu G. Robust sliding mode control for uncertain time-delay systems based on delta operator. *IEEE Transactions on Industrial Electronics* 2009; **56**(9):3646–3655.
20. Wang Z, Yang F, Ho DWC, Liu X. Robust H_∞ filtering for stochastic time-delay systems with missing measurements. *IEEE Transactions on Signal Processing* 2006; **54**(7):2579–2587.
21. Zhang H, Chen Q, Yan H, Liu J. Robust H_∞ filtering for switched stochastic system with missing measurements. *IEEE Transactions Signal Processing* 2009; **57**(9):3466–3474.
22. Yang G, Wang J. Non-fragile H_∞ control for linear systems with multiplicative controller gain variations. *Automatica* 2001; **37**(5):727–737.
23. Shu Z, Lam J, Xiong J. Non-fragile exponential stability assignment of discrete-time linear systems with missing data in actuators. *IEEE Transactions on Automatic Control* 2009; **54**(3):625–630.
24. Chen J, Yang C, Lien C, Horng J. New delay-dependent non-fragile H_∞ observer-based control for continuous time-delay systems. *Information Sciences* 2008; **178**(24):4699–4706.
25. Kim JH, Oh DCh. Robust and non-fragile H_∞ control for descriptor systems with parameter uncertainties and time delay. *International Journal of Control, Automation, and Systems* 2007; **5**(1):8–14.
26. Xie L, Fu M, De Souza CE. H_∞ control and quadratic stabilization of systems with parameter uncertainty via output feedback. *IEEE Transactions on Automatic Control* 1992; **37**(8):1253–1256.