Introduction to a Keplerian-Orbital-Element-based optimisation approach via Differential Dynamic Programming

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Abstract

One of the classic non-linear constrained optimal control problems in space applications involves the optimisation of low-thrust trajectories which are based on propulsion systems granting a greater final operational mass thanks to their high specific impulse despite for the higher time of flight.

One of the methods in solving such difficult control problem is Differential Dynamic Programming (DDP), which is based on the identification of optimal feedback control laws by the discretisation of the dynamics and the application of Bellman optimal principle. In a past work [1] a modified DDP algorithm for the optimisation of low-thrust trajectories was proposed where the problem is discretised in several decision steps, so that the optimisation process requires the solution of a great number of small problems. In [2] a new second-order algorithm was developed based on DDP, called Hybrid Differential Dynamic Programming, which maps the required derivatives recursively through first-order and second-order state transition matrices. More recently in [2] a Stochastic Differential Dynamic Programming was presented where random perturbations enter the dynamics of the problem and their expected values are computed by the unscented transform. In all these works the formulation of the dynamics and of the DDP procedure has always been in Cartesian coordinates and no attempt was made to exploit the orbital elements for such optimisation method.

In this talk an unconstrained low-thrust trajectory optimisation through a DDP approach based on Keplerian orbital elements is derived. Classic Keplerian motion is used to model the dynamics of the spacecraft with no addiction of orbital perturbations. The aim of the work is to translate the existing DDP procedures into orbital elements used as state variables which present the advantage to have a more physical interpretation of the orbital scenario together with a smooth dynamics, apart from the true anomaly variation. The cost function to be minimised is simply set to be the square of the magnitude of the control thrust. This talk introduces the first step for the definition of a new approach to solve non-linear constrained optimal control problems which can handle also more complex element-based dynamics making use of averaged perturbation techniques.

An interplanetary transfer to Mars is used as example to test the proposed method and a comparison of the results between the new algorithm and the classic Cartesian based DDP is carried out to enhance the differences and assess the performance.

References

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[2] Lantoine, G., and Russell, R.P., "A Hybrid Differential Dynamic Programming Algorithm for Constrained Optimal Control Problems. Part 1: Theory." J. Optimization Theory and Applications 154 (2012): 418-442. DOI:10.1007/s10957-012-0038-1

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