# The use of negative capacitances in piezoelectric resonant shunt

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Abstract—This document addresses the use of negative capacitances in piezoelectric resonant shunt. The main aims of the document are to show that it is possible to derive general analytical formulas for the optimization of the shunt and to explain how to practically build the shunt circuits.

# I. INTRODUCTION

The use of shunted piezoelectric actuators is really promising in the field of vibration mitigation in light structures. This approach relies on the connection between a properly designed electric impedance [1,2] and a piezoelectric actuator bonded to the vibrating structure. Several works are already available in the literature, describing how to shunt piezoelectric actuators (both benders and stacks) and achieve high damping performances. One possibility to increase the maximum achievable performance is the use of negative capacitances (NC) in the shunt circuit.



 $\begin{array}{l} FIGURE \ 1. \ Piezoelectric \ shunt \ with \ an \ nc \ in \ parallel \ (a), \ series \ (b) \\ and \ series \ parallel \ (c). \ -c_1 \ indicates \ an \ nc \ in \ parallel, \ -c_2 \ an \ nc \ in \\ series, \ and \ Z_{sh} \ the \ shunt \ impedance \ (made \ from \ either \ the \ series \ or \\ parallel \ of \ an \ inductance \ and \ a \ resistance) \end{array}$ 

Even if the use of NCs is already proved to be highly effective and reliable, a comprehensive analysis of this control approach is missing in the literature. Particularly, the use of resonant shunts (i.e. shunts where the electric impedance is made from a resistance and an inductance connected in either series or parallel) coupled to NCs is not deeply treated. Since this layout of the shunt impedance (i.e. resonant shunt coupled to NCs) offers high damping performances [3,4] for monomodal control, this work is specifically addressed to derive optimization formulas for this type of shunt. This allows to design optimal shunt impedances, as well as to foresee the consequent expected level of vibration attenuation.

Moreover, this study also allows to compare different layouts of the shunt impedance, highlighting which one is the best for a given specific engineering application. Indeed, the combination of a resonant impedance and NCs can have six different layouts because the inductance and the resistance can be linked in either series or parallel, and the NCs can be connected to the piezoelectric actuator in three different ways: series, parallel, and the newly introduced series+parallel (SP) [5,6] (FIGURE 1).

Finally, the practical implementation of the NCs is treated, showing how it is possible to avoid instability problems and increase the performances when electrical circuits not equivalent to pure NCs must be employed to physically build the NC circuit.

# II. THE COUPLING BETWEEN RESONANT SHUNT AND NC

The dynamic behavior of the electro-mechanical system (EMS) when NCs are used in the shunt can be described by means of equations which have the same mathematical form of those employed for piezoelectric shunt without NCs. This is possible by using the proper values of the variables which are affected by the presence of the NC (e.g. short/open-circuit natural frequencies). Among these variables, there is the modal electro-mechanical coupling coefficient  $k_i$  of the *i*-th mode which is artificially enhanced by the NC, leading to the enhanced modal electro-mechanical coupling coefficient  $\tilde{k}_i$  which has to be used in the EMS model. The parameter  $\tilde{k}_i$  represents the improvement achieved using NCs in the coupling between the mechanical and electrical parts of the whole system [5].

Relying on this equivalent formulation between EMSs with and without NCs, it is possible to derive the optimal values of *L* and *R* exploiting the approaches already used in the literature to tune resonant shunts without NCs [2,7], as well as tuned mass dampers (TMD) [8]. Indeed, the resonant shunt coupled to the piezoelectric actuator can be seen as the electrical equivalent of the TMD. Thus, using this analogy with the mechanical TMD, the referenced methods allow to estimate the optimal values of the eigenfrequency and damping ratio of the electrical circuit ( $\omega_e^{opt}$  and  $\xi_e^{opt}$ , respectively). From these values, it is then straightforward to derive the values of *L* and *R*  [4]. The expressions of  $\omega_e^{opt}$  and  $\xi_e^{opt}$  are provided in TABLE 1, where  $\omega_i^{sc}$  and  $\omega_i^{oc}$  are the short- (i.e.  $Z_{sh} = 0$ , see FIGURE 1) and open-circuit (i.e.  $Z_{sh} = \infty$ , see FIGURE 1) eigenfrequencies affected by the presence of the NCs, respectively. It is noticed that the expressions provided in TABLE 1 are valid for all the three possible layouts of the NC connection.

TABLE 1. EXPRESSIONS OF  $\omega_{e}^{opt}$  and  $\xi_{e}^{opt}$ 

	L and R in parallel	L and R in series
$\omega_{\rm e}^{\rm opt}$	$\sqrt{\frac{3(\omega_i^{\rm sc})^2 - (\omega_i^{\rm oc})^2}{2}}$	$\omega_i^{ m oc}$
ξ <sup>opt</sup> Se	$\frac{\sqrt{3}}{2} \sqrt{\frac{(\omega_i^{\rm oc})^2 - (\omega_i^{\rm sc})^2}{3(\omega_i^{\rm sc})^2 - (\omega_i^{\rm oc})^2}}$	$\frac{\sqrt{3}}{2} \sqrt{\frac{(\omega_i^{\text{oc}})^2 - (\omega_i^{\text{sc}})^2}{(\omega_i^{\text{oc}})^2 + (\omega_i^{\text{sc}})^2}}$



FIGURE 2. Attenuation in the area of the considered resonance for a system chosen as an example ( $k_i = 0.01$ ; however the curves do not change significantly changing the  $k_i$  value) and different values of the non-dimensional mechanical damping ratio  $\xi_i$ . Dashed line for *L* and *R* connected in parallel, and solid line for connection in series. The attenuation curves do not change changing the value of the original eigenfrequency  $\omega_i$  in short-circuit without NCs.

FIGURE 2 shows the attenuation (expressed in decibel) provided by the optimized shunt in the frequency range close to the resonance to be attenuated for a system chosen as an example. It is remembered that an NC in series is recommended to attenuate low-order modes, while the NC in parallel is advisable when high-order modes must be controlled. For this reason, the two NC layouts are not compared. The NCs in SP (that can be used for any mode order) provide attenuation levels between those shown in FIGURE 2 for the NCs in parallel and in series. It is interesting to evidence that, for high  $\tilde{k}_i$  values, the series connection between *L* and *R* provides attenuation levels higher than in the case of the parallel connection. Conversely, in case of low

values of  $\tilde{k}_i$ , the two connection types show similar attenuations.

# III. PRACTICAL IMPLEMENTATION OF THE NC

The NCs are practically built using operational amplifiers (OP-AMP). In some cases it is possible to use circuits which can be considered as simulating pure NCs, other times, due to practical problems, modified circuits must be employed [5]. These modified circuits behave like the parallel of an NC and a negative resistance (NR). This additional NR makes the behavior of the shunt circuit different from that of a pure NC at low frequency. This causes a worsening of the attenuation performance and instability problems. One approach to solve this problem is to add a compensation resistance in the OP-AMP circuit, which allows to better approximate the behavior of a pure NC [5].

# **IV. CONCLUSION**

This document has dealt with the coupling between NCs and resonant shunt. Optimisation of the shunt circuit, its corresponding attenuation performance and the practical implementation of the circuits are discussed.

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