Structural Identification of a Masonry Tower Based on Operational Modal Analysis

Carmelo Gentile,[1] Antonella Saisi,[1] and Alessandro Cabboi[2]

[1]*Department of Architecture, Built Environment, and Construction Engineering, Politecnico di Milano, Milan, Italy*

[2]*Department of Civil, Environmental, and Architectural Engineering, University of Cagliari, Cagliari, Italy*

1. INTRODUCTION

The structural assessment of a Cultural Heritage building is a step-by-step process that merges research methodologies and information from several disciplines. Although general rules that can be applied to all historic constructions are very difficult to define, it is generally agreed (Binda, Saisi, and Tiraboschi 2000) that the first phase of a correct diagnostic approach (i.e., the evaluation of the current health state or performance of the building) consists of the following tasks:

- Historic and documentary research and on site geometric survey, aimed at collecting all the essential information on the building geometry, its evolution, and the construction technology;
- Evaluation of the overall state of preservation by accurate visual inspection, that is generally complemented by survey of crack pattern and the identification and mapping of possible irregularities, different materials, and discontinuities; and
- Performing non-destructive tests (NDT) and/or minor destructive tests (MDT) on site and tests in the laboratory on sampled materials in order to evaluate the characteristics of the masonry.

As it is suggested also in current Italian Guidelines for the seismic risk mitigation of Cultural Heritage (Direttiva del Presidente del Consiglio dei Ministri [DPCM] 2011), the collected documentary and experimental information provide a first-level diagnosis, highlighting the overall state of preservation and the presence of local defects and vulnerabilities as well as the need of possible interventions, especially when local issues are detected. In this case, the design of strengthening interventions should be addressed by simplified local models, for example Giuffrè (1993), DPCM (2011), and Curti, Podestà, and Scandolo (2012).

In principle, the previously collected knowledge of global and local geometry, the characteristics of masonry texture, the construction details, and the mechanical characterization of the materials should be synthesized in the development of a finite element (FE) model of the structure. The FE model, in turn, should provide a second-level diagnosis since it could be used for evaluating the structural safety under service loads, predicting the performance under exceptional loads (such as earthquakes), and simulating the effects of structural modifications or repair interventions.

However, FE modeling of a historic structure is characterized by well-known issues:

- It is not simple to avoid errors resulting from inappropriate simplifying assumptions made in modeling such

Received September 24, 2013; accepted December 8, 2013.
Address correspondence to Carmelo Gentile, Department of Architecture, Built Environment, and Construction Engineering, Politecnico di Milano, 32 Piazza Leonardo da Vinci, 20133 Milan, Italy. E-mail: carmelo.gentile@polimi.it
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complicated structures; for example, excessive simplifications or "regularizations" of the actual geometry have generally to be avoided and the interaction with neighboring buildings has to be accounted for (Gentile and Saisi 2007);
- The correlation between the results of local tests (which indeed provide the mechanical characterization of the materials) and quantitative parameters to build up global structural capacity models is still an open issue (Binda, Saisi, and Tiraboschi 2000);
- The structural model of a historic structure, even when all the collected information is accurately represented, continues to involve significant uncertainties, for example, in the material properties (and their distribution) as well as in the boundary conditions. This aspect is especially critical for complex historic buildings evolved in different phases; and
- FE models are often used, even in refined non-linear analyses (e.g., Milani et al. 2012), without experimental validation and only occasionally the model validation is roughly performed by using few available local data (such as the stress level evaluated in few points through flat-jack tests).

A possible practice to obtain reliable linear models is based on performing ambient vibration modal tests and "forcing" the match between FE predictions and the measured modal parameters, as schematically shown in Figure 1. Ambient vibration testing (AVT) is a fully NDT, especially suitable to historic structures since the test is performed by just measur-ing the response in operational conditions. It is indeed true that the expected response of a historic building to ambi-ent excitation is quite low but this cannot be considered a real issue because highly sensitive and relatively inexpensive accelerometers are available on the market. Furthermore, a large number of operational modal analysis (OMA) techniques are available in the literature (Bendat and Piersol 1993, van Overschee and De Moor 1996, Brincker, Zhang, and Andersen 2000).

Hence, AVT and vibration-based structural identification (FE model calibration) of Cultural Heritage structures is emerging as a subject of great importance, although a limited number of complete investigations is reported in the literature, related to historic towers or minarets (Bennati, Nardini, and Salvatore 2005; Ivorra and Pallares 2006; Gentile and Saisi 2007; Peña et al. 2010; Ramos et al. 2010; Gentile and Saisi 2013), churches (Casarin and Modena 2008; De Matteis and Mazzolani 2010; Ramos et al. 2010) and monuments (Jaishi et al. 2003; Pau and Vestroni 2008; Aras et al. 2011).

This study mainly presents the methodology developed for the calibration of the numerical model of a historic bell tower in stonework masonry (Giampaolo 1960; Cazzani 1964), based on modal parameters. Two series of AVTs were performed on the tower (June 2007 and June 2008) and five vibration modes were clearly identified from both series of collected data by using an OMA technique based on the stochastic subspace identification (SSI) method (van Overschee and De Moor 1996).

The experimental investigation was complemented by the development of a linear FE model. Although the model accurately represented the geometry of the tower in its present condition and included all the information coming from visual inspection and on-site ND tests (Binda et al. 2012), the predicted dynamic characteristics exhibited major differences with the experimental results. The procedure proposed to identify the uncertain structural parameters of the model is based on the following steps: 1) prior identification of the uncertain parameters of the model; 2) sensitivity analysis, aimed at evaluating the minimum number of parameters, which are good candidates for the model tuning; 3) systematic manual tuning, aimed at estimating a model exhibiting an acceptable correlation with the experimental modal parameters (i.e., one-to-one correspondence with experimental mode shapes and limited dis-crepancies with respect to the observed resonant frequencies); 4) evaluation of the optimal values of the updating parame-ters by using a simple system identification technique (Douglas and Reid 1982); and 5) further improvement of the model by selecting an increased number of parameters and by repeating Step 4.

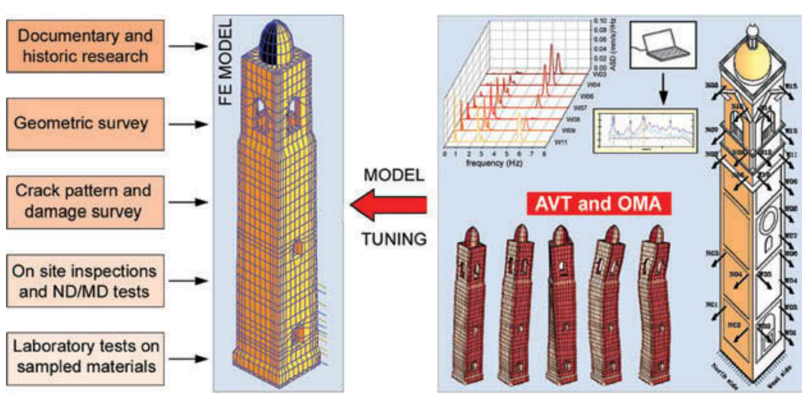

FIG. 1. Vibration-based validation of the finite element (FE) model of a historic building.

2. DESCRIPTION AND STATE OF PRESERVATION OF THE TOWER

The investigated bell tower (Figure 2), located in the small town of Arcisate (northern Italy), is approximately 37.0 m high and built in stonework masonry. The tower has squared plan, with sides of 5.8 m, and is connected to the church *Chiesa Collegiata* on the East side and partly on the South side. The church, dedicated to St. Vittore, dates to the 15th century and replaced a more ancient church, built in the 4th century (Giampaolo 1960) and modified in the 11th century. Probably the tower foundation dates back to the late Roman age, as well.

The first historic document concerning the tower is from the 16th century and reports St. Carlo Borromeo's request of access modification. Damages and further modifications occurred in time (Cazzani 1964). In 1702, a lightning storm damaged the top part of the bell tower, and, at the beginning of the 20th century, meaningful cracks were documented, especially on the masonry between the porch and the bell tower.

Seven orders of floors are present, with five of them being defined by masonry offsets at the corners and by corresponding sequences of small hanging arches marking the floor levels. The last two orders were probably added in the 18th century to host the bell trusses. The wall thickness decreases progressively along the height, from 135 cm at the ground level up to 65 cm at the top level (Binda et al. 2012).

Although extensive visual inspections and few sonic tests generally indicate that the stone masonry is relatively compact and of fairly good execution, the masonry texture appears locally often highly disordered and characterized by the local presence of vertical joints; in addition, due to insufficient stone interlocking and erosion of the mortar joints, it is difficult to distinguish clearly the cracks. The crack pattern (Figure 3) has been accurately surveyed also by using an aerial platform, which allowed close inspection of the external wall surface and detect the flaw paths. As shown in Figure 3, the tower exhibits long vertical cracks on every side, most of them cutting the

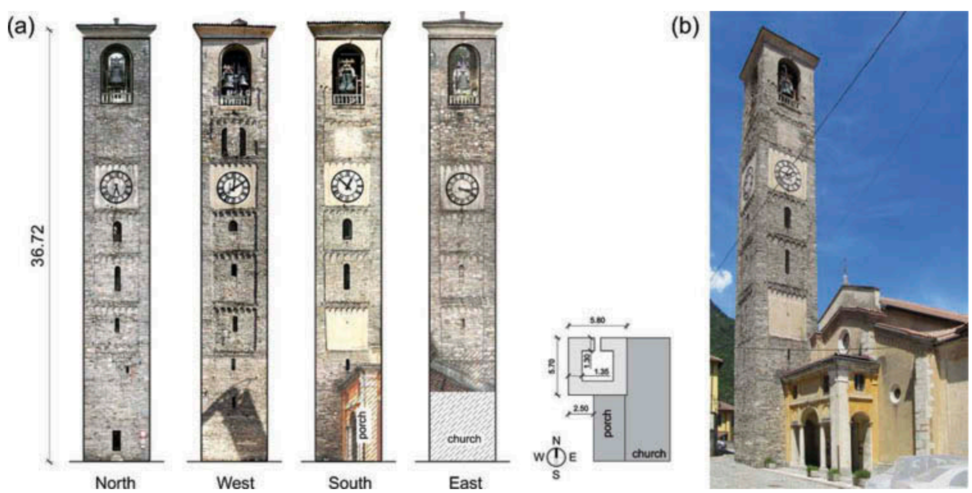

FIG. 2. The investigated tower: (a) fronts, and (b) general view.

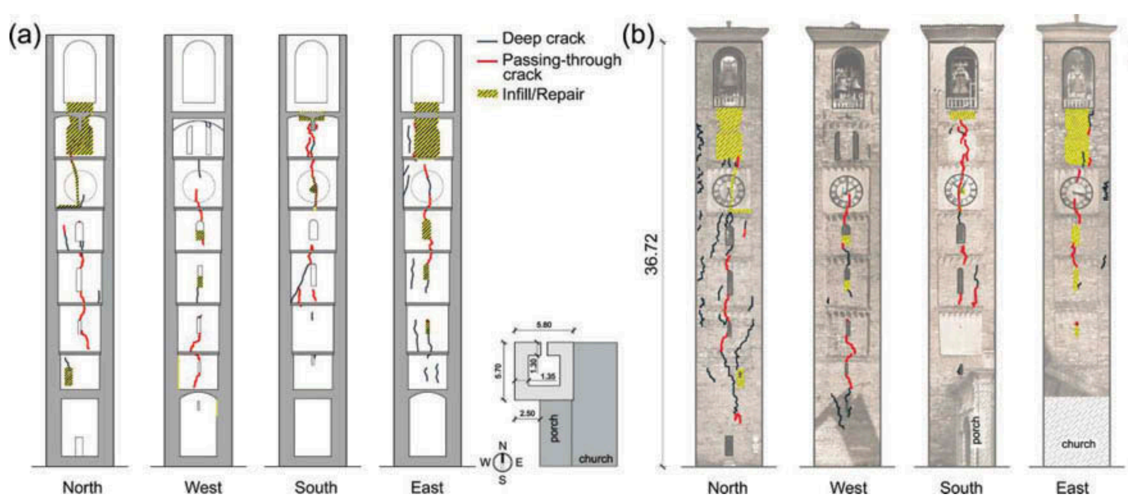

FIG. 3. Crack patterns on: (a) the vertical cross-sections, and (b) the fronts of the tower.

entire wall thickness and passing through the keystones of the arch window openings (Figure 4). These cracks are mainly distributed between the second and third order of the tower. Many superficial cracks are also diffused, particularly on the north and west fronts (Figure 3 and Figure 5a), which are not adjacent to the church.

Diffuse surface masonry discontinuities and changes of the masonry texture suggest several past modifications, that are not clearly datable as well as the construction of floors (Figure 5b). The visual inspection highlighted that the upper part of the tower, beneath the belfry level (Figure 5c), could be probably considered the most vulnerable, due to the widespread mortar erosion and the infilled openings, mainly on the east and north fronts; in addition, the infillings are often not properly linked to the surrounding load-bearing masonry.

3. AMBIENT VIBRATION TESTS AND DYNAMIC CHARACTERISTICS OF THE TOWER

3.1. Testing Procedures and Modal Identification

Two AVTs were conducted, on June 2007 and June 2008, using a 16-channel data acquisition system and WR 731A piezoelectric accelerometers (10 V/g sensitivity and 0.5 g peak acceleration). Each accelerometer was connected with a short cable (1 m) to a WR P31 power unit/amplifier, providing the constant current needed to power the accelerometer's internal amplifier, signal amplification, and selective filter-ing. The response of the tower was measured in 15 selected points, belonging to five different cross-sections along the height of the building, according to the sensor layout illustrated in Figure 6. Figure 7 shows the mounting of accelerome-ters 7-9 in the instrumented cross-section at level +22.78 m (Figure 6c).

In both tests, acceleration time-histories induced by ambient excitation were recorded for 3600 s. Examples of the acceleration time-histories recorded during the first test in the upper part of the tower are given in Figures 8a–c. It should be noticed that very low level of ambient excitation was present during the tests, with the maximum-recorded acceleration being always lower than 0.4 cm/s^2.

The modal identification was performed using a time window of 3600 s, in order to comply with the widely agreed recommendation of using an appropriate duration of the acquired time window (ranging between 1000 e 2000 times the fundamental period of the structure [e.g., Cantieni 2005]) to obtain accurate estimates of the modal parameters from OMA techniques. In fact, OMA methods assume that the excitation input is a zero mean Gaussian white noise and this assumption is as

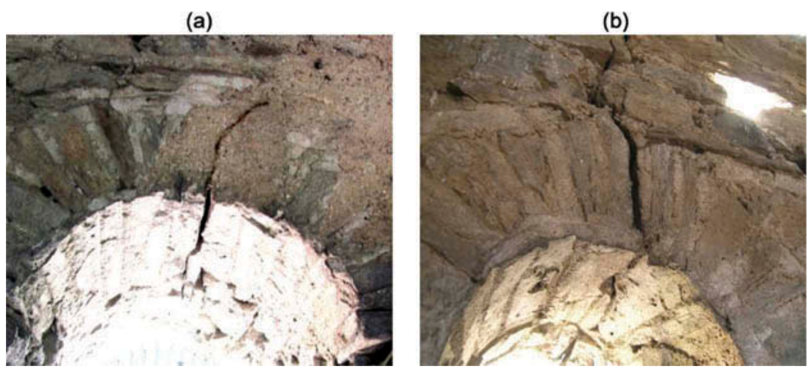

FIG. 4. Details of the passing-through cracks at the keystones of the arched openings.

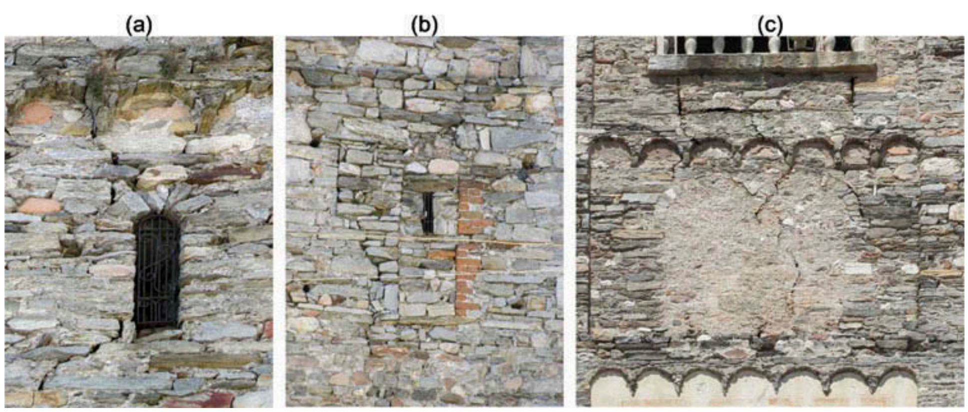

FIG. 5. (a, b) Details of cracked areas around openings at the first level of the north front; (c) details of the south front below the lodge, characterized by the presence of shaped discontinuities, mortar erosion and long passing-through cracks.

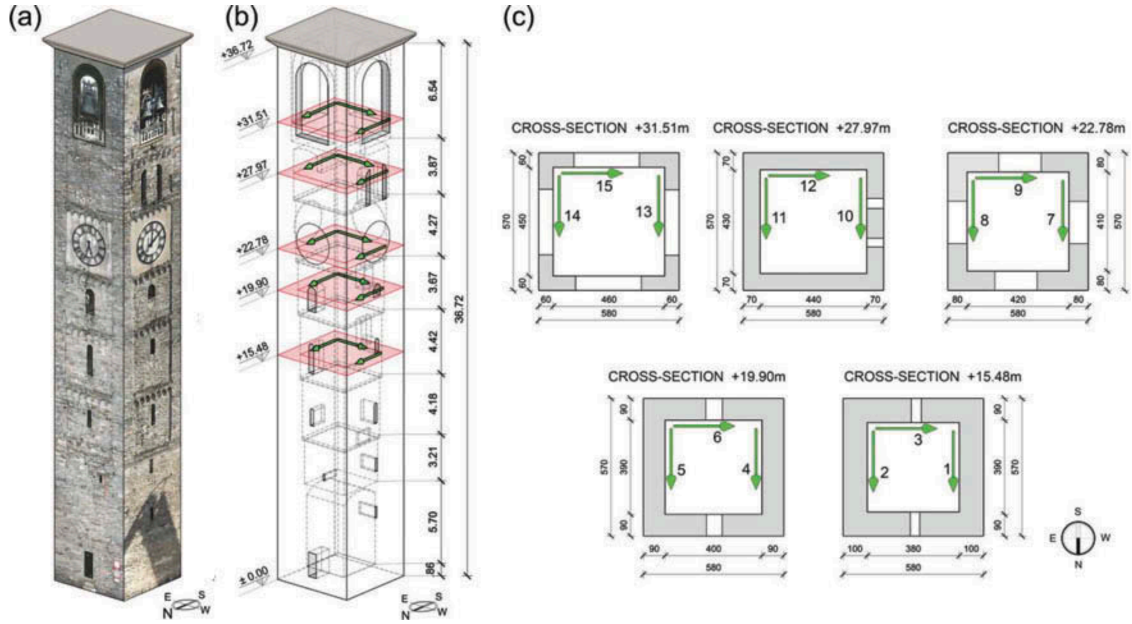

FIG. 6. (a) View of the tower; (b) sensor layout adopted in the dynamic tests; (c) position and numbering of the accelerometers (dimensions in m).

FIG. 7. Mounting of the accelerometers at test points (a) 7 and (b) 8–9.

closely verified as the length of the acquired time window is longer.

The sampling frequency was 200 Hz, which is much higher than that required for the investigated structure, as the significant frequency content of signals is below 6 Hz. Hence, low pass filtering and decimation were applied to the data before the use of the identification tools, reducing the sampling frequency from 200 Hz to 20 Hz; after decimation, the number of samples in each 1-hour record was of 72000, with a sampling interval of 0.05 s.

The extraction of modal parameters from ambient vibration data was carried out by using the data-driven SSI method (van Overschee and De Moor 1996) available in the commercial soft-ware ARTeMIS (SVS 2010); the natural frequency estimates have been verified also by inspecting the first singular value (SV) line of the spectral matrix, which is the mode indication function adopted in the frequency domain decomposition (FDD, Brincker, Zhang, and Andersen 2000) method.

3.2. Dynamic Characteristics of the Tower

Notwithstanding the very low level of ambient response (Figure 8) that existed during the tests, the application of the SSI technique to all collected data sets generally allowed to iden-tify five vibration modes in the frequency range of 0–6 Hz. The results of OMA in terms of natural frequencies can be summa-rized through the plots of Figures 9a–b, which refer to the two different tests. Each figure shows the first SV line of the spec-tral matrix and the stabilization diagram obtained by applying the FDD and the SSI technique to each collected dataset, respec-tively. The inspection of Figures 9a–b clearly highlights that the alignments of the stable poles in the stabilization diagram of the SSI method provides a clear indication of the tower modes and those alignments of stable poles correspond to well-defined local maxima in the first SV line of the FDD technique.

Table 1 summarizes the results obtained by applying the SSI method through the average and the standard deviation values of the natural frequencies (f , σ_f) and modal damping ratios (ζ , σ_ζ)

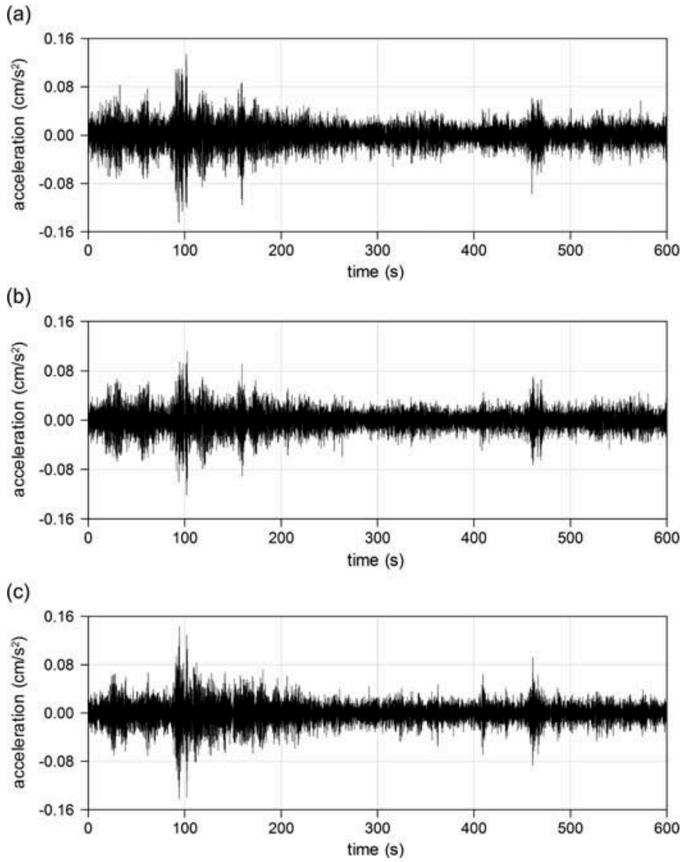

FIG. 8. Typical acceleration time series measured in June 2007 at test points (a) 10, (b) 11, and (c) 12.

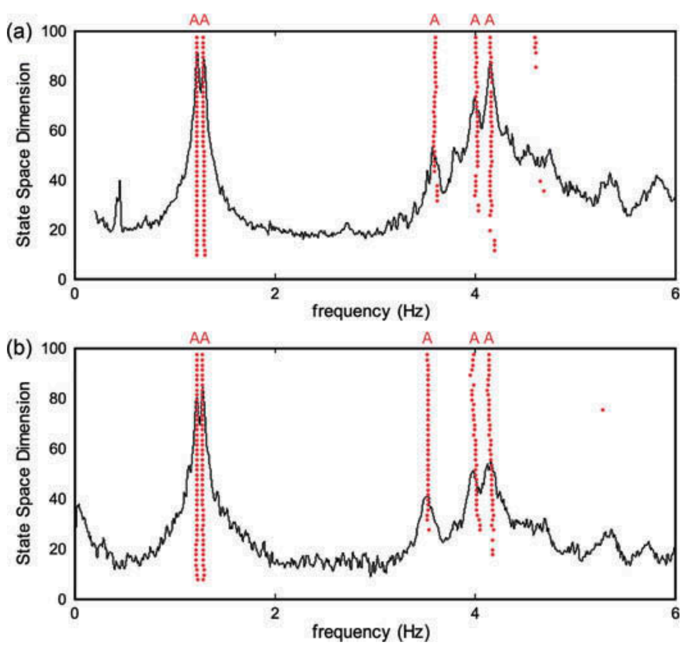

FIG. 9. First singular value line (FDD), alignments of stable poles (SSI) and automatic (A) identification of modal frequencies: (a) June 2007 and (b) June 2008.

identified by the SSI method. Table 1 (as well as Figures 9a–b) clearly shows that very similar estimates of the modal frequencies were obtained in the two tests. Furthermore, also the damping ratios of the lower three modes exhibit an excellent correspondence between the two AVTs. Since the mode shapes obtained from the two AVTs are very similar too, only the estimates from the test performed in June 2007 are presented in Figure 10. As it had to be expected, the identified modes can be classified as bending and torsion: dominant bending (B) modes were identified at 1.22 (B_1), 1.28 (B_2), 4.01 (B_3) and 4.16 Hz (B_4) while only one torsion mode (T_1) was identified at 3.60 Hz. It is worth noting that the dominant bending modes of the tower involve flexure practically along the diagonals. It is finally observed that in the correlation analysis with the predictions of numerical models, the results of the test performed on June 2007 were assumed as experimental reference because the modal estimates (Table 1) generally exhibit lower standard deviations.

4. FINITE ELEMENT MODELLING AND MODEL TUNING

4.1. Model Tuning Methodology

The model tuning procedure developed and applied to the present case study consists of the following steps:

1. Prior identification of the uncertain parameters of the numerical model;
2. Selection of the most sensitive uncertain parameters of the model, which are good candidates for the model tuning. In order to assess whether a certain parameter of the model would be identifiable from the measured quantities (so that ill conditioning of the inverse problem is avoided), the initial FE model was checked through a sensitivity analysis (e.g., Maia and Silva 1997). The sensitivity analysis computes the sensitivity coefficient as the rate of change of a particular response of the model with respect to a change in a structural parameter. Since the natural frequencies are assumed as reference responses, the sensitivity coefficients are defined as $s_{ik}=\partial f_i^{FEM}/\partial X_k$, where f_{i}^{FEM} ($i = 1, 2,\ldots, M$) is the i-th natural frequency of the model and X_k ($k = 1, 2,\ldots, N$) is the k-th parameter to be identified. The sensitivity coefficients were evaluated by a perturbation technique in the normalized form:

$$s_{i,k} = 100\frac{X_k}{f_i^{FEM}}\frac{\partial f_i^{FEM}}{\partial X_k} \qquad \text{(EQ1)}$$

representing the percentage change in mode frequency per 100% change in updating parameter;
3. After having defined a set of uncertain parameters, a systematic manual tuning (with respect to each parameter) is carried out, aimed at estimating a model exhibiting an

TABLE 1
Natural frequencies identified (SSI) in 2007 and 2008

	2007				2008			
Mode Identifier	f (Hz)	σ_f (Hz)	ζ (%)	σ_ζ (%)	f (Hz)	σ_f (Hz)	ζ (%)	σ_ζ (%)
B_1	1.216	0.0012	1.16	0.21	1.207	0.0045	1.15	0.27
B_2	1.283	0.0032	0.91	0.29	1.272	0.0024	0.88	0.27
T_1	3.598	0.0105	1.49	0.17	3.545	0.0160	1.55	0.56
B_3	4.009	0.0069	1.62	0.16	3.976	0.0099	1.34	0.45
B_4	4.156	0.0053	1.17	0.19	4.157	0.0118	1.78	0.48

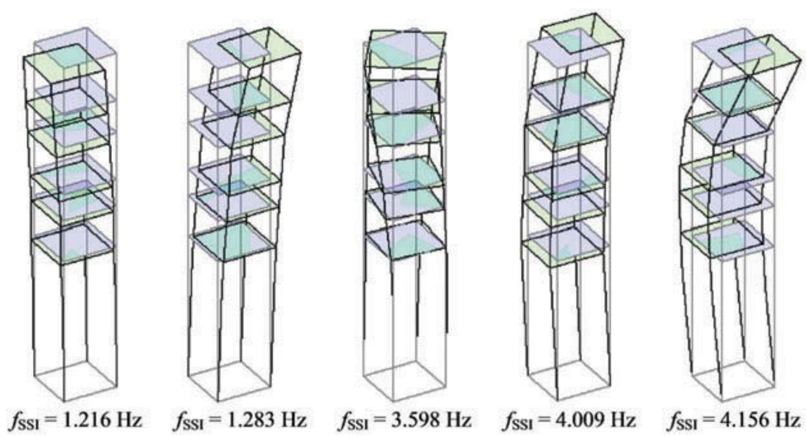

FIG. 10. Vibration modes identified from ambient vibration data (SSI, June 2007).

acceptable correlation with the experimental modal parameters (i.e., one-to-one correspondence with experimental modes and limited discrepancies with respect to the observed resonant frequencies). The correlation between the experimental modal frequencies f_i^{EXP} and the model predictions f_i^{FEM} is evaluated via the maximum (absolute) frequency discrepancy $DF_{\max}$:

$$DF_{\max} = \max\ (DF_{\mathrm{i}}) \qquad \text{(EQ2)}$$

$$DF_i = 100\left|\frac{f_i^{\mathrm{FEM}} - f_i^{\mathrm{EXP}}}{f_i^{\mathrm{EXP}}}\right| \qquad \text{(EQ3)}$$

and the average frequency discrepancy DF_{ave}:

$$DF_{\mathrm{ave}} = \frac{1}{M}\sum_{i=1}^{M} DF_i \qquad \text{(EQ4)}$$

4. If the manually tuned model tends to exhibit a good agreement with the experimental results, the optimal values of the updating structural parameters can be determined, by minimizing the difference between theoretical and experimental natural frequencies, through the procedure proposed in (Douglas and Reid 1982). According to this approach, the dependence of the natural frequencies of the model on the unknown structural parameters X_k (k = *1, 2,. . ., N*) is approximated around the current values of X_k, by the following:

$$f_i^*\ (X_1, X_2,\ \ldots,\ X_N) = \sum_{k=1}^{N}\left[A_{i,k}X_k + B_{i,k}X_k^2\right] + C_i \qquad \text{(EQ5)}$$

where f_i^* represents the approximation of the i-th frequency of the FE model. Once the set of approximating functions (5) has been established, the structural parameters of the model are evaluated by a least-square minimization of the difference between each f_i^* and its experimental counterpart $f_{\mathrm{i}}^{\mathrm{EXP}}$:

$$J = \sum_{i=1}^{M} w_i \varepsilon_i^2 \qquad \text{(EQ6a)}$$

$$\varepsilon_i = f_i^{\mathrm{EXP}} - f_i^*(X_1, X_2, \ldots, X_{\mathrm{N}}) \qquad \text{(EQ6b)}$$

where w_i is a weight constant. However, Equation (5) represents a reasonable approximation in a range, around the "base" value of the structural parameters $X_k{}^B$, limited by lower $X_k{}^L$ and upper values $X_k{}^U$ (k = *1,2,. . ., N*); thus, the coefficients A_{ik}, B_{ik}, C_i are dependent on both the base value of the structural parameters and the range in which these parameters can vary. The coefficients A_{ik}, B_{ik}, C_i are readily evaluated from *(2N+1)* finite element analyses (Douglas and Reid 1982), each with a different choice of the parameters: the first choice of the parameters corresponds to the base values; then each parameter is varied, one at time, from the base value to upper and lower limit, respectively. It is further noticed that, in principle, the quadratic approximation in Equation (5) is as better as the base values are closer to the solution; hence, the accuracy and stability of the optimal estimates should be carefully checked either by the complete correlation with the experimental data or by repeating the procedure with new base values; and

5. Further improvement (if necessary) of the model by selecting an increased number of structural parameters, checking their sensitivity and repeating Step 4.

4.2. Finite Element Modeling and Systematic Manual Tuning

As previously pointed out, a three-dimensional structural model (Figure 11) of the tower was developed (using the FE program Straus7), based on the available geometric survey. The tower was modeled by using eight-node brick elements. A relatively large number of finite elements have been used in the model, so that a regular distribution of masses could be obtained, and all the geometric variations and openings in the load-bearing walls could be reasonably represented. The model consists of 3475 solid elements with 17052 active degrees of freedom.

Since the geometry of the tower was accurately surveyed and described in the model, the main uncertainties are related to the characteristics of the material and boundary conditions. In order to reduce the number of uncertainties in the model calibration, the following initial assumptions were introduced: 1) homogeneous distribution of the masonry elastic properties; 2) the weight per unit volume of the masonry and the Poisson's ratio

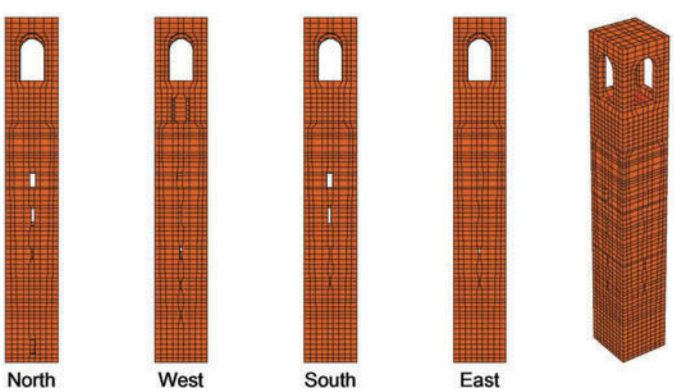

FIG. 11. Finite element model of the tower.

of the masonry were assumed as 17.0 kN/m^3 and 0.15, respectively; 3) the tower footing was considered as fixed since the soil-structure interaction is hardly involved at the low level of ambient vibrations that existed during the tests.

Under the above assumptions, sensitivity analysis pointed out that sensitive parameters were:

- The average Young's modulus E of stone masonry, which is the most sensitive parameter related to the frequency variation;
- The ratio $\alpha = G/E$, implying the assumption of orthotropic material and significantly affecting the natural frequency of the torsion mode; and
- The connection between the tower and the neighboring building (represented by linear nodal springs), which highly affects the bending mode shapes.

Hence, three steps of manual tuning were carried out. In the first model, referenced as FEM1, the stone masonry was assumed as a linear elastic and isotropic material, and the connection with the church was not taken into account. After tuning, the Young's modulus of stone masonry was assumed equal to 3.00 GPa, a value that is in good agreement with the results of the tests performed to characterize the masonry and reported in (Binda et al. 2012): more specifically, the average sonic velocity var-ied between 1620 and 2240 m/s^2 and the values measured by two double flat-jack tests at the base of the tower were larger than 3.00 GPa. The mode shapes and natural frequencies of FEM1 are reported in Figure 12. Furthermore, Table 2 summa-rizes the correlation between the modal frequencies of FEM1 (as well as of the models obtained in the next steps) and the experimental results, via the maximum frequency discrepancy $DF_{\max}$, and the average frequency discrepancy DF_{ave}.

FEM1 exhibits a highly imperfect correlation with the experimental results, since DF_{ave} is larger than 20% and $DF_{\max}$ is equal to 54.06%. Beyond that, FEM1 does not accurately represent the structural behavior of the tower since:

- the FEM1 model is much stiffer than the tower, with all the natural frequencies of the model signifi-cantly exceeding the experimental ones (Figure 12 and Table 2);
- the torsion mode T_1 of the model does not follow the experimental sequence, where the torsion mode is placed between two couples of bending modes (Fig. 10); on the contrary, the torsion mode follows two couples of bending modes in FEM1 model (Figure 12);
- the mode shapes of bending modes exhibit major differences with the experimental results: the predicted bending modes involve motion along the main north-south and east-west directions (Figure 12), whereas the identified modes involve bending along the diagonals (Figure 10).

It is important to underline that FEM1 would have been conceivably very similar to the model adopted in the structural

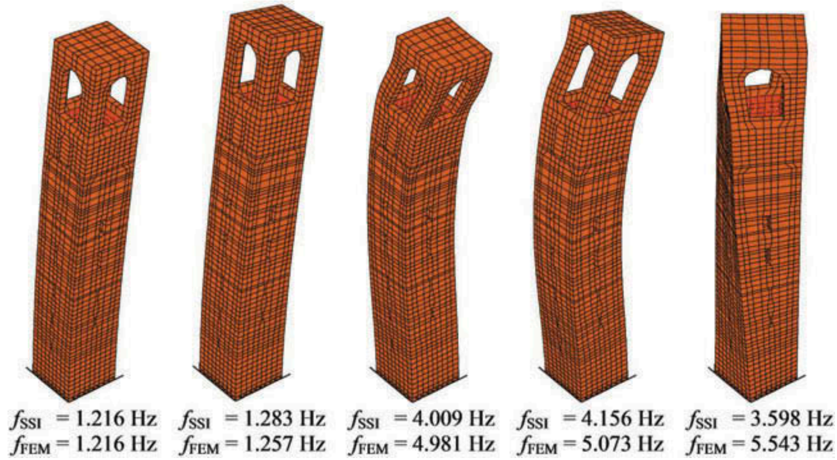

FIG. 12. Vibration modes of finite element model FEM1 (isotropic material, $E = 3.00$ GPa).

TABLE 2
Comparison between theoretical and experimental modal frequencies

	f (Hz)					
Mode Identifier	SSI (2007)	FEM1	FEM2	FEM3	FEM-DR1	FEM-DR2
B_1	1.216	1.216	1.154	1.216	1.216	1.220
B_2	1.283	1.257	1.174	1.259	1.257	1.282
T_1	3.598	5.543	3.382	3.570	3.597	3.592
B_3	4.009	4.981	4.170	4.427	4.438	4.057
B_4	4.156	5.073	4.224	4.516	4.522	4.145
	DF_{ave} (%)	20.70	5.05	4.36	4.31	0.42
	DF_{max} (%)	54.06	8.49	10.43	10.69	1.20

analysis of the tower without the information collected in AVT. On the contrary, the poor correlation between the FEM1 predictions and the actual dynamic characteristics of the tower suggests that some assumptions adopted in the model (such as the isotropic behavior of stone masonry and neglecting the connection with the neighboring building) need to be deeply revised.

In the second model, referenced as FEM2, an orthotropic elastic behavior was assumed for the stone masonry. After tuning, the average characteristics of the material were $E =$ 3.00 GPa and $G_{13} = G_{23} = 0.45$ GPa (corresponding to $\alpha = G_{13}/E = G_{23}/E = 0.15$). Figure 13 shows the modal characteristics of this second model, and highlights that the introduction of orthotropic elasticity dramatically improved the correlation with the experimental results. More specifically:

- the stiffness of the model significantly decreased, so that the average and maximum frequency discrepancies reduced from 20.70% to 5.05% and from 54.06% to 8.49%, respectively (Table 2);
- the torsion mode T_1 of the FEM2 model correctly follows the experimental sequence;
- nevertheless, the bending modes continue to exhibit flexure along the main north-south and east-west directions (Figure 13), differently from the observed modes which involve bending along the diagonals (Figure 10).

Introducing the effects of the connection between the tower and the church through a series of linear (nodal) springs of constant k, a third model FEM3 was obtained. In FEM3 model, the elastic characteristics of the masonry provided by FEM2 ($E =$ 3.00 GPa and $G_{13} = G_{23} = 0.45$ GPa) were not changed and only the parameter k was tuned in a pre-selected inter-val (1×10^4 kN/m $\leq k \leq 10\times10^4$ kN/m); after tuning, $k = 4\times10^4$ kN/m was assumed since this value tends to minimize the average frequency discrepancy DF_{ave}.

Figure 14 illustrates the dynamic characteristics of FEM3 model. As it had to be expected from previous investigations (see e.g. Gentile and Saisi 2007), the bending modes are now fully consistent with the experimental results. As reported in Table 2, the correlation between the FEM3 and the modal frequencies is fairly good for the first three modes ($DF_{max} < 2\%$),

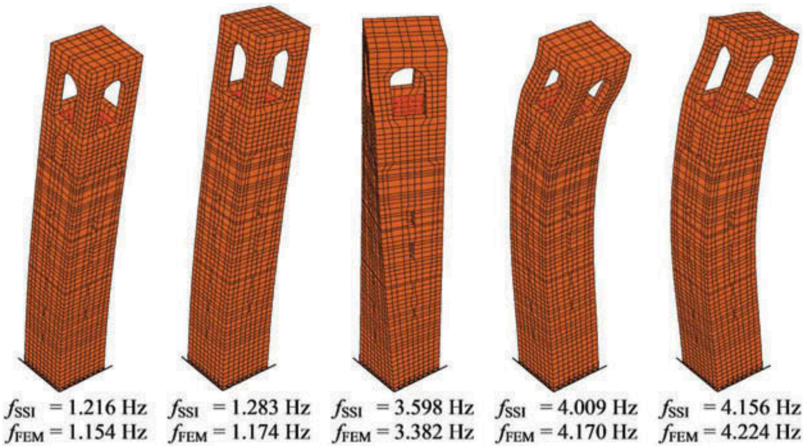

FIG. 13. Vibration modes of finite element model FEM2 (orthotropic material, $E = 3.00$ GPa, $G_{13}= G_{23}= 0.45$ GPa).

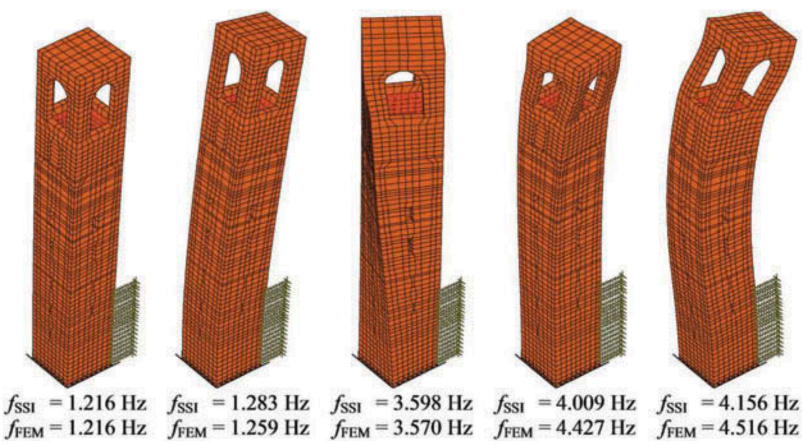

FIG. 14. Vibration modes of finite element model FEM3 (orthotropic material + springs).

whereas the discrepancies for the higher modes remains quite high (DF_{max}=10.43 % for the fourth mode).

4.3. Model Tuning By Structural Identification

At the end of manual tuning, a further refinement of the previously defined set of uncertain parameters (E, G_{13}=G_{23}, k) was obtained by minimizing the difference between theoretical and experimental natural frequencies (6) through the DR procedure (Douglas and Reid 1982). Table 3 summarizes the selected range of variation of each structural parameter, suggested by engineering judgment and the available tests on materials (Binda et al. 2012).

The structural identification procedure provided the following estimates of the parameters:

- $E = 3.00$ GPa
- $G_{13} = G_{23} = 0.46$ GPa
- $k = 3.76 \times 10^4$ kN/m

that are very similar to the ones of FEM3 (i.e. to the values obtained from systematic manual tuning). Consequently, the natural frequencies of current model FEM-DR1 (Table 2) continue to exhibit a satisfactory correlation for the lower modes as well as high differences for the higher bending modes (DF_{max}=10.69%).

The discrepancy is conceivably related to the simplified distribution of the model elastic properties. As suggested by the mode shapes, the higher bending modes depend on the elastic characteristics of the masonry in the upper part of the tower. In addition, as pointed out in Section 2, the upper region is characterized by a more evident mortar joint erosion and changes of the masonry texture including wide infilled openings on the East and North fronts, beneath the belfry level (Figure 3).

Hence, the distribution of the Young's modulus was updated and the tower was divided in two regions, with the masonry Young's modulus being assumed as constant within each zone. The two regions, denoted as I and II, correspond to the lower five levels of the building (E_I,h$\leq$ 26.0 m, including a large number of passing-through cracks, Figure 3) and the upper part (E_{II},h$>$ 26.0 m, including infilled/repaired areas and the belfry), respectively. The possible set of updating parameters includes E_I, E_{II}, α and k. It should be noticed that assuming only

TABLE 3
Structural parameters for finite element (FE) models identification

Structural parameter	Base value	Upper value	Base value	Optimal value FEM-DR1	Optimal value FEM-DR2
E (whole structure) [GPa]	2.0	4.0	3.0	3.0	–
E_{I} (height $\leq$ 26 m) [GPa]	2.0	4.0	3.0	–	2.97
E_{II} (height > 26 m) [GPa]	1.0	3.0	2.0	–	1.60
α	0.110	0.190	0.150	0.153	0.172
k [kN/m]	1.0×10^4	10.0×10^4	4.0×10^4	3.76×10^4	8.30×10^4

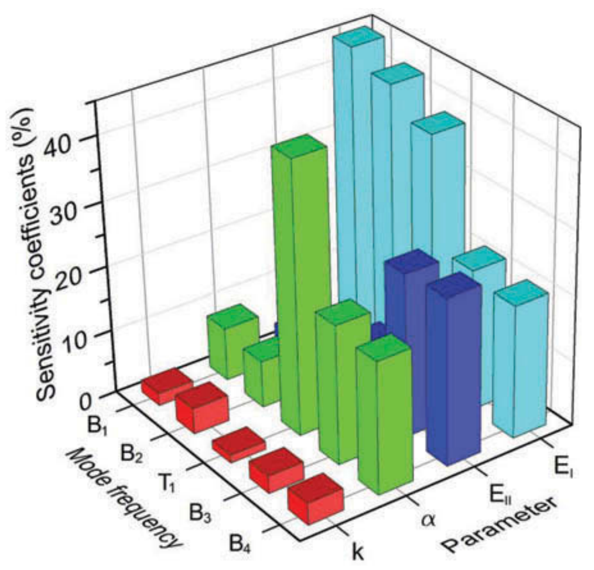

FIG. 15. Sensitivity coefficients.

one parameter α means that the ratio between the shear moduli in the two regions of the tower is equal to the ratio between the Young's moduli.

Before repeating the system identification process, a sensitivity analysis was carried out and the sensitivity coefficients are shown in Figure 15. The sensitivity coefficients indicate that the natural frequencies of all modes are highly affected by E_{I}, E_{II} and α and weakly depend on the stiffness k of springs, as well. Therefore, the inverse problem appears not affected by ill-conditioning and the investigated structural parameters are good candidates to be selected as updating parameters in a structural identification approach.

Table 3 summarizes the optimal estimates of the structural parameters obtained from the DR method, the base values and the assumed lower and upper limits. By examining the optimal values in Table 3, the following comments can be made:

- the optimal values of the elastic parameters in the lower region (E_{I}=2.97 GPa, $G^{I}{}_{13} = G^{I}{}_{23} = \alpha \times E_I = 0.172 \times 2.97 \cong 0.51$ GPa) are very similar to those provided by the FEM3 and FEM-DR1 models;
- the optimal estimates of the elastic parameters in the upper region (E_{II}=1.60 GPa, $G^{\mathrm{II}}{}_{13} = G^{\mathrm{II}}{}_{23} = \alpha \times E_{\mathrm{II}} = 0.172 \times 1.60 \cong 0.28$ GPa) turned out to be lower than the ones of in the lower part, reflecting the observed state of preservation;
- the stiffness of the springs becomes higher than in model FEM3 and FEM-DR1 in order to better fit the ratio between the natural frequencies of the first two modes.

Figure 16 shows the mode shapes of the FEM-DR2 updated model, corresponding to the experimental ones (Figure 10), and the correlation with the measured modal behavior. It should be noticed that FEM-DR2 model represents an excellent approximation of the real structure, with the maximum relative error between natural frequencies being larger than 1% only for mode B_3. Furthermore, also the correlation between mode shapes—estimated via the MAC (Allemang and Brown 1983) in Figure 16—is very good for the first two modes (with the MAC being larger than 0.96); for the higher modes, the MAC is in the range 0.711–0.845 so that appreciable average differences are detected and again related to the simplified distribution of the model elastic properties, which were held constant for large regions of the tower. However, the final FEM-DR2 model could surely be adopted to evaluate the structural safety under service or exceptional loads as well as a suitable baseline model for long-term dynamic monitoring (Gentile Saisi, and Cabboi 2012) of the tower.

5. CONCLUSIONS

A rational methodology, developed for the calibration of the numerical model of a historic masonry tower, has been presented in the paper. The proposed procedure is based on:

- Development of a FE model, accurately representing all the information collected on global and local geometry of the structure, characteristics of masonry texture and mechanical characterization of the materials; and
- Use of the modal parameters (i.e. natural frequencies and mode shapes) measured in ambient vibration tests to identify the uncertain parameters of the model.

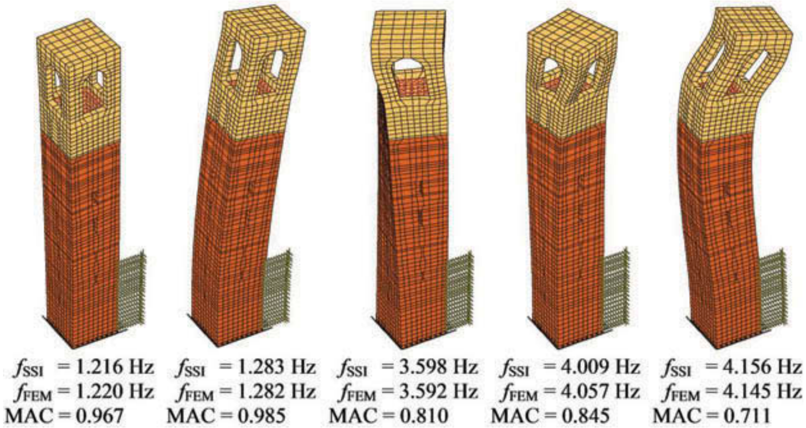

FIG. 16. Vibration modes of the optimal (updated) finite element model FEM-DR2.

The proposed methodology of model tuning consists in the appropriate combination of systematic manual tuning, sensitivity analysis and simple system identification algorithm.

Although the presented results refer to the investigated building, the following conclusions conceivably exhibit general value, at least for historic masonry towers:

- Notwithstanding the very low level of ambient vibrations that existed during the tests, AVT and OMA have proved to be effective tools for identifying the dynamic characteristics of key vibration modes, provided that appropriate and very sensitive acquisition chain (capable of capturing the "interesting" dynamics embedded in the noise) is used in the tests;
- Although the initial model FEM1 accurately represented the geometry of the tower in its present condition and included all the information coming from visual inspection and on-site ND tests, the model exhibited very poor correlation with the actual dynamic characteristics of the tower, with maximum differences between predicted and measured modal frequency ranging between 22.06% and 54.06%. Since the initial model FEM1 would have been adopted in structural analysis without the information provided by OMA, in the authors' opinion, vibration-based validation should be included in common diagnostic practice especially when the model has to be used to predict the non-linear behavior of the structure or to design repair interventions;
- The proposed methodology of vibration-based structural identification—involving systematic manual tuning and sensitivity analysis—seems appropriate (although slightly time-consuming) to simultaneously handle the uncertainties of a historic construction and the possible ill-conditioning of the inverse problem;
- The application of the proposed procedure provided a linear elastic model of the tower, representing an excellent approximation of the structure in its present condition. More specifically, the maximum difference between predicted and measured natural frequencies did not exceed 1.20% and fairly good correlation in terms of mode shapes was obtained, as well; and
- The results obtained in the case study indicate that the orthotropic elasticity assumption is more suitable to modeling stone masonry structures.

ACKNOWLEDGEMENTS

Sincere thanks are due to M. Antico, M. Cucchi (VibLab, Laboratory of Vibrations and Dynamic Monitoring of Structures, Politecnico di Milano) and PhD N. Gallino who assisted the authors in conducting the field tests.

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