Simplified Approaches to Design Medium-Rise Unbraced Steel Storage Pallet Racks. II: Fundamental Period Estimates

Claudio Bernuzzi[1]; Armando Gobetti[2]; Giammaria Gabbianelli[3]; and Marco Simoncelli[4]

Introduction

In the framework of a research project currently in progress in conjunction between the Technical University of Milan and the University of Pavia, attention has been paid to the reliability of simplified approaches for the design of steel storage pallet racks. These approaches are the focus of a two-part paper, the second part of which is the present paper. Part I, "Elastic Buckling Analysis," (Bernuzzi et al. 2015a) appraises the critical load multiplier for the sway buckling mode, α_{cr}, which governs the choice of the analysis method and the stability verification checks for static design. Furthermore, α_{cr} is of fundamental importance for seismic design, and its prediction through Horne's equation (Horne 1975) is proposed. As to the European approach {FEM 10.2.08 (Federation Européenne de Manutention 2010); prEN 16681 [European Committee for Standardization (CEN) 2013]; and EN 1998-1 (CEN 2004)}, the requirement to account for second-order effects is in fact associated with the maximum value of the interstory drift sensitivity coefficient θ, which corresponds to $1/\alpha_{cr}$, where θ is defined as

$$\theta = \frac{P_{\text{tot}} d_r}{V_{\text{tot}} h_{\text{LL}}} \quad (1)$$

where P_{tot} = total gravity load at and above the considered story in the seismic design situation, which corresponds to V_{Ed} in Eq. (5) of the companion paper; d_r = design interstory drift evaluated as the difference of the average lateral displacements at the top and bottom of the story under consideration and is calculated by means of linear elastic first-order elastic analysis, corresponding to $\delta_U - \delta_L$; V_{tot} = total seismic story shear, corresponding to the products between the frame imperfection angle ϕ and vertical loads; and H_{LL} = interstory height. On the basis of the θ value, lateral force (LFMA), modal response spectrum analysis (MRSA), or large displacement method of analysis (LDMA) can be adopted for seismic design.

For the assessment of the fundamental period of vibration, two simplified approaches are considered: Rayleigh's method (Chopra 2011; Clough and Penzien 1995) and an analytical base-displacement method proposed for semicontinuous racks [Rack Manufacturers Institute (RMI) 2012; FEMA 2005). Both approaches have been applied to the same set of racks previously presented in the companion paper (Bernuzzi et al. 2015a), to which reference can be made for all the input data related to the geometry of the racks and key components and the degree of flexural stiffness of both beam-to-column and base-plate joints. Fig. 1 presents the layout of the executed analysis and explains the symbols used in this paper to describe the main results. The outcomes of the present study allow a direct appraisal of the degree of accuracy of these methods. Improvements of the considered simplified approaches are proposed to both increase their level of reliability and also clearly define their limitations in terms of degree of accuracy when applied to racks. Finally, for research reproduction purposes, the "Appendix" summarizes all of the computation phases associated with the simplified approaches on a two-bay and four-load level pallet rack. The mechanical and geometrical data of the members and joints are reported together with the main output results related

[1]Associate Professor, Dept. of Architecture, Built Environment and Construction Engineering, Politecnico di Milano, Piazza Leonardo da Vinci, 32, 20133 Milano, Italy.

[2]Associate Professor, Dept. of Civil Engineering and Architecture, Università di Pavia, Via A. Ferrata, 3, 27100 Pavia, Italy.

[3]Ph.D. Student, Dept. of Civil Engineering and Architecture, Università di Pavia, Via A. Ferrata, 3, 27100 Pavia, Italy (corresponding author). E-mail: giammaria.gabbianelli@gmail.com

[4]Ph.D. Student, Dept. of Architecture, Built Environment and Construction Engineering, Politecnico di Milano, Piazza Leonardo da Vinci, 32, 20133 Milano, Italy.

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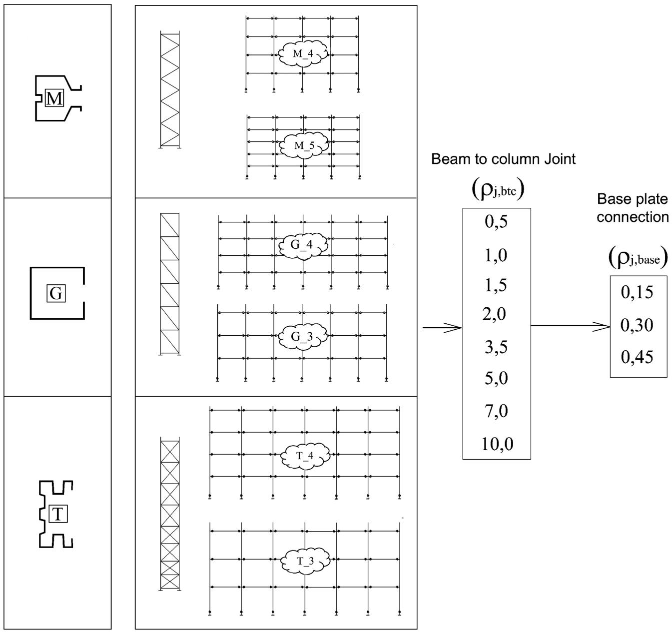

Fig. 1. Synopsis of numerical cases considered in parametric analysis

to the use of both six- and seven-degrees of freedom (DOFs) finite-element (FE) beam formulations.

Remarks on Rack Modeling for Seismic Design

As shown with reference to static design (Teh et al. 2004; Bernuzzi et al. 2014a, b, 2015b), the Wagner's effect term and shear center eccentricity are expected to significantly influence the dynamic behavior of racks. As a consequence, they cannot be neglected in seismic design. From well-established equations of motion (Chopra 2011; Clough and Penzien 1995), the free vibration behavior of an elastic multi-DOF system (no damping) undergoing small deformations and displacements (modal analysis) is governed by the system

$$[K]^E - \omega^2 [M]\{A\} = 0 \tag{2a}$$

where $[K]^E$ and $[M]$ represent the stiffness and mass matrices, respectively; and ω = natural frequency of vibrations.

The solutions are the values of ω_i [eigenvalue analysis of the system Eq. (2*a*)] and the associated modes of vibration $\{A\}_i$. The fundamental period of vibration T_1, i.e., the longest period, is evaluated by considering the minimum value of ω_i as

$$T_1 = \max\left\{\frac{2\pi}{\omega_i}\right\} = \frac{2\pi}{\min\{\omega_i\}} \tag{2b}$$

As Vöros (2004) clearly demonstrated in singly symmetric cross section members, the mass matrix $[M]$ has to be adequately formulated and considered in the structural analysis. As a consequence, matrix $[M]$ has been implemented in the *Śiva* FE analysis software (Bernuzzi and Gobetti 2014) by considering the presence of seven DOFs for each beam node. As expected from the basis of the theory of structures and clearly discussed by Trahair (1993), axial forces and moments acting on members change the natural frequencies/periods of vibration of columns, beams, and beam columns. With reference to a single degree-of-freedom system of mass M and stiffness K, the well-known fundamental period is obtained as $T_1 = 2\pi\sqrt{M/K}$. If second-order effects are neglected, T_1 is independent of the load condition, i.e., not influenced by the values of internal forces and bending and torsional moments, applied to the member; otherwise, when the effects of deformations are expected to be relevant for equilibrium and compatibility conditions, K will necessarily include the geometric stiffness contribution, strictly depending on the loads acting on the structure. By increasing the loads applied to the structure, the fundamental period of vibration also increases and tends to infinity when the buckling load is applied.

With reference to more flexible and complex semicontinuous unbraced structures, such as the racks in the downaisle direction, a second-order analysis is usually required for routine design. Dynamic rack properties are expected to be strictly dependent on the load condition. To correctly evaluate all of the modes of global vibration shapes, i.e., flexural, torsional, and lateral-torsional vibration, and any mutual interaction among them, the geometric stiffness matrix $[K]^G$ will be considered in the system [Eq. (2*a*)], which must be modified in

$$\{([K]^E + [K]^G) - \omega^2[M]\}\{A\} = 0 \tag{3}$$

Including second-order effects in the free vibration problem leads to determination of a set of eigenvalues and, as a consequence, a set of periods of vibration, strictly depending on the level of the applied loads because of the presence of the geometric stiffness matrix $[K]^G$.

Prediction of the Fundamental Period of Vibration

As previously mentioned, the most important parameter in the analysis and design of any structure subject to seismic load is the fundamental period of vibration (T_1) being used to define the design spectrum and select the method of analysis. Major standard codes [RMI MH 16.1 (RMI 2012); FEM 10.2.08 (Federation Européenne de Manutention 2010); and prEN 16681 (CEN 2013)] base the rack design on period T_1. As in traditional steel, steel-concrete composite, or concrete buildings, T_1 can be directly determined through a FE free vibration (modal) analysis, which, in the case of racks and other steel structures made by components with a singly symmetric cross section, must be developed, taking into account all of the contributions associated with the presence of the seventh DOF (cross section warping). For more traditional steel structures made by double-symmetric cross section hot-rolled members, significant research has been developed to predict the fundamental period of vibrations through very simplified equations on the basis of key geometrical data of the frame. In these cases, the available analysis approaches and associated design rules are very well-established, with suitable and reliable procedures proposed by seismic design codes. These approximated formulas for the T_1 period calculation are not admitted for racks. Furthermore, in accordance with the United States practice for routine rack design (RMI 2012), the traditional Rayleigh's method is recommended, and, as an alternative, a simplified displacement-based procedure (FEMA 2005) is admitted. Both of these approaches, which are summarized in the following sections, have been applied to the considered racks to evaluate their level of accuracy, especially with reference to the presence/absence of the seventh DOF to evaluate the set of lateral displacements.

Rayleigh's Method Applied to Racks

As previously mentioned, RMI (2012) and FEMA 460 (FEMA 2005) recommend Rayleigh's approach to assess the fundamental period of vibration. It is clearly declared that the fundamental period must be determined on the basis of the structural properties and deformation characteristics of the resisting elements in a properly substantiated analysis. With reference to the generic floor level i, identifying the associated vertical load, i.e., generally a dead load plus a consistent fraction of the pallet load, and the seismic lateral force with the terms W_i and F_i, respectively, the fundamental period T_1^R is approximated through Rayleigh's approach (Superscript R) as

$$T_1^R = 2\pi\sqrt{\frac{\sum W_i \delta_i^2}{g \sum F_i \delta_i}} \tag{4}$$

where g = acceleration because of gravity and is assumed equal to $9.81\ \mathrm{m/s^2}$; and δ_i expresses the total lateral displacement relative to the base evaluated by means of a first-order elastic analysis at level i, as computed using F_i, and is assumed equal to the resulting gravitational force on the i load level.

FEMA Method Applied to Racks

An alternative to Rayleigh's method, a very simplified expression to directly estimate the fundamental period of vibration could be used, which does not need any kind of structural analysis and therefore appears extremely convenient for preliminary design. In particular, in the "Appendix" of FEMA 460 (FEMA 2005), a simple analytical displacement-based model discussed in the literature (Filiatrault et al. 2006) is proposed to appraise the seismic behavior of storage racks in their downaisle direction. This procedure, which is used to evaluate the displacement demand and the consequent rack performance, enables the prediction of few parameters governing seismic design, such as the fundamental period of vibration, base shear, and top lateral displacement. The proposed simplified expressions are on the basis of assumptions, which are usually satisfied in the design practice of unbraced racks, such as

- Regularity of components and geometry (equal beam-to-upright connections, beams spaced relatively uniformly with height, and equal base-plate connections);
- Moment-resisting connections of the racks simultaneously experiencing similar rotations at all times;
- Inelastic deformations occurring only at the connection locations (beam-to-column and base-plate joints); and
- Overall seismic response reasonably captured through a single degree-of-freedom system, which corresponds to an assumed first downaisle mode.

Attention in this paper is focused on the sole prediction of the fundamental period T_1. With reference to the symbols presented in Fig. 2, the predicted T_1^F value can be expressed as

$$T_1^F = 2\pi\sqrt{\frac{\sum_{i=1}^{N_{\mathrm{LL}}} W_i h_i^2}{g\left[N_{\mathrm{btc}}\left(\frac{S_{j,\mathrm{btc}}\cdot K_b}{S_{j,\mathrm{btc}}+K_b}\right) + N_{\mathrm{base}}\left(\frac{S_{j,\mathrm{base}}\cdot K_u}{S_{j,\mathrm{base}}+K_u}\right)\right]}} \tag{5}$$

where g = already defined gravity acceleration; W_i = mass used in the calculation of the seismic force scaled by the effective horizontal seismic factor; h_i = height from the base to the center of gravity of the vertical load on level i; N_{LL} = number of story; $S_{j,\mathrm{btc}}$ and $S_{j,\mathrm{base}}$ = rotational stiffness of the beam-to-column and base-plate connections, respectively; N_{btc} and N_{base} = number of beam-to-column and base-plate connections, respectively; and K_b and K_u = beam and upright flexural stiffness, respectively, expressed as

$$K_b = \frac{6EI_b}{L_b} \tag{6a}$$

$$K_u = \frac{4EI_u}{h_{\mathrm{LL}}} \tag{6b}$$

where E = Young's modulus; I = second moment of area; L_b = beam length; h_{LL} = interstory height; and subscripts b and u are related to the beam and upright, respectively.

The $[(S_{j,\mathrm{btc}} \cdot K_b)/(S_{j,\mathrm{btc}} + K_b)]$ and $[(S_{j,\mathrm{base}} \cdot K_u)/(S_{j,\mathrm{base}} + K_u)]$ terms in Eq. (5) express the total rotational stiffness between the beam and upright and the upright end and industrial floor, respectively. Both terms are obtained by the sum in series of the rotational stiffness of the connections ($S_{j,\mathrm{btc}}$ and $S_{j,\mathrm{base}}$) and of the flexural stiffness of the members (I_b and I_u). Using the same

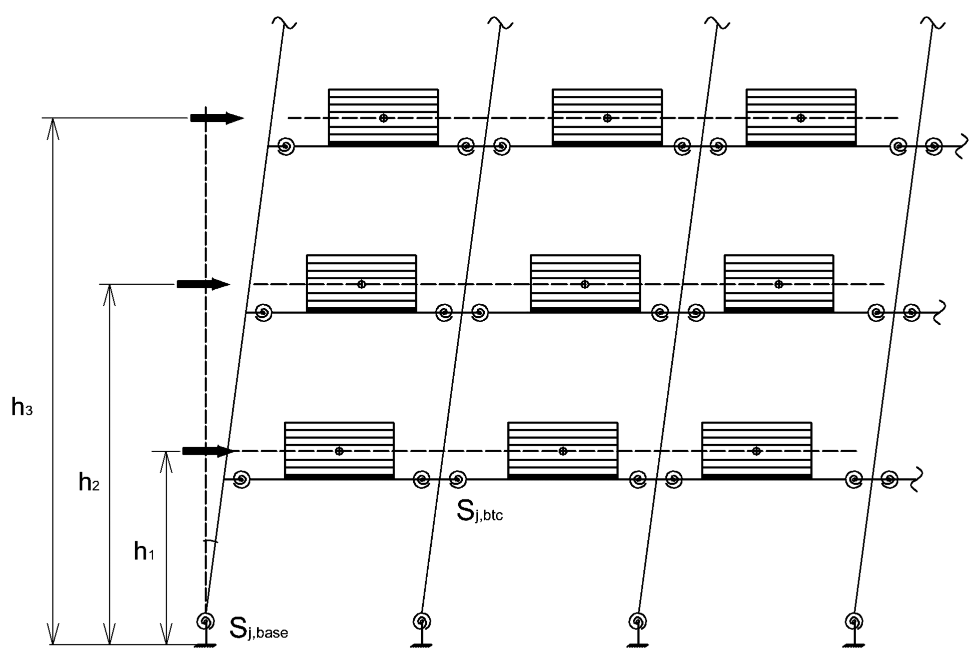

Fig. 2. Simplified model to assess fundamental rack period in accordance with FEMA's method

symbols already introduced to describe the rack parametric analysis in the companion paper, T_1^F can be expressed as

$$T_1^F = 2\pi \sqrt{\frac{\sum_{i=1}^{N_{\mathrm{LL}}} W_i h_i^2}{g\left[N_{\mathrm{btc}}\left(\frac{6\rho_{j,\mathrm{btc}}}{\rho_{j,\mathrm{btc}}+12}\right)\frac{EI_b}{L_b} + N_{\mathrm{base}}\left(\frac{60\rho_{j,\mathrm{base}}}{15\rho_{j,\mathrm{base}}+2}\right)\frac{EI_u}{h_{\mathrm{LL}}}\right]}} \quad (7)$$

Numerical Applications

As already mentioned, FE analysis packages offering only six-DOF beam element formulations are currently used for routine rack design, but in several cases, they appear inadequate, leading to overestimation of the level of safety or, similarly, greater evaluation of the load-carrying capacity values than what is effective. In the framework of the research phase focused on seismic design, attention was at first paid to quantify the errors when the effects associated with warping are neglected. For this purpose, reference can be made to Table 1, where the T_1^6/T_1^7 ratio is reported for all of the considered racks, where the Superscripts 6 and 7 are associated with the number of DOFs in the FE beam formulation. Furthermore, the domain containing all of these data is plotted in Fig. 3 versus the beam-to-column joint stiffness parameter $\rho_{j,\mathrm{btc}}$. Hence

- The T_1^6/T_1^7 ratio appears to be practically independent of the value of the base-plate joint stiffness.

Table 1. Approximation of the Fundamental Period of Vibration T_1, Neglecting Warping Effects (T_1^6/T_1^7) through Rayleigh's Method (T_1^{6-R}/T_1^7 and T_1^{7-R}/T_1^7) and FEMA Approach (T_1^F/T_1^7)

		$\rho_{j,\mathrm{base}} = 0.15$				$\rho_{j,\mathrm{base}} = 0.30$				$\rho_{j,\mathrm{base}} = 0.45$			
Racks	$\rho_{j,\mathrm{btc}}$	T_1^6/T_1^7	T_1^{6-R}/T_1^7	T_1^{7-R}/T_1^7	T_1^F/T_1^7	T_1^6/T_1^7	T_1^{6-R}/T_1^7	T_1^{7-R}/T_1^7	T_1^F/T_1^7	T_1^6/T_1^7	T_1^{6-R}/T_1^7	T_1^{7-R}/T_1^7	T_1^F/T_1^7
M_4	0.5	0.74	0.74	0.75	0.77	0.75	0.75	0.75	0.76	0.75	0.74	0.75	0.74
	1.0	0.82	0.82	0.84	0.85	0.83	0.82	0.84	0.84	0.82	0.84	0.85	0.85
	1.5	0.84	0.84	0.86	0.85	0.86	0.86	0.88	0.87	0.85	0.85	0.88	0.86
	2.0	0.87	0.86	0.89	0.85	0.86	0.85	0.88	0.85	0.87	0.86	0.89	0.86
	3.5	0.86	0.86	0.90	0.79	0.87	0.87	0.91	0.81	0.88	0.87	0.92	0.82
	5.0	0.87	0.87	0.92	0.76	0.88	0.87	0.92	0.78	0.88	0.88	0.93	0.79
	7.0	0.87	0.87	0.93	0.73	0.88	0.87	0.93	0.75	0.88	0.87	0.93	0.75
	10.0	0.87	0.86	0.92	0.68	0.88	0.87	0.94	0.71	0.86	0.86	0.93	0.71
M_5	0.5	0.80	0.79	0.80	0.78	0.81	0.80	0.80	0.75	0.80	0.79	0.80	0.73
	1.0	0.86	0.85	0.87	0.85	0.86	0.85	0.87	0.84	0.87	0.86	0.88	0.84
	1.5	0.88	0.87	0.90	0.87	0.88	0.87	0.89	0.85	0.89	0.88	0.90	0.86
	2.0	0.89	0.88	0.91	0.86	0.88	0.88	0.91	0.85	0.89	0.89	0.92	0.86
	3.5	0.90	0.89	0.93	0.83	0.90	0.89	0.94	0.84	0.89	0.88	0.93	0.83
	5.0	0.89	0.88	0.94	0.79	0.89	0.88	0.94	0.80	0.90	0.89	0.95	0.81
	7.0	0.88	0.88	0.94	0.76	0.89	0.88	0.94	0.77	0.89	0.88	0.95	0.77
	10.0	0.89	0.87	0.95	0.73	0.88	0.87	0.95	0.74	0.88	0.87	0.95	0.74
G_3	0.5	0.80	0.79	0.80	0.83	0.82	0.81	0.81	0.80	0.83	0.82	0.83	0.80
	1.0	0.87	0.86	0.87	0.94	0.88	0.87	0.88	0.92	0.88	0.88	0.89	0.91
	1.5	0.88	0.88	0.90	0.97	0.89	0.88	0.90	0.95	0.89	0.89	0.90	0.95
	2.0	0.89	0.88	0.90	0.97	0.89	0.88	0.91	0.96	0.90	0.89	0.91	0.95
	3.5	0.89	0.89	0.92	0.95	0.91	0.90	0.93	0.96	0.91	0.90	0.93	0.96
	5.0	0.90	0.89	0.94	0.92	0.89	0.89	0.93	0.92	0.90	0.90	0.94	0.94
	7.0	0.90	0.89	0.94	0.89	0.90	0.89	0.95	0.91	0.90	0.89	0.95	0.91
	10.0	0.89	0.88	0.94	0.85	0.89	0.88	0.94	0.87	0.90	0.89	0.95	0.88
G_4	0.5	0.79	0.79	0.80	0.76	0.77	0.76	0.77	0.70	0.81	0.80	0.80	0.71
	1.0	0.85	0.84	0.87	0.86	0.86	0.86	0.88	0.85	0.87	0.87	0.88	0.84
	1.5	0.86	0.85	0.88	0.89	0.89	0.88	0.91	0.89	0.88	0.87	0.88	0.87
	2.0	0.87	0.87	0.90	0.91	0.88	0.87	0.90	0.90	0.89	0.88	0.90	0.89
	3.5	0.88	0.87	0.92	0.90	0.89	0.89	0.93	0.91	0.90	0.89	0.93	0.91
	5.0	0.88	0.88	0.94	0.89	0.89	0.88	0.93	0.89	0.89	0.88	0.93	0.89
	7.0	0.87	0.87	0.94	0.86	0.88	0.88	0.94	0.87	0.89	0.88	0.94	0.87
	10.0	0.88	0.87	0.95	0.84	0.88	0.88	0.95	0.85	0.88	0.88	0.95	0.86
T_3	0.5	0.83	0.83	0.84	0.90	0.85	0.85	0.86	0.89	0.84	0.84	0.85	0.86
	1.0	0.89	0.89	0.90	0.96	0.90	0.90	0.91	0.96	0.89	0.88	0.90	0.94
	1.5	0.91	0.91	0.92	0.96	0.91	0.91	0.92	0.97	0.90	0.90	0.91	0.95
	2.0	0.91	0.91	0.93	0.95	0.91	0.91	0.93	0.95	0.92	0.91	0.94	0.96
	3.5	0.92	0.91	0.93	0.89	0.93	0.92	0.94	0.91	0.93	0.92	0.94	0.92
	5.0	0.93	0.92	0.95	0.86	0.92	0.91	0.94	0.87	0.93	0.93	0.96	0.89
	7.0	0.92	0.92	0.95	0.81	0.93	0.92	0.95	0.84	0.93	0.92	0.95	0.84
	10.0	0.92	0.92	0.95	0.77	0.93	0.92	0.95	0.79	0.93	0.93	0.96	0.81
T_4	0.5	0.84	0.83	0.84	0.85	0.84	0.83	0.84	0.82	0.83	0.83	0.84	0.79
	1.0	0.89	0.88	0.89	0.92	0.88	0.88	0.89	0.89	0.90	0.89	0.90	0.90
	1.5	0.90	0.89	0.91	0.92	0.91	0.91	0.92	0.92	0.90	0.90	0.91	0.91
	2.0	0.92	0.91	0.93	0.92	0.92	0.92	0.93	0.93	0.92	0.91	0.93	0.91
	3.5	0.93	0.93	0.95	0.89	0.93	0.93	0.95	0.90	0.93	0.92	0.94	0.89
	5.0	0.94	0.93	0.95	0.86	0.94	0.93	0.95	0.87	0.94	0.93	0.96	0.87
	7.0	0.93	0.93	0.95	0.82	0.93	0.93	0.95	0.83	0.93	0.93	0.95	0.84
	10.0	0.94	0.93	0.96	0.79	0.93	0.93	0.96	0.80	0.94	0.93	0.96	0.81

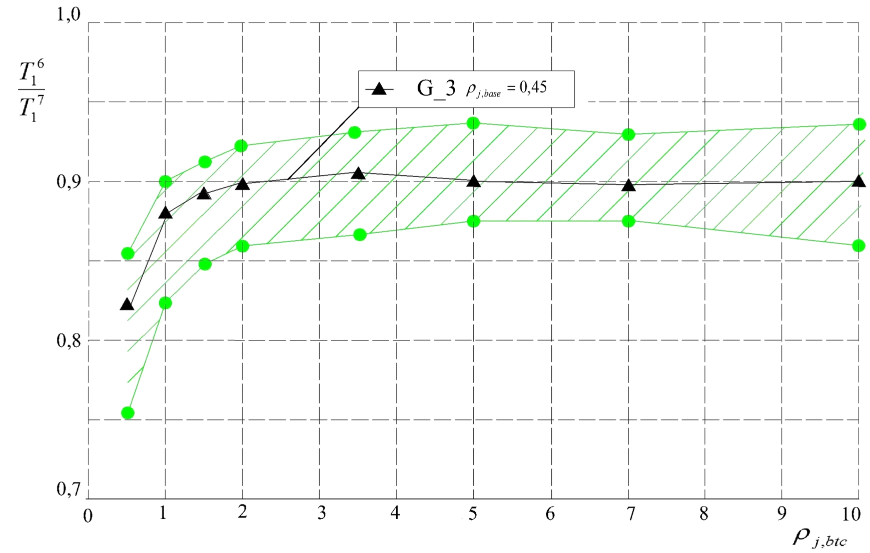

Fig. 3. Influence of warping on fundamental period of vibration

- The trend of all of the T_1^6/T_1^7 versus $\rho_{j,\mathrm{btc}}$ curves is quite similar and, as an example, Fig. 3 plots the curve related to Rack G_3 with $\rho_{j,\mathrm{base}} = 0.45$. The lowest values of the ratio are in correspondence to $\rho_{j,\mathrm{btc}} = 0.5$. By increasing the beam-to-column joint stiffness, the error in predicting T_1^7 decreases quickly, and approximately for $\rho_{j,\mathrm{btc}} \geq 2.0$, it results approximately constant.
- Neglecting the presence of the warping degree of freedom, i.e., adopting a classical six-DOF FE beam formulation, the fundamental period of vibration is significantly underestimated up to 26% for racks with more flexible joints. By increasing $\rho_{j,\mathrm{btc}}$, the T_1^6/T_1^7 ratio also increases, but it is always lower than unity and its upper limit is 0.94.

Rayleigh's Method Applied to Racks

The fundamental period of vibration has been predicted through Rayleigh's method by using both the set of first-order displacement obtained from a FE elastic analysis using both a six- and seven-DOF beam formulation, hereinafter identified as T_1^{6-R} and T_1^{7-R}, respectively. To allow a direct appraisal of the degree of accuracy of Rayleigh's method, Table 1 reports the T_1^{6-R}/T_1^7 and T_1^{7-R}/T_1^7 ratios. Hence

- The fundamental period of vibration is always significantly underestimated by the method, independently of the adopted FE beam formulation.
- With reference to the use of the traditional beam element formulation, T_1^{6-R} is practically equal to the one determined through a six-DOF FE modal analysis and the very limited differences are not greater than 2%. This confirms the validity of the approach when applied to structural systems composed of doubly symmetric cross section members. Furthermore, T_1^{6-R} is significantly lower than the fundamental rack period, up to 0.74 times, and by increasing the degree of stiffness of the joints, the T_1^{6-R}/T_1^7 ratio increases up to 0.93.
- All of the periods that were predicted considering warping DOF are in general not significantly different from the ones obtained through a six-DOF beam modal analysis for the lowest values of $\rho_{j,\mathrm{btc}}$. By increasing the degree of stiffness of beam-to-column joints, the level of accuracy of the prediction increases as well, and this trend is moderately dependent on the degree of stiffness of the base plate connections. The use of a seven-DOF beam element formulation allows slight improvement of the prediction, especially for $\rho_{j,\mathrm{btc}} > 2.0$. The T_1^{7-R}/T_1^7 ratio is always lower than unity but ranges between 0.75 and 0.96.
- A direct comparison between T_1^{6-R} and T_1^{7-R} shows that the predicted period through seven-DOF beam element formulation is slightly greater than T_1^{6-R}, owing to the greater lateral deformability associated with the coupling between the flexure and torsion in the seven-DOF FE beam formulation. The greatest differences can be noted for the $M\_$ and $G_$racks. In the case of $\rho_{j,\mathrm{btc}} \leq 3.5$, the T_1^{7-R}/T_1^{6-R} ratio ranges from unity up to 1.09. Otherwise, if the T_3 and T_4 racks are considered, the influence of the seventh DOF on displacements is more limited and never greater than 4%.

Fig. 4 presents the domains associated with the T_1^{6-R}/T_1^7 and T_1^{7-R}/T_1^7 ratios versus the beam-to-column stiffness parameter $\rho_{j,\mathrm{btc}}$. These domains, which were obtained from the values contained in Table 1, are directly overlapped to allow a direct appraisal of the accuracy obtained through seven- and six-DOF FE beam formulations. In general, the accuracy is improved when warping DOF is considered, which corresponds to a lower distance from unity. Furthermore, in correspondence to a $\rho_{j,\mathrm{btc}}$ of approximately greater than 2.0, the amplitude of the seven-DOF domain decreases, whereas for the six-DOF prediction, it remains approximately constant. Fig. 5 presents the cumulated distribution of the T_1^{6-R}/T_1^7 and T_1^{7-R}/T_1^7 ratios with the value corresponding to the 95% fractile values. As already discussed, the results are quite different in terms of accuracy, as it appears from the distribution curves plotted in the same figure. The use of Rayleigh's approach does not appear adequate for design purposes, which in many cases are too inaccurate, despite the fact that 95% of fractile values are quite close to unity and are approximately equal to 0.93 and 0.95 when warping DOF is neglected or considered, respectively.

FEMA Method Applied to Racks

Attention has been paid to the prediction of the fundamental period through the FEMA approach (T_1^F), and all of the values of the computed ratio T_1^F/T_1^7 are reported in Table 1 and plotted in Fig. 6. A moderate influence of the base-plate joint stiffness can be noted; however, it is slightly greater than the one observed for the Rayleigh's approach. By increasing $\rho_{j,\mathrm{base}}$, the T_1^F/T_1^7 ratio

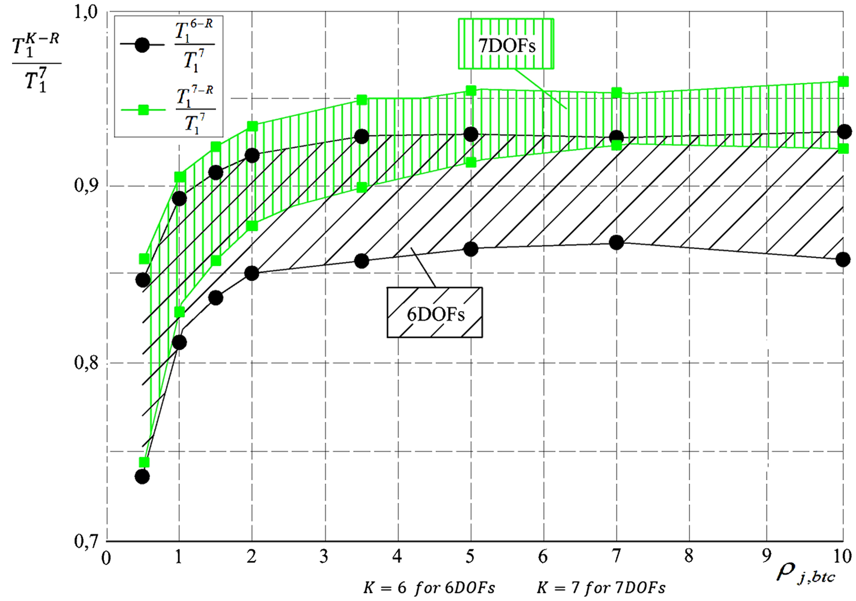

Fig. 4. Accuracy of Rayleigh's method neglecting T_1^{6-R}/T_1^7 or considering T_1^{7-R}/T_1^7 in presence of seventh warping DOF

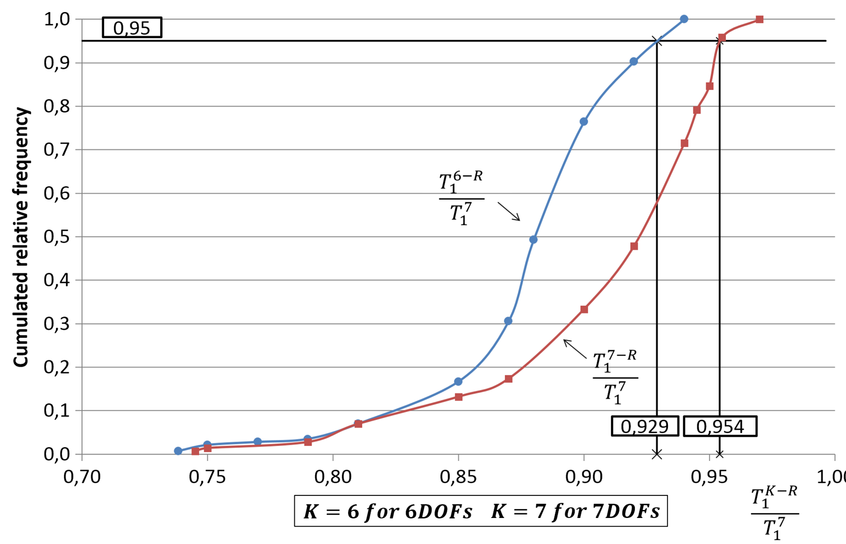

Fig. 5. Distribution of cumulated relative frequency of T_1^{6-R}/T_1^7 and T_1^{7-R}/T_1^7 for all considered racks

increases as well, and differences from the limit cases of $\rho_{j,\text{base}} = 0.15$ and 0.45 are never greater than 6%. The trend of all of these curves is significantly different from the ones corresponding to Rayleigh's method. The T_1^F/T_1^7 ratio increases rapidly, starting from the lowest values of $\rho_{j,\text{btc}}$, remains constant, and then decreases moderately for the greatest values of $\rho_{j,\text{btc}}$. In correspondence with the lowest values of $\rho_{j,\text{btc}}$, the results of the two prediction methods are similar. By increasing $\rho_{j,\text{btc}}$, the inaccuracy of the FEMA method increases significantly, leading to relevant errors, which are up to 21% for $\rho_{j,\text{btc}} = 10.0$.

Improvements for the Fundamental Period Prediction

As it appears from the previous sections, the degree of accuracy of both Rayleigh's and FEMA's approaches appears to be not properly adequate for a practical design purpose when applied to racks. As an example of a direct comparison between the results associated with the considered prediction approaches, reference can be made to Fig. 7, where the T_1^{6-R}/T_1^7, T_1^{7-R}/T_1^7, and T_1^F/T_1^7 ratios are plotted versus $\rho_{j,\text{base}}$. A total of two different racks have been considered, M_4 with $\rho_{j,\text{base}} = 0.15$ and T_4 with $\rho_{j,\text{base}} = 0.45$, which correspond to the cases with the best and the worst T_1 prediction, respectively. A more accurate prediction can be obtained through the use of T_1^{7-R}, but the period obtained from the modal analysis (T_1^7) is, however, always underestimated and, as a result, rack design approaches assume seismic loads greater than the ones associated with a correct assessment of the rack flexibility. At the same time, rack response is more rigid, and hence the influence of second-order effects is remarkably underestimated without any evidence that the design is however conservative. This is quite in contrast to what occurs in the case of static loading. The predicted critical load multiplier (α_{cr}) is generally underestimated,

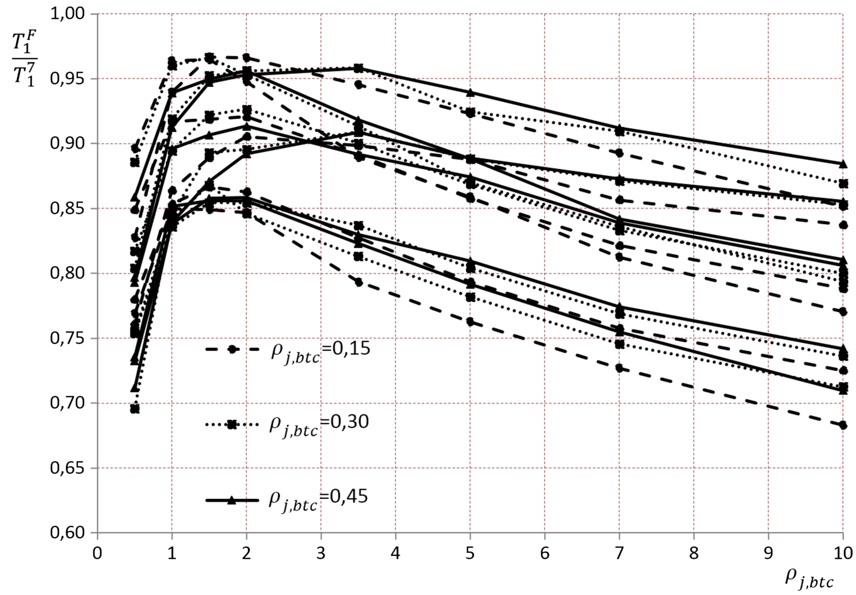

Fig. 6. Accuracy of FEMA's method to predict fundamental period of vibration

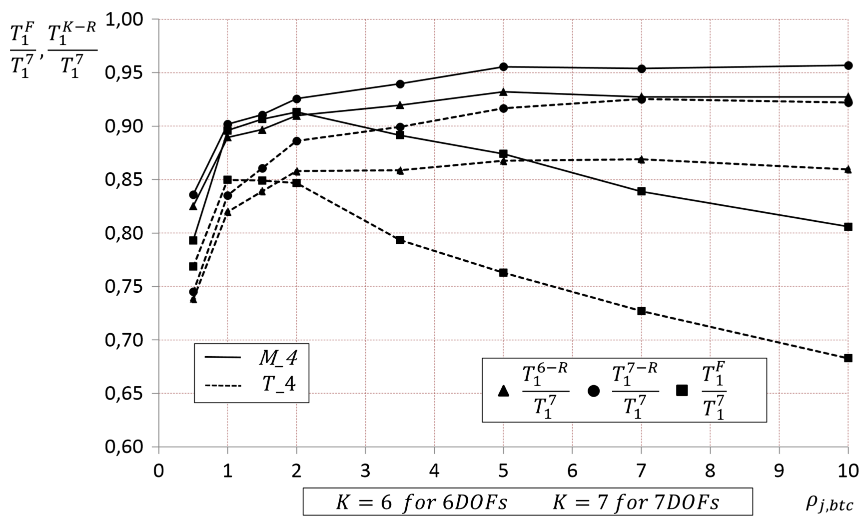

Fig. 7. Accuracy of considered prediction method for M_4 racks with $\rho_{j,\mathrm{btc}} = 0.15$ and T_4 racks with $\rho_{j,\mathrm{btc}} = 0.45$

but the use of Horne's method always leads to a quite conservative design. It is evident that the discrepancy between the approximated and numerical value of the fundamental period seems in part attributable to the influence of the effects of the deformations on very flexible structures, such as pallet racks. As a consequence, the authors tried to improve the accuracy of both the T_1 prediction approaches, considering second-order effects, as discussed subsequently.

The use of the set of second-order lateral displacements in Rayleigh's method should be more appropriate, but a nonnegligible problem is the solution of the system of the algebraic equation associated with the structural analysis. The geometric stiffness matrix is significantly influenced by the values of the internal forces and especially by the bending moments because of the horizontal force (F_i) imposed at each load level. Racks are usually unbraced in the downaisle direction and hence are characterized by a great flexibility to lateral loads. As a consequence, refined large deflection analysis FE formulations are required, as Bakker and Pekoz (2003) clearly stated. It is easy to imagine that for this load condition, second-order effects are expected to be dominant because of the very high values of the horizontal loads, out of the range of practical interest, which hamper the obtainment of the convergence of the solution of the algebraic system associated with the structural analysis. A reasonable alternative seems to consider, in a very simplified way, the second-order effects amplifying first-order displacements through the amplifying factor typically used for the amplified sway moment method [EN 1993-1-1 (CEN 2005)], which is distinguished in β^{6A} and β^{7A} if a six- or seven-DOF set of displacements is used, respectively. In particular, the fundamental period should be predicted by taking into account the second-order effects

$$T_1^{K-RA} = (\beta^{KA})^{\psi} \cdot T_1^{K-R} = \left(\frac{1}{1 - \frac{1}{\alpha_{\mathrm{cr}}^{K-H}}} \right)^{\psi} \cdot T_1^{K-R} \qquad (8)$$

where $\alpha_{\mathrm{cr}}^{K-H}$ = elastic critical load multiplier estimated by Horne's method; Superscript K indicates the number of DOFs considered in the FE beam element formulation; and ψ = suitable numerical coefficient.

The first attempt was done by assuming $\psi = 0.50$, which corresponds to the direct amplification of lateral displacement δ in Eq. (5), but the predicted values were significantly greater than the fundamental period obtained from a seven-DOF FE modal analysis, T_1^7, up to 25%. As a consequence, a sensitivity analysis has been carried out, and $\psi = 0.25$ has been identified as the best value to be used to obtain a quite satisfactory degree of accuracy. To allow an appraisal of the degree of accuracy of Rayleigh's method, both the ratios T_1^{7-RA}/T_1^7 and T_1^{6-RA}/T_1^7 have been evaluated and presented in Table 2. Hence

- The T_1^{6-RA} and T_1^{7-RA} values are greater than the corresponding T_1^{6-R} and T_1^{7-R} ones, as expected, where $\sqrt[0.25]{\beta^{KA}}$ is greater than unity. It results that T_1^{7-RA} is slightly greater than T_1^{6-RA} (up to 6%), and the degree of accuracy is significantly improved by the proposed amplification factor.
- With reference to the six-DOF formulation, the fundamental period is in general underestimated, primarily with reference to the greatest values of $\rho_{j,\mathrm{btc}}$ (up to 10% for $M\_$ and $G_$racks and 5% for $T_$racks). In a very limited number of cases, i.e., only for M_4 with $\rho_{j,\mathrm{btc}} = 0.5$, T_1^{6-RA} is moderately greater than T_1^7 by up to 4%.
- Considering the amplified period obtained through the seven-DOF set of displacements, the errors decrease significantly. Generally, T_1^{7-RA} is lower than T_1^7, and differences are generally not greater than 5%. In a very limited number of cases, the fundamental period is overestimated and the

Table 2. Approximation of the Fundamental Period of Vibration T_1, Including Second-Order Effects in Rayleigh's Method (T_1^{6-RA}/T_1^7 and T_1^{7-RA}/T_1^7) and FEMA Approach (T_1^{FA}/T_1^7)

		$\rho_{j,\mathrm{base}} = 0.15$			$\rho_{j,\mathrm{base}} = 0.30$			$\rho_{j,\mathrm{base}} = 0.45$		
Racks	$\rho_{j,\mathrm{btc}}$	T_1^{6-RA}/T_1^7	T_1^{7-RA}/T_1^7	T_1^{FA}/T_1^7	T_1^{6-RA}/T_1^7	T_1^{7-RA}/T_1^7	T_1^{FA}/T_1^7	T_1^{6-RA}/T_1^7	T_1^{7-RA}/T_1^7	T_1^{FA}/T_1^7
M_4	0.5	1.04	1.06	0.90	1.02	1.04	0.85	1.00	1.02	0.82
	1.0	0.97	1.00	0.95	0.97	0.99	0.93	0.98	1.01	0.93
	1.5	0.95	0.98	0.94	0.97	1.00	0.94	0.95	0.99	0.92
	2.0	0.95	0.99	0.92	0.94	0.98	0.91	0.95	0.99	0.92
	3.5	0.93	0.98	0.85	0.93	0.98	0.86	0.94	0.99	0.87
	5.0	0.93	0.99	0.81	0.93	0.98	0.83	0.93	0.99	0.83
	7.0	0.93	0.99	0.77	0.92	0.98	0.78	0.92	0.99	0.79
	10.0	0.91	0.99	0.72	0.92	0.99	0.75	0.90	0.98	0.74
M_5	0.5	0.99	1.00	0.88	0.98	0.99	0.83	0.97	0.98	0.80
	1.0	0.97	0.99	0.93	0.96	0.98	0.90	0.97	0.99	0.90
	1.5	0.96	0.99	0.93	0.95	0.98	0.91	0.96	0.99	0.91
	2.0	0.95	0.99	0.92	0.94	0.98	0.90	0.95	0.99	0.90
	3.5	0.94	0.99	0.87	0.94	1.00	0.87	0.93	0.99	0.86
	5.0	0.93	0.99	0.83	0.92	0.99	0.83	0.93	1.00	0.84
	7.0	0.92	0.99	0.78	0.91	0.99	0.79	0.91	0.99	0.80
	10.0	0.91	0.99	0.75	0.90	0.99	0.76	0.90	0.99	0.76
G_3	0.5	0.96	0.97	0.93	0.97	0.97	0.89	0.98	0.98	0.87
	1.0	0.97	0.98	1.04	0.97	0.98	1.00	0.98	0.99	0.98
	1.5	0.96	0.98	1.05	0.96	0.98	1.02	0.96	0.98	1.01
	2.0	0.95	0.98	1.04	0.95	0.97	1.02	0.95	0.98	1.01
	3.5	0.94	0.98	1.00	0.94	0.99	1.01	0.95	0.99	1.00
	5.0	0.93	0.98	0.97	0.92	0.97	0.97	0.94	0.99	0.98
	7.0	0.93	0.99	0.93	0.93	0.99	0.94	0.93	0.98	0.95
	10.0	0.92	0.98	0.88	0.91	0.98	0.90	0.92	0.99	0.91
G_4	0.5	0.96	0.99	0.86	0.92	0.94	0.77	0.96	0.99	0.78
	1.0	0.94	0.98	0.95	0.96	0.98	0.92	0.97	0.99	0.91
	1.5	0.93	0.97	0.96	0.96	0.99	0.96	0.95	0.96	0.93
	2.0	0.93	0.98	0.97	0.94	0.97	0.95	0.94	0.96	0.95
	3.5	0.92	0.98	0.94	0.93	0.99	0.95	0.94	0.99	0.95
	5.0	0.92	0.99	0.93	0.92	0.98	0.92	0.92	0.98	0.92
	7.0	0.90	0.98	0.89	0.91	0.98	0.90	0.91	0.99	0.90
	10.0	0.90	0.99	0.86	0.91	0.99	0.88	0.91	0.99	0.88
T_3	0.5	0.96	0.98	0.99	0.97	0.99	0.96	0.96	0.98	0.92
	1.0	0.97	0.98	1.04	0.97	0.99	1.02	0.96	0.98	0.99
	1.5	0.97	0.99	1.03	0.97	0.99	1.02	0.96	0.97	1.00
	2.0	0.96	0.99	1.01	0.96	0.98	1.00	0.96	0.99	1.00
	3.5	0.95	0.98	0.93	0.95	0.99	0.95	0.95	0.98	0.95
	5.0	0.96	1.00	0.89	0.95	0.98	0.90	0.96	0.99	0.92
	7.0	0.95	0.99	0.84	0.95	0.99	0.86	0.95	0.98	0.87
	10.0	0.95	0.99	0.80	0.95	0.99	0.82	0.95	0.99	0.83
T_4	0.5	0.98	1.00	0.94	0.97	0.99	0.89	0.96	0.98	0.85
	1.0	0.97	0.99	0.99	0.96	0.98	0.96	0.97	0.99	0.95
	1.5	0.96	0.98	0.98	0.97	0.99	0.97	0.96	0.98	0.95
	2.0	0.97	0.99	0.97	0.98	0.99	0.97	0.96	0.98	0.96
	3.5	0.97	0.99	0.93	0.97	0.99	0.94	0.96	0.98	0.92
	5.0	0.97	0.99	0.89	0.96	0.99	0.90	0.97	0.99	0.90
	7.0	0.96	0.99	0.85	0.96	0.98	0.86	0.96	0.99	0.86
	10.0	0.96	0.99	0.81	0.95	0.99	0.82	0.95	0.98	0.83

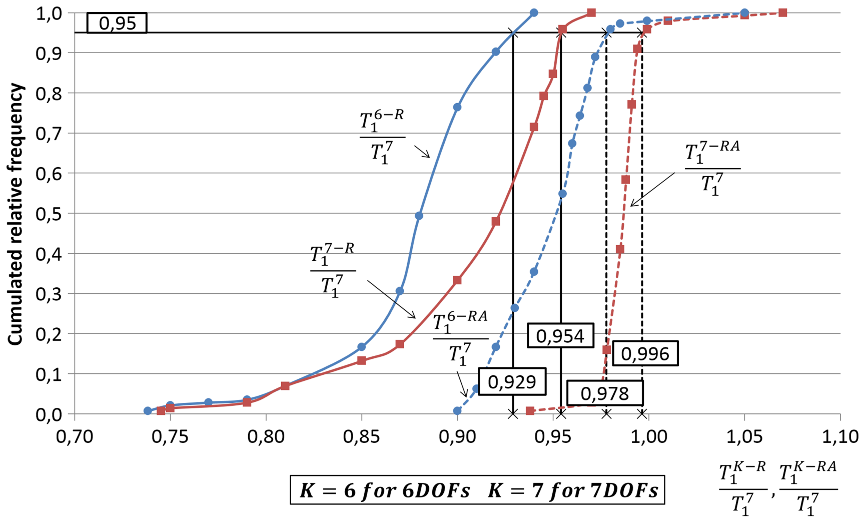

Fig. 8. Distribution of cumulated relative frequency of T_1^{6-R}/T_1^7, T_1^{7-R}/T_1^7, T_1^{6-RA}/T_1^7, and T_1^{7-RA}/T_1^7 for all considered racks

maximum error is 6% for the M_4 rack, with $\rho_{j,\text{btc}} = 0.5$ and $\rho_{j,\text{base}} = 0.5$.

As a summary of the results associated with Rayleigh's equation, reference can be made to Fig. 8, which is related to the cumulated distribution of the error in the period prediction, in particular, the period obtained through the use of the set of displacements associated with both six and seven DOFs (T_1^{6-R} and T_1^{7-R}) already plotted in Fig. 5, and can be directly compared with T_1^{6-RA} and T_1^{7-RA}. The nonnegligible increment of the degree of accuracy associated with the use of the multiplier $\sqrt[0.25]{\beta^{6A}}$ or $\sqrt[0.25]{\beta^{7A}}$ can be directly appraised by the nonnegligible translation of the amplified distribution toward unity and the increase of the associated slope. Furthermore, the 95% fractile values (equal to 0.978 and 0.996 for six and seven DOFs, respectively) also confirm that the proposal to slightly modify Rayleigh's method seem to be adequate for design purposes.

Similarly to what was proposed to account for the actual set of displacements on flexible racks for Rayleigh's method, for the FEMA approach, an improvement is also required to increase the accuracy in the T_1 prediction. Owing to the great flexibility of the rack frames in the downaisle loads, second-order effects have already been introduced in the discussion of the method by Filiatrault et al. (2006) to evaluate the top lateral displacement of the racks. In particular, a second-order amplification factor β_α has been proposed, with reference to the deformed rack under the gravity loads caused by the pallet weights. The expression is

$$\beta_\alpha = \frac{\sum_{i=1}^{N_{\text{LL}}} W_i h_i \left(\frac{S_{j,\text{btc}}+K_b}{S_{j,\text{btc}}K_b}\right)}{\left[N_{\text{btc}} + N_{\text{base}}\left(\frac{S_{j,\text{base}}K_u}{S_{j,\text{btc}}K_b}\right)\left(\frac{S_{j,\text{btc}}+K_b}{S_{j,\text{base}}+K_u}\right)\right]} \tag{9a}$$

In accordance with the symbols previously introduced that related to the current study, the multiplier β_α can be expressed as

$$\beta_\alpha = \frac{\left[\sum_{i=1}^{N_{\text{LL}}} W_i h_i\right] \frac{L}{EI_b}\left(\frac{\rho_{j,\text{btc}}+12}{6\rho_{j,\text{btc}}}\right)}{\left[N_{\text{btc}} + N_{\text{base}}\frac{\rho_{j,\text{base}}}{\rho_{j,\text{btc}}}\frac{10 I_u L_b}{I_b h_{\text{LL}}}\left(\frac{\rho_{j,\text{btc}}+12}{15\rho_{j,\text{base}}+2}\right)\right]} \tag{9b}$$

The authors' proposal is to also include the term β_α in the expression of the fundamental period of vibration by substituting the amplified value $\beta_\alpha \delta_i$ with the value of the generic displacement δ_i in Eq. (5). The modified fundamental period results, as a consequence, in

$$T_1^{FA} = T_1^F \cdot \sqrt{\beta_\alpha} \tag{10}$$

Table 2 presents the T_1^{FA}/T_1^7 ratio. The values of T_1^{FA} are significantly greater than the corresponding T_1^F, and this appears to be practically independent from the degree of stiffness of the base column. The trend of the $T_1^{FA} - \rho_{j,\text{btc}}$ relationships is similar to the $T_1^F - \rho_{j,\text{btc}}$ ones previously described, but it is shifted up ($T_1^{FA} > T_1^F$); hence, the errors are significantly reduced, especially for lower values of $\rho_{j,\text{btc}}$. The use of the improved approach leads to overestimation of the period in a very limited number of cases, but the error is never greater than 5%, confirming its efficiency for routine rack design. In Figs. 9 and 10, the distribution of the relative and cumulated frequency, respectively, of both the T_1^{FA}/T_1^7 and T_1^F/T_1^7 ratios is proposed for all of the considered cases. A direct comparison between the relative frequency curves shows that in correspondence with the most important concentration of the T_1^F/T_1^7 values, i.e., in the range between 0.82 and 0.95, the values of the relative frequency associated with the T_1^{FA}/T_1^7 ratio are now more reduced. Otherwise, the number of occurrences increases remarkably when T_1^{FA}/T_1^7 tends to unity. Influence of the second-order effects and efficiency of the proposed improvements are also confirmed by the nonnegligible translation of the associated cumulated relative frequency distribution curve toward unity. Moreover, the 95% fractile value is significantly increased from 0.96 to 1.02.

Concluding Remarks

A two-part paper on steel storage pallet racks summarizes the research outcomes to evaluate the level of accuracy of the simplified approaches that were developed for traditional steel frames composed of doubly symmetric cross section hot-rolled members. The uprights of the racks present the key feature to have the cross section with only one axis of symmetry, and their design approaches do not appear to appropriately consider all of the effects associated with the eccentricity between the shear center and cross section centroid.

In the companion paper (Bernuzzi et al. 2015a), attention was focused on the static design, and prediction of the elastic critical

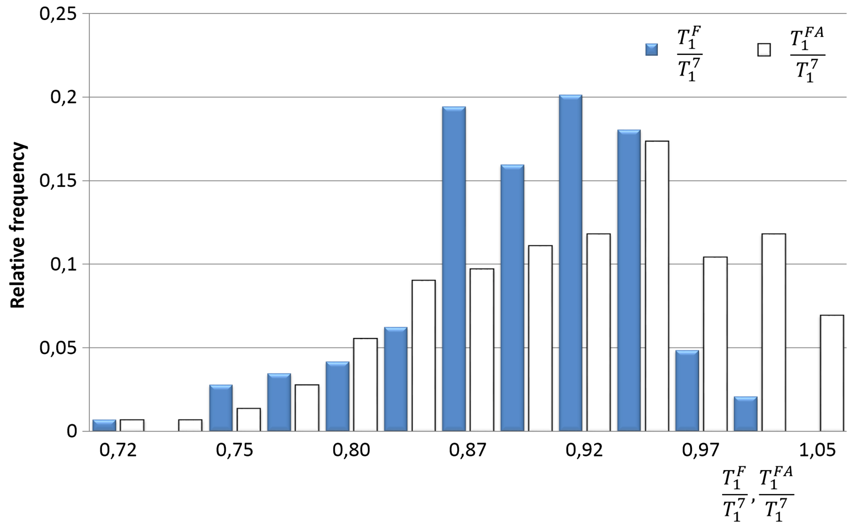

Fig. 9. Distribution of relative frequency of T_1^{FA}/T_1^7 and T_1^F/T_1^7 for all considered racks

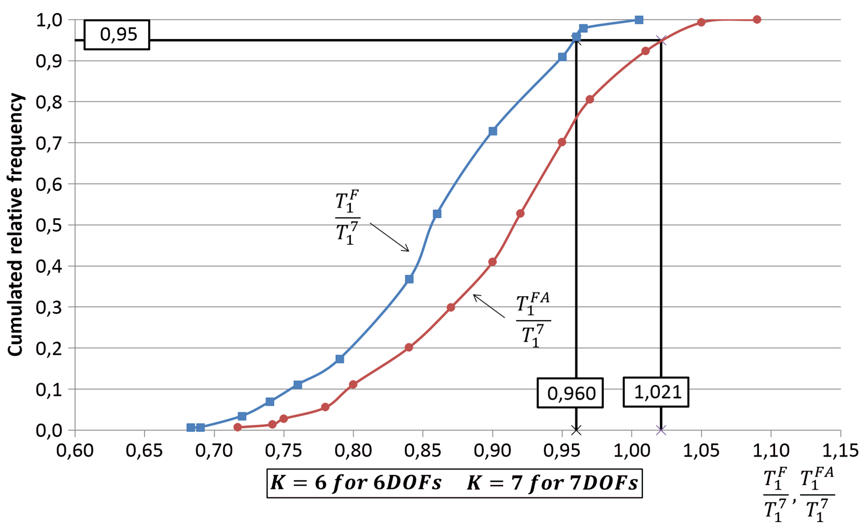

Fig. 10. Distribution of cumulated relative frequency of T_1^{FA}/T_1^7 and T_1^F/T_1^7 for all considered racks

load multiplier was discussed with reference to a wide range of cases of practical interest for the routine design of medium-rise pallet racks. These same cases have been considered in the present paper, which deals with seismic design. The fundamental period of vibration obtained from a modal analysis using a seven-DOF FE beam formulation (T_1^7) has been predicted through Rayleigh's and FEMA approaches. The first formulation has been applied by using the set of horizontal displacements arising from a first-order elastic analysis in the case of both six- and seven-DOF FE beam formulations. Despite a slightly greater degree of accuracy guaranteed by considering the presence of Wagner's constants and shear center eccentricity in the stiffness matrices in the analysis, the fundamental period of vibration is significantly underestimated if warping is considered, which leads to the assumption of a more rigid rack behavior and, as a consequence, underestimation of lateral deformability and second-order effects. Conversely, the authors' suggestion is to use the set of second-order displacements obtained through a suitable amplifying factor. This leads to a more accurate prediction, ranging from the period between $0.90T_1^7$ and $1.04T_1^7$ when warping is neglected and between $0.95T_1^7$ and $1.05T_1^7$ when warping is considered. Furthermore, the alternative use of the direct approach proposed by FEMA has been discussed, and its results indicate a nonsatisfactory degree of accuracy for rack design purposes. The authors propose to consider second-order effects through a suitable coefficient, β_α, depending on frame geometry and rack components, obtaining, in this case, an important reduction of errors in the prediction of the fundamental period of vibration. Finally, research outcomes show that both the improved proposed approaches seem very promising for routine rack design in the absence of more refined FE analysis packages, but further numerical cases are required to better define the correction factors to be recommended for practical design.

Appendix. Benchmark for the Simplified Approaches

This Appendix is a benchmark for the application of the simplified approaches presented in this paper and the companion paper

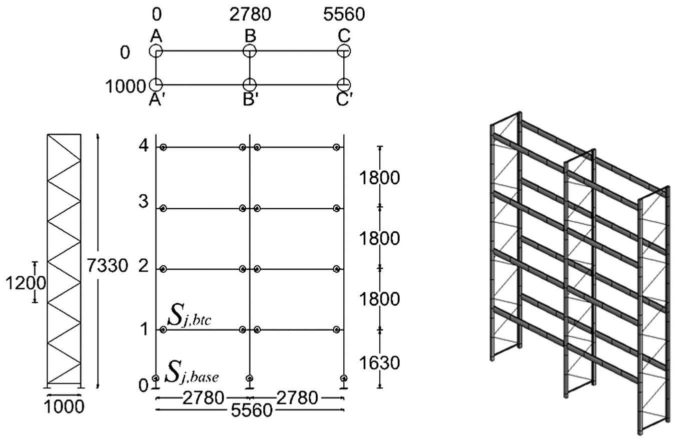

Fig. 11. Geometry of considered rack; all dimensions are in millimeters

([Bernuzzi et al. 2015a](#)). Because of the impossibility of directly presenting all of the data related to the commercial racks considered in the numerical analysis, reference can be made to the following contents that concern the routine application of the considered methods.

The two-bay and four-load level rack in Fig. 11 has been considered. Pallet beams in Fig. 12 are assumed to be connected to the uprights through a semirigid connection having a rotational stiffness $S_{j,\mathrm{btc}}$ of 400 kNm/rad. Base plate connections have been assumed as a semirigid joint, with a rotational stiffness $S_{j,\mathrm{base}}$ of 800 kNm/rad. Table 3 reports the geometry of the cross sections.

All of the components have been assumed to belong to Class 3 in accordance with the European criteria for cross section classification [EN 1993-1-1 (CEN 2005)]. Young's modulus E has been assumed to be equal to 210,000 MPa, and Poisson's coefficient ν is equal to 0.3.

Procedure for Static Design

The sole case of fully loaded racks was considered, with pallet units interested by a uniform load on pallet beams. The value of the applied uniform load is 4.00 kN/m. The overall frame imperfections equal to 0.0033 rad in terms of out of plumb (ϕ) of the uprights in both the cross-aisle and downaisle directions have been considered contemporaneously and were simulated through horizontal forces concentrated on each floor level.

Parameter	Value
Distributed load (q)	4.00 kN/m
L_b	2.78 m
h_{LL}	1.80 m
ϕ	0.0033
N_{LL}	4

The critical load multiplier α_{cr} obtained by buckling analysis with *Śiva* software is

$$\alpha_{\mathrm{cr}}^{6} = 6.27$$

$$\alpha_{\mathrm{cr}}^{7} = 5.78$$

where Superscripts 6 and 7 are related to the six- and seven-DOF FE beam formulation.

The displacements obtained by first-order analysis with *Śiva* software are

Node	δ^6 (mm)	δ^7 (mm)	δ^6/δ^7
C1	0.899	0.900	1.00
C2	1.976	2.115	0.93
C3	2.762	3.006	0.92
C4	3.147	3.529	0.89

The critical load multiplier α_{cr} obtained by Horne's method is

$$\alpha_{\mathrm{cr}}^{6-H} = 5.58$$

$$\alpha_{\mathrm{cr}}^{7-H} = 4.94$$

The final ratios are

Ratio	Value
$\alpha_{\mathrm{cr}}^{6}/\alpha_{\mathrm{cr}}^{7}$	1.085
$\alpha_{\mathrm{cr}}^{6-H}/\alpha_{\mathrm{cr}}^{7}$	0.965
$\alpha_{\mathrm{cr}}^{7-H}/\alpha_{\mathrm{cr}}^{7}$	0.854

Procedure for Seismic Design

The sole case of fully loaded racks was considered with pallet units interested by a uniform load on pallet beams. The value of the applied uniform load is 4.00 kN/m, like in the previous section. The overall frame imperfections, which are equal to 0.0033 rad in terms

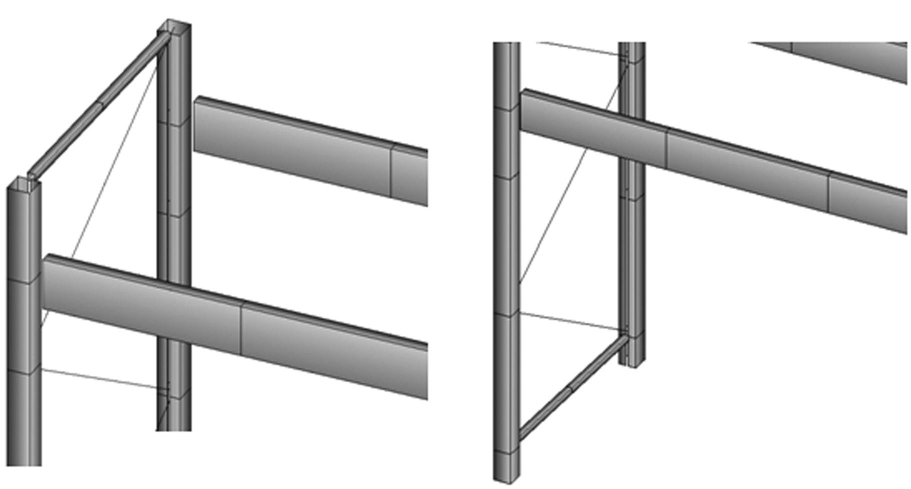

Fig. 12. Particular of Node A: top and bottom of external uprights

Table 3. Cross Section Geometry of the Rack Components

Parameter	Upright	Pallet beam	Lacing
Height (mm)	75	166	30
Width (mm)	74.5	40	30
Lip (mm)	32.5	—	—
Thickness (mm)	2.0	1.3	3.0
A (mm^2)	563	527	299
I_y (mm^4)	489,296	172,381	34,645
I_x (mm^4)	509,851	1,639,699	34,645
I_t (mm^4)	771.53	525,172	61,523
I_w (mm^6)	1,509,751,119	178,028,881	7,392
x_O (mm)	80.3	0.00	0.00
α_z	8,700.5	—	—
α_x	0.00	—	—
α_y	168.31	—	—
α_w	−0.261	—	—

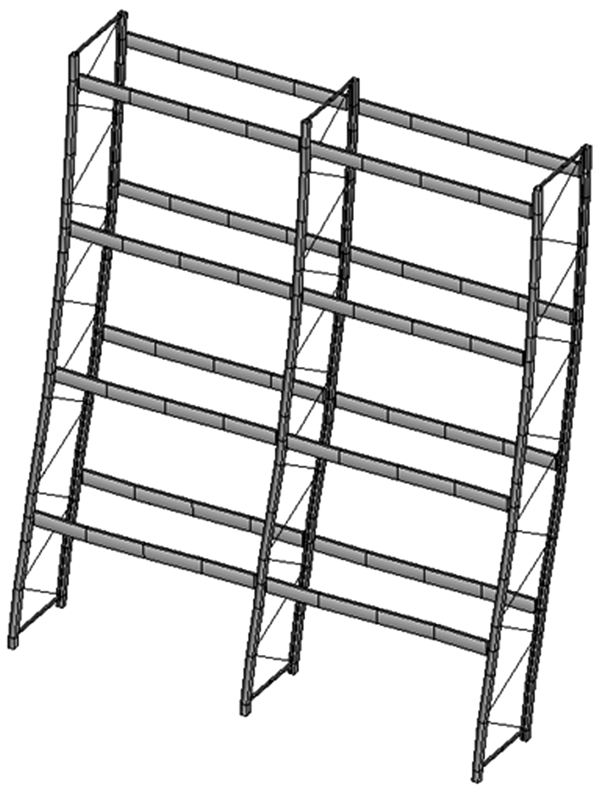

Fig. 13. Example of deformed shape associated with fundamental period of vibration (T_1)

of out of plumb (ϕ) of the uprights in both the cross-aisle and downaisle directions, have been considered contemporaneously and simulated through horizontal forces concentrated on each floor level. The fundamental period obtained by analysis with *Śiva* software in Fig. 13 is

$$T_1^6 = 1.70$$

$$T_1^7 = 1.96$$

For the use of Rayleigh's method, at each level floor, a horizontal force has been applied, which is obtained as

$$F_i^R = q \cdot 5.56 \cdot 2 = 44.48 \text{ kN} \quad (11)$$

Rayleigh's lateral displacements obtained with *Śiva* software are

Node	δ^6 (mm)	δ^7 (mm)	δ^6/δ^7
C1	247.549	247.501	1.00
C2	549.677	587.771	0.94
C3	765.481	837.675	0.91
C4	881.611	975.943	0.90

For FEMA's procedure, the principal data are reported as

Parameter	Value
N_{btc}	32
N_{base}	6
I_b	1.639×10^{-6} m^4
I_u	5.0985×10^{-7} m^4
ϕ	0.0033
N_{LL}	4
K_b [Eq. (6*a*)]	743.17 kNm
K_u [Eq. (6*b*)]	262.75 kNm

The fundamental period obtained by Rayleigh's lateral displacements [Eq. (4)] and with the FEMA procedure [Eq. (5)] is

Period	Value
T_1^{6-R}	1.69
T_1^{7-R}	1.77
T_1^F	1.49

The amplification of the fundamental period [Eq. (8)] is

Period	Value
T_1^{6-RA}	1.76
T_1^{7-RA}	1.86
T_1^{FA}	1.58

The final ratios are

Ratio	Value
T_1^6/T_1^7	0.87
T_1^{6-R}/T_1^7	0.86
T_1^{7-R}/T_1^7	0.90
T_1^F/T_1^7	0.76
T_1^{6-RA}/T_1^7	0.90
T_1^{7-RA}/T_1^7	0.95
T_1^{FA}/T_1^7	0.81

Notation

The following symbols are used in this paper:

A = eigenvector matrix, cross-sectional area;
B = bimoment;
d = interstory drift;
E = Young's modulus;
F = shear force, seismic lateral force;
G = shear modulus;
g = acceleration of gravity;
H = height;
h = height;
I = second moment of area;
K = flexural stiffness, stiffness matrix;
L = length;
M = moment, mass matrix;
min = minimum;
N = axial force;
P = gravity load;
q = distributed uniform load;
S = beam-to-column joint stiffness, base-plate joint stiffness;

T = period of vibration;
V = seismic story shear;
W = vertical load;
x = symmetry axis of cross section, distance between centroid and shear center;
y = nonsymmetry axis of cross section;
α = load multiplier, Wagner's coefficients;
β = multiplier for second-order effects;
δ = lateral displacement;
θ = interstory drift sensitive coefficient;
ρ = adimensional stiffness;
ϕ = out of plumb; and
ω = dynamic eigenvalue, sectorial area.

Subscripts

1 = first;
b = beam;
base = base-plate connection;
btc = beam-to-column connection;
cr = critical;
Ed = design value;
i = i-esim;
j = initial node of beam element, joint;
k = final node of beam element;
L = lower;
LL = load level;
max = maximum;
O = position of the centroid;
r = relative;
t = Saint Venant's torsion;
tot = total;
U = upper;
u = upright;
w = warping;
x = symmetry axis of cross section;
y = nonsymmetry axis of cross section;
z = longitudinal axis of beam element; and
α = load multiplier.

Superscripts

A = amplification;
E = elastic stiffness matrix;
F = FEMA method;
G = geometric stiffness matrix;
H = Horne's method;
K = index used to identify six- or seven-DOF FE beam formulation;
R = Rayleigh's method;
ψ = suitable number;
6 = analysis with beam element formulation having six DOFs per node; and
7 = analysis with beam element formulation having seven DOFs per node.

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