Finite-time H_∞ Control for Switched Systems with Time-varying Delay using Delta Operator Approach

Chen Qin, Zhengrong Xiang*, and Hamid Reza Karimi

Abstract: Finite-time H_∞ control for switched systems with time-varying delay using delta operator approach is investigated in this paper. Firstly, by using the average dwell time approach and delta operator theory, sufficient conditions for H_∞ finite-time boundedness of the underlying systems are derived. Then a state feedback controller is proposed such that the resulting closed-loop system is H_∞ finite-time bounded. All the obtained results are formulated in terms of linear matrix inequalities (LMIs). Finally, an example is presented to show the validity of the proposed results.

Keywords: Average dwell time, delta operator, finite-time boundedness, H_∞ performance, switched systems, time-varying delays.

1. INTRODUCTION

A switched system is a hybrid dynamic system that is composed of a family continuous-time or discrete-time subsystems with certain maps for switching among them. Many dynamical systems can be modeled as switched systems [1-4], and many important achievements and remarkable researches have been developed [5,6].

The delta operator which creates the rapprochement between continuous- and discrete-time systems, and establishes the natural framework to investigate the behavior of discrete-time systems in the fast sampling limit has been investigated by Goodwin and Middleton in [7]. The delta operator is defined by

$$\delta x(t) = \begin{cases} dx(t)/dt, & T = 0, \\ (x(t+T) - x(t))/T, & T \neq 0, \end{cases}$$

where T is the sampling period. The transformations between shift operator and delta operator transfer function models were investigated in [8]. The delta operator can avoid the numerical instability problems caused by the conventional shift operator when fast sampling, and also can improve the performance of adaptive algorithms. In addition, it was shown in [9] that a technique was developed to obtain an approximate delta operator system for a given continuous system. Since then, more researches based on delta operator have been highlighted [10-13]. To mention a few, stability of uncertain systems was established in [14]. Recently, robust control for delta operator systems was investigated in [15-18]. In [19], robust stabilization problem for discrete-time systems with time-varying delays was discussed. Some results on filter and observer design of delta operate systems were obtained in [20,21]. It should be pointed out that the aforementioned results are on Lyapunov stability.

Finite-time stability is another stability concept which admits that the state does not exceed a certain bound during a fixed finite-time interval. The early results on finite-time stability date back to the 1950s, when it was introduced in the Russian literature. It should be emphasized that a finite-time stable system may not be Lyapunov stable, and finite-time stability is more useful to study the behavior of the system within a finite interval. Recently, a few results on finite-time stability of switched systems have been given in [22,23]. Some results on finite-time H_∞ control for switched systems have been reported in the literature [24-29]. However, to the best of our knowledge, there are few results available on finite-time stability and finite-time boundedness of delta operator switched systems with time-varying delay, and this is the motivation for our study.

In this paper, the finite-time H_∞ controller design problem for delta operator switched systems with time-varying delay is considered. The main contributions of this paper can be summarized as follows: 1) The definition of H_∞ finite-time boundedness is extended to delta operator switched systems with time-varying delay; 2) Sufficient conditions for the existence of H_∞ finite-time boundedness of the underlying systems are given through constructing a new Lyapunov-Krasovskii functional candidate and using the average dwell time approach; 3) By virtue of linear matrix inequality approach, a state feedback controller is designed to guarantee that the closed-loop delta operator switched system is H_∞ finite-time bounded.

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The paper is organized as follows. In Section 2, the formulation of the considered systems and some corresponding definitions and lemmas are given. The H_∞ finite-time boundedness analysis and control are developed for the underlying systems in Section 3. A numerical example is provided to illustrate the proposed results in Section 4 and concluding remarks are presented in Section 5.

Notations: In this paper, $A > 0\ (\geq 0)$ means that the matrix A is positive (nonnegative) definite; A^T is the transpose of matrix A; R^n represents the n-dimensional real vector space; $R^{m\times n}$ stands for the set of all $(m\times n)$-dimensional real matrices; $l_2[k_0,\infty)$ represents the space of square summable functions on $[k_0,\infty)$; $diag\{\cdots\}$ refers to a block-diagonal matrix; I is an identity matrix of an appropriate dimension; $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimum and maximum eigenvalues of a matrix, respectively; $\|\cdot\|_2$ means the Euclidean norm. The symbol * represents the symmetric term in a symmetric matrix.

2. PROBLEM FORMULATION AND PRELIMINARIES

Consider the following delta operator switched system with time-varying delay:

$$\begin{cases}\delta x(k) = \hat{A}_{\sigma(k)}(k)x(k) + \hat{A}_{d\sigma(k)}(k)x(k-\tau(k)) \\ \qquad + \hat{B}_{\sigma(k)}(k)u(k) + D_{\sigma(k)}w(k), \\ z(k) = C_{\sigma(k)}x(k) + G_{\sigma(k)}w(k), \\ x(\theta) = \varphi(\theta),\ \theta = -\bar{\tau}, -\bar{\tau}+1, \ldots, 0, \end{cases} \tag{1}$$

where $x(k)\in R^n$ represents the state vector; $u(k)\in R^m$ means the control input; $z(k)\in R^l$ is the controlled output; $w(k)\in R^w$ denotes the disturbance input satisfying

$$\sum_{k=0}^{\infty} w^T(k)w(k) < d^2, \quad d \geq 0. \tag{2}$$

$\sigma(k):[0,\infty)\to \underline{N} = \{1,2,\cdots,N\}$ is the switching signal with N being the number of subsystems. $\varphi(\theta)$ denotes the discrete vector-valued initial function. $\tau(k)$ stands for the time-varying delay satisfying $0 \leq \underline{\tau} \leq \tau(k) \leq \bar{\tau}$ for known constants $\underline{\tau}$ and $\bar{\tau}$. k represents the time $t = kT$ and $T > 0$ is the sampling period. C_i, D_i and G_i, $i\in \underline{N}$, are constant matrices with appropriate dimensions. $\hat{A}_i(k)$, $\hat{A}_{di}(k)$ and $\hat{B}_i(k)$ are uncertain real-valued matrices with proper dimensions and are briefly denoted by $\hat{A}_i$, $\hat{A}_{di}$ and $\hat{B}_i$, respectively. $\hat{A}_i$, $\hat{A}_{di}$ and $\hat{B}_i$ can be expressed as follows:

$$\begin{aligned}[\hat{A}_i \quad \hat{A}_{di} \quad \hat{B}_i] &= [A_i \quad A_{di} \quad B_i] \\ &\quad + H_i F_i(k)[E_{ai} \quad E_{adi} \quad E_{bi}],\end{aligned} \tag{3}$$

where A_i, A_{di}, B_i, H_i, E_{ai}, E_{adi} and E_{bi} are known real constant matrices with suitable dimensions and the uncertain time-varying matrix $F_i(k)$ satisfies

$$F_i^T(k)F_i(k) \leq I. \tag{4}$$

Next, we will give some definitions and lemmas which will be essential in our later development for the following switched system:

$$\begin{cases}\delta x(k) = A_{\sigma(k)}x(k) + A_{d\sigma(k)}x(k-\tau(k)) + D_{\sigma(k)}w(k), \\ z(k) = C_{\sigma(k)}x(k) + G_{\sigma(k)}w(k), \\ x(\theta) = \varphi(\theta),\ \theta = -\bar{\tau}, -\bar{\tau}+1, \ldots, 0. \end{cases} \tag{5}$$

Definition 1 [26]**:** (Finite-time stability) Given positive constants T_f, η_1 and η_2 with $\eta_1 < \eta_2$, a positive definite matrix R and a switching signal $\sigma(k)$, if $\forall k\in[0,T_f)$, we have

$$\sup_{-\bar{\tau}\leq\theta\leq 0}\{x^T(\theta)Rx(\theta)\} \leq \eta_1 \Rightarrow x^T(k)Rx(k) < \eta_2,$$

then switched system (5) with $w(k)\equiv 0$ is said to be finite-time stable with respect to $(\eta_1,\eta_2,T_f,R,\sigma(k))$. If the above condition is satisfied for any switching signal $\sigma(k)$, system (5) with $w(k)\equiv 0$ is said to be uniformly finite-time stable with respect to (η_1,η_2,T_f,R).

Definition 2 [26]**:** (Finite-time boundedness) Given positive constants T_f, η_1 and η_2 with $\eta_1 < \eta_2$, a positive definite matrix R and a switching signal $\sigma(k)$, if $\forall k\in[0,T_f)$, one has

$$\sup_{-\bar{\tau}\leq\theta\leq 0}\{x^T(\theta)Rx(\theta)\} \leq \eta_1 \Rightarrow x^T(k)Rx(k) < \eta_2,$$

then switched system (5) is said to be finite-time bounded with respect to $(\eta_1,\eta_2,d,T_f,R,\sigma(k))$. If the above condition is satisfied for any switching signals $\sigma(k)$, system (5) is said to be uniformly finite-time bounded with respect to (η_1,η_2,d,T_f,R).

Definition 3 [29]**:** (weighted H_∞ finite-time boundedness) For a given time constant T_f, η_1 and η_2 with $\eta_1 < \eta_2$ and a positive definite matrix R, switched system (5) is said to be H_∞ finite-time bounded with respect to $(\eta_1,\eta_2,d,T_f,R,\sigma(k))$, if the following conditions are satisfied:

1) Switched system (5) is finite-time bounded with respect to $(\eta_1,\eta_2,d,T_f,R,\sigma(k))$;

2) Under zero-initial condition, i.e., $\varphi(\theta) = 0$, $\theta = -\bar{\tau}, -\bar{\tau}+1,\ldots,-1,0$, it holds that

$$\sum_{k=0}^{T_f-1}(1-T\alpha)^{-2k}\|z(k)\|^2 \leq \gamma^2\sum_{k=0}^{T_f-1}\|w(k)\|^2, \tag{6}$$

where $\alpha < 0$, $\gamma > 0$ and $w(k)$ satisfies (2).

Definition 4 [16]**:** For any switching signal $\sigma(k)$ and $k_2 > k_1 \geq 0$, let $N_\sigma(k_1,k_2)$ denote the switching number of $\sigma(k)$ over the interval $[k_1,k_2)$. For given $\tau_a > 0$ and $N_0 \geq 0$, if the inequality

$$N_\sigma(k_1,k_2) \leq N_0 + \frac{k_2-k_1}{\tau_a}$$

holds, then the positive constant τ_a is called the average dwell time and N_0 is called the chattering bound. As commonly used in the literature, we choose $N_0 = 0$ in this paper.

Lemma 1 [16]: For a given matrix $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix}$, where S_{11} and S_{22} are square matrices, the following conditions are equivalent:

(i) $S < 0$;

(ii) $S_{11} < 0,\ S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$;

(iii) $S_{22} < 0,\ S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$.

Lemma 2 [16]**:** Let U, V, W and X be real matrices of appropriate dimensions with X satisfying $X = X^T$, then for all $V^T V \le I$,

$$X + UVW + W^T V^T U^T < 0,$$

if and only if there exists a scalar ε such that

$$X + \varepsilon UU^T + \varepsilon^{-1} W^T W < 0.$$

Lemma 3 [18]**:** For any time function $x(t)$ and $y(t)$, the delta operator has the following property

$$\begin{aligned} \delta(x(t)y(t)) &= \delta(x(t))y(t) + x(t)\delta(y(t)) \\ &\quad + T\delta(x(t))\delta(y(t)), \end{aligned} \tag{7}$$

where T is the sampling period.

The aim of the paper is to find a class of switching signals $\sigma(k)$ and design a state feedback controller $u(k) = K_{\sigma(k)} x(k)$ for delta operator switched system (1) such that the corresponding closed-loop system is H_∞ finite-time bounded.

3. MAIN RESULTS

3.1. Finite-time stability and boundedness

In this section, we focus on finite-time boundedness of system (5).

Theorem 1: For given positive constants T_f, R, η_1 and η_2 satisfying $\eta_1 < \eta_2$, and a constant $\alpha < 0$, if there exist positive scalars λ_g, $g = 1,2,3,4$, and positive definite symmetric matrices P_i, Y_i and S_i, $i \in \underline{N}$, with appropriate dimensions, such that

$$\begin{bmatrix} \Omega_i & P_i A_{di} + TA_i^T P_i A_{di} & P_i D_i + TA_i^T P_i D_i \\ * & TA_{di}^T P_i A_{di} - (1 - T\alpha)^{\underline{\tau}} S_i & TA_{di}^T P_i D_i \\ * & * & TD_i^T P_i D_i - \frac{1}{T} Y_i \end{bmatrix} \le 0, \tag{8}$$

$$\chi < \lambda_1 \eta_2, \tag{9}$$

where

$$\Omega_i = P_i A_i + A_i^T P_i + TA_i^T P_i A_i + \alpha_i P_i + (\overline{\tau} - \underline{\tau} + 1) S_i,$$

$$\chi = (1 - T\alpha)^{T_f} ((\lambda_2 + T(\overline{\tau}^2 - \overline{\tau}\underline{\tau} + \overline{\tau})(1 - T\alpha)^{\overline{\tau}} \lambda_3)\eta_1 + (\frac{d^2 \lambda_4}{1 - T\alpha})),$$

$$\lambda_1 R \le P_i \le \lambda_2 R, \quad S_i \le \lambda_3 R, \quad Y_i \le \lambda_4 I,$$

then under any switching signal $\sigma(k)$ with the following average dwell time scheme

$$\tau_a > \tau_a^* = \frac{T_f \ln \mu}{\ln(\lambda_1 \eta_2) - \ln \chi}, \tag{10}$$

system (5) is finite-time bounded with respect to $(\eta_1, \eta_2, d, T_f, R, \sigma(k))$, where $\mu \ge 1$ satisfies

$$P_i \le \mu P_j, \quad S_i \le \mu S_j, \quad \forall i, j \in \underline{N}. \tag{11}$$

Proof: Choose the following Lyapunov-Krasovskii functional candidate

$$V_i(k) = V_{i1}(k) + V_{i2}(k) + V_{i3}(k), \quad \forall i \in \underline{N}, \tag{12}$$

where

$$V_{i1}(k) = x^T(k) P_i x(k),$$

$$V_{i2}(k) = T \sum_{s=k-\tau(k)}^{k-1} (1 - T\alpha)^{(k-s-1)} x^T(s) S_i x(s),$$

$$V_{i3}(k) = T \sum_{l=-\overline{\tau}+1}^{-\underline{\tau}} \sum_{s=k+l}^{k-1} (1 - T\alpha)^{(k-s-1)} x^T(s) S_i x(s).$$

Taking the delta operator manipulations of Lyapunov functional candidate $V_i(k)$ along the trajectory of system (5) with $w(k) \equiv 0$, by Lemma 3 we have

$$\begin{aligned} \delta V_{i1}(k) &= \delta(x^T(k) P_i x(k)) \\ &= \delta(x^T(k) P_i) x(k) + x^T(k) P_i \delta(x(k)) \\ &\quad + T\delta(x^T(k) P_i) \delta(x(k)) \\ &= x^T(k) P_i (A_i x(k) + A_{di} x(k - \tau(k)) + D_i w(k)) \\ &\quad + (A_i x(k) + A_{di} x(k - \tau(k)) + D_i w(k))^T P_i x(k) \\ &\quad + T(A_i x(k) + A_{di} x(k - \tau(k)) \\ &\quad + D_i w(k))^T P_i (A_i x(k) \\ &\quad + A_{di} x(k - \tau(k)) + D_i w(k)) \\ &= \begin{bmatrix} x(k) \\ x(k - \tau(k)) \\ w(k) \end{bmatrix}^T \overline{\Omega}_i \begin{bmatrix} x(k) \\ x(k - \tau(k)) \\ w(k) \end{bmatrix}, \end{aligned} \tag{13}$$

where

$$\overline{\Omega}_i = \begin{bmatrix} P_i A_i + A_i^T P_i + TA_i^T P_i A_i & P_i A_{di} + TA_i^T P_i A_{di} & P_i D_i + TA_i^T P_i D_i \\ * & TA_{di}^T P_i A_{di} & TA_{di}^T P_i D_i \\ * & * & TD_i^T P_i D_i \end{bmatrix},$$

$$\begin{aligned} \delta V_{i2}(k) &= \frac{1}{T}(V_{i2}(k+1) - V_{i2}(k)) \\ &= \frac{1}{T}(T \sum_{s=k+1-\tau(k+1)}^{k+1-1} (1 - T\alpha)^{(k-s)} x^T(s) S_i x(s) \end{aligned}$$

$$-T\sum_{s=k-\tau(k)}^{k-1}(1-T\alpha)^{(k-s-1)}x^T(s)S_ix(s))$$

$$\begin{aligned}
&\le -T\alpha\sum_{s=k-\tau(k)}^{k-1}(1-T\alpha)^{(k-s-1)}x^T(s)S_ix(s)\\
&+x^T(k)S_ix(k)\\
&-(1-T\alpha)^{\underline{\tau}}x^T(k-\tau(k))S_ix(k-\tau(k))\\
&+\sum_{s=k+1-\overline{\tau}}^{k-\underline{\tau}}(1-T\alpha)^{(k-s)}x^T(s)S_ix(s),
\end{aligned}\tag{14}$$

$$\begin{aligned}
\delta V_{i3}(k)&=\frac{1}{T}(V_{i3}(k+1)-V_{i3}(k))\\
&=-T\alpha\sum_{l=-\overline{\tau}+1}^{-\underline{\tau}}\sum_{s=k+l}^{k-1}(1-T\alpha)^{(k-s-1)}x^T(s)S_ix(s)\\
&+(\overline{\tau}-\underline{\tau})x^T(k)S_ix(k)\\
&-\sum_{s=k+1-\overline{\tau}}^{k-\underline{\tau}}(1-T\alpha)^{(k-s)}x^T(s)S_ix(s).
\end{aligned}\tag{15}$$

Combining (13)-(15), we have

$$\begin{aligned}
&\delta V_i(k)+\alpha V_i(k)\\
&=\begin{bmatrix}x(k)\\x(k-\tau(k))\\w(k)\end{bmatrix}^T\overline{\Omega}_i\begin{bmatrix}x(k)\\x(k-\tau(k))\\w(k)\end{bmatrix}\\
&+\alpha x^T(k)P_ix(k)+(\overline{\tau}-\underline{\tau}+1)x^T(k)S_ix(k)\\
&-(1-T\alpha)^{\overline{\tau}}x^T(k-\tau(k))S_ix(k-\tau(k))\\
&\le\begin{bmatrix}x(k)\\x(k-\tau(k))\\w(k)\end{bmatrix}^T\tilde{\Omega}_i\begin{bmatrix}x(k)\\x(k-\tau(k))\\w(k)\end{bmatrix},
\end{aligned}\tag{16}$$

where

$$\tilde{\Omega}_i=\begin{bmatrix}\Omega_i & P_iA_{di}+TA_i^TP_iA_{di} & P_iD_i+TA_i^TP_iD_i\\ * & TA_{di}^TP_iA_{di}-(1-T\alpha)^{\underline{\tau}}S_i & TA_{di}^TP_iD_i\\ * & * & TD_i^TP_iD_i\end{bmatrix},$$

$$\Omega_i=P_iA_i+A_i^TP_i+TA_i^TP_iA_i+\alpha_iP_i+(\overline{\tau}-\underline{\tau}+1)S_i.$$

According to (8) and (16), we can easily obtain

$$\delta V_i(k)+\alpha V_i(k)\le\frac{1}{T}w^T(k)Y_iw(k).\tag{17}$$

It follows from (17) that

$$\begin{aligned}
\delta V_i(k)=\frac{V_i(k+1)-V_i(k)}{T}&\le-\alpha V_i(k)+\frac{1}{T}w^T(k)Y_iw(k)\\
\Rightarrow V_i(k+1)-V_i(k)&\le-\alpha TV_i(k)+w^T(k)Y_iw(k)\\
\Rightarrow V_i(k+1)&\le(1-\alpha T)V_i(k)+w^T(k)Y_iw(k).
\end{aligned}\tag{18}$$

Let $0<k_1<\cdots<k_q$ denote the switching instants of $\sigma(k)$ over the interval $[0,T_f)$. For $k\in[k_p,k_{p+1})$, one obtains from (18) that

$$\begin{aligned}
V_{\sigma(k)}(k)&\le(1-T\alpha)^{(k-k_p)}V_{\sigma(k_p)}(k_p)\\
&+\sum_{s=k_p}^{k-1}(1-T\alpha)^{(k-s-1)}w^T(s)Y_{\sigma(k_p)}w(s).
\end{aligned}\tag{19}$$

Consider the following piecewise Lyapunov functional candidate for system (5)

$$V(k)=V_{\sigma(k)}(k)=V_{\sigma(k_p)}(k),\quad\forall k\in[k_p,k_p+1),\quad p=0,1,\cdots,q.\tag{20}$$

From (11), we can obtain

$$V_{\sigma(k_p)}(k_p)\le\mu V_{\sigma(k_p^-)}(k_p^-),\quad p=0,1,\cdots,q.\tag{21}$$

From (19), (21) and Definition 4, we can obtain

$$\begin{aligned}
V_{\sigma(k)}(k)&\le(1-T\alpha)^{(k-k_q)}V_{\sigma(k_q)}(k_q)\\
&+\sum_{s=k_q}^{k-1}(1-T\alpha)^{(k-s-1)}w^T(s)Y_{\sigma(k_q)}w(s)\\
&\le\mu(1-T\alpha)^{(k-k_q)}V_{\sigma(k_q^-)}(k_q^-)\\
&+\sum_{s=k_q}^{k-1}(1-T\alpha)^{(k-s-1)}w^T(s)Y_{\sigma(k_q)}w(s)\\
&\le\dots\\
&\le\mu^{N_\sigma(0,k)}(1-T\alpha)^kV_{\sigma(0)}(0)\\
&+\mu^{N_\sigma(0,k)}\sum_{s=0}^{k_1-1}(1-T\alpha)^{(k-s-1)}w^T(s)Y_{\sigma(0)}w(s)\\
&+\cdots+\sum_{s=k_q}^{k-1}(1-T\alpha)^{(k-s-1)}w^T(s)Y_{\sigma(k_q)}w(s)\\
&=\mu^{N_\sigma(0,k)}(1-T\alpha)^kV_{\sigma(0)}(0)\\
&+\sum_{s=0}^{k-1}\mu^{N_\sigma(s,k)}(1-T\alpha)^{(k-s-1)}w^T(s)Y_{\sigma(s)}w(s)\\
&\le\mu^{\frac{k}{\tau_a}}(1-T\alpha)^{T_f}V_{\sigma(0)}(0)\\
&+\mu^{N_\sigma(0,k)}\sum_{s=0}^{k-1}(1-T\alpha)^{(T_f-1)}w^T(s)Y_{\sigma(s)}w(s)\\
&\le\mu^{\frac{T_f}{\tau_a}}((1-T\alpha)^{T_f}V_{\sigma(0)}(0)\\
&+(1-T\alpha)^{(T_f-1)}\sum_{s=0}^{k-1}w^T(s)Y_{\sigma(s)}w(s)).
\end{aligned}\tag{22}$$

Considering that $\lambda_1R\le P_i\le\lambda_2R$, $S_i\le\lambda_3R$ and $Y_i\le\lambda_4I$, $\forall i\in\underline{N}$, it yields that

$$\begin{aligned}
V_{\sigma(k)}(k)&=x^T(k)P_{\sigma(k)}x(k)\\
&+T\sum_{s=k-\tau(k)}^{k-1}(1-T\alpha)^{(k-s-1)}x^T(s)S_{\sigma(k)}x(s)\\
&+T\sum_{l=-\overline{\tau}+1}^{-\underline{\tau}}\sum_{s=k+l}^{k-1}(1-T\alpha)^{(k-s-1)}x^T(s)S_{\sigma(k)}x(s)
\end{aligned}$$

$$\begin{aligned}&\geq x^T(k)P_{\sigma(k)}x(k)\\&= x^T(k)R^{\frac{1}{2}}(R^{-\frac{1}{2}}P_{\sigma(k)}R^{-\frac{1}{2}})R^{\frac{1}{2}}x(k) \qquad (23)\\&\geq \lambda_1 x^T(k)Rx(k),\end{aligned}$$

$$\begin{aligned}V_{\sigma(k)}(k) &\leq \mu^{\frac{T_f}{\tau_a}}(1-T\alpha)^{T_f}(V_{\sigma(0)}(0)\\&\quad+\sum_{s=0}^{k} w^T(s)Y_{\sigma(s)}w(s)) \qquad (24)\\&\leq \mu^{\frac{T_f}{\tau_a}}(1-T\alpha)^{T_f}(V_{\sigma(0)}(0)+d^2\lambda_4),\end{aligned}$$

$$\begin{aligned}V_{\sigma(0)}(0) &\leq \lambda_2 x^T(0)Rx(0)+T(\bar{\tau}^2-\bar{\tau}\underline{\tau}+\bar{\tau})(1\\&\quad-T\alpha)^{\bar{\tau}}\lambda_3 \sup_{-\tau\leq\theta\leq 0}\{x^T(\theta)Rx(\theta)\} \qquad (25)\\&\leq (\lambda_2+T(\bar{\tau}^2-\bar{\tau}\underline{\tau}+\bar{\tau})(1-T\alpha)^{\bar{\tau}}\lambda_3)\eta_1.\end{aligned}$$

Combining (23)-(25), we obtain

$$\lambda_1 x^T(k)Rx(k) \leq \mu^{\frac{T_f}{\tau_a}}\chi, \qquad (26)$$

where

$$\chi = (1-T\alpha)^{T_f}((\lambda_2+T(\bar{\tau}^2-\bar{\tau}\underline{\tau}+\bar{\tau})(1-T\alpha)^{\bar{\tau}}\lambda_3)\eta_1 +(\frac{d^2\lambda_4}{1-T\alpha})).$$

Substituting (10) into (26), we have

$$x^T(k)Rx(k) \leq \eta_2.$$

According to Definition 2, system (5) is finite-time bounded with respect to $(\eta_1,\eta_2,d,T_f,R,\sigma(k))$.

The proof is completed.

Remark 1: When $\mu=1$ in (11), which leads to $P_i=P_j$, $S_i=S_j$, $\forall i,j\in\underline{N}$, and $\tau_a^*=0$ by (10), system (5) has a common Lyapunov-Krasovskii functional candidate and the switching signal can be arbitrary.

Remark 2: Compared with the existing results on finite-time boundedness of switched systems [24-26], the results derived in this paper is based on delta operate theory, and the proposed Lyapunov-Krasovskii functional candidate is dependent on the sampling time *T*. Furthermore, the proposed results unify some existing results of finite-time boundedness into the delta operator framework.

3.2. H_∞ performance analysis

The following subsection will consider the problem of H_∞ finite-time boundedness of system (5).

Theorem 2: For given positive constants T_f, R, η_1 and η_2 satisfying $\eta_1<\eta_2$, and constants $\alpha<0$ and $\gamma>0$, if there exist positive scalars λ_g, $g=1,2,3$, and positive definite symmetric matrices P_i and S_i, $i\in\underline{N}$, with appropriate dimensions, such that

$$\begin{bmatrix}\Theta_{i1} & P_iA_{di}+TA_i^TP_iA_{di} & \Theta_{i2}\\ * & TA_{di}^TP_iA_{di}-(1-T\alpha)^{\underline{\tau}}S_i & TA_{di}^TP_iD_i\\ * & * & \Theta_{i3}\end{bmatrix}\leq 0, \qquad (27)$$

$$\chi<\lambda_1\eta_2, \qquad (28)$$

where

$$\begin{aligned}&\Theta_{i1}=P_iA_i+A_i^TP_i+TA_i^TP_iA_i+\alpha_iP_i+(\bar{\tau}-\underline{\tau}+1)S_i+C_i^TC_i,\\&\Theta_{i2}=P_iD_i+TA_i^TP_iD_i+T^{-1}C_i^TG_i,\\&\Theta_{i3}=TD_i^TP_iD_i+T^{-1}G_i^TG_i-T^{-1}\gamma^2I,\\&\chi=(1-T\alpha)^{T_f}((\lambda_2+T(\bar{\tau}^2-\bar{\tau}\underline{\tau}+\bar{\tau})(1-T\alpha)^{\bar{\tau}}\lambda_3)\eta_1\\&\quad+(\frac{d^2\lambda_4}{1-T\alpha})),\\&\lambda_1R\leq P_i\leq\lambda_2R,\quad S_i\leq\lambda_3R,\quad T\gamma^2=\lambda_4,\end{aligned}$$

then under any switching signal $\sigma(k)$ with the following average dwell time scheme

$$\tau_a>\tau_a^*=\max\left\{\frac{T_f\ln\mu}{\ln(\lambda_1\eta_2)-\ln\chi},\frac{\ln\mu}{\ln(1-T\alpha)}\right\}, \qquad (29)$$

system (5) is H_∞ finite-time bounded with respect to $(\eta_1,\eta_2,d,T_f,R,\sigma(k))$, where $\mu\geq 1$ satisfies (11).

Proof: Setting $Y_i=\gamma^2TI$ in Theorem 1, (8) can be directly derived from (27). We can obtain from (27), (28) and (11) that system (5) is finite-time bounded with respect to (η_1,η_2,d,T_f,R).

Choosing the Lyapunov-Krasovskii functional candidate (12) and following the proof line of Theorem 1, we can get

$$\begin{aligned}&\delta V_i(k)+\alpha V_i(k)+T^{-1}z^T(k)z(k)-T^{-1}\gamma^2w^T(k)w(k)\\&=\delta V_i(k)+\alpha V_i(k)-T^{-1}\gamma^2w^T(k)w(k)\\&\quad+T^{-1}(Cx(k)+Gw(k))^T(Cx(k)+Gw(k)) \qquad (30)\\&=\begin{bmatrix}x(k)\\x(k-\tau(k))\\w(k)\end{bmatrix}^T\tilde{\Theta}_i\begin{bmatrix}x(k)\\x(k-\tau(k))\\w(k)\end{bmatrix},\end{aligned}$$

where

$$\tilde{\Theta}_i=\begin{bmatrix}\Theta_{i1} & P_iA_{di}+TA_i^TP_iA_{di} & \Theta_{i2}\\ * & TA_{di}^TP_iA_{di}-(1-T\alpha)^{\bar{\tau}}S_i & TA_{di}^TP_iD_i\\ * & * & \Theta_{i3}\end{bmatrix}.$$

It follows from (27) and (30) that

$$\begin{aligned}V_{\sigma(k)}(k)&\leq(1-T\alpha)^{(k-k_p)}V_{\sigma(k_p)}(k_p)\\&\quad-\sum_{s=k_p}^{k-1}(1-T\alpha)^{(k-s-1)}\Lambda(s), \qquad (31)\end{aligned}$$

where $\Lambda(s)=\|z(s)\|^2-\gamma^2\|w(s)\|^2$.

Following the proof line of (22), for any $k\in[0,T_f)$, we can get

$$V_{\sigma(k)}(k) \le \mu^{N_\sigma(0,k)}(1-T\alpha)^k V_{\sigma(0)}(0) - \sum_{s=0}^{k-1}\mu^{N_\sigma(s,k)}(1-T\alpha)^{(k-s-1)}\Lambda(s). \tag{32}$$

Under the zero initial condition, we have

$$\sum_{s=0}^{k-1}\mu^{N_\sigma(s,k)}(1-T\alpha)^{(k-s-1)}\|z(s)\|^2 < \gamma^2\sum_{s=0}^{k-1}\mu^{N_\sigma(s,k)}(1-T\alpha)^{(k-s-1)}\|w(s)\|^2. \tag{33}$$

Multiplying both sides of (33) by $\mu^{-N_\sigma(0,k)}$ leads to

$$\sum_{s=0}^{k-1}\mu^{-N_\sigma(0,s)}(1-T\alpha)^{(k-s-1)}\|z(s)\|^2 \le \gamma^2\sum_{s=0}^{k-1}\mu^{-N_\sigma(0,s)}(1-T\alpha)^{(k-s-1)}\|w(s)\|^2. \tag{34}$$

From (29), we have

$$\mu^{-N_\sigma(0,s)} \ge (1-T\alpha)^{-s}. \tag{35}$$

Then, we can obtain

$$\sum_{s=0}^{k-1}(1-T\alpha)^{-s}(1-T\alpha)^{(k-s)}\|z(s)\|^2 \le \gamma^2\sum_{s=0}^{k-1}(1-T\alpha)^{(k-s)}\|w(s)\|^2. \tag{36}$$

Let $k = T_f$, then multiplying both sides of (36) by $(1-T\alpha)^{-T_f}$ leads to

$$\sum_{s=0}^{T_f-1}(1-T\alpha)^{-2s}\|z(s)\|^2 \le \gamma^2\sum_{s=0}^{T_f-1}\|w(s)\|^2. \tag{37}$$

According to Definition 3, we can conclude that the theorem is true.

The proof is completed.

3.3. Finite-time H_∞ control

Considering system (1) under the state feedback controller $u(k) = K_{\sigma(k)}x(k)$, the corresponding closed-loop system is given by

$$\begin{cases} \delta x(k) = (\hat{A}_{\sigma(k)}(k) + \hat{B}_{\sigma(k)}(k)K_{\sigma(k)})x(k) \\ \qquad + \hat{A}_{d\sigma(k)}(k)x(k-d(k)) + D_{\sigma(k)}w(k), \\ z(k) = C_{\sigma(k)}x(k) + E_{\sigma(k)}w(k), \\ x(\theta) = \varphi(\theta), \quad \theta = -\overline{\tau}, -\overline{\tau}+1, \ldots, 0. \end{cases} \tag{38}$$

Theorem 3: Consider system (1), for given positive constants T_f, R, η_1 and η_2 satisfying $\eta_1 < \eta_2$, and constants $\alpha < 0$ and $\gamma > 0$, if there exist positive scalars ε_i and λ_g, $g = 1,2,3$, positive definite symmetric matrices X_i and Q_i, and any matrices W_i, $i \in \underline{N}$, with appropriate dimensions, such that

$$\begin{bmatrix} \Xi_i & A_{di}X_i & D_i & T(A_iX_i + B_iW_i)^T & X_iC_i^T & \varepsilon_iH_i & (E_{ai}X_i + E_{bi}W_i)^T \\ * & -(1-T\alpha)^{\underline{\tau}}Q_i & 0 & TX_iA_{di}^T & 0 & 0 & X_iE_{adi}^T \\ * & * & -T^{-1}\gamma^2 I & TD_i^T & G_i^T & 0 & 0 \\ * & * & * & -TX_i & 0 & \varepsilon_iTH_i & 0 \\ * & * & * & * & -TI & 0 & 0 \\ * & * & * & * & * & -\varepsilon_iI & 0 \\ * & * & * & * & * & * & -\varepsilon_iI \end{bmatrix} \le 0, \tag{39}$$

$$\chi < \lambda_1\eta_2, \tag{40}$$

where

$$\Xi_i = (A_iX_i + B_iW_i) + (A_iX_i + B_iW_i)^T + \alpha_iX_i + (\overline{\tau} - \underline{\tau} + 1)Q_i,$$

$$\chi = (1-T\alpha)^{T_f}((\lambda_2 + T(\overline{\tau}^2 - \overline{\tau}\underline{\tau} + \overline{\tau})(1-T\alpha)^{\overline{\tau}}\lambda_3)\eta_1 + (\frac{d^2\lambda_4}{1-T\alpha})),$$

$$\lambda_1R \le X_i^{-1} \le \lambda_2R, \quad X_i^{-1}Q_iX_i^{-1} \le \lambda_3R, \quad T\gamma^2 = \lambda_4,$$

then under any switching signal $\sigma(k)$ with the average dwell time scheme (29), the closed-loop system (38) is H_∞ finite-time bounded with respect to $(\eta_1, \eta_2, d, T_f, R, \sigma(k))$, where $\mu \ge 1$ satisfies

$$X_i \le \mu X_j, \quad Q_i \le \mu Q_j, \quad \forall i, j \in \underline{N}. \tag{41}$$

Proof: Replacing A_i and A_{di} in (27) with $\hat{A}_i + \hat{B}_iK_i$ and $\hat{A}_{di}$, we can get

$$\begin{bmatrix} \hat{\Theta}_i & P_i\hat{A}_{di} + T(\hat{A}_i + \hat{B}_iK_i)^TP_i\hat{A}_{di} & P_iD_i + T(\hat{A}_i + \hat{B}_iK_i)^TP_iD_i + T^{-1}C_i^TG_i \\ * & T\hat{A}_{di}^TP_i\hat{A}_{di} - (1-T\alpha)^{\underline{\tau}}S_i & T\hat{A}_{di}^TP_iD_i \\ * & * & TD_i^TP_iD_i + T^{-1}G_i^TG_i - T^{-1}\gamma^2I \end{bmatrix} \le 0, \tag{42}$$

where

$$\hat{\Theta}_i = P_i(\hat{A}_i + \hat{B}_iK_i) + (\hat{A}_i + \hat{B}_iK_i)^TP_i + \alpha_iP_i + T(\hat{A}_i + \hat{B}_iK_i)^TP_i(\hat{A}_i + \hat{B}_iK_i) + (\overline{\tau} - \underline{\tau} + 1)S_i + T^{-1}C_i^TC_i.$$

By Lemma 1, (42) is equivalent to

$$\begin{bmatrix} \hat{\Xi}_i & P_i\hat{A}_{di} & P_iD_i & T(\hat{A}_i+\hat{B}_iK_i)^T & C_i^T \\ * & -(1-T\alpha)^{\underline{\tau}}S_i & 0 & T\hat{A}_{di}^T & 0 \\ * & * & -T^{-1}\gamma^2 I & TD_i^T & G_i^T \\ * & * & * & -TP_i^{-1} & 0 \\ * & * & * & * & -TI \end{bmatrix} \le 0, \quad (43)$$

where

$$\hat{\Xi}_i = P_i(\hat{A}_i+\hat{B}_iK_i)+(\hat{A}_i+\hat{B}_iK_i)^T P_i + \alpha_i P_i + (\overline{\tau}-\underline{\tau}+1)S_i.$$

Using $diag\{P_i^{-1} \quad P_i^{-1} \quad I \quad I \quad I\}$ to pre- and post-multiply the left term of (43), respectively, we can obtain

$$\begin{bmatrix} \Xi_{1i} & \hat{A}_{di}P_i^{-1} & D_i & \Xi_{3i} & P_i^{-1}C_i^T \\ * & \Xi_{2i} & 0 & TP_i^{-1}\hat{A}_{di}^T & 0 \\ * & * & -T^{-1}\gamma^2 I & TD_i^T & G_i^T \\ * & * & * & -TP_i^{-1} & 0 \\ * & * & * & * & -TI \end{bmatrix} \le 0, \quad (44)$$

where

$$\Xi_{1i} = (\hat{A}_i+\hat{B}_iK_i)P_i^{-1} + P_i^{-1}(\hat{A}_i+\hat{B}_iK_i)^T + \alpha_i P_i^{-1} + (\overline{\tau}-\underline{\tau}+1)P_i^{-1}S_iP_i^{-1},$$

$$\Xi_{2i} = -(1-T\alpha)^{\underline{\tau}} P_i^{-1}S_iP_i^{-1},$$

$$\Xi_{3i} = TP_i^{-1}(\hat{A}_i+\hat{B}_iK_i)^T.$$

Denote $Q_i = P_i^{-1}S_iP_i^{-1}$, $X_i = P_i^{-1}$ and $W_i = K_iX_i$, the n substituting (3) into (44) and applying Lemmas 1 and 2, (44) and (41) is equivalent to (39) and (11), respectively.

The proof is completed.

Based on Theorem 3, we are now in a position to present an effective algorithm for constructing the desired controller.

Algorithm 1:

Step 1: Input the system matrices.

Step 2: Choose the parameters $\alpha < 0$ and $\gamma > 0$. By solving (39) and (40), one can get the solutions of ε_i, W_i, X_i and Q_i.

Step 3: From $K_i = W_iX_i^{-1}$ with the obtained W_i and X_i, one can compute K_i.

Step 4: Compute μ and τ_a^* by (29) and (41).

4. NUMERICAL EXAMPLE

In this section, a numerical example will be presented to demonstrate the validity of the proposed results.

Consider system (1) with parameters as follows:

$$A_1 = \begin{bmatrix} -4 & 1 \\ 1 & -3 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} 0.2 & 0.1 \\ 0.2 & 0.1 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} -0.5 & -0.1 \\ 0.3 & -0.2 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 0.1 & -0.1 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 0.1 \\ -0.2 \end{bmatrix},$$

$$G_1 = 0.2, \quad H_1 = \begin{bmatrix} 0.05 \\ -0.05 \end{bmatrix}, \quad E_{a1} = \begin{bmatrix} -0.01 \\ 0.03 \end{bmatrix}^T,$$

$$E_{ad1} = \begin{bmatrix} 0.03 \\ -0.1 \end{bmatrix}^T, \quad E_{b1} = \begin{bmatrix} 0.02 \\ -0.01 \end{bmatrix}^T, \quad A_2 = \begin{bmatrix} -5 & 2 \\ 2 & -3 \end{bmatrix},$$

$$A_{d2} = \begin{bmatrix} 0.1 & 0.2 \\ 0.1 & 0.2 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -0.6 & -0.2 \\ 0.4 & -0.3 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 0.1 & 0 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0.2 \\ -0.1 \end{bmatrix}, \quad G_2 = 0.1,$$

$$H_2 = \begin{bmatrix} 0.07 \\ -0.1 \end{bmatrix}, \quad E_{a2} = \begin{bmatrix} 0.06 \\ -0.03 \end{bmatrix}^T,$$

$$E_{ad2} = \begin{bmatrix} 0.01 \\ -0.03 \end{bmatrix}^T, \quad E_{b2} = \begin{bmatrix} -0.04 \\ 0.01 \end{bmatrix}^T,$$

$$F_1(k) = F_2(k) = \sin(k), \quad \overline{\tau} = 1, \quad \underline{\tau} = 0.$$

Taking $\eta_1 = 1$, $\eta_2 = 5$, $T = 0.25$, $\alpha = -0.4$, $\gamma = 1.2$, $d = 0.1$ and $R = I$, and solving (39) and (40) in Theorem 3 lead to

$$X_1 = \begin{bmatrix} 4.4082 & 0.0142 \\ 0.0142 & 4.5037 \end{bmatrix}, \quad Q_1 = \begin{bmatrix} 3.7646 & -0.0188 \\ -0.0188 & 3.7844 \end{bmatrix},$$

$$X_2 = \begin{bmatrix} 4.3821 & 0.0688 \\ 0.0688 & 4.4141 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 3.7734 & -0.0272 \\ -0.0272 & 3.7869 \end{bmatrix},$$

$$\lambda_1 = 0.2219, \quad \lambda_2 = 0.2311, \quad \lambda_3 = 0.2032,$$

and the state feedback gain matrices can be given as follows:

$$K_1 = \begin{bmatrix} -5.2693 & 3.0743 \\ -3.0873 & -5.1986 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -5.9941 & 3.8432 \\ -1.6713 & -1.3216 \end{bmatrix}.$$

According to (41), we have $\mu = 1.0278$. Then from (29), we get $\tau_a^* = 1.2950$. Choosing $\tau_a = 2$, the simulation results are shown in Figs. 1-2, where the initial conditions are $x(0) = [1 \quad 0]^T$, $x(k) = [0 \quad 0]^T$, $k = -1$, and the exogenous disturbance input is $w(k) = 0.05e^{-0.5k}$.

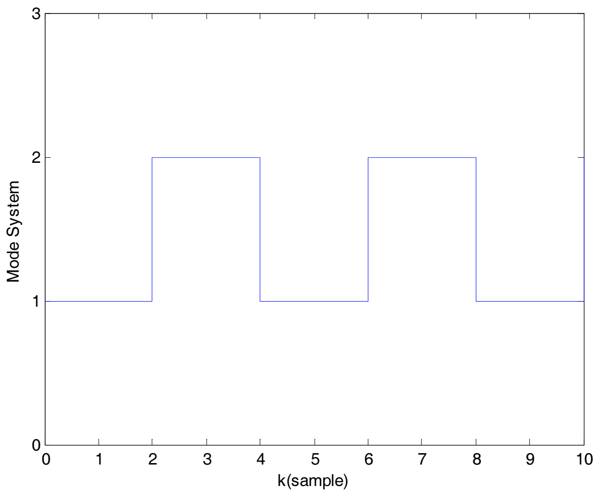

Fig. 1. Switching signal.

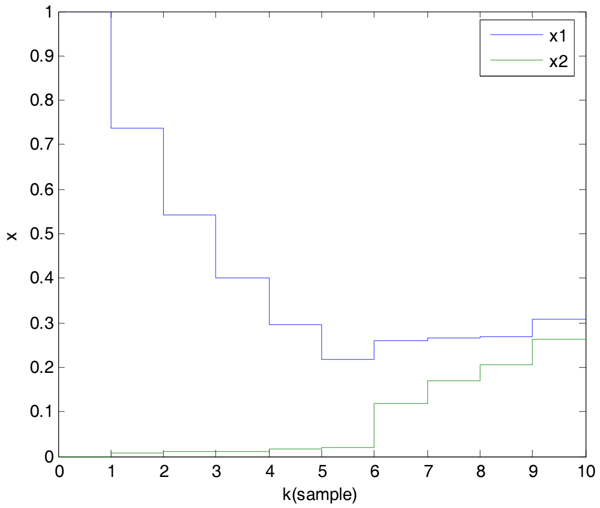

Fig. 2. State response of the closed-loop system.

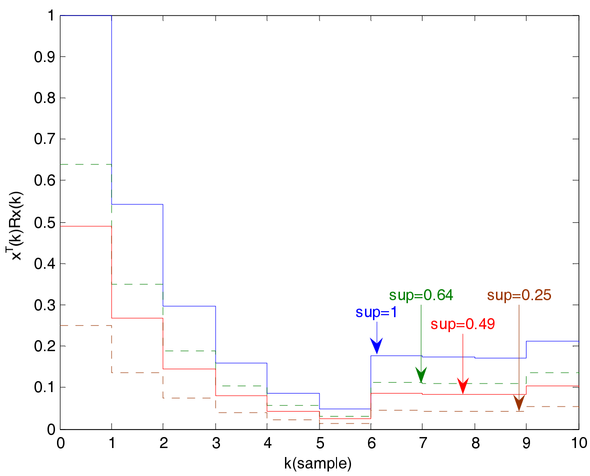

Fig. 3. The evolution of $x^T(k)Rx(k)$.

Fig. 1 depicts the switching signal. The state trajectory of the closed-loop system is shown in Fig. 2. Furthermore, the evolution of $x^T(k)Rx(k)$ is plotted in Fig. 3, where "sup" denotes the value of $\sup_{-\bar{\tau}\le\theta\le 0}\{x^T(\theta)Rx(\theta)\}$. It is easy to see that the closed-loop system is finite-time bounded with respect to $(1,5,0.1,10,I,\sigma(k))$. This demonstrates the effectiveness of the proposed method.

5. CONCLUSIONS

This paper has dealt with the finite-time H_∞ control for switched systems with time-varying delay using delta operator approach. Sufficient conditions for finite-time stability and H_∞ finite-time boundedness of the underlying systems are given. The desired state feedback controller is designed such that the closed-loop system is H_∞ finite-time bounded. Finally, a numerical example illustrates the effectiveness of the proposed method. In our further work, we will extend the proposed results to switched nonlinear systems or switched systems with Markov jump framework [30-32].

REFERENCES

[1] W. Zhang, M. S. Branicky, and S. M. Phillips, "Stability of networked control systems," *IEEE Control Systems, Magazine*, vol. 21, no. 1, pp. 84-99, February 2001.

[2] P. P. Varaiya, "Smart Cars on smart roads: problems of control," *IEEE Trans. on Automatic Control*, vol. 38, no. 2, pp. 195-207, February 1993.

[3] C. Tomlin, G. J. Pappas, and S. Sastry, "Conflict resolution for air traffic management: a study in multi-agent hybrid systems," *IEEE Trans. on Automatic Control*, vol. 43, no. 4, pp. 509-521, April 1998.

[4] B. Lennartson, M. Tittus, and B. Egardt, "Hybrid systems in process control," *IEEE Control Systems, Magazine*, vol. 16, no. 5, pp. 45-56, October 1996.

[5] B. Niu and J. Zhao, "Robust H_∞ control of uncertain nonlinear switched systems using constructive method," *International Journal of Control, Automation and Systems*, vol. 10, no. 3, pp. 481-489, June 2012.

[6] N. H. El-Farra and P. D. Christofides, "Coordinating feedback and switching for control of spatially distributed processes," *Computers and Chemical Engineering*, vol. 28, no. 1-2, pp. 111-128, January 2004.

[7] G. C. Goodwin, R. L. Leal, D. Q. Mayne, and R. H. Middleton, "Rapprochement between continuous and discrete model reference adaptive control," *Automatica*, vol. 22, no. 2, pp. 199-207, March 1986.

[8] R. H. Middleton and G. C. Goodwin, "Improved finite word length characteristics in digital control using delta operators," *IEEE Trans. on Automatic Control*, vol. 31, no. 11, pp. 1015-1021, November 1986.

[9] C. P. Neuman, "Transformation between delta and forward shift operator transfer function models," *IEEE Trans. on Systems, Man and Cybernetics*, vol. 23, no. 1, pp. 295-296, January/February 1993.

[10] D. J. Zhang and C. W. Yang, "Delta-operator theory for feedback control system-a survey," *Control Theory & Applications*, vol. 15, no. 2, pp.153-160, February 1998.

[11] J. Wu, S. Chen, G. Li, R. H. Istepanian, and J. Chu, "Shift and delta operator realizations for digital controllers with finite word length considerations," *IEEE Proc.-Control Theory and Applications*, vol. 147, no. 6, pp. 664-672, November 2000.

[12] R. Vijayan, H. V. Poor, J. B. Moore, and G. C. Goodwin, "A Levinson-type algorithm for modeling fast sampled data," *IEEE Trans. on Automatic Control*, vol. 36, no. 3, pp. 314-321, March 1991.

[13] W. Ebert, "Optimal filtered predictive control a delta operator approach," *Systems & Control Letters*, vol. 42, no. 1, pp. 69-80, January 2001.

[14] C. B. Soh, "Robust stability of discrete-time systems using delta operators," *IEEE Trans. on*

Automatic Control, vol. 36, no. 3, pp. 377-380, March 1991.

[15] Z. Xiang, Q. Chen, and W. Hu, "Robust H_∞ control of interval systems using the delta operator," *Proc. of the 4th World Congress on Intelligent Control and Automation*, Shanghai, pp. 1731-1733, June 2002.

[16] Z. Xiang and R. Wang, "Robust H_∞ control for a class of uncertain switched systems using delta operator," *Trans. of the Institute of Measurement and Control*, vol. 32, no. 3, pp. 331-344, June 2010.

[17] S. Chen, R. H. Istepanian, J. Wu, and J. Chu, "Comparative study on optimizing closed loop stability bounds of finite-precision controller structures with shift and delta operators," *Systems & Control Letters*, vol. 40, no. 3, pp. 153-163, July 2000.

[18] J. Xing, R. Wang, P. Wang, and Q. Yang, "Robust control for a class of uncertain switched time delay systems using delta operator," *Proc. of 12th International Conf. Control Automation Robotics & Vision*, Guangzhou, pp. 518-523, December 2012.

[19] J. Qiu, Y. Xia, H. Yang, and J. Zhang, "Robust stabilization for a class of discrete-time systems with time-varying delays via delta operators," *IET Control Theory & Applications*, vol. 2, no. 1, pp. 87-93, January 2008.

[20] X. Q. Xiao and Z. Lei, "Delay-dependent robust l_2-l_∞ filter design for uncertain delta-operator time delay systems," *International Journal of Control, Automation and Systems*, vol. 9, no. 3, pp. 611-615, June 2011.

[21] S. Li, Z. Xiang, and H. R. Karimi, "Mixed l_-/l_1 fault detection observer design for positive switched systems with time-varying delay via delta operator approach," *International Journal of Control, Automation and Systems*, vol. 12, no. 4, pp. 709-721, August 2014.

[22] L. L. Hou, G. D. Zong, and Y. Q. Wu, "Finite-time control for discrete-time switched systems with time delay," *International Journal of Control, Automation, and Systems*, vol. 10, no. 4, pp. 855-860, August 2012.

[23] Y. Shen and H. Liu, "Finite-time stabilization of switched time-delay system via dynamic output feedback control," *Mechanical Engineering and Technology, Advances in Intelligent and Soft Computing*, vol. 125, pp. 523-528, November 2012.

[24] Z. Xiang, Y. N. Sun, and M. S. Mahmoud, "Robust finite-time H_∞ control for a class of uncertain switched neutral systems," *Communication in Nonlinear Science Numerical Simulation*, vol. 17, no. 4, pp. 1766-1778, April 2012.

[25] W. Xiang and J. Xiao, "H_∞ finite-time control for switched nonlinear discrete-time systems with norm-bounded disturbance," *Journal of the Franklin Institute*, vol. 348, no. 2, pp. 331-52, March 2011.

[26] H. Liu and Y. Shen, "H_∞ finite-time control for switched linear systems with time-varying delay," *Intelligent Control and Automation*, vol. 2, no. 3, pp. 203-213, February 2011.

[27] H. Liu, Y. Shen, and X. Zhao, "Asynchronous finite-time H_∞ control for switched linear systems via mode-dependent dynamic state-feedback," *Nonlinear Analysis: Hybrid Systems*, vol. 8, pp. 109-120, May 2013.

[28] H. Song, L. Yu, D. Zhang, and W. A. Zhang, "Finite-time H_∞ control for a class of discrete-time switched time-delay systems with quantized feedback," *Communications in Nonlinear Science and Numerical Simulation*, vol. 17, no. 12, pp. 4802-4814, December 2012.

[29] X. Lin, H. Du, S. Li, and Y. Zhou, "Finite-time stability and finite-time weighted L_2-gain analysis for switched systems with time-varying delay," *IET Control Theory & Applications*, vol. 7, no. 7, pp. 1058-1069, May 2013.

[30] H. Shen, S. Xu, J. Lu, and J. Zhou, "Passivity-based control for uncertain stochastic jumping systems with mode-dependent round-trip time delays," *Journal of the Franklin Institute*, vol. 349, no. 5, pp. 1665-1680, June 2012.

[31] H. Shen, J. H. Park, L. Zhang, and Z. G. Wu, "Robust extended dissipative control for sampled-data Markov jump systems," *International Journal of Control*, vol. 87, no. 8, pp. 1549-1564, August 2014.

[32] Z. G. Wu, P. Shi, H. Su, and J. Chu, "Asynchronous l_2-l_∞ filtering for discrete-time stochastic Markov jump systems with randomly occurred sensor nonlinearities," *Automatica*, vol. 50, no. 1, pp. 180-186, January 2014.