

ROBUST UNKNOWN INPUT OBSERVER DESIGN FOR LINEAR UNCERTAIN TIME DELAY SYSTEMS WITH APPLICATION TO FAULT DETECTION

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ABSTRACT

In this paper, a novel approach is proposed to design a robust fault detection observer for uncertain linear time delay systems. The system is composed of both norm-bounded uncertainties and exogenous signals (noise, disturbance, and fault) which are considered to be unknown. The main contribution of this paper is to present unknown input observer (UIO)-based fault detection system which shows the maximum sensitivity to fault signals and the minimum sensitivity to other signals. Since the system contains uncertainty terms, an H_∞ model-matching approach is used in design procedure. The reference residual signal generator system is designed so that the fault signal has maximum sensitivity while the exogenous signals have minimum sensitivity on the residual signal. Then, the fault detection system is designed by minimizing the estimation error between the reference residual signal and the UIO residual signal in the sense of H_∞ norm. A sufficient condition for the existence of such a filter is exploited in terms of certain linear matrix inequalities (LMIs). Application of the proposed method in a numerical example and an engineering process are simulated to demonstrate the effectiveness of the proposed algorithm. Simulation results show the validity of the proposed approach to detect the occurrence of faults in the presence of modeling errors, disturbances, and noise.

Key Words: Unknown input observer, fault detection, time delay, linear matrix inequality, uncertainty.

1. INTRODUCTION

Recently, many research studies have been devoted to the realm of fault detection due to the problems arising from the occurrence of faults in control systems. Three approaches have been developed in the literature to detect the occurrence of faults: model-based, signal-based, and knowledge-based approaches. Although the second and third approaches show appropriate performance in real world applications, the model-based techniques are more popular among the control scientific community than other approaches.

The model-based fault detection (FD) approaches are developed based on the mathematical model of the plant. Since the real model of a system is hardly realizable in practice, robust schemes of model based FD have been exploited in the last decade [1–4]. Among these various techniques, the applications of observers have been widely reported in the literature [5,6].

The existence of model uncertainty and exogenous signals such as noise and disturbance necessitates an observer design procedure which needs to be robust against the uncer-

tainty and exogenous signal as well as to be sensitive enough to detect faults. In other words, the residual signal produced by the observer should be independent from the uncertainty and the exogenous signal. Therefore, an optimization approach should be employed in the design procedure. In this regard, the performance index has been defined in H_1/H_∞ or H_2/H_∞ which provides both robust and optimal solutions. In [7–9], a design procedure which uses an optimal residual generator as a reference model has been developed. Then a residual generator is designed for a system with model uncertainty such that the error between the optimal residual signal and residual signal is minimized. The solution of this optimization problem has been handled by a linear matrix inequality (LMI) tool. To solve the problem of H_1/H_∞ and H_2/H_∞ , much other research has been conducted [10–15].

Time delay exists in many industrial systems; hence, enormous research activities have been carried out to investigate problems arising from delay in systems. For example, in [16], the authors present a fuzzy H_∞ controller for system with state delay. The problem of robust stability for uncertain delay fuzzy systems and application of fuzzy controller in active suspension system with actuator delay and fault have been studied in [17] and [18], respectively. Another interesting research field is the design of an optimal observer for fault detection purposes [19–21]. The main idea of designing an optimal observer for systems consisting of both model uncertainty and exogenous signals is similar to those used in delay-free approaches. However, the definition of the performance

Manuscript received September 3, 2012; revised March 7, 2013; accepted May 29, 2013.

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index is different. In [22,23], the authors designed a fault detection scheme for uncertain delay systems. The optimal residual generator system has been designed based on a model-matching approach. The same approach has been used in [24], however, an iterative algorithm has been employed to solve the optimization procedure. The problem of fault detection for linear systems with time delay and nonlinear perturbation has been investigated in [25] and for Master-Slave systems in [26]. In [27] the authors presented an H_∞ filter for faulty input signal estimation and controlled the system. The problem of fault isolation was investigated in [28] using a bank of residual generator filters in which each filter had been designed by H_∞ optimization. Recently, enormous research efforts have been devoted to the problem of fault detection in network control systems [29,30].

In the previous works, the Luenberger observer or filter has been utilized as an observer in the residual generator systems. Unknown input observer (UIO) is another class of observer which has been widely studied in the literature [31–34]. The problem of fault diagnosis for time delay systems using UIO is investigated in [35] for the first time, however, the model uncertainty and optimal design are not considered. In [36], UIO has been designed for state estimation in the presence of noise and uncertainty, without considering time delay. Although the UIO has been used in many fault detection problems in real world applications [37,38], UIO-based fault detection for uncertain time delay systems has not been comprehensively studied to date. Motivated by these considerations, in the current work, an optimal fault detection filter for linear uncertain time delay systems using unknown input observer is developed. A reference residual generator system is designed to have maximum sensitivity to the fault and minimum sensitivity to the noise. Then, the residual generator system is designed by model-matching techniques. The FD system is designed without resorting to any model transformations and bounding techniques for some cross terms.

The organization of this paper is as follows: In Section 2, system descriptions and problem formulation are presented. The main result is presented in Section 3. Efficacy of the proposed methods is shown by a simulation results in Section 4. Finally, concluding remarks are given in Section 5.

II. PROBLEM FORMULATION

In many industrial applications such as mechanical, electrical, meteorological, chemical, economic, and biological systems, nonlinearity and sources of time delay exist in the mode [39,40]. Therefore, a linearized model of these systems around an expected operation point is considered. However, there are always some discrepancies between the real dynamics of the system and the linearized model. These differences arise from system uncertainties, as a consequence

of neglecting dynamics, and changes in system parameters. Therefore, the following linear uncertain system with additive disturbances and time delay is considered to represent the model described:

$$\begin{aligned}\dot{x}(t) &= (A + \Delta A(t))x(t) + (A_d + \Delta A_d)x(t - \tau) \\ &\quad + (B + \Delta B(t))u(t) + Ed(t) + F_x f(t) + R_n n(t) \quad (1) \\ y(t) &= Cx(t) + F_y f(t) + Dn(t)\end{aligned}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $y(t) \in \mathbb{R}^p$ is the output vector, $u(t) \in \mathbb{R}^q$ is the input vector, $d(t) \in \mathbb{R}^m$ is an unknown scalar function representing the disturbance that belongs to $\mathbb{L}_2^m(0, \infty)$, $f(t) \in \mathbb{R}^f$ denotes the faults and $n(t) \in \mathbb{R}^r$ represents the noise.

Note that $\Delta A(t)$, $\Delta B(t)$, and $\Delta A_d(t)$ are the norm bounded time-varying uncertainties of the matrices A , B , and A_d , respectively. $\tau \geq 0$ is a constant time delay. The characteristics of uncertainty matrices are assumed to belong to:

$$\begin{aligned}\Omega_1 &= \{\Delta A(t) | \Delta A(t) = M_1 \Sigma_1(t) N_1, \Sigma_1^T(t) \Sigma_1(t) \leq I\} \\ \Omega_2 &= \{\Delta A_d(t) | \Delta A_d(t) = M_2 \Sigma_2(t) N_2, \Sigma_2^T(t) \Sigma_2(t) \leq I\} \quad (2) \\ \Omega_3 &= \{\Delta B(t) | \Delta B(t) = M_3 \Sigma_3(t) N_3, \Sigma_3^T(t) \Sigma_3(t) \leq I\}\end{aligned}$$

It is generally acknowledged [39] that a full order UIO for a class of time delay systems can be represented as follows:

$$\begin{aligned}\dot{z}(t) &= Fz(t) + Gz(t - \tau) + Hu(t) + K_{1z}y(t) + K_{2z}y(t - \tau) \\ \hat{x}(t) &= z(t) + L_1y(t)\end{aligned} \quad (3)$$

where $\hat{x}(t)$ is the estimated state vector. Therefore, the dynamic of $\hat{x}(t)$ is governed by:

$$\begin{aligned}\dot{\hat{x}}(t) &= F\hat{x}(t) + G\hat{x}(t)(t - \tau) + Hu(t) \\ &\quad + L_1\dot{y}(t) + L_2y(t) + L_3y(t - \tau)\end{aligned} \quad (4)$$

where F , G , H and L_1 are the observer matrices with $L_2 = K_{1z} - FL_1$ and $L_3 = K_{2z} - GL_1$. The observer matrices will be computed such that the disturbance and input are decoupled from the estimation error which is defined by $e(t) = x(t) - \hat{x}(t)$. The observer described by (4) is illustrated in Fig. 1. The state estimation error (5) is obtained from (1), (3), and (4) as:

$$\begin{aligned}\dot{e}(t) &= Fe(t) + Ge(t)(t - \tau) \\ &\quad + ((I - L_1C)A - L_2C - F)x(t) \\ &\quad + ((I - L_1C)A_d - L_3C - G)x(t - \tau) \\ &\quad + ((I - L_1C)B - H)u(t) + (I - L_1C)Ed(t) \quad (5) \\ &\quad + ((I - L_1C)F_x - L_2F_y)f(t) - L_1F_y\dot{f}(t) - L_3F_yf(\tau - t) \\ &\quad + ((I - L_1C)R - L_2D)n(t) - L_1D\dot{n}(t) - L_3Dn(\tau - t) \\ &\quad + ((I - L_1C)\Delta A(t))x(t) + ((I - L_1C)\Delta A_d(t))x(\tau - t) \\ &\quad + ((I - L_1C)\Delta B(t))u(t).\end{aligned}$$

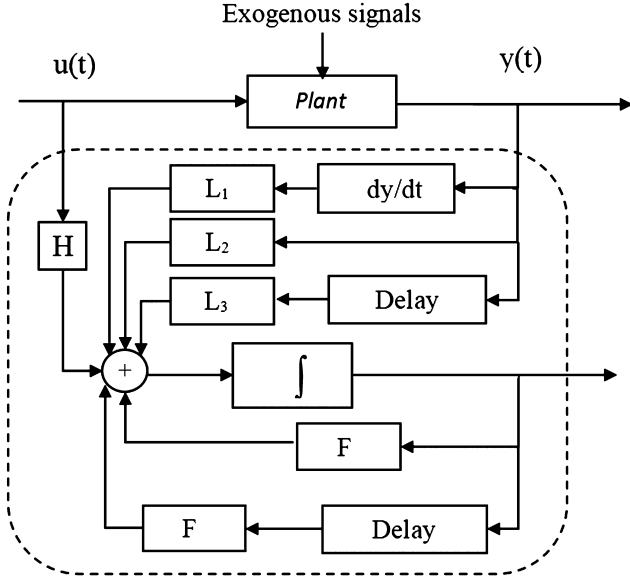


Fig. 1. Structure of a full-order UIO with delay.

In the absence of uncertainties and faults, it has been shown that the observer defined in (3) and (4) is an UIO for the system described by (1) if the following conditions are satisfied [36]:

- C1. $\dot{e}(t) = Fe(t) + Ge(t - \tau)$ is asymptotically stable.
- C2. $F = (I - L_1C)A - L_2C$.
- C3. $G = (I - L_1C)A_d - L_3C$.
- C4. $H = (I - L_1C)B$.
- C5. $(I - L_1C)E = 0$.

where 0 denotes the null matrix with compatible dimensions. Using these conditions, and considering the following definitions,

$$\begin{aligned}
 T &= (I - L_1C) \\
 \bar{F} &= [-L_1F_y \quad TF_x - L_2F_y \quad -L_3F_y] \\
 \bar{R} &= [-L_1D \quad TR - L_2D \quad -L_3D] \\
 \bar{f} &= [\bar{f}^T(t) \quad f^T(t) \quad f^T(t - \tau)]^T \\
 \bar{n} &= [\bar{n}^T(t) \quad n^T(t) \quad n^T(t - \tau)]^T
 \end{aligned} \tag{6}$$

the state estimation error dynamic (5) can be represented as

$$\begin{aligned}
 \dot{e}(t) &= Fe(t) + Ge(t - \tau) + \bar{F}\bar{f} + \bar{R}\bar{n} \\
 &\quad + T\Delta A(t)x(t) + T\Delta A_d(t)x(\tau - t) + T\Delta B(t)u(t).
 \end{aligned} \tag{7}$$

In order to use an UIO for fault detection purposes, it is required to define a residual signal. The definition of the

residual signal is based on the difference between the measured and estimated outputs by the following equation:

$$\begin{aligned}
 \hat{y}(t) &= C\hat{x}(t) \\
 r(t) &= V(y(t) - \hat{y}(t)) = VCe(t) + VK_1\bar{f}(t) + VK_2\bar{n}(t)
 \end{aligned} \tag{8}$$

where $K_1 = [0 \quad F_y \quad 0]$ and $K_2 = [0 \quad D \quad 0]$. Now, the objective of robust fault detection problems is to minimize the performance index defined in (9) for all classes of model uncertainty belong to Ω_i .

$$J_r = \max_{(\Delta A, \Delta B, \Delta A_d) \in \Omega_i} \frac{\|G_{r[d\bar{n}]\|_\infty}{\|G_{r[\bar{f}]\|_\infty} \tag{9}$$

In general, this performance index is minimized using H_∞ model matching approach which minimizes the difference between the residual signal $r(t)$ and the reference residual signal $r_f(t)$ in the presence of the worst case disturbance signals. One approach to design the residual reference is to compute the residual signal in the absence of uncertainties. In this case, the reference residual signal is designed such that the fault signals have the maximum effect and disturbance and unknown inputs have the minimum influence on the reference residual signal. The reference residual system is defined as:

$$\begin{aligned}
 \dot{e}_f(t) &= F^*e_f(t) + G^*e_f(t - \tau) + \bar{F}^*\bar{f} + \bar{R}^*\bar{n} \\
 r_f(t) &= V^*Ce_f(t) + V^*K_1\bar{f}(t) + V^*K_2\bar{n}(t)
 \end{aligned} \tag{10}$$

Considering these issues, the problem of robust fault detection is to compute observer matrices such that the overall system is asymptotically stable and the following performance index is minimized.

$$J_{r_e} = \sup_{(\Delta A, \Delta B, \Delta A_d) \in \Omega_i} \frac{\|r_e\|_2}{\|w\|_2} \tag{11}$$

where

$$\begin{aligned}
 r_e(t) &= r(t) - r_f(t) \\
 w &= [u^T \quad \bar{f}^T \quad d^T \quad \bar{n}^T]^T
 \end{aligned} \tag{12}$$

Denoting $\xi(t) = [e^T(t) \quad e_f^T(t) \quad x^T(t)]^T$, we obtain an augmented system as follows:

$$\begin{aligned}
 \dot{\xi}(t) &= (\bar{A} + \Delta\bar{A})\xi(t) + (\bar{A}_d + \Delta\bar{A}_d)\xi(t - \tau) + (\bar{B}_w + \Delta\bar{B}_w)w(t) \\
 r_e(t) &= \bar{C}\xi(t) + \bar{D}w(t)
 \end{aligned} \tag{13}$$

where

$$\begin{aligned}
\tilde{A} &= \begin{bmatrix} F & 0 & 0 \\ 0 & F^* & 0 \\ 0 & 0 & A \end{bmatrix} \quad \tilde{A}_d = \begin{bmatrix} G & 0 & 0 \\ 0 & G^* & 0 \\ 0 & 0 & A_d \end{bmatrix} \\
\tilde{B}_d &= \begin{bmatrix} 0 & \bar{F} & 0 & \bar{R} \\ 0 & \bar{F}^* & 0 & \bar{R}^* \\ B & K_3 & E & K_4 \end{bmatrix} \quad \Delta\tilde{A} = \begin{bmatrix} 0 & 0 & T\Delta A \\ 0 & 0 & 0 \\ 0 & 0 & \Delta A \end{bmatrix} \\
\Delta\tilde{A}_d &= \begin{bmatrix} 0 & 0 & T\Delta A_d \\ 0 & 0 & 0 \\ 0 & 0 & \Delta A_d \end{bmatrix} \\
\Delta\tilde{B}_w &= \begin{bmatrix} T\Delta B & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \Delta B & 0 & 0 & 0 \end{bmatrix} \quad \tilde{C}_w = [VC \quad -V^*C \quad 0] \\
\tilde{D}_w &= [0 \quad VK_1 - V^*K_1 \quad 0 \quad 0] \\
K_3 &= [0 \quad F_x \quad 0] \quad K_4 = [0 \quad R \quad 0].
\end{aligned} \tag{14}$$

Furthermore, from (2) the uncertain terms $\Delta\tilde{A}$, $\Delta\tilde{A}_d$, and $\Delta\tilde{B}_w$ can be expressed by:

$$\begin{aligned}
\Delta\tilde{A} &= \tilde{M}_1 \Sigma_1(t) \tilde{N}_1 = \begin{bmatrix} TM_1 \\ 0 \\ M_1 \end{bmatrix} \Sigma_1(t) [0 \quad 0 \quad N_1] \\
\Delta\tilde{A}_d &= \tilde{M}_2 \Sigma_2(t) \tilde{N}_2 = \begin{bmatrix} TM_2 \\ 0 \\ M_2 \end{bmatrix} \Sigma_2(t) [0 \quad 0 \quad N_2] \\
\Delta\tilde{B}_w &= \tilde{M}_3 \Sigma_3(t) \tilde{N}_3 = \begin{bmatrix} TM_3 \\ 0 \\ M_3 \end{bmatrix} \Sigma_3(t) [N_3 \quad 0 \quad 0 \quad 0]
\end{aligned} \tag{15}$$

Before proceeding further, we introduce the following lemmas, which will be used in subsequent developments.

Lemma 1 [39]. C.4 is solvable iff the following relation holds:

$$\text{rank}(CE) = m, \quad m \leq p \tag{16}$$

and the general solution of C4 can be calculated by:

$$\begin{aligned}
L_1 &= E(CE)^+ + Y[I - CE(CE)^+] \\
&= \Psi_1 + Y\Psi_2
\end{aligned} \tag{17}$$

where Y is an arbitrary matrix with appropriate dimensions.

Lemma 2. Suppose that M , N , and $\Sigma(t)$ are compatible and $\Sigma^T(t)\Sigma(t) \leq I$, then there exists a scalar $\varepsilon > 0$ such that (18) holds.

$$M\Sigma(t)N + N^T\Sigma^T(t)M^T \leq \varepsilon MM^T + \varepsilon^{-1}N^TN \tag{18}$$

III. MAIN RESULTS

3.1 The reference model section

The choice of reference model is an important key in designing robust fault detection filters for linear uncertain time-delay systems. In this paper, a reference model for a class of observer with unknown inputs is developed. The reference residual signal can be written as the sum of two signals, $r_{f\bar{n}}(t)$ and $r_{f\bar{f}}(t)$. The former represents the effect of exogenous signal (noise) on the reference residual signal and the latter represents the effect of state faults on the reference residual signal. Hence, the reference model should be chosen such that the effect of exogenous signals on the reference residual signal is minimized, while the effect of fault signal is maximized. These two tasks are described mathematically by:

$$\begin{aligned}
\|T(r_{f\bar{n}}, \bar{n})\|_\infty &\leq \alpha \\
\|T(r_{f\bar{f}}, \bar{f})\|_\infty &\geq \beta
\end{aligned} \tag{19}$$

where $T(\cdot, \cdot)$ is the transfer function between two signals. The following two theorems provide conditions which ensure the asymptotic stability of (10) and increase the sensitivity of reference residual signal on faults while decrease the sensitivity of reference residual signal to the noise.

Theorem 1. For given $\alpha > 0$, if there exist symmetric positive definite matrices P, Q, Z^* , matrices Φ_1^*, Φ_2^* and Φ_3^* such that following LMI holds:

$$\begin{aligned}
\begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} \\ * & \Xi_{22} & 0 \\ * & * & \Xi_{33} \end{bmatrix} &< 0 \\
\Xi_{11} &= PF^* + F^{*T}P + Q + C^T Z^* C \\
\Xi_{12} &= PG^* \quad \Xi_{13} = P\bar{R}^* + C^T Z^* K_2 \\
\Xi_{22} &= -Q \quad \Xi_{33} = -\alpha^2 I + K_3^T Z^* K_2
\end{aligned} \tag{20}$$

where

$$\begin{aligned}
PF^* &= P(A - \Psi_1 CA) - \Phi_1^*(\Psi_2 CA) - \Phi_2^* C \\
PG^* &= P(A_d - \Psi_1 CA_d) - \Phi_1^*(\Psi_2 CA_d) - \Phi_3^* C \\
P\bar{R}^* &= \begin{bmatrix} (-P(\Psi_1 D) - \Phi_1^*(\Psi_2 D))^T \\ (P(R - \Psi_1 CR) - \Phi_1^*(\Psi_2 CR) - \Phi_2^* D)^T \\ (-\Phi_3^* D)^T \end{bmatrix}^T
\end{aligned} \tag{21}$$

then the system (22) is asymptotically stable and $\|T(r_{f\bar{n}}, \bar{n})\|_\infty \leq \alpha$

$$\begin{aligned}\dot{e}_{f\bar{n}}(t) &= F^*e_{f\bar{n}}(t) + G^*e_{f\bar{n}}(t-\tau) + \bar{R}^*\bar{n} \\ r_{f\bar{n}}(t) &= V^*Ce_{f\bar{n}}(t) + V^*K_2\bar{n}(t)\end{aligned}\quad (22)$$

Furthermore, the UIO matrices are obtained by considering C.2 to C.5, and, $Y^* = P^{-1}\Phi_1^*$, $L_2 = P^{-1}\Phi_2^*$, $L_3 = P^{-1}\Phi_3^*$, $V^* = (Z^*)^{1/2}$.

Proof. Definition of $\|T(r_{f\bar{n}}, \bar{n})\|_\infty \leq \alpha$ is equivalent to $J_{r_{f\bar{n}}} : \int_0^\infty (r_{f\bar{n}}^T(t)r_{f\bar{n}}(t) - \alpha^2\bar{n}^T(t)\bar{n}(t))dt \leq 0$. Now consider the following Lyapunov–Krasovskii function $V(t) = e_{f\bar{n}}^T(t)Pe_{f\bar{n}}(t) + \int_{t-\tau}^t e_{f\bar{n}}^T(s)Qe_{f\bar{n}}(s)ds$. Then we have:

$$\begin{aligned}J_{r_{f\bar{n}}} &= \int_0^\infty (r_{f\bar{n}}^T(t)r_{f\bar{n}}(t) - \alpha^2\bar{n}^T(t)\bar{n}(t) + \dot{V}(t))dt \\ &\quad + V(0) - V(\infty)\end{aligned}\quad (23)$$

Assuming $r_{f\bar{n}} = 0$ for $t \in [-\tau, 0]$. Since $V(\infty) > 0$, we have:

$$J_{r_{f\bar{n}}} \leq \int_0^\infty (r_{f\bar{n}}^T(t)r_{f\bar{n}}(t) - \alpha^2\bar{n}^T(t)\bar{n}(t) + \dot{V}(t))dt \quad (24)$$

Taking derivation from $V(t)$ and considering (22) yields

$$J_{r_{f\bar{n}}} \leq \int_0^\infty \begin{bmatrix} e_{f\bar{n}}(t) \\ e_{f\bar{n}}(t-\tau) \\ \bar{n} \end{bmatrix}^T \Xi \begin{bmatrix} e_{f\bar{n}}(t) \\ e_{f\bar{n}}(t-\tau) \\ \bar{n} \end{bmatrix} dt \quad (25)$$

where

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} \\ * & \Xi_{22} & 0 \\ * & * & \Xi_{33} \end{bmatrix}$$

$$\Xi_{11} = PF^* + F^{*T}P + Q + C^TV^*V^*C$$

$$\Xi_{12} = PG^* \Xi_{13} = P\bar{R}^* + C^TV^*V^*K_2$$

$$\Xi_{22} = -Q \quad \Xi_{33} = -\alpha^2I + K_2^TV^*V^*K_2$$

Hence, $\Xi \leq 0$ implies $J_{r_{f\bar{n}}} \leq 0$. By assuming $Z^* = V^{*T}V^*$, the LMI (20) is concluded from (25). The inequality (20) (without considering (21)) includes nonlinearity terms which leads the LMI to be infeasible. To overcome this problem, define $\Phi_1^* = PY^*$, $\Phi_2^* = PL_2^*$, $\Phi_3^* = PL_3^*$. Using C2, C3 and (17) it can be seen that (21) makes the LMI feasible.

Theorem 2. For given $\beta > 0$, if there exist symmetric positive definite matrices P, Q, Z^* , matrices Φ_1^*, Φ_2^* , and Φ_3^* such that following LMI holds:

$$\begin{aligned}\begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} \\ * & \Xi_{22} & 0 \\ * & * & \Xi_{33} \end{bmatrix} &< 0 \\ \Xi_{11} &= PF^* + F^{*T}P + Q - C^TZ^*C + I + \delta I \\ \Xi_{12} &= -PG^* \quad \Xi_{13} = -P\bar{F}^* + C^TZ^*K_1 \\ \Xi_{22} &= -Q + I + \delta I \quad \Xi_{33} = \beta^2I - K_1^TZ^*K_1 - \varepsilon^2\delta I\end{aligned}\quad (26)$$

where

$$\begin{aligned}PF^* &= P(A - \Psi_1CA) - \Phi_1^*(\Psi_2CA) - \Phi_2^*C \\ PG^* &= P(A_d - \Psi_1CA_d) - \phi_1^*(\Psi_2CA_d) - \Phi_3^*C \\ P\bar{F}^* &= \begin{bmatrix} (-P(\Psi_1F_y) - \Phi_1^*(\Psi_2F_y))^T \\ (P(F_x - \Psi_1CF_x) - \Phi_1^*(\Psi_2CF_x) - \Phi_2^*F_y)^T \\ (-\Phi_3^*F_y)^T \end{bmatrix}^T\end{aligned}\quad (27)$$

then the system (22) is asymptotically stable and $\|T(r_{f\bar{f}}, \bar{f})\|_\infty \geq \beta$

$$\begin{aligned}\dot{e}_{f\bar{f}}(t) &= F^*e_{f\bar{f}}(t) + G^*e_{f\bar{f}}(t-\tau) + \bar{F}^*\bar{f} \\ r_{f\bar{f}}(t) &= V^*Ce_{f\bar{f}}(t) + V^*K_1\bar{f}(t)\end{aligned}\quad (28)$$

Furthermore, the UIO matrices are obtained by considering C.2 to C.5, and, $Y^* = P^{-1}\Phi_1^*$, $L_2 = P^{-1}\Phi_2^*$, $L_3 = P^{-1}\Phi_3^*$, $V^* = (Z^*)^{1/2}$.

Proof. The definition of $\|T(r_{f\bar{f}}, \bar{f})\|_\infty \geq \beta$ is equivalent to $J_{r_{f\bar{f}}} : \int_0^\infty (r_{f\bar{f}}^T(t)r_{f\bar{f}}(t) - \beta^2\bar{f}^T(t)\bar{f}(t))dt \geq 0$. Now consider the following Lyapunov–Krasovskii function $V(t) = e_{f\bar{f}}^T(t)Pe_{f\bar{f}}(t) + \int_{t-\tau}^t e_{f\bar{f}}^T(s)Qe_{f\bar{f}}(s)ds$. Then we have:

$$\begin{aligned}J_{r_{f\bar{f}}} &= \int_0^\infty (r_{f\bar{f}}^T(t)r_{f\bar{f}}(t) - \beta^2\bar{f}^T(t)\bar{f}(t) - \dot{V}(t))dt \\ &\quad - V(0) + V(\infty)\end{aligned}\quad (29)$$

Assuming $r_{f\bar{f}} = 0$ for $t \in [-\tau, 0]$. Since $V(\infty) > 0$, we have:

$$J_{r_{f\bar{f}}} \geq \int_0^\infty (r_{f\bar{f}}^T(t)r_{f\bar{f}}(t) - \beta^2\bar{f}^T(t)\bar{f}(t) - \dot{V}(t))dt \quad (30)$$

Taking derivation from $V(t)$ and considering (28) yields

$$J_{r_{f\bar{f}}} \geq \int_0^\infty \begin{bmatrix} e_{f\bar{f}}(t) \\ e_{f\bar{f}}(t-\tau) \\ \bar{f} \end{bmatrix}^T \Xi \begin{bmatrix} e_{f\bar{f}}(t) \\ e_{f\bar{f}}(t-\tau) \\ \bar{f} \end{bmatrix} dt \quad (31)$$

with

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} \\ * & \Xi_{22} & 0 \\ * & * & \Xi_{33} \end{bmatrix}$$

$$\Xi_{11} = -PF^* - F^{*T}P - Q + C^T V^* V^* C$$

$$\Xi_{12} = PG^* \Xi_{13} = -P\bar{F}^* + C^T V^* V^* K_1$$

$$\Xi_{22} = Q \Xi_{33} = -\beta^2 I + K_1^T V^* V^* K_1$$

Hence, $\Xi \geq 0$ implies $J_{r_{f\bar{f}}} \geq 0$. $\Xi \geq 0$ is equivalent to (32) by considering $Z^* = V^* V^*$.

$$\begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} \\ * & \Xi_{22} & 0 \\ * & * & \Xi_{33} \end{bmatrix} \leq 0 \quad (32)$$

$$\begin{aligned} \Xi_{11} &= PF^* + F^{*T}P + Q - C^T Z^* C \\ \Xi_{12} &= -PG^* \Xi_{13} = -P\bar{F}^* + C^T Z^* K_1 \\ \Xi_{22} &= -Q \Xi_{33} = \beta^2 I - K_1^T Z^* K_1 \end{aligned}$$

The inequality (32) is conservative and cannot be solved using the LMI toolbox, because the term $\beta^2 I + K_1^T Z^* K_1$ cannot be less than zero for all $\beta > 0$ due to the structure of K_1 . To tackle this point, the S-procedure [41] is utilized. To this end, the constraints (33) have been considered with (32) which results in (26).

$$\begin{aligned} \|e_{f\bar{f}}(t)\|^2 + \|e_{f\bar{f}}(t-\tau)\|^2 &\geq 0 \\ \|e_{f\bar{f}}(t)\|^2 + \|e_{f\bar{f}}(t-\tau)\|^2 &\geq \varepsilon^2 \|\bar{f}(t)\|^2 \end{aligned} \quad (33)$$

To overcome the infeasibility of (26) the same variables definition, (27), as Theorem 1 are used.

Remark 1. To solve the LMI (26), the fixed small value of ε is chosen and the LMI is solved. If the feasible solution is not found, ε is increased until a feasible solution is achieved. It is noted that the value of ε should be selected as small as possible. The upper bound of ε is a design parameter and it is selected by trial and error procedure. Our observations verify this point that increasing the value of ε leads to achieve the bigger value of β , but results in increasing the matrix Z^* . As a consequence, the constraint should be considered along with (26) to restrict Z^* from being large. This aim can be achieved by following LMI condition

$$Z^* - \lambda_{Z^*}^2 I \leq 0 \quad (34)$$

where λ_{Z^*} is a design parameter.

Corollary 1. The system (10) is asymptotically stable and satisfies (19) if there exist symmetric positive matrices P , Q , Z^* matrices Φ_1^* , Φ_2^* , and Φ_3^* such that the LMIs (20), (26) and (34) are simultaneously held.

Remark 2. It is desirable to obtain a reference residual system having maximum sensitivity to the fault and minimum

sensitivity to the exogenous signal. This objective can be formulated by the performance index defined in the form of $\inf \frac{\alpha}{\beta}$. To this end, the following iterative optimization method is used. The procedures of this method are as follows:

1. Choose appropriate values of α , β and λ_{Z^*} .
2. Set small value of ε .
3. Solve LMIs (20), (26), and (34) by increasing ε until its maximum value to find a feasible solution of P , Q , Z^* , Φ_1^* , Φ_2^* , and Φ_3^* .
4. Increase β , decrease α , and go to Step 3. Continue this procedure until the feasible solution cannot be found for LMIs (20), (26), and (34).

3.2 The UIO design

As mentioned before, the aim of robust fault detection is to design an observer which detects the occurred fault in the presence of exogenous signals and system uncertainty. To this end, Theorem 4 is presented to guarantee the overall system (13) asymptotically stable and to hold performance index (9) for a given $\gamma > 0$.

Theorem 3. For a given $\gamma > 0$, the following system is asymptotically stable and $\|v(t)\|_2 \leq \|u(t)\|_2$:

$$\begin{aligned} \dot{\chi}(t) &= (\tilde{A} + \Delta\tilde{A})\chi(t) + (\tilde{A}_d + \Delta\tilde{A}_d)\chi(t-\tau) + (\tilde{B}_u + \Delta\tilde{B}_u)u(t) \\ v(t) &= \tilde{C}\chi(t) + \tilde{D}u(t) \end{aligned} \quad (35)$$

if there exist symmetric positive matrices P , Q , and constants ε_1 , ε_2 , ε_3 such that the LM [S_{ij}]₇₇ < 0 holds, where

$$\begin{aligned} s_{11} &= P(\tilde{A}) + \tilde{A}^T P + Q + \varepsilon_1^{-1} \tilde{N}_1^T \tilde{N}_1, s_{12} = P\tilde{A}_d \\ s_{13} &= P\tilde{B}_u, s_{14} = \tilde{C}^T, s_{15} = P\tilde{M}_1, \\ s_{16} &= P\tilde{M}_2, s_{17} = P\tilde{M}_3 \\ s_{22} &= -Q + \varepsilon_2^{-1} \tilde{N}_2^T \tilde{N}_2, s_{33} = -\gamma^2 I + \varepsilon_3^{-1} \tilde{N}_3^T \tilde{N}_3 \\ s_{34} &= \tilde{D}^T, s_{44} = -I, s_{55} = -\varepsilon_1^{-1}, s_{66} = -\varepsilon_2^{-1}, \\ s_{77} &= -\varepsilon_3^{-1}, \text{otherwise } s_{ij} = 0 \end{aligned} \quad (36)$$

Proof. Define the following Lyapunov–Krasovskii function:

$$V(t) = \chi^T(t)P\chi(t) + \int_{t-\tau}^t \chi^T(s)Q\chi(s)ds \quad (37)$$

The performance index $\|v(t)\|_2 \leq \|u(t)\|_2$ can be written as follows:

$$\begin{aligned} J_v &= \int_0^\infty v^T(t)v(t) - \gamma^2 u^T(t)u(t) + \dot{V}(t) dt \\ &\quad + V(0) - V(\infty) \end{aligned} \quad (38)$$

Assuming $\chi(t) = 0$ for $t \in [-\tau, 0]$. Since $V(\infty) > 0$, we have:

$$J_v \leq \int_0^\infty v^T(t)v(t) - \gamma^2 u^T(t)u(t) + \dot{V}(t) dt \quad (39)$$

Taking derivation from (37), considering (35) and (39) yields:

$$J_v \leq \int_0^\infty \begin{bmatrix} v(t) \\ v(t-\tau) \\ u(t) \end{bmatrix}^T \Xi \begin{bmatrix} v(t) \\ v(t-\tau) \\ u(t) \end{bmatrix} dt \quad (40)$$

with

$$\begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} \\ * & \Xi_{22} & 0 \\ * & * & \Xi_{33} \end{bmatrix}$$

$$\Xi_{11} = P(\tilde{A} + \Delta\tilde{A}) + (\tilde{A} + \Delta\tilde{A})^T P + Q + \tilde{C}^T \tilde{C}$$

$$\Xi_{12} = P(\tilde{A}_d + \Delta\tilde{A}_d) \quad \Xi_{13} = P(\tilde{B}_u + \Delta\tilde{B}_u) + \tilde{C}^T \tilde{D}$$

$$\Xi_{22} = -Q, \quad \Xi_{33} = -\gamma^2 I + \tilde{D}^T \tilde{D}$$

then, $\Xi < 0$ implies $J_v < 0$ Moreover, the inequality $\Xi < 0$ can be written as:

$$\begin{bmatrix} P\tilde{A} + \tilde{A}^T P + Q & P\tilde{A}_d & P\tilde{B}_u \\ * & -Q & 0 \\ * & * & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} \tilde{C}^T \\ 0 \\ \tilde{D}^T \end{bmatrix} I \begin{bmatrix} \tilde{C} & 0 & \tilde{D} \end{bmatrix} \\ + \begin{bmatrix} P\Delta\tilde{A} + \Delta\tilde{A}^T P & P\Delta\tilde{A}_d & P\Delta\tilde{B}_u \\ * & 0 & 0 \\ * & * & 0 \end{bmatrix} < 0 \quad (41)$$

Using Lemma 2, one can write the following inequality

$$\begin{bmatrix} P\Delta\tilde{A} + \Delta\tilde{A}^T P & P\Delta\tilde{A}_d & P\Delta\tilde{B}_u \\ * & 0 & 0 \\ * & * & 0 \end{bmatrix} \leq \\ + \tilde{M} \begin{bmatrix} \varepsilon_1 I & 0 & 0 \\ 0 & \varepsilon_2 I & 0 \\ 0 & 0 & \varepsilon_3 I \end{bmatrix} \tilde{M}^T \\ + \tilde{N}^T \begin{bmatrix} \varepsilon_1 I & 0 & 0 \\ 0 & \varepsilon_2 I & 0 \\ 0 & 0 & \varepsilon_3 I \end{bmatrix}^{-1} \tilde{N} \quad (42)$$

Considering (42) and using Schur compliment, (41) leads to (36). This completes the proof.

Theorem 4. For a given $\gamma > 0$, if there exist symmetric positive definite matrices $P_1, P_2, P_3, Q_1, Q_2, Q_3$, matrices $\Phi_1,$

Φ_2, Φ_3, V , and constants $\varepsilon_1, \varepsilon_2, \varepsilon_3$, such that the following LMI $[s_{ij}]_{14 \times 14}$ holds, then the overall system (13) is asymptotically stable and $J_{re} < \gamma$. The observer matrices are calculated by considering (3), (4), C2 to C5 and $Y = P^{-1}\Phi_1, L_2 = P^{-1}\Phi_2, L_3 = P^{-1}\Phi_3$. The LMI coefficients are defined as:

$$\begin{aligned} s_{1,1} &= P_1 F + F^T P + Q_1, s_{1,4} = P_1 G, s_{1,8} = P_1 \bar{F} \\ s_{1,10} &= P_1 \bar{R}, s_{1,11} = C^T V^T, s_{1,12} = P_1 (T M_1) \\ s_{1,13} &= P_1 (T M_2), s_{1,14} = P_1 (T M_3) \\ s_{2,2} &= P_2 F^* + F^{*T} P_2 + Q_2, s_{2,5} = P_2 G^*, s_{2,8} = P_2 \bar{F}^* \\ s_{2,10} &= P_2 \bar{R}^*, s_{2,11} = -C^T V^{*T} \\ s_{3,3} &= P_3 A + A^T P_3 + Q_3 + \varepsilon_1^{-1} N_1^T N_1, s_{3,6} = P_3 A_d \\ s_{3,7} &= P_3 B, s_{3,8} = P_3 K_3, s_{3,9} = P_3 E \\ s_{3,10} &= P_3 K_4, s_{3,12} = P_3 M_1, s_{3,13} = P_3 M_2 \\ s_{3,14} &= P_3 M_3, s_{4,4} = -Q_1, s_{5,5} = -Q_2 \\ s_{6,6} &= -Q_3 + \varepsilon_2^{-1} N_2^T N_2, s_{7,7} = -\gamma^2 I + \varepsilon_3^{-1} N_3^T N_3 \\ s_{8,8} &= -\gamma^2 I, s_{8,11} = K_1^T V^T - K_1^T V^{*T} \\ s_{9,9} &= -\gamma^2 I, s_{10,10} = -\gamma^2 I, s_{10,11} = K_2^T V^T - K_2^T V^{*T} \\ s_{11,11} &= -I, s_{12,12} = -\varepsilon_1^{-1} I, s_{13,13} = -\varepsilon_2^{-1} I \\ s_{14,14} &= -\varepsilon_3^{-1} I, \text{ otherwise } s_{i,j} = 0 \end{aligned} \quad (43)$$

where

$$\begin{aligned} P_1 T &= P_1 (I - \Psi_1 C) - \Phi_1 (\Psi_2 C) \\ P_1 F &= P_1 (A - \Psi_1 C A) - \Phi_1 (\Psi_2 C A) - \Phi_2 C \\ P_1 G &= P_1 (A_d - \Psi_1 C A_d) - \Phi_1 (\Psi_2 C A_d) - \Phi_3 C \\ P_1 \bar{F} &= \begin{bmatrix} (-P_1 (\Psi_1 F_y) - \Phi_1 (\Psi_2 F_y))^T \\ (P_1 (F_x - \Psi_1 C F_x) - \Phi_1 (\Psi_2 C F_x) - \Phi_2 F_y)^T \\ (-\Phi_3^* F_y)^T \end{bmatrix} \\ P_1 \bar{R} &= \begin{bmatrix} (-P_1 (\Psi_1 D) - \Phi_1 (\Psi_2 D))^T \\ (P_1 (R - \Psi_1 C R) - \Phi_1 (\Psi_2 C R) - \Phi_2 D)^T \\ (-\Phi_3^* D)^T \end{bmatrix} \end{aligned} \quad (44)$$

Proof. In Theorem 3 assumes that $P = \text{diag}(P_1, P_2, P_3)$ and $Q = \text{diag}(Q_1, Q_2, Q_3)$. Then, using system dynamic (13), it can be seen that s_{ij} are the same as (43). Without considering (44), the inequality (43) includes nonlinear terms which lead the LMI to be infeasible. To overcome this problem, define $\Phi_1 = P Y, \Phi_2 = P L_2, \Phi_3 = P L_3$ Using C2, C3 and (17) it can be seen that (44) makes the obtained LMI feasible.

Remark 3. It is noted that our approach is different from that in the references [14] and [36] in the following perspectives:

- (a) The system structure in [14] and [36] do not consider the time-delays, *i.e.*, the results in [14] and [36] cannot be directly applied to the system under consideration in this paper.

- (b) The class of observer considering in the current work is different from the one in [14].
- (c) The proposed conditions in Theorems 1–4 are obtained without resorting to any model transformations and bounding techniques for some cross terms, thus reducing the conservatism in the derivation of the stability conditions.

Remark 4. The designed steps for FD in (1) can be summarized as:

1. Design residual reference system using Remark 2,
2. Apply Theorem 4 to obtain UIO matrices.

In addition, it is worth noting that the number of variables to be determined in each step are $2n^2 + p^2 + 3n \times p$ and $6n^2 + p^2 + 3n \times p + 3$, respectively.

3.3 Evaluating the residual signal using threshold

To take a decision about the occurrence of a fault, an appropriate level threshold should be selected. According to (8), the residual signal for fault-free system satisfies the following equation:

$$\begin{aligned} \|r_f(t)\|_2 &= \|r_{\bar{n}}(t) + r_u(t)\|_2 \leq \|r_{\bar{n}}(t)\|_2 + \|r_u(t)\|_2 \\ &\leq J_{th,\bar{n}} + J_{th,u} \end{aligned} \quad (45)$$

where

$$\begin{aligned} J_{th,\bar{n}} &= \sup_{(\Delta A, \Delta B, \Delta A_d) \in \Omega_i} \|r_{\bar{n}}(t)\|_2 \\ J_{th,u} &= \sup_{(\Delta A, \Delta B, \Delta A_d) \in \Omega_i} \|r_u(t)\|_2 \end{aligned} \quad (46)$$

The value of $\|r_{\bar{n}}(t)\|_2$ can be computed offline and under the assumption that $\bar{n} \in \mathbb{L}_2$, we have $\sup_{(\Delta A, \Delta B, \Delta A_d) \in \Omega_i} \|r_{\bar{n}}(t)\|_2 = M_{\bar{n}}$. Since the signal u is supposed to be known online, the value of $J_{th,u}$ can be determined online by

$$J_{th,u} = \gamma_u \|u(t)\|_2 \quad (47)$$

with

$$\gamma_u = \sup_{(\Delta A, \Delta B, \Delta A_d) \in \Omega_i} \frac{\|r_u(t)\|_2}{\|u(t)\|_2},$$

where γ_u can be computed by using Theorem 3. Therefore the threshold value can be evaluated by:

$$J_{th} = \gamma_u \|u(t)\|_2 + M_{\bar{n}}. \quad (48)$$

IV. SIMULATION RESULTS

In order to investigate the effectiveness of the proposed algorithm, two cases including a numerical example, and real

world application are considered. Simulation results for each case presented in this section.

4.1 Numerical example

As a first example, let us consider a numerical example to make the design procedure more clear. Consider a system which is defined by (1) with the following matrices:

$$A = \begin{bmatrix} -3.8 & 1.5 & -0.5 \\ 0.5 & -3 & 1 \\ -0.3 & 0.7 & -2.4 \end{bmatrix}, \tau = 1$$

$$A_d = \begin{bmatrix} 0.4 & 0.1 & -0.2 \\ 0.1 & -0.8 & 0.2 \\ 0.7 & -0.1 & 0.5 \end{bmatrix}, B = \begin{bmatrix} 0.1 \\ 0.2 \\ -0.4 \end{bmatrix}$$

$$F_x = \begin{bmatrix} 0.6 \\ -0.5 \\ 0.4 \end{bmatrix}, E = \begin{bmatrix} -0.4 \\ 0.1 \\ -0.3 \end{bmatrix}, F_y = \begin{bmatrix} 0.2 \\ 0.8 \\ -1.2 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, R = \begin{bmatrix} 0.1 \\ 0.2 \\ -0.4 \end{bmatrix}, D = \begin{bmatrix} 0.9 \\ 0.2 \\ 0.7 \end{bmatrix}$$

$$M_1 = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.1 \end{bmatrix}, M_2 = \begin{bmatrix} 0.1 \\ 0 \\ -0.1 \end{bmatrix}, M_3 = \begin{bmatrix} -0.1 \\ 0.2 \\ 0.1 \end{bmatrix}$$

$$N_1 = [0 \quad 0.1 \quad 0.3], N_2 = [0.1 \quad 0 \quad 0], N_3 = 0.1$$

The first step to design the fault detection system is to solve the LMIs (20), (26) and (34) in Theorem 1 and Theorem 2. The Yalmip toolbox is used to solve the LMIs [42]. To start the iterative optimization method presented in Remark 1, initial values $\alpha_{int} = 3$ and $\beta_{int} = 0$ are selected and the upper value of ε is chosen 0.1. Using this procedure, the following results are obtained:

$$V^* = \begin{bmatrix} 7.20 & -5.38 & -6.81 \\ -5.38 & 17.77 & 1.46 \\ -6.81 & 1.46 & 10.28 \end{bmatrix}$$

$$\Phi_1^* = \begin{bmatrix} -60.20 & -247.58 & -371.78 \\ -247.58 & 458.01 & 323.29 \\ -371.78 & 323.29 & -91.56 \end{bmatrix}$$

$$\Phi_2^* = \begin{bmatrix} 261.98 & -313.81 & -233.12 \\ -313.81 & 643.13 & 257.94 \\ -233.12 & 257.94 & 234.02 \end{bmatrix}$$

$$\Phi_3^* = \begin{bmatrix} -5.27 & -0.41 & -1.09 \\ -0.41 & 1.30 & 0.77 \\ -1.09 & 0.77 & 0.29 \end{bmatrix}, \alpha = 2.4, \beta = 0.6$$

Using these values, the LMI (43) is solved and the following observer dynamic matrices are obtained:

$$F = \begin{bmatrix} -5.07 & 2.13 & -2.37 \\ -0.53 & -1.67 & 0.53 \\ -3.84 & 3.71 & -4.75 \end{bmatrix}$$

$$G = \begin{bmatrix} 1.51 & 2.42 & -1.85 \\ 0.06 & -0.34 & -0.07 \\ 1.66 & 3.47 & -2.03 \end{bmatrix}$$

$$K_{1z} = \begin{bmatrix} 1.22 & -23.81 & -18.70 \\ -0.14 & 0.54 & 0.57 \\ 1.71 & -28.97 & -23.02 \end{bmatrix}, H = \begin{bmatrix} 0.42 \\ 0.23 \\ 0.07 \end{bmatrix}$$

$$K_{2z} = \begin{bmatrix} -3.05 & -1.87 & -2.91 \\ 0.08 & -0.31 & -0.20 \\ -3.76 & -1.64 & -3.04 \end{bmatrix}$$

$$L_1 = \begin{bmatrix} -0.10 & 4.18 & 2.86 \\ -0.30 & 0.50 & 0.26 \\ -0.71 & 5.80 & 3.90 \end{bmatrix}$$

$$V = \begin{bmatrix} 1.53 & 1.54 & -2.13 \\ 1.54 & 4.30 & -3.49 \\ -2.12 & -3.49 & 4.07 \end{bmatrix}$$

To show the effectiveness of the designed FD system, two types of fault are exerted on the system. In both cases, the step disturbance signals exerted on the system from 2 to 4 s. The noise signal is assumed to be white Gaussian noise with power 0.005, and the uncertainty $\Sigma_i(t)$ is considered a sinusoidal signal. In the first case, an abrupt fault, shown in Fig. 2, occurs from 4 to 6 s. The residual signals are shown in Fig. 3. It can be seen that the residual signals change when the fault occurs, however the residual signals show no sensitivity to the external disturbance. The value of threshold J_{th} is presented in Fig. 4. This figure indicates that the fault is detected rapidly and the difference between the threshold for a faulty system and a fault-free system is high enough to detect the occurrence of a fault in the system.

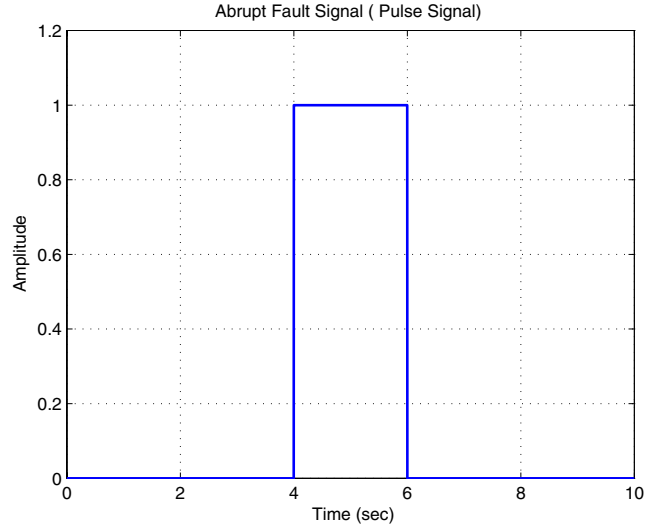


Fig. 2. Abrupt fault occurs in $t = 4$ s.

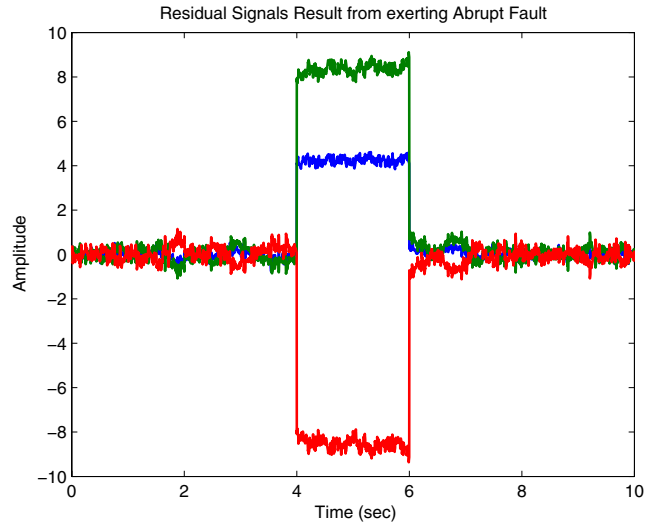


Fig. 3. The residual signals of the designed FD system for the case in which abrupt fault is exerted.

In the second case, an incipient fault, shown in Fig. 5, is considered. The residual signals, and value of threshold are depicted in Fig. 6 and Fig. 7, respectively. It can be seen that the residual signals show enough sensitivity to the occurrence of the fault.

In contrast to the first case, it can be seen that the value of the threshold in Fig. 4 changes faster than Fig. 7 which results from this point that $f(t)$, $f(t - \tau)$ and $\dot{f}(t)$ affect on the residual signal. Since the derivation of abrupt faults is larger than incipient faults, the residual signals are more sensitive to abrupt faults.

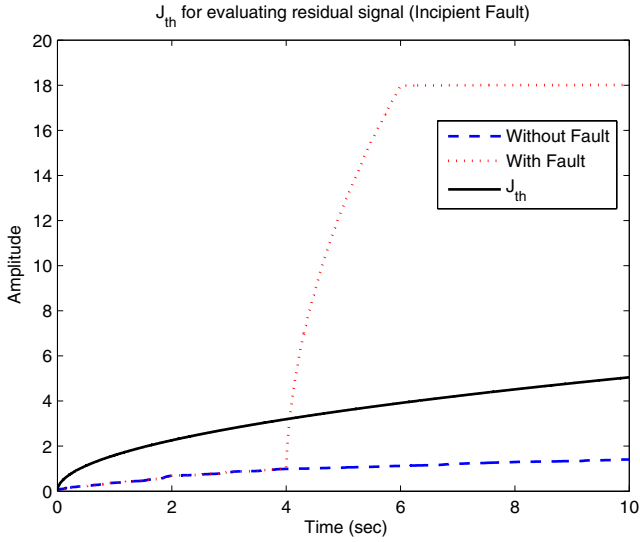


Fig. 4. Evaluating residual signals using defined threshold (Abrupt fault).

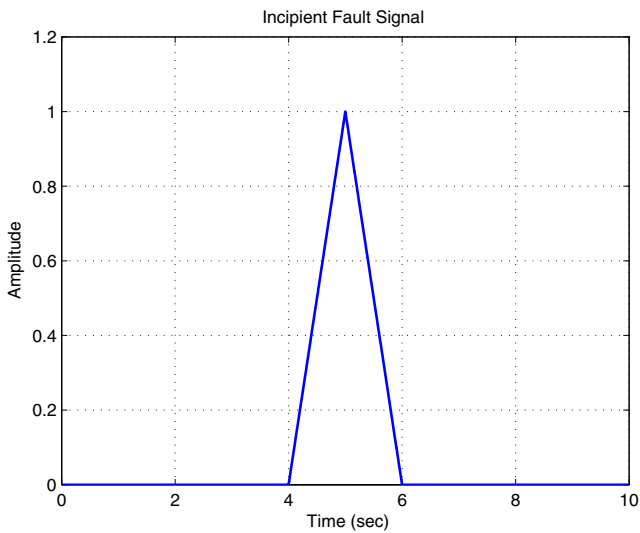


Fig. 5. Incipient fault.

4.2 Fault detection of an engineering process

As a second example an engineering system is considered in this sub-section to investigate application of the proposed method in the real world. To this end, the proposed approach is adopted to detect faults of the Williams–Otto process [40]. Since this system has many characteristics of typical chemical process, it has been frequently studied in chemical engineering literature. The schematic diagram of the Williams–Otto process is shown in Fig. 8. In this figure, F_A , F_B , F_P are feed rate of material A, B and valuable product, respectively. The F_{w_1} and F_{w_2} are undesirable by-products. The dynamic equation of this chemical process is nonlinear.

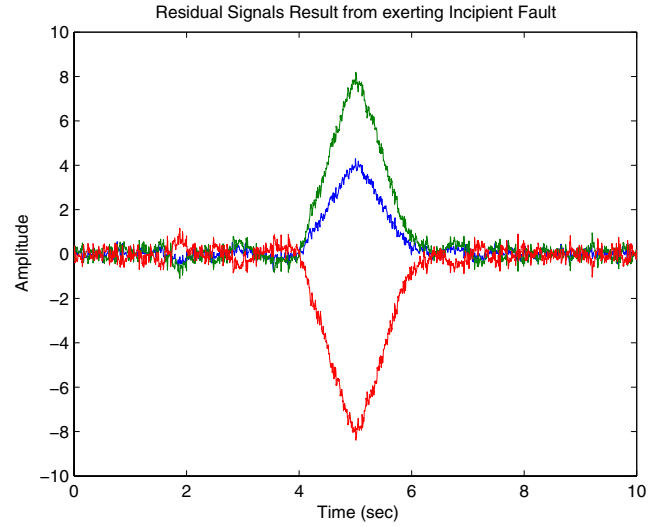


Fig. 6. The residual signals of the designed FD system for the case in which incipient fault is exerted.

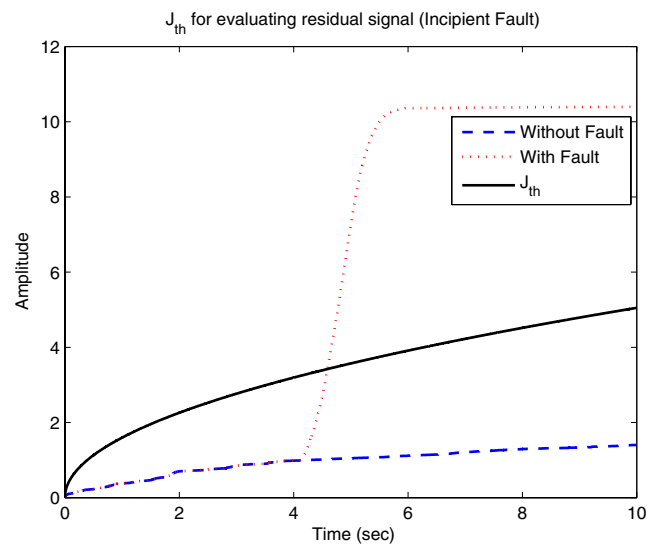


Fig. 7. Evaluating residual signals using defined threshold (Incipient fault).

However, the linear model is appropriate for determination of proportion of the feed rate F_A and F_B at the desired operating point. To make the linear model similar to (1), we consider disturbance, fault, and noise signals. In addition, the model uncertainties are considered to have 5 percentage variation of nominal matrices. The value of the matrices are:

$$A = \begin{bmatrix} -4.93 & -1.01 & 0 & 0 \\ -3.20 & -5.30 & -12.8 & 0 \\ 6.40 & 0.347 & -32.5 & -1.04 \\ 0 & 0.833 & 11 & -3.96 \end{bmatrix}, \tau = 1$$

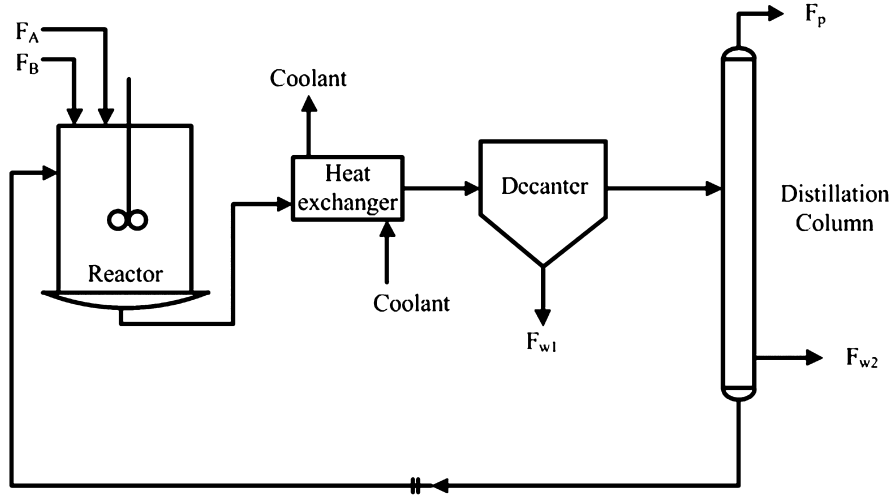


Fig. 8. Schematic diagram of Williams–Otto process [40].

$$A_d = \begin{bmatrix} 1.92 & 0 & 0 & 0 \\ 0 & 1.92 & -12.8 & 0 \\ 0 & 0 & 1.87 & 0 \\ 0 & 0 & 0 & 0.724 \end{bmatrix}$$

$$F = \begin{bmatrix} -27.66 & -8.88 & -12.95 & -1.30 \\ -64.05 & -26.58 & -39.75 & -8.72 \\ 13.98 & 3.60 & 3.63 & -0.71 \\ -52.67 & -18.61 & -27.75 & -4.97 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, F_x = \begin{bmatrix} 1 \\ -0.5 \\ 0.3 \\ 0.1 \end{bmatrix}, E = \begin{bmatrix} 0.1 \\ -0.2 \\ -1 \\ 0.5 \end{bmatrix}$$

$$G = \begin{bmatrix} 13.86 & 1.41 & -0.83 & -1.99 \\ 34.37 & 5.60 & -2.28 & -4.74 \\ -7.00 & -0.18 & 0.36 & 1.06 \\ 27.65 & 3.33 & -1.71 & -3.93 \end{bmatrix}$$

$$C = I_4, F_y = \begin{bmatrix} 0.2 \\ 0.5 \\ -0.1 \\ 0.4 \end{bmatrix}, R = \begin{bmatrix} -0.2 \\ 0.2 \\ 0.4 \\ 0.1 \end{bmatrix}$$

$$K_{1z} = \begin{bmatrix} 273.08 & 35.73 & -46.65 & -97.27 \\ 767.27 & 102.04 & -133.95 & -273.50 \\ -115.39 & -14.67 & 18.99 & 41.05 \\ 567.30 & 74.63 & -97.63 & -202.12 \end{bmatrix}$$

$$D = \begin{bmatrix} 0.9 \\ 0.2 \\ 0.7 \end{bmatrix}, M_1 = 0.1A, M_2 = 0.1A_d, M_3 = 0.1B$$

$$H = \begin{bmatrix} 6.86 & 0.75 \\ 16.97 & 3.02 \\ -3.48 & -0.09 \\ 13.68 & 1.79 \end{bmatrix}$$

$$N_1 = I_4, N_2 = I_4, N_3 = I_2$$

The linear model has four states $x_1, x_2, x_3,$ and x_4 which represents variations in the weight composition of the raw materials A and B, of intermediate product, and of the desired product respectively. The control inputs u_1 and u_2 are equal to $\delta F_A/V_R$ and $\delta F_B/V_R$ where V_R is the volume of reactor and δF_i are derivation of feed rate which are considered equal to 2.628 and 0.45 respectively [40]. Following the same procedure as Section 4.1, LMIs (20), (26), and (34) have been solved by $\varepsilon = 0.01, \lambda_{z^*} = 33$ which result in $\alpha = 0.1, \beta = 400$. Then, the observer dynamic matrices are obtained as:

$$K_{2z} = \begin{bmatrix} -81.50 & -9.76 & 7.97 & 31.13 \\ -241.65 & -28.78 & 23.60 & 90.74 \\ 31.27 & 3.79 & -3.06 & -12.33 \\ -172.47 & -20.62 & 16.86 & 65.49 \end{bmatrix}$$

$$L_1 = \begin{bmatrix} -5.86 & -0.75 & 0.65 & 2.39 \\ -16.97 & -2.02 & 1.79 & 5.77 \\ 3.48 & 0.09 & 0.70 & -1.24 \\ -13.86 & -1.79 & 1.35 & 5.72 \end{bmatrix}$$

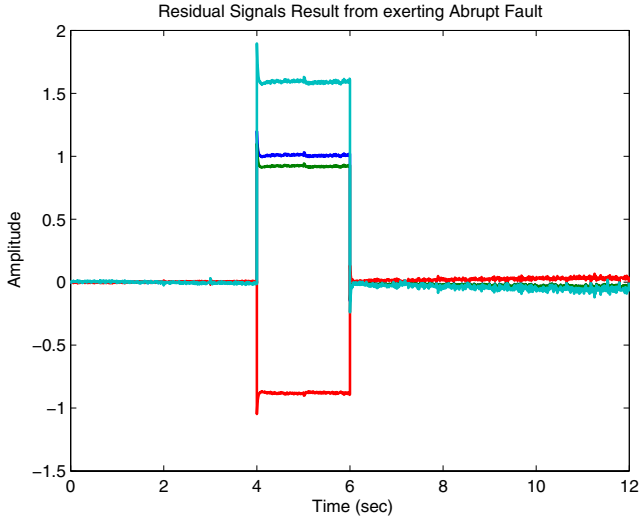


Fig. 9. Residual signals of designed FD system for Williams–Otto process (abrupt fault).

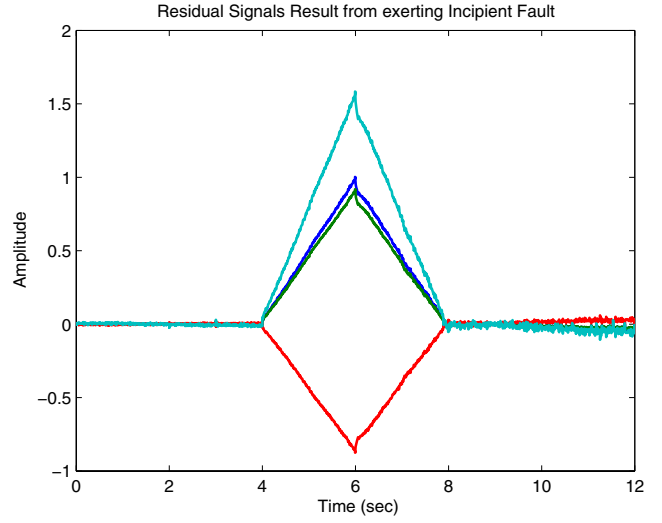


Fig. 11. Residual signals of designed FD system for the Williams–Otto process (incipient fault).

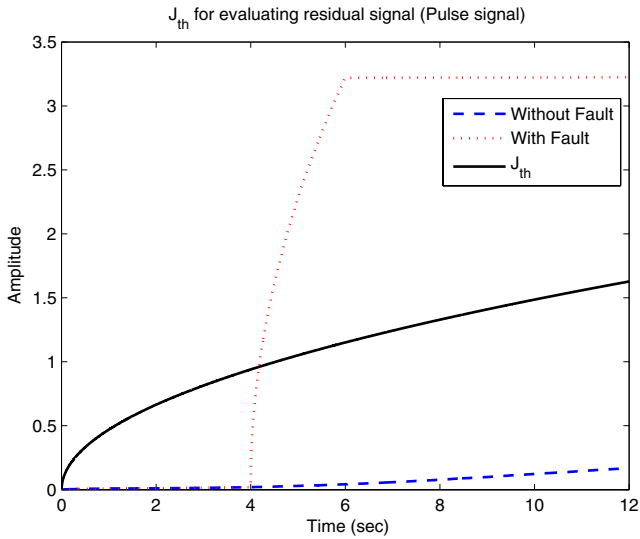


Fig. 10. Evaluating residual signals using defined threshold (abrupt fault).

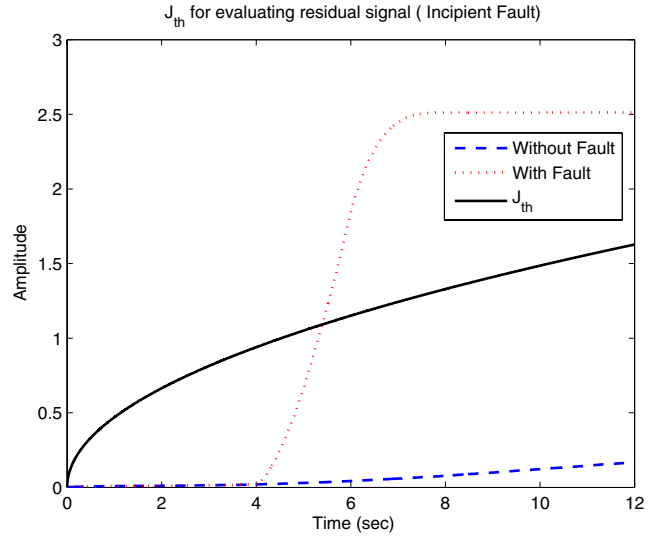


Fig. 12. Evaluating residual signals using defined threshold (incipient fault).

V. CONCLUSION

In both cases, disturbance and uncertainty are considered as in Section 4.1. The noise signal is assumed to be white Gaussian noise with power $1e^{-7}$. Two types of fault signals are simulated as in Section 4.1, however the amplitude of them are chosen equal to amplitude of input signal of process. Residual signals and value of threshold are shown in Fig. 9 and Fig. 10 for an abrupt fault, and in Fig. 11 and Fig. 12 for an incipient fault. For both cases, we see that residual signals are sensitive enough to the fault. The occurrence of the fault can be determined using threshold.

In this paper, a novel approach is developed to design fault detection system for uncertain linear time delay systems. The main contribution of paper is the design of an optimal fault detection system using an unknown input observer. To this end, the effects of noise signals and uncertainty terms are minimized in the residual signal while the effect of disturbance is completely decoupled. Since the influence of both fault signals and their derivations are maximized in the residual signal, the fault detection system shows more sensitivity to the occurrence of faults. In order to verify the

performance of the proposed method, a numerical example and an engineering system are investigated. The simulation results confirm the robustness and effectiveness of the proposed scheme for fault detection in the presence of model uncertainty and external disturbances. Based on the result obtained, the presented work can be extended to other systems such as descriptor systems, and stochastic systems with jump. Moreover, developing the same approach for non-linear systems will be interesting for future work.

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